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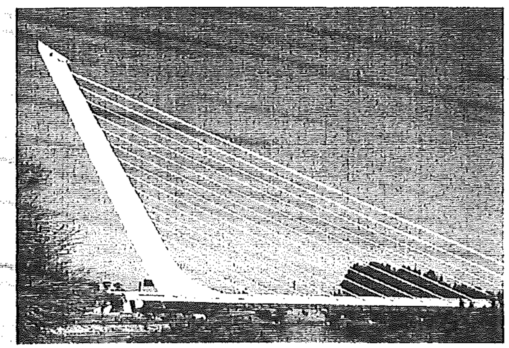

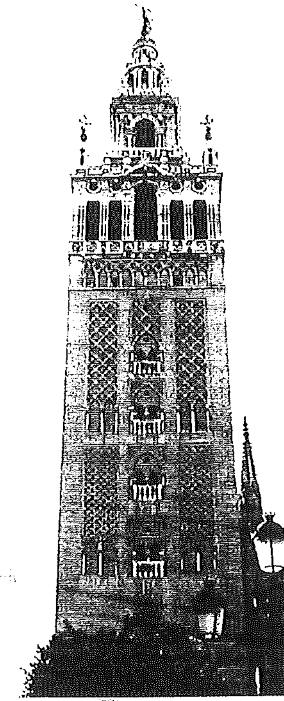

Sevilla 2002
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- -Microsoft WINDOWS 98 or later
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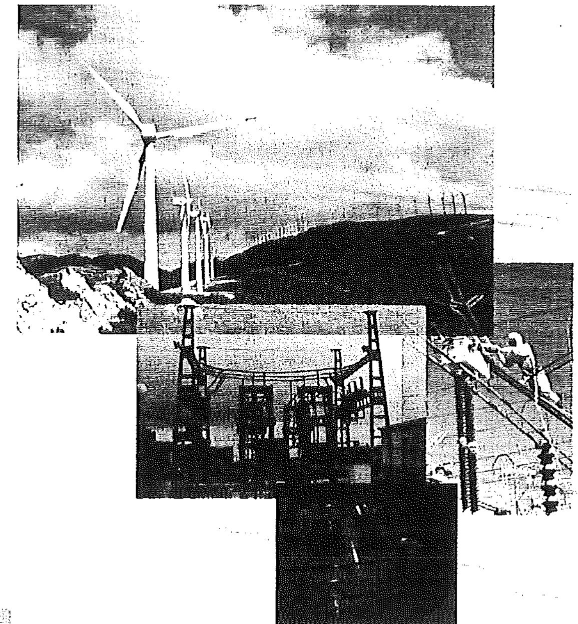

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Extended Lyapunov Functions for Detailed Power System Models

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Abstract - The main concern of this paper is the proposal of Extended Lyapunov Functions for power system models which do not have Lyapunov Functions in the usual sense. Extended Lyapunov Functions are proposed for a single-machine-infinite-bus-system considering line losses and with both the classical model and the one-axis-model for the generator. An Extended Lyapunov Function is also proposed for multimachine systems taking transfer conductances in consideration.

Keywords - Invariance Principle, Transient Stability, Direct Methods, Energetic methods, Lyapunov Functions

1 Introduction

DIRECT methods have been shown to be suitable for stability analysis of power systems on real time. Among these methods, Lyapunov's ideas associated to LaSalle's Invariance Principle have been used to estimate the stability region of power systems. In the last decades, many authors have addressed the problem of estimating the stability regions and these studies culminated with the development of the BCU method [1] [3] [4] [7] [8], which nowadays is considered the most efficient direct method for studying transient stability. In spite of these advances, there exist many obstacles in the application of direct methods to the assessment of stability in real power systems. The main one is that direct methods are still improper to deal with more realistic models. In fact this obstacle is intimately related to the problem of finding a suitable Lyapunov Function associated to those models.

