São Paulo J. Math. Sci.
DOI 10.1007/s40863-017-0071-y

The representation dimension of Artin algebras

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Abstract In the seventies of the last century, M. Auslander introduced the notion of representation dimension of an algebra with the objective of having a measure of the complexity of its module category. In the last 20 years, there was a renewal interest in studying such a dimension. Our aim here is to survey the last developments concerning it.

Keywords Representation dimension · Module theory · Representations of algebras

Mathematics Subject Classification 16G70 · 16G20 · 16E10

1 Introduction

Throughout this paper all algebras A are finite dimensional algebras over an algebraically closed field k unless otherwise stated. For such an algebra A, we denote by mod$A$ the category of finitely generated left A-modules, while ind$A$ will denote a

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Dedicated to Prof. Antonio Paques on the occasion of his 70th birthday.

The author acknowledge partial support from CNPq (Project PQ 306940/2015-9). During the time this paper was written, the author was enjoying a sabbatical year at the Institute of Advanced Studies of USP (Brazil).

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Published online: 14 August 2017

subcategory of mod$A$ containing a representative of each isomorphism class of indecomposable A-module.

The use of homological techniques has been very useful in the representation theory of algebras. In general, we use homological invariants to *measure* how far an algebra or a module is from having *good properties*. This is the basic idea behind the concept of *representation dimension*.

Introduced by Auslander in his famous Quenn's Notes in the seventies [10], the study of such dimension had a renewed interest about twenty years ago after a talk given by Reiten in the *International Conference on Representations of Algebras-1998* held in Bielefeld, Germany. The Bielefeld group also organized, in 2008, a *Workshop on Representation Dimension* [14] which helped the development of the subject.

However, despite significant advances, the basic question still remains valid: *What does this dimension really mean?* Auslander's idea was that such a dimension would measure how far the algebra A is to be representation-finite, but the examples we have nowadays indicate us that the complexity behind this dimension is not so easily seen. The idea of this survey is to collect some of the recent results on the subject and show possible directions of investigation. Surely, we will not be able to cover all the aspects investigated in the last years, but we will concentrated in a couple of directions, not only because of personnal taste but also because we do believe they have been more succesfull. In any case, we include in the bibliography some papers which can give some clues of further lines of investigations.

This survey is organized as follows. In the Sect. 2, we recall basic notions of homological algebras that are needed in the remaining of the paper. In Sect. 3, we recall some classes of algebras which are defined using some homological properties. Section 4 is devoted to the definition and basic facts os the representation dimension while in Sect. 5 we discuss some algebras having representation dimension at most three. In Sect. 6, we briefly discuss the question of tameness. Finally, in Sect. 7, we comment a recent result using trisections, we do believe this technique can be further explored.

For basic notions of Representation Theory we refer the reader to the books [7,11].

2 Preliminaries

In this section we will establish some further notations and basic facts used along the paper. As mentioned in the introduction, A will stand for an algebra.

If we decompose A as a sum of indecomposable projective modules $A = P_1^{n_1} \oplus \cdots \oplus P_t^{n_t}$ where $P_i \not\cong P_j$ whenever $i \neq j$ and $n_i > 0$ for each i, then the algebras A and $B = \mathrm{End}_A(P_1 \oplus \cdots \oplus P_t)$ are Morita equivalent, that is, mod$A$ and mod$B$ are equivalent categories. That is why we can assume that the original algebra A is basic, that is, with $n_i = 1$ for each i in the above decomposition.

Denote by D the standard duality. So, DA, seen as an A-module, is the sum of one copy of each indecomposable injective module.

Given a module M, denote by add$M$ the full subcategory of mod$A$ containing the direct sums of direct summands of M.

Also, for a module M, $\mathrm{pd}_A M$ and $\mathrm{id}_A M$ indicate, respectively its projective and its injective dimension. The global and the finitistic dimensions of an algebra A are, respectively,

$$\mathrm{gl.dim}A = sup\{\mathrm{pd}_A M : M \in \mathrm{mod}A\}$$
$$\mathrm{fin.dim}A = sup\{\mathrm{pd}_A M : M \in \mathrm{mod}A \text{ and } \mathrm{pd}_A M < \infty\}$$

We also recall the following definitions. We say that a subcategory $\mathcal{C}$ of $\mathrm{ind}A$ is covariantly finite provided for each $X \in \mathrm{ind}A$ there exists a morphism $f_X : X \longrightarrow Y$ with Y in $\mathcal{C}$ such that any morphism from X to a module in $\mathcal{C}$ factors through f_X (such f_X is called right $\mathcal{C}$-approximation). Dually for contravariantly finite.

