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Modeling multicriteria group decision making as games from enhanced pairwise comparisons

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ABSTRACT

This research aimed to replace the pairwise comparison function used in Leoneti [2016] with a new function inspired on the exponential model of prospect theory and to perform a comparative analysis to verify the hypothesis that a better performance of the utility function would be possible from the use of a component of judgment better theoretically grounded. A field study consisting of solving three cases in a group was proposed to evaluate the number of matches of the method regarding the decision negotiated by the group, and the number of times that, when there was a match, the group maintained its decision after voting. From an empirical evaluation with a hundred students that participated in the field study, the main hypothesis of this research was confirmed, which is that the use of a pairwise comparison function adhering to a decision and judgment theory increases the predictive capacity of the utility function proposed in Leoneti [2016].

1. Introduction

Utility functions are mathematical artifacts that are used for modeling human perception based on the principles of rationality [1]. These functions have been used in many areas of science, with particular interest within game theory [1,2]. Most of such functions are defined as $\pi : \mathbb{R}_+ \rightarrow [0,\ 1]$, where the domain represents the stimulus, usually a monetary measure, and the image the agent's perception. According to Bernoulli, the utility functions should be in line with two principles: (i) the Weber-Fechner's law between the physical magnitude of a stimulus and the intensity of the stimulus perception; and (ii) the Pascal rules of rational choice [3,4]. The joint adoption of these two principles can be mathematically modelled by the means of a marginally decreasing concave function. Consequently, the most immediate mathematical models that would be suitable for modeling utility functions are the power, logarithmic and exponential functions. In his earliest studies on the subject of rational choice Bernoulli adopted a logarithmic mathematical function in the form $v(d) = \lambda \log d$, while Gabriel Cramer proposed, on the same subject, the use of a power function in the form $v(d) = \lambda d^{0.5}$ [3,4]. Both functions are unbounded and were eventually associated to the St. Peterburg paradox.[1] In this sense, the use of exponential utility functions had also been investigated [5]. It should be noted that these functions are settled in a $\mathbb{R}^2$ space and, therefore, those mathematical models are commonly interpreted as the utility function itself.

Recently, Leoneti [6] proposed a utility function defined as $\pi : \mathbb{R}_+^{a \times c_i} \rightarrow [0, 1]$ to model multicriteria problems as a non-cooperative game, the branch of game theory in which prior communication or agreements cannot be enforced [2], for solving the strategic interaction that occurs between a agents in the course of establishing their preferred alternative, from a set S with b alternatives, based on their respective c_i evaluation criteria, where $i = 1,\ldots,a$. This utility function takes into account

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[1] The St. Peterburg paradox is the situation where an individual makes choices that are not in accordance with the rational choice according to Pascal's assumptions. Particularly, the paradox is related to the situation where an individual would prefer a certain sum of money rather playing a lottery that has infinite expected value. By means of the logarithmic function, Bernoulli had demonstrated a way for solving this paradox. However, not completely, since unbounded functions may also present the same effects.

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the preferences of the agents regarding the multiple criteria under evaluation for providing a measure of perception that can be used for modeling the game.[2] Examples of applications of this utility function can be found in the literature. Cuoghi and Leoneti [11] applied the utility function to model the complex scenario of a group decision of choosing the Belo Monte hydroelectric plant in Brazil. Araujo and Leoneti [12], in turn, used the utility function to model and assess the stability of the Brazilian gas and oil exploration and production regulation model. Gimon and Leoneti [13] expanded the application of the utility function to processes that include repeated interactions between agents. Finally, Ziotti and Leoneti [14] have demonstrated that the use of the utility function as a structured group decision making method can increase the chances of agreements' implementation.

The utility function [6] structure has two components: (i) a judgment component, where the individual's rationality is modeled based on the multicriteria procedure of pairwise comparisons; and (ii) a decision component, where the values of the pairwise comparisons are amalgamated to establish the payoffs for each arrangement between the agent's initial alternative (*status-quo*) and the alternatives proposed by his/her counterparts. The structure resulting from the modeling is similar to a non-cooperative game and can be summarized by the tuple $\langle A,S, \prec_i\rangle$, where A is the set of a agents, S is the set of b alternatives, and $\prec_i$ are the preferences of each agent $i = 1,\ldots,a$ with respect to all alternatives' arrangements in the set of S with the remaining $j = 1,\ldots,a$ agents with $j \neq i$, given by the equation

$$\pi_i\left(s_i, s_{j\neq i}\right) = \varphi(s_i, IA_i)\prod_{i\neq j, j=1}^{a}\varphi\left(s_i, s_j\right).\varphi\left(s_j, IA_i\right) \quad (1)$$

where s_i is the alternative of the agent i (his/her *status-quo*), $s_{j\neq i}$ are the alternatives proposed by his/her counterparts with $j = 1,\cdots,a,$ and $j \neq i$, IA_i is the ideal alternative of the agent i (the alternative with the best scores for each of the c_i criteria under his/her evaluation, with $i = 1,\ldots, a$ the number of agents), and the Phi function provides the rationale of the judgment component, which is based on pairwise comparisons, according to the equation

$$\varphi\left(s_i, s_j\right) = \left[\frac{\alpha_{s_is_j}}{\| s_j\|}\right]^{\delta} cos\theta_{s_is_j},\ and\delta = \begin{cases} 1, & if\alpha_{s_is_j} \leq \| s_j \| \\ -1, & otherwise \end{cases} \quad (2)$$