Finding Lyapunov Functions for power systems is a difficult task which has been challenging engineers for several decades. Unfortunately, there is no systematic method for obtaining these functions, and the success of a process of trial and error exclusively depends on the personal experience.

In general, in order to find a Lyapunov Function for power systems, many simplifications are usually made. Machines are modeled as constant electromotive forces behind the transient reactances. Loads are modeled as constant impedances, the network is reduced to the electromotive force nodes and the transfer conductances are neglected in the reduced model.

Recently, an extension of the LaSalle's Invariance Principle has been proposed [17] [18]. This extension relaxes some of the requirements on the auxiliary function which is commonly called Lyapunov Function. In this extension, the derivative of the auxiliary function can be positive in some bounded regions of the state space and, for distinction purposes, it is called, in this case, Extended Lyapunov Function.

The advantage of this extension is that a greater number of problems, that could not be solved by the usual invariance principle, now can be treated by this theory. Furthermore, it is easier to find an Extended Lyapunov Function than a Lyapunov Function in the usual sense.

The main concern of this paper is to provide Lyapunov Functions in the extended sense for power system models which do not have a Lyapunov Function in the usual sense. First of all, the usual Invariance Principle is reviewed and the extension of the Invariance Principle is presented. In the sequence, the problem of one-machine-infinite-bus system is studied. Using the classical model for the machine and considering losses in the transmission line, an Extended Lyapunov Function is proposed. For the same system, a Lyapunov Function in the usual sense is proposed considering the one-axis model for the generator and neglecting line losses. Considering the one-axis model and taking into account line losses, an Extended Lyapunov Function is proposed. In these three cases, the proposed Lyapunov Functions are used to estimate the attraction area of the respective systems.

In the second part of this paper, an Extended Lyapunov Function is proposed for a multimachine system model taking into consideration the transfer conductances. This function can be used for attraction area estimation and is based on a solid theoretical background.

2 The Invariance Principle

This section starts by reviewing the usual Invariance Principle [11] [12] [13] [14]. Consider the following autonomous differential equation:

$$\dot{x} = f(x) \tag{1}$$

Theorem 2.1 Let $V_{FL} : \mathbb{R}^n \to \mathbb{R}$ and $f : \mathbb{R}^n \to \mathbb{R}^n$ be C^1 functions. Let $L > 0$ be a constant such that $\Omega_L = \{x \in \mathbb{R}^n : V_{FL}(x) < L\}$ is bounded. Suppose that $\dot{V}(x) \leq 0$ for every $x \in \Omega_L$ and define $E := \{x \in \Omega_L : \dot{V}(x) = 0\}$. Let B be the largest invariant set contained in E. Then every solution of (1) starting in Ω_L converges to B as $t \to \infty$.

In this work, more general results than the previous one are presented. They require less restrictive conditions and allow the possibility of the derivative of V_{FL} to be positive in some regions. The advantage of these results is that it is easier to find the function V_{FL} and some quite complicated problems can be treated as well.

Theorem 2.2 (The Extended Invariance Principle): Let $V_{FL} : \mathbb{R}^n \to \mathbb{R}$ and $f : \mathbb{R}^n \to \mathbb{R}$ be C^1 functions. Let $L \in \mathbb{R}$ be a constant such that $\Omega_L = \{x \in \mathbb{R}^n : V_{FL}(x) < L\}$ is bounded. Let $C := \{x \in \Omega_L : \dot{V}(x) > 0\}$, suppose that $sup_{x \in C} V_{FL}(x) = l < L$. Define $\bar{\Omega}_l = \{x \in \mathbb{R}^n : V_{FL}(x) \leq l\}$ and $E := \{x \in \Omega_L : \dot{V}_{FL}(x) = 0\} \bigcup \bar{\Omega}_l$. Let B be the largest invariant set of (1) contained in E. Then every solution of (1) starting in Ω_L converges to the invariant set B, as $t \to \infty$.