3 Algebras defined by homological properties

We shall recall here some classes of algebras which are defined in terms of homological properties. Not surprisingly, since the notion of representation dimension is based also on some homological property (see next section), the representation dimension of the classes we are going to discuss are easier to calculate.

3.1 Tilted algebras

Introduced by Happel and Ringel [22] upon a work of Brenner and Butler [13], the class of tilted algebras plays nowadays a very important role in the representation theory of algebras. The overall idea here is to transfer informations from the module category of some class of algebras we are familiar with to some other class which we have control by means of functors between the corresponding module categories.

So, the idea is to consider an A-module T with some special properties and such that the algebras A and $B = \mathrm{End}_A T$ have module categories, although not equivalent, with a good connection between them. One of such special module is called tilting.

Definition 3.1 A module T is tilting if

(a) $\mathrm{pd}_A T \leq 1$.
(b) $\mathrm{Ext}^1_A(T, T) = 0$.
(c) There exists a short exact sequence $0 \to A \to T_1 \to T_2 \to 0$ with $T_i \in \mathrm{add}T$.

A trivial example of tilting module is given by the regular A-module ${}_A A$.

Let now T be a tilting A-module and consider $B = \mathrm{End}_A T$. One can consider torsion pairs $(\mathcal{T}(T), \mathcal{F}(T))$ in $\mathrm{mod}A$ and $(\mathcal{X}(T), \mathcal{Y}(T))$ in $\mathrm{mod}B$ as follows

- $\mathcal{T}(T) = \{M \in \mathrm{mod}A : \mathrm{Ext}^1_A(T, M) = 0\}$
- $\mathcal{F}(T) = \{M \in \mathrm{mod}A : \mathrm{Hom}_A(T, M) = 0\}$
- $\mathcal{X}(T) = \{N \in \mathrm{mod}B : T \otimes_B N = 0\}$
- $\mathcal{Y}(T) = \{N \in \mathrm{mod}B : \mathrm{Tor}^B_1(T, N) = 0\}$

The so-called Brenner Butler Theorem proven in [13] establishes that $\mathrm{Hom}_A(T, -)$ is an equivalence between $(\mathcal{T}(T)$ and $\mathcal{Y}(T))$ while $\mathrm{Ext}^1_A(T, -)$ is an equivalence between $\mathcal{F}(T)$ and $\mathcal{X}(T)$. A particular important case occurs when the tilting module T is splitting, that is, when the torsion pair $(\mathcal{X}(T), \mathcal{Y}(T))$ splits. Consequently, each indecomposable B-module lies either in ($\mathcal{X}(T)$ or in $\mathcal{Y}(T)$). This is the case when the algebra A is hereditary.

Definition 3.2 (*Happel–Ringel* [22]) If A is a hereditary algebra and T is tilting, then $\mathrm{End}_A(T)$ is called a tilted algebra.

We recall the following result.

Proposition 3.1 *If A is tilted, then gl.dim$A \leq 2$ and for each indecomposable module M, $pd_A M \leq 1$ or $id_A M \leq 1$.*

For further informations on tilted algebras, we refer the reader to [7].

3.2 Quasitilted algebras

Notice that the above proposition indicates that tilted algebras have good homological properties, that is, its global dimension is low and each indecomposable module is *close* to either a projective or to an injective module. In [21], Happel et al. studied a generalization of the construction of tilted algebras and introduced the so-called quasitilted algebra. One can defined such a class of algebras using the above proposition.

Definition 3.3 An algebra A is quasitilted if gl.dim$A \leq 2$ and for each indecomposable module M, $\mathrm{pd}_A M \leq 1$ or $\mathrm{id}_A M \leq 1$.