where, $\alpha_{s_is_j} = \| s_i \| cos\theta_{s_is_j}$ is the scalar projection of the vector s_i onto the vector s_j, $\theta_{s_is_j}$ is the angle between the two vectors, and $\| s_j\| = \sqrt{(s_j^1)^2 + (s_j^2)^2 + \ldots + (s_j^{c_i})^2}$ is the norm of the respective vector with c_i components (each of the c_i criteria under evaluation by the agent i, with $i = 1,\ldots,a$ the number of agents). The image of the Phi function varies between zero and one (due to the conditional δ), meaning that the closer to one, the more similar the alternatives are. Graphically, the structure of the decision component, formed from the amalgamation of the judgment components, can be interpreted from Fig. 1.

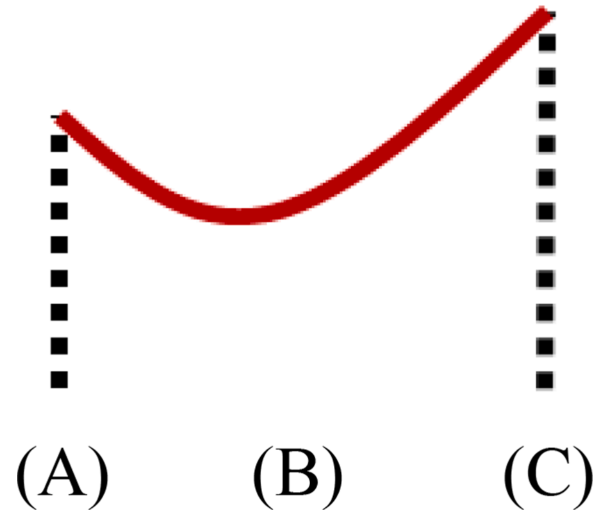

Fig. 1. Structure of the decision component.

In the structure of the decision component, we have that: (A) represents the comparison of the initial alternative of an agent (*status quo*) with his/her ideal alternative; (B) represents the comparison between the agent's initial alternative (*status quo*) and the alternative proposed by his/her counterparty; and (C) represents the comparison between the alternative proposed by the counterparty with the agent's ideal alternative, all of which are calculated from the judgment component, which is the pairwise comparison function. It should be noted that the absolute comparison of alternatives provided by the judgment components A and C is relativized by the judgment component B, with the aim of incorporating the endowment effect into the model, which is a trading cost given the major valuation that an agent may give to a good when it is part of his/her *status quo*. For instance, supposing two different alternatives that are equidistant to the ideal alternative, then the judgment component represented by B is going to include a cost for trading them. It is also notable that the structure of the judgment component itself incorporates into its model the framing effect, which is the different perception that an agent may have in circumstances of gains or losses. As a result, the utility function in [6] incorporates into its decision and judgment analysis the principles that are part of the most complete structure of rational choice, which is the prospect theory by Kahneman and Tversky's [15]. Summarily, while utility theory requires solely a marginally decreasing concave function as its mathematical model, prospect theory demands similar structure for specifically modeling the perception of gains. Moreover, it assumes the empirically verified evidence that human beings present more aversion to losses than prone to gains, which results in a "s" shaped curve with an amplified convex part related to losses and a concave part related to gains, the latter being similar as the utility theory.

However, as highlighted in Leoneti [6], the component judgment is a pairwise comparison function that was modeled, inspired on [16], in terms of the scalar projection between the alternatives, represented by vectors in the orthogonal algebraic space considered. In this sense, the research aim is to replace that pairwise comparison function used in Leoneti [6] by a mathematical function shaped on the proposal of prospect theory to verify the hypothesis that a better performance of the utility function would be achieved from the use of a component of judgment better theoretically grounded. Among the immediate possibilities for such replacement, it has been chosen the exponential function, since it is bounded, invariant under linear transformation, and has constant absolute risk aversion, which is usually associated to better performance for modeling different types of behavior in relation to risk [17]. For modeling such exponential function, it has been selected as a start point the exponential function proposed by Leoneti & Gomes [18] that was used for providing the ExpTODIM method, a new version of the TODIM method (acronym of the Portuguese expression *TOmada de Decisão Interativa e Multicritério)* [19,20], which is a multicriteria decision-making method based on the principles of prospect theory.

To verify the performance level that the different judgement's components would lead the utility function, a field study was proposed

[2] Other approaches have also been developed for modelling multicriteria games, including the first attempt by Shapley [7] and Blackwell [8] as vector payoff games (or multicriteria games), and other recent approaches that use soft set theory for the same purpose as described and reviewed in [9,10]. The justification for a new methodology as the one presented in this research is the fact that, differently from the mentioned approaches, here the pairwise comparisons provide the utility functions themselves, which is a different procedure of the mentioned approaches, in which those functions should be elicitated.

based on Leoneti and Sessa [21]. The field study consisted of solving three cases in group, where, in their initial phase, five volunteers individually evaluate each case in the form of a decision matrix with five alternatives and eight criteria each, and, in a group phase, negotiate in order to convince the counterparts to adopt his/her preferred alternatives as the group decision. The decision matrices refer to three different cases: (i) the choice of a travel destination; (ii) the choice of a language school; and (iii) the choice of a CEO to a company, proposed by Ziotti and Leoneti [14]. A group of a hundred students from different undergraduate and graduate courses from the University of São Paulo Campus in Ribeirão Preto participated in the field study, totaling twenty sessions.