Moreover if $x_o \in \bar{\Omega}_l$ then $\varphi(t, x_o) \in \bar{\Omega}_l$ for every $t \geq 0$ and $\varphi(t, x_o)$ tends to the largest invariant set of (1) contained in $\bar{\Omega}_l$. ■

For a proof see [17] and for more general results see [18].

Remark 2.1 *In many cases, the set* $\left\{x \in \Omega_L : \dot{V}_{FL}(x) = 0\right\}$ *is contained into the set* $\bar{\Omega}_l$. *In these cases* $E = \bar{\Omega}_l$ *is an estimate of the attractor and* Ω_L *is an estimate of the attraction area or stability region.*

3 Extended Lyapunov Functions for SMIB Systems

3.1 *Classical Model*

Consider the SMIB system of Figure 1 where a synchronous machine is connected to an infinite bus through a transmission line with losses.

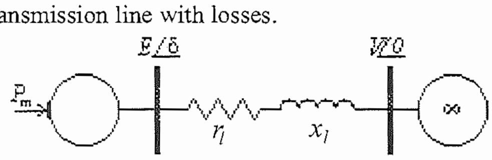

Figure 1: Single Machine Infinite Bus System

Modeling the generator as a constant electromotive force behind the transient reactance, this system can be mathematically described by the following pair of differential equations:

$$\begin{aligned} \dot{\delta} &= \omega \\ \frac{H}{\pi f_o}\dot{\omega} &= P_m - E^2 G + E E_\infty B sin\delta + E E_\infty G cos\delta - T\omega \end{aligned} \tag{2}$$

where δ and ω are respectively the rotor angle and the generator frequency deviation from the synchronous frequency, P_m is the input mechanical power, H is the inertia constant, E is the electromotive force, E_∞ is the voltage magnitude at the infinite bus, T is the damping coefficient and $G + jB$ is the admittance of the equivalent transmission line. For notation simplicity, let us rewrite the SMIB differential equations as:

$$\begin{aligned} \dot{\delta} &= \omega \\ \frac{H}{\pi f_o}\dot{\omega} &= P - Csin\delta - Dcos\delta - T\omega \end{aligned} \tag{3}$$

where $P = P_m - E^2 G$, $C = -E E_\infty B$ and $D = -E E_\infty G$. Alternatively, one can write equation (3) in a shorter form as:

$$\begin{aligned} \dot{\delta} &= \omega \\ \frac{H}{\pi f_o}\dot{\omega} &= P_l(\delta) - T\omega \end{aligned} \tag{4}$$

where $P_l(\delta) = P - Csin\delta - Dcos\delta$.

Although this model incorporates line losses, this system has a general Lyapunov Function in the usual sense given by:

$$V_{FL}(\delta, \omega) := \frac{H}{\pi f_o}\frac{\omega^2}{2} - P\delta - Ccos\delta + Dsin\delta + \alpha \tag{5}$$

where α is an arbitrary constant. It is easy to show that the derivative of V along the orbits is given by:

$$\dot{V}_{FL} = -T\omega^2 \leq 0 \tag{6}$$

which is a negative semi-definite function. The function V_{FL} satisfies the requirements of the usual Invariance Principle and as consequence this function can be used to study the stability of this system in the usual way.

In spite of that, a new function will be proposed and the Extended Invariance Principle will be used to study the stability of this system. Our purpose is to illustrate the application of the extension of the Invariance Principle and to prepare the ideas to solve other problems which do not present a Lyapunov Function in the usual sense.

For this purpose, consider the following function:

$$\begin{aligned} W(\delta, \omega) \;\; := \;\; & \frac{H}{\pi f_o}\frac{\omega^2}{2} - P\delta - C\cos\delta \\ & -\beta\omega\left(P - C\sin\delta - D\cos\delta\right) + \alpha \end{aligned} \tag{7}$$

where β is a parameter to be adjusted and α is an arbitrary constant. Our goal is to show that this function satisfies the requirements of Theorem 2.2.