Quasitiled algebras includes the tilted but also the canonical algebras (see [30]). In their study of such class of algebras, Happel et al. introduced the following two subcategories (called the sides of the modules categories):

$$\mathcal{L}_A = \{M \in \text{ ind}A : \text{ pd}_A N \leq 1 \quad \text{for each predecessor } N \text{ of } M\}$$
$$\mathcal{R}_A = \{M \in \text{ ind}A : \text{ id}N \leq 1 \quad \text{for each successor } N \text{ of } M\}$$

3.3 Shod algebras

We observe that the two defining conditions for quasitilted algebras are independent, and among them, the second one seems to be the more important. Having this in mind, we introduced the notion of shod algebras in a joint work with Lanzilotta [16]

Definition 3.4 An algebra A is shod (for small homological dimension) if for each indecomposable module M, $\mathrm{pd}_A M \leq 1$ or $\mathrm{id}_A M \leq 1$.

Shod algebras have global dimension at most 3. We also mention the following characterization of shod algebras using the sided subcategories of the module category.

Proposition 3.2 (Coelho–Lanzilotta [16]) *An algebra A is shod if and only if $\mathcal{L}_A \cup \mathcal{R}_A = indA$*

3.4 Further classes of algebras

Along the years, other classes of algebras were introduced using the side subcategories as defining properties or, in the case of the sided glued algebras, as characterising properties. We mention some of them:

- Laura algebras (Assem–Coelho [3]) An algebra is called laura provided $\mathcal{L}_A \cup \mathcal{R}_A$ is cofinite in ind$A$ (also called double gluing of tilted algebras). In an independent work, Reiten and Skowroński also studied such a class of algebras [29] under the name generalized double tilted algebras.
- Right glued algebras (Assem–Coelho [2]) We say that A is a right glued algebra provided $\mathcal{L}_A$ is cofinite in ind$A$.
- Left glued algebras (Assem–Coelho [2]) We say that A is a left glued algebra if $\mathcal{R}_A$ is cofinite in ind$A$.
- ada algebras (Assem–Castonguay–Lanzilotta–Vargas [1]) An algebra A is said to be an ada algebra provided $A \oplus DA \in \mathcal{L}_A \cup \mathcal{R}_A$.

For further details on these classes of algebras, we refer the reader to the survey [4].

4 The representation dimension

As mentioned in the introduction, the representation dimension of an algebra A was introduced by Auslander in the earlier 70s with the intention of establishing a measure of the complexity of the module category mod$A$. Contrary to Auslander's initial belief, this dimension does not seem to have a straightforward relation with the representation type of A, but still it has some connections. For the rest of this section, A will indicate an algebra.

4.1 Definition of the representation dimension

We first recall that an A-module M is a *generator–cogenerator* of mod$A$ if it contains all the indecomposable projective modules and all the indecomposable injective modules, that is, $A \oplus DA \in \text{add}M$. Clearly, then, given a module L, there exist a monomorphism $L \longrightarrow M_1$ and an epimorphism $M_2 \longrightarrow L$, with $M_1, M_2 \in \text{add}M$.

Definition 4.1 The representation dimension of an algebra A is the number

$$\text{rep.dim } A = \inf \{\text{gl.dim } (\text{End}_A(M)) : M \text{ generator-cogenerator}\}$$

A generator-cogenerator M such that rep.dim$A$ = gl.dim(End$_A M$) is also called an Auslander generator.

We first observe that if A is semisimple, then rep.dim$A = 0$. On the other hand, if A is not semisimple, then rep.dim$A \geq 2$. Auslander's belief on the connection between such a dimension and the representation type of the algebra is based in the following result.

Theorem 4.1 [10] *An algebra has representation dimension at most 2 if and only if it is representation-finite.*

4.2 Auslander lemma

Upon the interest of such measure was renewed, the idea behind Theorem 4.1 inspired a criterium for the calculation or the representation dimension of an algebra. Versions of such criterium appear, independently, in the works [17,19,34], all of them based in the Auslander's proof of his theorem. We mention the following version.

Lemma 4.2 (Auslander lemma) *Let M be an Auslander generator for A and $d \geq 0$. Then rep.dim $A \leq d+2$ if and only if for each A-module X, there exists an exact sequence*

$$0 \longrightarrow M_d \longrightarrow \cdots \longrightarrow M_1 \longrightarrow M_0 \longrightarrow X \longrightarrow 0$$

with $M_i \in addM$ such that the sequence

$$0 \longrightarrow (M, M_d) \longrightarrow \cdots \longrightarrow (M, M_0) \longrightarrow (M, X) \longrightarrow 0$$

remains exact.