2. Preliminaries

2.1. The ExpTODIM method and its PHI function

In the early 1990s, Gomes & Lima [19,20] used the prospect theory principles to propose a multicriteria decision-making method. In its original version, the so called TODIM method used a Phi function in the form of a power function based on the value function of the cumulative prospect theory, which calculates the difference between an initial reference value and its respective final value, including an amplification parameter and the powers of the respective concave and convex functions.

The original version of the Phi function of the TODIM method was adapted by different researchers that found the necessity of adjusting its mathematical structure for better adherence to the principles of prospect theory. The studies by Gomes & González [22], Lourenzutti & Krohling [23], Lee & Shih [24] and Llamazares [25] are available in the literature and address this subject by using the same structure of power functions. Instead of using a power function as it used in these studies, or using a unbonded function as the logarithmic one, Leoneti & Gomes [18] introduced a new mathematical structure to the Phi function of TODIM method, which is based on the exponential function. In the modeling of the new exponential function, the authors have been chosen the basis 10, with relation to the logarithm from Napier and Briggs, to make easier numeric calculations within a decimal numeric system [26]. The proposed exponential Phi function can be seen in the equation

$$\varphi_j(s_{ij}, s_{kj}) = \begin{cases} w_j\left(1 - 10^{-\rho\left|s_{ij}-s_{kj}\right|}\right)\ if\left(s_{ij} - s_{kj}\right)\rangle 0 \\ 0\ if\left(s_{ij} - s_{kj}\right) = 0 \\ -w_j\lambda\left(1 - 10^{-\rho\left|s_{ij}-s_{kj}\right|}\right)\ if\left(s_{ij} - s_{kj}\right)\langle 0 \end{cases} \quad (3)$$

where w_j are the weights of each respective criteria, λ is the amplification parameter commonly used to adjust the different responses regarding gains and losses, generally close to twice as much for losses according to the findings of Tversky and Kahneman [27], and $\rho \in \mathbb{N}^*$ indicates how significant the decision is according to the perception of the decision makers. Assuming a low value to ρ indicates that the curvature of the function would be smoother, which would make its shape very similar to the one of the original Phi function. When assuming higher values to ρ it will make the curvature of the function more accentuate, which will make the function very sensitive to small variations. According to the authors [18], this feature aims to adequate the function to the theory of agent's sensitivity to risk in decision making as presented by De Giorgi & Hens [28]. Furthermore, the authors claimed necessary to make it adherent to recent findings on organization behavior studies that indicated that the shape of utility function may differs among decision makers due the heterogeneity involved in different strategic decisions [29]. Leoneti & Gomes [18], then, proposed the parameter ρ as being discrete and varying between [1-5], with 1 indicating the agent is little sensitive to the result and 5 meaning that the agent is extremely sensitive to the result. This choice was justified mainly by the ease of obtaining an estimated value from a five-level Likert scale, a scientifically accepted and validated manner to measure subjectivity [30]. The new version of the TODIM method with its Phi function replaced by the exponential function was named Exponential TODIM, the ExpTODIM method.

The steps of the ExpTODIM method can be summarized as follows: (i) standardizing the criteria used for evaluating the alternatives; (ii) calculating the Phi function for each criterion based on a pairwise comparison for all alternatives; (iii) calculating the dominance, which is the absolute difference between gains and losses of each alternative from the sum of the values of the Phi function for all criteria; (iv) calculating the alternatives' performance from the sum of the dominance calculations and using a linear standardization function to assign the value zero to the alternative with the worst performance, one to the alternative with the best performance, and intermediate values between zero and one for the others; and (v) ordering the alternatives based on performance's standardized values. Considering a decision matrix $\{x_{ij}\}$ with $i = 1, \ldots, b$ alternatives and $j = 1, \ldots, c$ criteria, the steps for the application of the ExpTODIM method are detailed below.

Step 1: standardize the criteria using the sum linear standardization technique

$$s_{ij} = \frac{x_{ij}}{\sum_{i=1}^{c} x_{ij}},\ \text{if benefit criteria} \quad (4)$$

$$s_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{c} \frac{1}{x_{ij}}},\ \text{if cost criteria} \quad (5)$$

Step 2: using the Eq. (3), calculate the Phi function for each criterion $j = 1, \ldots, c$ in the pairwise comparisons between the alternatives $i = 1, \ldots, b$ and $k = 1, \ldots, b$ for $\forall(i,k)$

Step 3: determine the dominance

$$\delta_i(s_i, s_k) = \sum_{j=1}^{c} \varphi_j(s_i, s_k) \quad (6)$$

Step 4: calculate the performance of each alternative based on the sum of the dominance and standardize the values between 0 and 1.

$$\xi_i = \frac{\sum_{k=1}^{s} \delta_i(s_i, s_k) - \min \sum_{k=1}^{s} \delta_i(s_i, s_k)}{\max \sum_{k=1}^{s} \delta_i(s_i, s_k) - \min \sum_{k=1}^{s} \delta_i(s_i, s_k)} \quad (7)$$

Step 5: order the alternatives according to the ξ_i values

The authors demonstrated that the ExpTODIM achieved the best performance through a comparative analysis that included the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS)[3] and other available versions of TODIM method in the literature.