Calculating the derivative of W along the orbits we obtain:

$$\begin{aligned} \dot{W} \;\; := \;\; & -\left(T - \beta\left(C\cos\delta - D\sin\delta\right)\right)\omega^2 + \\ & + \frac{\beta\pi f_o T}{H}\left(P - C\sin\delta - D\cos\delta\right)\omega - \\ & - \frac{\beta\pi f_o}{H}\left(P - C\sin\delta - D\cos\delta\right)^2 - D\cos(\delta)\omega \end{aligned} \tag{8}$$

which is equivalent to:

$$\begin{aligned} -\dot{W} := & \begin{bmatrix} P_l(\delta) \\ \omega \end{bmatrix}^T \begin{bmatrix} \frac{\beta\pi f_o}{H} & -\frac{\beta\pi f_o T}{2H} \\ -\frac{\beta\pi f_o T}{2H} & T - \beta(C\cos\delta - D\sin\delta) \end{bmatrix} \begin{bmatrix} P_l(\delta) \\ \omega \end{bmatrix} \\ & + D\cos(\delta)\omega \end{aligned} \tag{9}$$

Note that this function is composed by a quadratic term plus the term $D\cos(\delta)\omega$. Parameter β can be chosen in order to make the quadratic term positive definite. Applying the Silvester's Criteria one can easily find that this is certainly guaranteed if

$$0 < \beta < \frac{T}{C + D + \frac{\pi f_o T^2}{4H}}$$

In this way, only the term $D\cos(\delta)\omega$ will be responsible for generating regions where the derivative of W is positive. Once parameter β has been chosen, it is necessary to find a real number L such that the conditions required in Theorem 2.2 are satisfied. These conditions are:

- The set Ω_L must be bounded;
- $l = sup_{x \in C} V_{FL}(x) < L$.

In the following example, these conditions are checked numerically.

Example 3.1 *Consider the SMIB system of Figure 1 with* $P_l = 1.0$, $C = 2.0$, $D = 0.10$, $T = 0.10$ *and* $H = 9.425$. *The level curves of* W *are depicted in Figure 2 for* $\alpha = 2.2551$ *and* $\beta = 0.0093$. *The constant* α *was chosen in order to make the energy of the post-fault stable equilibrium equal to zero. The regions where the derivative of* W *is positive are small bounded sets which are shown in black in Figure 2. One of them is close to the unstable equilibrium point. The another set is close to the stable equilibrium point and corresponds to the set* C *of Theorem 2.2.*

The maximum value of W in $\bar{C}$ defines the set $\bar{\Omega}_l$ which is an attractor estimate, i.e. all the solutions starting into the stability region will enter in this attractor estimate in a finite time. In this example, one finds numerically $l = 0.07$. In order to estimate the stability region or attraction area of the attractor we must choose the largest number L of Theorem 2.2 such that the conditions of Theorem 2.2 are satisfied. In practice, we must guarantee that $\bar{\Omega}_L$ does not intercept the region close to the unstable equilibrium point where the derivative is positive. In this example $L = 1.34$ is found numerically. Figure 2 illustrates the attractor and the stability region estimates.

Suppose a solid three-phase short-circuit occurs at the terminal generator bus. The estimated critical clearing time obtained with this new energy function belongs to the interval $(0.358, 0.359s)$. This estimate is very close to the estimated critical clearing time obtained with the conventional Lyapunov Function V which belongs to the interval $(0.362, 0.363s)$. As expected, these estimates are a little conservative because the stability region estimate is contained into the real stability region. The critical clearing time obtained by simulation belongs to the interval $(0.393, 0.394s)$. Figure 2 shows the trajectories of the fault and post-fault system for a clearing time equal to 0.358s.