In view of the characterization of the algebras with representation dimension at most two, the natural interest was to describe those with representation dimension 3. The following corollary gives a version of Auslander lemma for the case of algebras with representation dimension at most 3.

Corollary 4.3 *Let A be a representation-infinite algebra. Then rep.dim$A = 3$ if and only if there exists an Auslander generator M such that for each A-module X, there exists a short exact sequence*

$$0 \longrightarrow M_1 \longrightarrow M_0 \longrightarrow X \longrightarrow 0$$

with $M_i \in addM$ such that

$$0 \longrightarrow (M, M_1) \longrightarrow (M, M_0) \longrightarrow (M, X) \longrightarrow 0$$

remains exact.

Such a result gives a possible strategy to show that a representation-infinite algebra has representation dimension equal to 3. It is enough to find an Auslander generator M in such a way that, rephrasing the result, a minimal right add$M$-approximation of each $X \in$ mod$A$ has kernel lying in add$M$. Most of the proofs of results of classes of algebras having representation dimension 3 are based in the exhibition of a module with these properties.

4.3 Basic facts on the representation dimension

One could first wonder if the representation dimension could be infinite or which numbers it actually assumes.

In [25], Iyama has shown that the representation dimension is always finite. In fact, he has shown that given any A-module N, there exists $N' \in \operatorname{mod}A$ such that the algebra $\operatorname{End}_A(N \oplus N')$ has finite global dimension (in fact, it is quasi-hereditary). So, if one starts with $N = A \oplus \mathrm{D}A$, the result follows easily.

An interesting connection with the so-called finitistic conjecture arises from the work of Igusa and Todorov [24]. Recall that such a conjecture states that the finitistic dimension fin.dim$A$ is finite. Although it has been proven for several important classes of algebra, it is still an open (and important) problem. Igusa and Todorov has shown the following.

Proposition 4.4 (Igusa–Todorov [24]) *The finitistic conjecture holds for algebras with representation dimension at most 3.*

By the time such result was proven, there were no examples of algebras with representation dimension greater than 3 and so, by a short period of time, the above result seems to be a good strategy for proving the finitistic conjecture. However, Rouquier has shown in [32] that the representation dimension of a vector space of dimension $n \geq 1$ is $n + 1$, and so, in particular, there is no bound on this dimension.

5 Algebras with representation dimension at most three

We shall discuss here some classes of algebras which have the representation dimension at most three. The first results in this direction were shown in the original paper by Auslander. However, we shall discuss them in the context of a nice result proven by Ringel [31] concerning torsionless algebras.

5.1 Torsionless-finite algebras

We say that an algebra is torsionless-finite provided there are only finitely many, up to isomorphism, indecomposable modules which are submodules of projective modules. Torsionless-finite algebras include, clearly, the hereditary ones, and the right glued algebras as defined above.

Theorem 5.1 (Ringel [31]) *Torsionless-finite algebras have representation dimensions at most three.*

The proof of such result is an example of application of the Auslander lemma. Ringel shows first that for a torsionless-finite algebra, there are only finitely many, up to isomorphism, indecomposable modules which are quotients of an injective. Given that, he considers the module

$$M = (\oplus \text{torsionless modules}) \oplus (\oplus \text{quotients of injectives})$$

and shows that in case A is representation-infinite, then it satisfies the property required in the Corollary 4.3, giving the result.

As a consequence, we have the following classes of algebras (we mention the works where the results were first proven):

- Hereditary algebras (Auslander [10]).
- Algebras with $J^2 = 0$ (Auslander [10]).
- Algebras with $J^n = 0$ and A/J^{n-1} representation-finite (Auslander [10]). As a consequence, minimal representation-infinite algebras have representation-dimension equal to three.
- Algebras stably equivalent to hereditary algebras [34].
- Left and right glued algebras (Coelho–Platzeck [17]). We recall that the right glued algebras (defined in Sect. 3) are the algebras having the support $\mathrm{Supp}(\mathrm{Hom}_A(-, A))$ finite and, so, they are clearly torsionless-finite. By duality, one gets the result for the left glued algebras.
- Special biserial algebras (Erdmann–Holm–Iyama–Schröer [19]).

