RT-MAE-9110

RATIONALITY IN RUBIN'S SENSE

by

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Palavras Chaves: Utility - Probability
(Key words)

Classificação AMS: 90A10
(AMS Classification) 60A05

- Maio de 1991 -

1.2 - Rationality in Rubin's Sense

In 1738 Daniel Bernoulli presented the St. Petersburg paradox and suggested maximization of *moral* expectation as a principle for decision-making. Ever since then, many different systems of axioms of "rationality" which imply maximization of expected utility have been proposed. More specifically, such systems of axioms typically refer to a "preference" relation among actions which is taken as a primitive object and also to a "belief" relation among states of nature which are unknown (and, therefore, random). Results establishing the existence (and, in a sense, unicity) of utility and probability functions which represent numerically the preference and belief relations are then derived. In this century, Ramsey (1926), de Finetti (1937) and Savage (1954) have the better known examples of such axiomatic developments. All these axiomatic approaches obtain Bayesian behavior - i.e., maximization of expected utility with respect to subjective probability - as a necessary condition for rationality. (There is also the work of von Neumann and Morgenstern [1947], who considered "objective" probabilities to integrate utility. See Section 3.1.1) . In this Section, we discuss the most recent system of axioms which has been proposed, as an attempt by Rubin (1987) to obtain a consistent non-Bayesian approach. In order to do so, Rubin proposed a system of axioms as weak as possible but he "failed" , as Bayesian behavior is once again obtained. However, Rubin does not obtain a separation between utility and probability, so the "unconditional" (not necessarily "expected") utility is an integral of the conditional utilities with respect to "weights" which do not necessarily form a probability measure. These "weights" are then obtained as a mathematical consequence of the existence of unconditional utility, as opposed to being a consequence of the axioms of "probability". As Rubin points out, the possible non-separability of utility and "prior" does not alter qualitatively the conclusion , which still calls for maximization of the unconditional utilities. Or, in his own words , "it is not necessary for coherent behavior to require a separation in order to remain Bayesian ". The use of a system of weights instead of a prior distribution is even less relevant when used (as approximations) in the actual elicitation of (theoretical and subjective) relations of preference and belief. Rubin also reminds us that nothing prevents the use of separate utilities and a probability measure, or the postulation of additional axioms implying such a separation. However, his weak system of axioms does not obtain it as a necessary consequence.

The primitive objects in Rubin's formulation are a set of actions A and a *choice-set function* C. defined on all subsets of the action space. As opposed to most other axiomatic approaches, "preference" relations are not taken as primitive objects. Instead, Rubin has axioms for the choice-set function. On the other hand, such a function assigns to every subset E of A a "choice" $C(E)$ contained in the set $H(E)$ of "random combinations" of elements of E. This is because Rubin makes the possibility of randomization primary, based on the fact that it is always possible for the rational person to randomize between two or more rigid actions (by use of a device which he considers "random"). This fact also voids the meaning of "preference". Therefore, Rubin is allowing the previous existence of a "roulette wheel" which enables the person to randomize between elements of A. Some authors (Krantz et al. [1971]) dispute this assumption since it implies the previous existence of "probabilities" for the roulette wheel. As suggested by Fishburn (1981), this objection could be refuted by enlarging the action space in such a way that it would contain the mixtures. (Operationally, one would still need the wheel to *realize* a mixed decision).

Rubin's first axiom demands that a subset E having 1, 2, or 3 elements has a non-empty choice-set value. The second axioms demands that a restriction of any subset E which maintains previous choices has the obvious consequences. The third axiom is very similar to Basu's (1975) Weak Conditionality Principle, which states that pre-randomization of experiments is irrelevant to data analysis. Rubin calls the third axiom the "nature isn't against you" axiom and demands that choices for several problems to be presented at random should be made independently. Rubin also adds to this list two structural axioms dealing with infinite subsets and continuity of choices. This modest list of axioms implies the existence and uniqueness - up to positive affine transformations - of a numerical utility function which represents the choice-set function. These are called the *conditional* utility functions. The next step is to consider a fixed choice problem for every *state of nature* ω in a set Ω and an overall choice problem for uncertain ω. The addition of a sixth axiom, which is equivalent to Savage's (1954) Sure-Thing Principle, suffices for the existence of a class of *unconditional* utility functions. (In section 1.3 we prove the possibility of always requiring the sixth axiom. The proof implicitly requires the availability of the "roulette wheel"). Rubin's unconditional utility is a finitely additive integral of the conditional utility functions as functions on Ω.

As one of the motivations for the weakening of the axioms, Rubin refers to the fact that passing from a fixed state of nature to an uncertain state of nature is formally equivalent to passing from one decision-maker to a group who must make decisions. The general impossibility of comparing different persons' utility scales would therefore imply the non-separability of unconditional utility into a conditional utility and a prior. This provides the heuristic meaning of non-separability. Indeed, having $U_i(\cdot)$ as a utility function conditional on a state of nature i, any $\alpha_i U_i(\cdot) + \beta_i$ is an equivalent conditional utility function in the sense that it characterizes the same choice-set function C (provided $\alpha_i > 0$). Hence, as Rubin shows, an unconditional utility function $U(\cdot)$, *even if a proper prior probability measure h is assumed* (not the case in Rubin's system), must satisfy

$$U(\cdot) = \sum_i h_i \alpha_i U_i(\cdot),$$

and by letting $w_i = h_i \alpha_i$, it is seen that the w_i do not necessarily form a probability measure. The situation where i labels a (Bayesian) person is related to Arrow's Impossibility Theorem (1951) about the construction of a group utility function. We discuss this further in Chapter 2, which deals with group decision problems.

Rubin's interpretation of rationality might explain his seemingly distinct (from de Finetti's) approach to statistical problems. Rubin declares himself to be a *prior* Bayesian, while de Finetti's "conditional" ideas - in favor of what became somehow formalized as the Likelihood Principle [Sections 1.4-1.6] - don't allow us to call him anything else but a *posterior* Bayesian. It seems that there is more to this difference than just the mathematical possibility of non-equivalence between the normal and extensive forms of analysis (see

Piccinato [1974]). Rubin mentions "true" priors (in [1977]) and makes the point that an infinitely fast and free computer might be necessary to obtain an exact conditional utility function. So Rubin considers a *theoretical* and "true" unconditional utility which exists for a person who follows the axioms. This person, however, does not know his own utility function. Such a point of view understandably makes one attentive to robustness against a declared, "untrue" prior : the prior Bayesian approach. De Finetti, on the other hand, has a much less rigid - even if more radical - interpretation of probability ; he writes on page 218 of (1951): "The probability does not correspond to a self proclaimed rational belief, but to the effective personal belief of anyone". And on page 220: "... the a priori probabilities...are expressions of one's own belief, which cannot be unknown to oneself..." . De Finetti understands a prior as an actual degree of belief and, therefore, as a valid starting point. This makes a posterior probability more believable (robust): the posterior Bayesian point of view. In the contrast we made there is no mention of the likelihood function (which, again, could be "true" or not) that generates the posterior probability. However, as seen in Section 1.1, under the predictivist point of view there is conceptually no need to separate prior and likelihood.

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