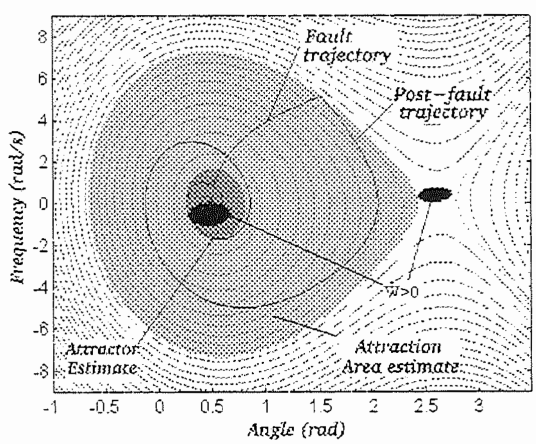

Figure 2: *Level Curves of W*

3.2 One-Axis Model

Consider again the SMIB system of Figure 1 where a synchronous machine is connected to an infinite bus through a transmission line. Using the one-axis model for the generator, this system can be mathematically described by the following differential equations:

$$\dot{\delta} = \omega \tag{10}$$

$$\frac{H}{\pi f_o}\dot{\omega} = P_l(\delta, E'_q) - T\omega \tag{11}$$

$$\frac{\tau'_{do}}{X_d - X'_d}\dot{E}'_q = \frac{1}{X_d - X'_d}E_{fd} + \frac{X'_q}{R^2 + X'_d X'_q}|V|\cos\delta - \frac{R^2 + X_d X'_q}{(R^2 + X'_d X'_q)(X_d - X'_d)}E'_q + \frac{R}{R^2 + X'_d X'_q}|V|\sin\delta \tag{12}$$

where

$$\begin{aligned} P_l(\delta, E'_q) = \; & P_m + \frac{(R^2 - X'_d X'_q)(X'_d - X'_q)}{2(R^2 + X'_d X'_q)^2}|V|^2 \sin 2\delta \\ & - \frac{R(R^2 + X'^2_q)}{(R^2 + X'_d X'_q)^2}E'^2_q + \frac{R X'_q (X'_d - X'_q)}{(R^2 + X'_d X'_q)^2}|V|^2 \sin^2\delta \\ & - \left(\frac{2R^2(X'_d - X'_q)}{(R^2 + X'_d X'_q)^2} + \frac{X'_q}{R^2 + X'_d X'_q}\right)E'_q|V|\sin\delta \\ & + \left(\frac{2R X'_q (X'_q - X'_d)}{(R^2 + X'_d X'_q)^2} + \frac{R}{R^2 + X'_d X'_q}\right)E'_q|V|\cos\delta \\ & + \frac{R X'_q (X'_d - X'_q)}{(R^2 + X'_d X'_q)^2}|V|^2 \cos^2\delta, \end{aligned}$$

$R = r_a + r_l$ is the armature plus the line resistance, $X'_d = x'_d + x_l$ and $X'_q = x'_q + x_l$ are respectively the transient reactances of direct and quadrature axis plus the line reactance, $X_d = x_d + x_l$ and $X_q = x_q + x_l$ are respectively the synchronous reactances of direct and quadrature axis plus the line reactance, E'_q is the electromotive force and E_{fd} is the voltage applied to the field winding.

Finding a Lyapunov Function for this system is not a trivial task. For this purpose define a new variable ε'_q and consider the following change of variables:

$$\varepsilon'_q = ln\,(E'_q)$$

This change of variables is well defined while $E'_q \neq 0$. Neglecting line losses and armature resistance, this system has a general Lyapunov Function in the usual sense given by:

$$\begin{aligned} V_{FL} \; & = \frac{H}{\pi f_o}\omega^2 - P_m\delta - \frac{1}{2}|V|^2\frac{(X'_d - X'_q)}{2X'_d X'_q}\cos 2\delta - \\ & \frac{e^{\varepsilon'_q}|V|}{X'_d}\cos\delta + \frac{1}{2}\frac{X_d}{X'_d(X_d - X'_d)}e^{2\varepsilon'_q} - \frac{1}{X_d - X'_d}E_{fd}e^{\varepsilon'_q} + \alpha \end{aligned} \tag{13}$$