5.2 Laura algebras

Recall, from Sect. 3, that an algebra A is laura if $\mathcal{L}_A \cup \mathcal{R}_A$ is cofinite. The class of laura algebras includes the tilted, the quasitilted, the shod and the glued algebras. Combining the results of Assem, Platzeck and Trepode [6] and Oppermann [28] one gets that the representation dimension of laura algebras are at most three.

The first step was given in [6] where the authors showed that tilted algebras have representation dimension at most three and Corollary 4.3 was used. The Auslander generator which gives the result is the module

$$M = A \oplus \mathrm{D}A \oplus [\text{complete slice}]$$

(tilted algebras are characterized by the existence of a complete slice. We are not going to enter in details here, but a complete slice has only a finite number of indecomposable summands, and M is, then, a finitely generated module).

Also, in the same paper [6], the authors show that if A is a laura algebra which is not quasi-tilted, then the representation dimension is at most three. For this, Corollary 4.3 is used again. Without entering into details, a possible Auslander generator is as follows:

$$\begin{aligned} M = A \oplus \mathrm{D}A \oplus [\text{Ext-injectives in } \mathcal{L}_A] \oplus \\ \oplus [\text{Ext-projectives in } \mathcal{R}_A] \oplus [\text{indecomposable not in } \mathcal{L}_A \cup \mathcal{R}_A] \end{aligned}$$

The remaining case, that is, for the quasi-tilted case, was shown by Oppermann [28]. Here, it is one of the few situation known where the proof is not based in the exhibition of a specific Auslander generator satisfying Corollary 4.3.

5.3 Piecewise hereditary algebras

In [23], Happel and Unger considered again the problem of relating the representation dimension of an algebra A and of the algebra $B = \mathrm{End}_A T$ where T is a tilting module. If A is hereditary, then B is tilted and, as we saw above, the situation is clear. The idea here is to consider a general algebra A but still restricting the tilting module to a splitting one. Their main result is

Theorem 5.2 (Happel–Unger [23]) *Let T be a splitting tilting A module and $B = End_A T$. If rep.dim$A \leq 3$, then rep.dim$B \leq$ rep.dim$A$.*

We quote Happel–Unger [23]: The module category of the endomorphism algebra of a splitting tilting module is "smaller", so the main result indicates, at least in this situation, that the representation dimension is related to the complexity of the module category. In this way we underline Auslander's original intention when defining representation dimension.

As a nice application of the above result, Happel and Unger proved that the representation dimension of a piecewise hereditary algebra is at most three. Recall that an algebra A is piecewise hereditary if the derived category $\mathrm{D}^b(A)$ of the category mod$A$ is equivalent as a triangulated category to $\mathrm{D}^b(\mathcal{H})$ for some hereditary abelian category $\mathcal{H}$.

It is worthwhile mentioning that in a joint work with Happel and Unger [15], we have proven a special case of the above application, namely the so-called iterated tilted algebras. The techniques used there, however, are different and we do believe it can be further explored in different contexts.

5.4 Some other classes

We also mention the following classes of algebras with representation dimension at most three:

- Trivial extensions of hereditary algebras (Coelho–Platzeck [17])
- Cluster-concealed algebras (Gonzalez Chaio–Trepode [20]).

6 Tameness

As already observed, the (initially) expected straightforward connection between representation type and representation dimension was not true. As seen above, a hereditary algebra has representation dimension either two (if it is representation-finite) or three (otherwise). The usual dicothomy tame-wild is not reflected by the representation dimension, at least in this, and many other, cases. However, the result so far indicates that the following question has a positive answer.

Question: *Do tame algebras have representation dimension at most three?*

Positive answers were given in some important cases. Among the cases we mentioned above, we stress the class of special biserial algebras (result proven by Erdmann et al. in [19]).

More recently, Assem et al. proved in [8] that self-injective algebras of euclidean type has representation dimension at most three. Recall that an algebra is called self-injective provided the set of indecomposable projective modules coincide with the set of indecomposable injective ones. Previously, Bocian et al. have proven the result for the particular case of self-injective algebras socle equivalent to a weakly symmetric algebra of euclidean type.