2.2. The utility function with an enhanced pairwise comparison function

Recall that, according to Leoneti [6], possible disadvantages in the calculation of $\pi_i(s_i, s_{i\neq j})$ would be related to the limitations of the original pairwise comparison function, which was modeled from linear algebra and not based on a specific theory of decision and judgment. In this sense, the exponential function from ExpTODIM method was chosen as a starting pointing to replace the original one presented in equation 2.

However, it should be noticed that the logical structure of the utility function $\pi_i(s_i, s_{i\neq j})$ involves a multiplicative aggregation. Thus, originally, if one of the factors of the utility function is close to zero (low similarity between the comparison of any pair of alternatives given by the Phi equation), then $\pi_i(s_i, s_{i\neq j})$ would tends to zero, which means that only alternatives similar to each other and to the ideal alternative would

[3] The TOPSIS method was proposed by Hwang & Yoon [31] and is based on the comparison of alternatives within a Euclidean space. The TOPSIS method defines an index of similarity (or relative proximity) for the ideal positive alternative and dissimilarity (or relative distance) for the negative ideal alternative. Then a value function merges into an index the distance to the positive and negative ideal alternatives, where the higher is D_i^- and the lower is $D_i^+ + D_i^-$, the better the alternative is going to be ranked.

receive values close to one. In this respect, due to the fact of having a minimum and a supreme, the exponential function can be conveniently modified so as to be defined in the bonded range from zero to one from a simple algebraic modification. Consequently, it was added to each conditional function of Eq. (3) the amplification factor λ and then divided this sum by the denominator $(1+\lambda)$, this being sufficient to make the image of the function $\varphi_j(s_i, s_k)$ remain between zero and one, as can be seen in the Equation

$$\varphi_j(s_{ij}, s_{kj}) = \begin{cases} \dfrac{w_j\left(1-10^{-\rho|s_{ij}-s_{kj}|}\right)+\lambda}{(1+\lambda)} \; if\,(s_{ij}-s_{kj})\rangle 0 \\ \dfrac{\lambda}{(1+\lambda)} \; if\,(s_{ij}-s_{kj})=0 \\ \dfrac{-\lambda w_j\left(1-10^{-\rho|s_{ij}-s_{kj}|}\right)+\lambda}{(1+\lambda)} \; if\,(s_{ij}-s_{kj})\langle 0 \end{cases} \tag{8}$$

In addition to adjusting the image range of the function, it was also necessary to adapt the concept of the ideal alternative. The ideal alternative, as presented in Leoneti [6], has the characteristic of having the best scores from each evaluation criteria so that the highest values of the pairwise comparisons are close to one when the alternatives are close to the ideal alternative. It should be noted, consequently, that the ideal alternative was modeled in the sense of distances in a Euclidian space. Note that for the exponential function, which has its structure based on differences, when the alternatives are similar, the value will tend to zero, in its original version, and to $\frac{\lambda}{(1+\lambda)}$, in its adjusted version. Therefore, it becomes necessary the adoption of the concept of dominance, rather than ideal alternative, for providing a measure of the alternative performance. Here, a central tendency measure from the comparison between a given alternative (*status quo*) with all other alternatives was used. It has been assumed, according to Simon [32], that a satisfactory outcome would be more adequate as a social outcome than an optimal one. This is especially convenient in the scenario of multiple criteria being optimizing with different preferences. Therefore, instead of using the best value of each criterion for the composition of the ideal alternative, the reference alternative started to be calculated directly from the average of the comparison of each alternative with the others. In other words, for an agent i and its *status quo* alternative s_i, the value is $\delta_i\overline{(s_i)} = \sum_{k=1}^{b}\frac{\delta_i(s_i,s_k)}{b}$, which is the average value of the pairwisecomparison values of alternative s_i with all other b alternatives under consideration.

Additionally, in the utility function presented in Leoneti [6] alternatives that are different make the value of the utility function tends to zero while alternatives that are similar make that its value tends to one. On opposite, in the Phi function presented in Eq. (3) the value zero is assumed when the maximum perception of loses in the exchange between the alternatives is found and one when the maximum perception of gains is found. In this sense, as a last adaptation, it was necessary to transpose the payoff matrix. Therefore, the factors of the utility function $\pi_i(s_i, s_{i\neq j})$ presented in Eq. (1) started to be according to the function presented in Eq. (8), which hence onwards is named adjusted utility function, as shown in the equation

$$\pi_i(s_i, s_{i\neq j}) = \delta_i\overline{(s_i)} \prod_{i\neq j, j=1}^{a} \delta_i(s_i, s_j)\delta_i\overline{(s_j)} \tag{9}$$

where δ_i are the dominance calculated from the pairwise comparison provided by the Phi function based on the exponential model of prospect theory as presented in ExpTODIM method with the due adjustments.