where α is an arbitrary constant. It is easy to show that the derivative of V along the orbits is given by:

$$\dot{V}_{FL} = -\frac{\tau'_{do}}{X_d - X'_d}\left(\dot{\widehat{e^{\varepsilon'_q}}}\right)^2 - T\omega^2 \leq 0 \tag{14}$$

which is a negative semi-definite function. Function V satisfies the requirements of the usual Invariance Principle and as consequence this function can be used to study the stability of this system in the usual way.

Example 3.2 *Consider the SMIB system of Figure 1 with $P_m = 1.0$, $H = 6$, $T = 0.08$, $\tau'_{do} = 5$, $E_{fd} = 1.92$, $V = 1$, $r_a = 0$, $x'_d = 0.2$, $x'_q = 0.4$, $x_d = 0.9$, $x_q = 0.8$, $r_l = 0$ and $x_l = 0.5$. The projected level curves of V_{FL} are depicted in two figures for $\alpha = 3.2625$. Again the constant α was chosen in order to make the energy of the post-fault stable equilibrium equal to zero. In Figure 3, E'_q is fixed and is equal to 1.3. In Figure 4, ω is fixed and is equal to -0.8. In order to estimate the attraction area of the stable equilibrium point, we must choose the largest number L of Theorem 2.1 such that Ω_L is bounded. In this example $L = 0.4081$ is found numerically. The shaded regions in Figures 3 and 4 shows the intersection of the attraction area estimate onto the respective planes. The estimated critical clearing time obtained with this energy function belongs to the interval $(0.127; 0.128s)$. As expected, this estimate is a little conservative because the attraction area estimate is contained into the real one. The* *critical clearing time obtained by simulation belongs to the interval* $(0.281, 0.282s)$. *Figures 3 and 4 show the projected trajectories of the fault and post-fault systems for a clearing time equal to* $0.127s$.

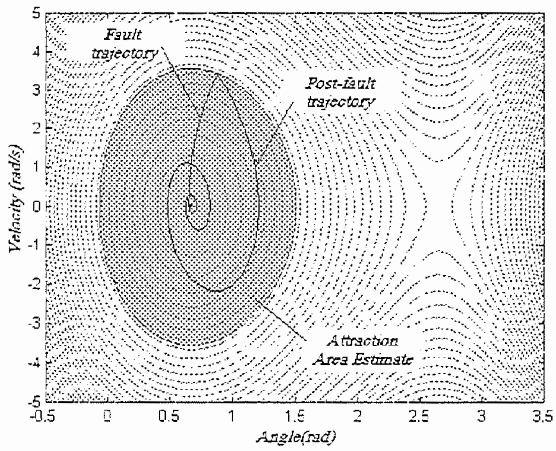

Figure 3: Level Curves of W. E'_q is fixed and equal to 1.3

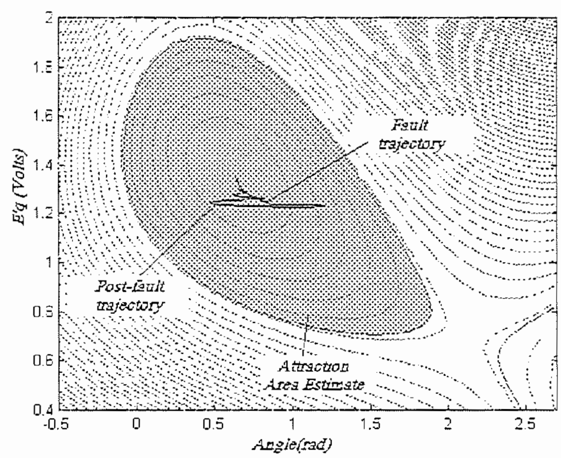

Figure 4: Level Curves of W. ω is fixed and equal to -0.8

When line losses and armature resistance are taken into consideration, this system does not have a Lyapunov Function in the usual sense. In spite of that, a new function, which is an extended Lyapunov Function, will be proposed and the extended Invariance Principle will be used to study the stability of this system. For this purpose, consider the following function:

$$
\begin{aligned}
W \quad = & \tfrac{H}{\pi f_o}\omega^2 - P_m\delta - \tfrac{1}{X_d - X'_d}E_{fd}e^{\varepsilon'_q} - \\
& \tfrac{X'_q}{R^2+X'_dX'_q}e^{\varepsilon'_q}|V|\cos\delta - \tfrac{R}{R^2+X'_dX'_q}e^{\varepsilon'_q}|V|\sin\delta - \\
& \tfrac{RX'_q(X'_d-X'_q)}{2(R^2+X'_dX'_q)}|V|^2\left(\delta + \tfrac{\sin 2\delta}{2}\right) + \\
& \tfrac{RX'_d(X'_d-X'_q)}{2(R^2+X'_dX'_q)}|V|^2\left(\delta - \tfrac{\sin 2\delta}{2}\right) + \\
& \tfrac{(R^2-X'_dX'_q)(X'_d-X'_q)}{4(R^2+X'_dX'_q)^2}|V|^2\cos 2\delta + \\
& \tfrac{R^2+X_dX'_q}{(R^2+X'_dX'_q)(X_d-X'_d)}e^{2\varepsilon'_q} - \beta\omega P_l(\delta, E'_q) + \alpha
\end{aligned} \tag{15}
$$

where β is a parameter to be adjusted and α is an arbitrary constant. Our goal is to show that this function satisfies the requirements of Theorem 2.2. Calculating the derivative of W along the orbits we obtain:

$$
\begin{aligned}
-\dot{W} \quad = & \begin{bmatrix} \widehat{e^{\varepsilon'_q}} \\ P_l(\delta, E'_q) \\ \omega \end{bmatrix}^T A \begin{bmatrix} \widehat{e^{\varepsilon'_q}} \\ P_l(\delta, E'_q) \\ \omega \end{bmatrix} \\
& + \tfrac{2R^2(X'_d-X'_q)}{(R^2+X'_dX'_q)^2}e^{\varepsilon'_q}|V|\omega\sin\delta \\
& - \tfrac{2RX'_q(X'_q-X'_d)}{(R^2+X'_dX'_q)^2}e^{\varepsilon'_q}|V|\omega\cos\delta \\
& + \tfrac{R(R^2+X'^2_q)}{(R^2+X'_dX'_q)^2}e^{2\varepsilon'_q}\omega
\end{aligned} \tag{16}
$$

where

$$
A = \begin{bmatrix} A_{11} & 0 & A_{13} \\ 0 & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}
$$