It is worthwhile to mention here that, using similar methods, Assem et al. extend in [9] this result for selfinjective algebras os wild tilted type.

7 A recent result

We shall discuss now a recent developed technique which seems to be very useful in many cases. The proofs of the results in this section can be found in [5].

7.1 Trisections

The idea is to relate the representation dimension of an algebra A with those given by left and the right support algebras defined upon some special subcategories. As a consequence, one gets that the representation dimensions of algebras of type ada or which is a Nakayama oriented pullback is at most three. A new proof for laura algebras is also a consequence of this technique. The strategy is to split the module category into pieces and calculate the representation dimension of the algebras associated to them. We start with some definitions.

Definition 7.1 A trisection in ind$A$ is a triple $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of disjoint subcategories such that

(a) $\text{ind}A = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$.
(b) $\text{Hom}_A(\mathcal{C}, \mathcal{B}) = \text{Hom}_A(\mathcal{C}, \mathcal{A}) = \text{Hom}_A(\mathcal{B}, \mathcal{A}) = 0$.

Given a subcategory $\mathcal{C}$ of ind$A$, the (left) support algebra ${}_{\mathcal{C}}A$ is the endomorphism algebra of the direct sum of the isoclasses of the indecomposable projective modules lying in $\mathcal{C}$. Dually, we define the (right) support algebra $A_{\mathcal{C}}$.

7.2 The results

The trisection we are going to consider is somehow relates with the usual trisection induced by the side subcategories $\mathcal{L}_A$ and $\mathcal{R}_A$.

Theorem 7.1 (Assem–Coelho–Wagner [5]) *Let A be a representation-infinite algebra and* $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ *be a trisection in ind$A$. Assume $\mathcal{B}$ finite.*

(a) *If $\mathcal{C} \subseteq \mathcal{R}_A$ and add$\mathcal{C}$ is covariantly finite, then*

$$rep.dim A = max\{3, rep.dim_{\mathcal{A}} A\}.$$

(b) *If* $\mathcal{A} \subseteq \mathcal{L}_A$ *and add*$\mathcal{A}$ *is contravariantly finite, then*

$$rep.dimA = max\{3, rep.dimA_{\mathcal{C}}\}.$$

A second very useful result is the following.

Theorem 7.2 (Assem–Coelho–Wagner [5]) *Let A be a representation-infinite algebra and* $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ *be a trisection in indA. If*

(a) $(\mathcal{A} \cup indA_{\mathcal{C}})^c$ *is finite and* $indA_{\mathcal{C}}$ *is closed under successors; or*
(b) $(ind_{\mathcal{A}}A \cup \mathcal{C})^{\rfloor}$ *is finite and* $ind_{\mathcal{A}}A$ *is closed under predecessors.*

Then,

$$rep.dimA \leq max\{rep.dim_{\mathcal{A}}A, rep.dimA_{\mathcal{C}}\}.$$

For the proofs of both results, we construct, using Auslander generators of the smaller algebras, corresponding modules in the larger one. To show that they are Auslander generators, Corollary 4.3 is again an essential tool.

7.3 Consequences

One first consequence one gets is a new proof for the laura case, that is, such algebras have representation dimension at most three.

A second consequence is that any representation-infinite algebra A which lies in add$(\mathcal{L}_A \cup \mathcal{R}_A)$ satisfies, as well, rep.dim$A \leq 3$. In particular, this is true for ada algebras (see Sect. 3 for a definition).

It is worthwhile mentioning that the our initial motivation was the calculation of the representation dimension of the pullback R of, say, $A \longrightarrow B$ and $C \longrightarrow B$ in terms of the representation dimension of the algebras A, B and C. In general, one easily loses control on the module category when considers a pullback as above and, so, it is natural to consider such a construction with some special extra conditions. We are not going to give the precise definition of a Nakayama oriented pullback here since it is very technical, but the idea behind it is to have a control in the way the categories of ind$A$ and ind$C$ (and so, clearly, of ind$B$) sits inside the category ind$R$. We mention the following result.

Corollary 7.3 (Assem–Coelho–Wagner [5]) *Let R be the Nakayama oriented pullback of* $A \longrightarrow B$ *and* $C \longrightarrow B$*. Then*

$$rep.dimR \leq max\{rep.dimA, rep.dimC\}.$$

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