Now, the application of the adapted utility function can be summarized in five steps: (i) proposition of a decision matrix (alternatives *versus* criteria), which is standardized based on the preferences of each agent involved; (ii) calculation of the pairwise comparison measure between the alternatives based on each possible arrangement of the group's decision using the judgment component based on the exponential model of prospect theory; (iii) creation of the payoff tables using the values generated by the arrangement of all pairwise alternatives comparisons through the decision component; (iv) search for a solution within the generated payoff tables; and (v) finally, if there are multiple solutions, the alternatives involved are subject to an ordering by a social welfare function, and this order is subsequently evaluated by the decision makers for the selection of the alternative preferable to the group. Considering a decision matrices $\{x_{ij}\}_l$, where $i = 1, \ldots, b$ alternatives, $j = 1, \ldots, c_l$ criteria, and $l = 1, \ldots, a$ is the number of agents, the steps for the application of the utility function are detailed below.

Step 1: for each agent $l = 1, \ldots, a$, standardize the criteria using the sum linear standardization technique

$$\{s_{ij}\}_l = \frac{\{x_{ij}\}_l}{\sum_{i=1}^{c_l}\{x_{ij}\}_l}, \text{ if benefit criteria} \tag{10}$$

$$\{s_{ij}\}_l = \frac{\frac{1}{\{x_{ij}\}_l}}{\sum_{i=1}^{c_l}\frac{1}{\{x_{ij}\}_l}}, \text{ if cost criteria} \tag{11}$$

Step 3: for each agent $l = 1, \ldots, a$, based on an exponential mathematical model, calculate the Phi function for each criterion $j = 1, \ldots, c_l$ in the pairwise comparisons between the alternatives $i = 1, \ldots, b$ and $k = 1, \ldots, b$

$$\varphi_j(\{s_{ij}\}_l, \{s_{kj}\}_l) = \begin{cases} \dfrac{\{w_j\}_l\left(1-10^{-\rho|\{s_{ij}\}_l-\{s_{kj}\}_l|}\right)+\lambda}{(1+\lambda)} \; if\,(\{s_{ij}\}_l-\{s_{kj}\}_l)\rangle 0 \\ \dfrac{\lambda}{(1+\lambda)} \; if\,(\{s_{ij}\}_l-\{s_{kj}\}_l)=0 \\ \dfrac{-\lambda\{w_j\}_l\left(1-10^{-\rho|\{s_{ij}\}_l-\{s_{kj}\}_l|}\right)+\lambda}{(1+\lambda)} \; if\,(\{s_{ij}\}_l-\{s_{kj}\}_l)\langle 0 \end{cases} \tag{12}$$

for $\forall(i,k)$ where $\{w_j\}_l$ is the weighting of criterion $j = 1, \ldots, c_l$, requiring that $\sum_{j=1}^{c_l}\{w_j\}_l = 1$

Step 3: determine the dominance

$$\delta_i(\{s_i\}_l, \{s_k\}_l) = \sum_{k=1}^{s} \varphi_i(\{s_i\}_l, \{s_k\}_l) \tag{13}$$

Step 4: using the adjusted utility function presented in Eq. (9), generate the payoff tables for each agent $l = 1, \ldots, a$ calculating all b^a possible arrangements in the form $\{S\}_1 \times \{S\}_2 \times \ \ldots \ \times \{S\}_a$

Step 5: An equilibrium solution concept is applied to the payoff tables for finding the equilibria solutions to the decision game. Among the possible equilibrium solution concept for solving non-cooperative games, it can be used the Nash equilibrium, which is a particular case of the General Metarationality (GMR) equilibrium [33]. Other possible equilibrium solution concepts, according to Hipel & Fang [33], are Symmetric Metarationality (SMR), Sequential Stability (SEQ), Limited-Move Stability (L_h), and Non-Myopic Stability (N-M). If more than one equilibrium is found, a social welfare function is also applied for selecting or ranking the equilibria. Alternatively, the concept of consensus solution can be applied by finding the highest average among the payoffs of the a agents for the cases where $s_1 = s_2 = \ldots = s_a$

3. Methodology

Following the steps of the experiment described by Leoneti and de Sessa [21] with the cases proposed by Ziotti and Leoneti [14], a hundred students from the Ribeirão Preto campus of the University of São Paulo were invited to participate in the resolution of three cases in a session with five participants each. The cases were focusing on different perspectives of time, where the first had a short run perspective, related to the choice of a travel destination, the second a medium run perspective, related to the enrollment at a language school, and the third a long run perspective, which was the choice of a CEO to a company [14]. At the beginning of each session, the volunteers read the free and informed consent form and agreed to participate in the study. The volunteers were also aware to the fact that it would not be possible to participate in another session. Then, a presentation was carried out to contextualize the applicability of the study and its objectives. The instructions and rules that each volunteer had to follow throughout the session were also introduced: (i) to not communicate in the individual phase; (ii) to interpretate the case as real as possible; and (iii) to know that there would not be veto power. After the end of the presentation, the volunteers completed an identification form to collect personal data such as name, age, email, undergraduate/graduate course, and course's semester. Subsequently, the volunteers had to read and analyze, one at a time, the context of the three cases and their respective decision matrix to order criteria and alternatives according to their preferences. Each case had a briefly introduction that involved the knowledge that the volunteer was about to join a group decision making and was represented by a decision matrix with five alternatives that should be evaluated using eight benefit (B) and/or cost (C) criteria, with discrete and/or continuous scales. For instance, for the first case, the contextualization was "In order to attract and retain customers, a travel agency creates a promotion and a group of people won an 'all-included' travel by the agency. The conditions are: the winners must travel together and the agency will cover the hotel (including breakfast) and travel expenses. Congratulations, you are one of the lucky ones! Considering that every winner has, at least, 12 days of vacation, you have to negotiate with the other agents the travel destination" [14]. The decision matrix of this case can be seen in Table 1, while the other decision matrices are shown in Tables 2, and 3, respectively.

Each session was divided into different phases. Firstly, the volunteers evaluated the criteria and provided a ranking for them. The preference order of the criteria was transformed into weighting vectors by using the Rank Order Centroid (ROC) method to be used as input for each of the methods to be compared. Subsequently, 15 min for negotiation was allowed, which was also the available time for calculating the results from a Microsoft Excel® spreadsheet that contained the steps of the utility function in Eq. (1). At the ending of negotiation, the disclosure of the group's agreement was made simultaneously by the presentation of the solution provided by the application of the Nash Equilibrium solution concept to the payoff tables generated by the utility function from Leoneti [6]. The Nash equilibria was found by means of an exhaustive search which was programmed within the same spreadsheet environment. It should be noted that, when more than one equilibrium was found, the selected solution was the one with the highest average of payoffs. Finally, after considering the negotiated agreement and the solution provided by the method, each participant presented their final decision in a secret manner by means of a ballot, as proposed by Ziotti and Leoneti [14], for verifying the commitment with the group's agreement.

Using the data gathered within the sessions, the methods were then compared. The versions of the methods were named as: UF+NE, utility function (Eq. (1)) and Nash equilibrium solution (the one with the highest average when in the case of a not singular solution); UF+Co, utility function (Eq. (1)) and a consensus solution[4]; AUF+NE, adjusted utility function (Eq. (9)) and Nash equilibrium solution (the one with the highest average when in the case of a not singular solution); and AUF+Co, adjusted utility function (Eq. (9)) and a consensus solution. For the application of the adjusted utility function, the λ parameter was defined as proposed in Tversky and Kahneman [16], that is, $\lambda = 2.25$, and the value of the parameter ρ was proposed as an intermediate value on the scale between [1-5], therefore, $\rho = 3$, as suggested by Leoneti & Gomes [18]. In addition to the four methods described, a classic method of supporting group decision-making, proposed by Jean-Charles de Borda in the 19th century, known as the Borda count [34], was added for comparison purposes. All methods were programmed within the same Microsoft Excel® spreadsheet environment.

Finally, in order to evaluate the performance of the methods, the following performance indicators were used: (i) the number of matches of the method regarding the agreement negotiated by the group; and (ii) the number of times that, when there was a match, the group maintained its decision after voting.

4. Results and discussion

The methods presented in the previous section were evaluated comparatively based on the performance indicators, whose results can be seen in Table 4.

It is noticed that among all the compared methods, the Borda's count method achieved the highest number of matches between the agreement of the group and the method's solution, achieving 42 correct matches. However, among the matches of this method, only in 26 times the decision was maintained after the manifestation of the voting. It should be noted that among the 60 negotiations, the group's decision was maintained 31 times after the voting phase, with this method identifying only 26 of them. Thus, despite a high number of matches, Borda's count was not able to find all alternatives that would reduce the chance of breaking the contract. It was expected, since this voting system is a method for identifying the winner and, not necessarily, to find for the best social outcome. On this aspect, it is noteworthy that the methods that involve the use of the exponential Phi function in its structure achieved the best performances. Furthermore, the adjusted utility function using the ExpTODIM's exponential function achieved 38 matches, the second largest among the methods, and identified 27 of the times that the decision was maintained by the group after the voting phase. Regarding the original utility function and the adjusted utility function, three more matches were found by the latter, when the highest average among the Nash equilibria found was selected, and seven more matches when consensus solutions were considered. It is noteworthy therefore, that the main hypothesis of this research was confirmed, that is, that the use of a Phi function based on a grounded theory of judgment and decision increases the performance of the utility function proposed in Leoneti [6], which used a Phi function that was not based on a specific judgment and decision theory. It can also be highlighted the fact that the method AUF+Co does not apply any equilibrium solution concept, which makes it simpler for searching the solution, which is also singular.

Finally, a supplementation analysis was performed between the methods that obtained the best performance in terms of the number of matches for the alternative chosen by the group and the method's solution, represented here by the Borda's count and the AUF+Co method. Firstly, it was evaluated the scenario when the matches from AUF+Co were supplemented by the matches from Borda's count. Then, the matches from Borda's count were supplemented by the matches from AUF+Co. It was verified that the methods supplemented the matches of each other with the same amplitude, totaling 53 matches out of 60 possible, including all 31 the agreements that the group maintained after the voting phase. Consequently, it should be emphasized that, considering that for finding the solution by the AUF+Co method it is only necessary the ranking of the initial criteria, this group decision making method can initiate the group decision making process by only requiring the order of criteria to the decision makers. In the case that the method doesn't match the group's decision, the Borda's count method can be applied in the sequence, for which the order of the alternative to the decision makers is required. In other words, for when the consensus solution was not found by the group, Borda's count can be used to assist in the group decision-making process. In the context of the simulations presented here, such procedure was able to predict approximately 88% of the final outcome. This result is in accordance with the results of

[4] The concept of consensus solution adopted was the selection of the alternative with the highest average among the payoffs of the agents for the cases where $s_1 = s_2 = \ldots = s_a$

Table 1
Decision matrix of the first case: travel destination.

	B: Hotel rating	C: Travel time (hours)	B: Stay duration (days)	C: Exchange rate (R $)	B: Shopping	B: Cultural attractions	B: Nature	B: Infrastructure
Destination A	2.5	8	4	0.90	5	3	7	8
Destination B	3.5	2.5	6	3.10	9	7	3	6
Destination C	3	4	7	4.70	4	5	9	7.5
Destination D	5	13	5	3.30	3	9	6	7
Destination E	4	16	8	1.10	6	8	5	5

Source: adapted from Ziotti and Leoneti [14].

Table 2
Decision matrix of the second case: language school.

	C: Distance in km	C: Teaching material (R$)	C: Class size (students)	B: Weekly hours/ class	B: Infrastructure	B: School reputation	B: Additional activities	B: Course quality
School A	14	450	15	2	10	6	4	9
School B	7	650	12	3	9	8	5	8
School C	16	590	4	2.5	8	7	9	7
School D	6.5	570	8	6	5	8	6	8
School E	10	300	18	4	7	9	8	5

Source: adapted from Ziotti and Leoneti [14].

Table 3
Decision matrix of the third case: new CEO.

	B: Professional training (years)	B: Years in the company	B: Years working in the area	B: Leadership positions	B: Ethics	B: Adaptation to change	B: Commitment	B: Professional influence
CEO A	8	7	7	3	10	4	6	9
CEO B	7	4	10	3	9	9	4	8
CEO C	6	6	15	4	8	7	7	4
CEO D	5	8	12	7	7	8	8	4
CEO E	4	10	9	6	5	7	10	7

Source: adapted from Ziotti and Leoneti [14].

Table 4
Comparison between methods.

Method	Matches (out of 60 cases)	Percentage	Decision maintained among the matches	Percentage
UF+NE	31	52%	20	65%
UF+Co	31	52%	22	71%
AUF+NE	34	57%	25	74%
AUF+Co	38	63%	27	71%
Borda's count	42	70%	26	62%

previous researches [14].

5. Conclusions

From an empirical evaluation, the main hypothesis of this research was confirmed, that is, that the use of a pairwise comparison function adhering to a decision and judgment theory could increase the predictive capacity of the utility function proposed in Leoneti [6]. An advantage identified was the use of the concept of consensus solution instead of the concept of equilibrium to find the group's solutions. While equilibrium solutions reduce the space for finding solutions, for example, reducing the initial number of alternatives to only those that are part of some identified equilibrium, decisions by consensus seem to be in greater agreement with the individual's point of view on the group. It can generate two practical advantages since there would not be the necessity of searching for equilibria by the use of some specific algorithmic neither to dealt with the equilibrium selection problem when in the case of a non-singular solution.

On the other hand, the total number of correct matches with the group's decision was not greater than the number of correct matches from Borda's count method, a well-known voting technique. It could lead to the equivocated conclusion that it would be better to apply a voting technique rather than the use of a group decision making method for supporting group decision making. However, it should be noted that the prediction made by the adjusted utility function is based solely on the preference of individuals over the criteria, not requiring alternatives' preferences, as in voting procedures. That proportionate a decreasing resistance on the search for consensus in group decision making, evidenced by the higher number of correct matches that were maintained after the final confirmation in comparison to the performance of Borda' count method to the same indicator. It could be explained due to the fact that when individuals focus their preferences on the alternatives it diminishes the chances of trading them in the search of a consensus.

That result presents interesting managerial implications, since that the joint use of the adjusted utility function with the Borda's count method was able to provide correct predictions of about 88% of the cases. In this sense, within a process of group decision making where more than one objective is under analysis, the results suggest that starting the process with the criteria evaluation with the proposition of a solution through the application of the adjusted utility function and, in the case of none consensus found, the application of the Borda's count method, can lead the group to a high level of convergence for reaching a solution. Consequently, the advantage of starting the process by the criteria point of view with the eventual treatment of the lack of consensus by a well-known voting technique diminish the chances of contract break, which is an important aspect for the group's decision implementation.

Future research can evaluate the results presented in this paper by means of comparisons among different types of multicriteria decision making methods, particularly the ones with its structures developed for supporting group decision making. Furthermore, field studies with different decision matrices with different number of criteria could be tested in future works.

Author statement

Alexandre Bevilacqua Leoneti: Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Resources; Supervision; Validation; Visualization; Writing; Review & editing. **Luiz Flavio Autran Monteiro Gomes:** Conceptualization; Formal analysis; Investigation; Methodology; Validation; Visualization; Writing; Review & editing.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Corrigendum

Corrigendum to modeling multicriteria group decision making as games from enhanced pairwise comparisons [Operations Research Perspectives 8 (2021) 100194]

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In the paper by [1], we have noticed a typo in the Eq. (6), where the index j is missing at s_i and s_k within function φ, then it should read as:

$$\delta_i(s_i, s_k) = \sum_{j=1}^{c} \varphi_j\left(s_{ij}, s_{kj}\right) \tag{6}$$

We also have noticed a typo in the Eq. (7), where the upper limit of the sum operators should be b, then it should read as:

$$\xi_i = \frac{\sum_{k=1}^{b}\delta_i(s_i, s_k) - \min\sum_{k=1}^{b}\delta_i(s_i, s_k)}{\max\sum_{k=1}^{b}\delta_i(s_i, s_k) - \min\sum_{k=1}^{b}\delta_i(s_i, s_k)} \tag{7}$$

We have noticed the lack of the factor $\delta_i(s_i, s_i)$ in the Eq. (9) as it is pointed out in the paper in "the comparison between a given alternative (status quo) with all other alternatives" (note that in the Eq. 1 $\varphi_i(s_i, s_i) = 1$). Then, the correct form of such the Eq. (9) therefore should be:

$$\pi_i\left(s_i, s_{i\neq j}\right) = \delta_i\overline{\left(s_i\right)}\delta_i(s_i, s_i)\prod_{i\neq j, j=1}^{a}\delta_i\left(s_i, s_j\right)\delta_i\overline{\left(s_j\right)} \tag{9}$$

We have also noticed the lack of the factor λw_j in the Eq. (12), as it is pointed out in the paper that for the case of multiple criteria then $\sum_{j=1}^{c_l}\{w_j\}_l = 1$, and, therefore, this factor guarantees that the value of dominance be between zero and one. Then, the equation should read as:

$$\varphi_j\left(\left\{s_{ij}\right\}_l, \left\{s_{kj}\right\}_l\right) = \begin{cases} \dfrac{\left\{w_j\right\}_l\left(1 - 10^{-\rho\left|\left\{s_{ij}\right\}_l - \left\{s_{kj}\right\}_l\right|}\right) + \lambda w_j}{(1+\lambda)} \; if\left(\left\{s_{ij}\right\}_l - \left\{s_{kj}\right\}_l\right) > 0 \\ \dfrac{\lambda w_j}{(1+\lambda)} \; if\left(\left\{s_{ij}\right\}_l - \left\{s_{kj}\right\}_l\right) = 0 \\ \dfrac{-\lambda\left\{w_j\right\}_l\left(1 - 10^{-\rho\left|\left\{s_{ij}\right\}_l - \left\{s_{kj}\right\}_l\right|}\right) + \lambda w_j}{(1+\lambda)} \; if\left(\left\{s_{ij}\right\}_l - \left\{s_{kj}\right\}_l\right) < 0 \end{cases} \tag{12}$$

We finally have noticed typos in the Eq. (13), where the index j is missing at s_i and s_k within function φ, the upper limit of the sum operator should be c_l, and the index of the sum operator should be j, then it should read as:

$$\delta_i\left(\left\{s_i\right\}_l, \left\{s_k\right\}_l\right) = \sum_{j=1}^{c_l}\varphi_j\left(\left\{s_{ij}\right\}_l, \left\{s_{kj}\right\}_l\right) \tag{13}$$

We additionally point out that the calculus of dominance can be alternatively performed in the following way:

Step 1: for each agent $l = 1, \ldots, a$, standardize the criteria using the linear standardization sum technique

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$$\{s_{ij}\}_l = \frac{\{x_{ij}\}_l}{\sum_{i=1}^{c_l}\{x_{ij}\}_l}, \text{ if benefit criteria} \tag{10}$$

$$\{s_{ij}\}_l = \frac{\frac{1}{\{x_{ij}\}_l}}{\sum_{i=1}^{c_l}\frac{1}{\{x_{ij}\}_l}}, \text{ if cost criteria} \tag{11}$$

Step 2: for each agent $l = 1, \ldots, a$, based on an exponential mathematical model, calculate the Phi function for each criterion $j = 1, \ldots, c_l$ in the pairwise comparisons between the alternatives $i = 1, \ldots, b$ and $k = 1, \ldots, b$

$$\varphi_j\left(\{s_{ij}\}_l, \{s_{kj}\}_l\right) = \begin{cases} \{w_j\}_l\left(1 - 10^{-\rho\left|\{s_{ij}\}_l - \{s_{kj}\}_l\right|}\right) \; if\left(\{s_{ij}\}_l - \{s_{kj}\}_l\right) > 0 \\ 0 \; if\left(\{s_{ij}\}_l - \{s_{kj}\}_l\right) = 0 \\ -\lambda\{w_j\}_l\left(1 - 10^{-\rho\left|\{s_{ij}\}_l - \{s_{kj}\}_l\right|}\right) \; if\left(\{s_{ij}\}_l - \{s_{kj}\}_l\right) < 0 \end{cases} \tag{12a}$$

for $\forall(i, k)$ where $\{w_j\}_l$ is the weighting of criterion $j = 1, \ldots, c_l$, requiring that $\sum_{j=1}^{c_l}\{w_j\}_l = 1$

Step 3: determine the dominance

$$\delta_i\left(\{s_i\}_l, \{s_k\}_l\right) = \frac{\left[\sum_{j=1}^{c_l}\varphi_j\left(\{s_{ij}\}_l, \{s_{kj}\}_l\right)\right] + \lambda}{(1 + \lambda)} \tag{13a}$$

Step 4: using the adjusted utility function presented in Eq. (9), generate the payoff tables for each agent $l = 1, \ldots, a$ calculating all b^a possible arrangements in the form $\{S\}_1 \times \{S\}_2 \times \ \ldots \ \times \{S\}_a$

Reference

[1] Leoneti AB, Gomes LFAM. Modeling multicriteria group decision making as games from enhanced pairwise comparisons. Operat Res Perspect 2021:100194.