$$
\begin{aligned}
A_{11} \quad & = \tfrac{\tau'_{do}}{X_d - X'_d} \\
A_{22} \quad & = \tfrac{1}{2\frac{H}{\pi f_o}}\beta \\
A_{33} \quad & = \beta\Bigg[-\left(\tfrac{X'_q}{R^2+X'_dX'_q} + \tfrac{2RX'_q(X'_q-X'_d)}{R^2+X'_dX'_q}\right)e^{\varepsilon'_q}|V|\cos\delta \\
& - \left(\tfrac{2R^2X'_q(X'_d-X'_q)}{(R^2+X'_dX'_q)^2} + \tfrac{R}{R^2+X'_dX'_q}\right)e^{\varepsilon'_q}|V|\sin\delta \\
& - \left(\tfrac{2RX'_q(X'_d-X'_q)}{(R^2+X'_dX'_q)^2} + \tfrac{2RX'_d(X'_d-X'_q)}{(R^2+X'_dX'_q)^2}\right)|V|^2\cos\delta\sin\delta \\
& + \tfrac{(R^2-X'_dX'_q)(X'_d-X'_q)}{(R^2+X'_dX'_q)^2}|V|^2\cos 2\delta\Bigg] + T \\
A_{13} \quad & = \tfrac{\beta}{2}\Bigg[\left(-\tfrac{X'_q}{R^2+X'_dX'_q} - \tfrac{2R^2(X'_d-X'_q)}{(R^2+X'_dX'_q)^2}\right)|V|\sin\delta \\
& + \left(\tfrac{R}{R^2+X'_dX'_q} - \tfrac{2RX'_q(X'_q-X'_d)}{(R^2+X'_dX'_q)^2}\right)|V|\cos\delta \\
& - \tfrac{2R(R^2+X'^2_d)}{(R^2+X'_dX'_q)^2}(e^{\varepsilon'_q})^2\Bigg] \\
A_{23} \quad & = -\tfrac{1}{4\frac{H}{\pi f_o}}\beta d
\end{aligned}
$$

Similarly to the classical model case, the derivative of W is composed by a quadratic term plus two additional terms. The parameter β can be chosen in order to make the quadratic term positive definite. Applying the variation of constants formula to the differential equation of E'_q one finds that E'_q, keeps bounded along the orbits. More precisely, the following estimate is obtained:

$$
\begin{aligned}
E'_q < \quad E'_{qest} = & |E'_q(0)| + \left|\tfrac{E_{fd}}{\tau'_{do}}\right| + \left|\tfrac{(x_d - x'_d)x'_q}{\tau'_{do}(r_a^2 + x'_dx'_q)}V\right| \\
& + \left|\tfrac{(x_d - x'_d)r}{\tau'_{do}(r_a^2 + x'_dx'_q)}V\right|
\end{aligned}
$$

Using this estimate and the Silvester's criteria it is possible to find a number β which guarantees that the matrix A is positive definite.

Example 3.3 *Consider the SMIB system of Figure 1 with* $P_m = 1.0$, $H = 6$, $T = 0.08$, $\tau'_{do} = 5$, $E_{fd} = 1.92$, $V = 1$, $r_a = 0.002$, $x'_d = 0.2$, $x'_q = 0.4$, $x_d = 0.9$, $x_q = 0.8$, $r_l = 0.04$ *and* $x_l = 0.5$. *The level curves of* W *are depicted in Figures 5 and 6 for* $\alpha = 3.3072$ *and* $\beta = 0.0117$. *Again the constant* α *was chosen in order to make the energy of the post-fault stable equilibrium equal to zero. In Figure 5,* E'_q *is fixed and is equal to* 1.3. *In Figure 6,* ω *is fixed and is equal to* -0.8. *The regions where the derivative of* W *is positive are small bounded sets which are shown in black in Figures 5 and 6. One of them is close to the stable equilibrium point, and corresponds to the set* C *of Theorem 2.2. The maximum value of* W *in* $\bar{C}$ *defines the set* $\bar{\Omega}_l$ *which is an attractor estimate, i.e. all the solutions starting into the stability region* *will enter in this attractor estimate in a finite time. To estimate the stability region or attraction area of the attractor, we must choose the largest number L of Theorem 2.2 such that the conditions of Theorem 2.2 are satisfied. In this example* $L = 0.3996$ *is found numerically. Figures 5 and 6 illustrate the attractor and the stability region estimates.*

Suppose a solid three-phase short-circuit occurs at the terminal generator bus. The estimated critical clearing time obtained with this new energy function belongs to the interval $(0.143, 0.144s)$. *As expected, these estimates are a little conservative because the stability region estimate is contained into the real stability region. The critical clearing time obtained by simulation belongs to the interval* $(0.333, 0.334s)$. *Figures 5 and 6 show the projected trajectories of the fault and post-fault system for a clearing time equal to* $0.143s$

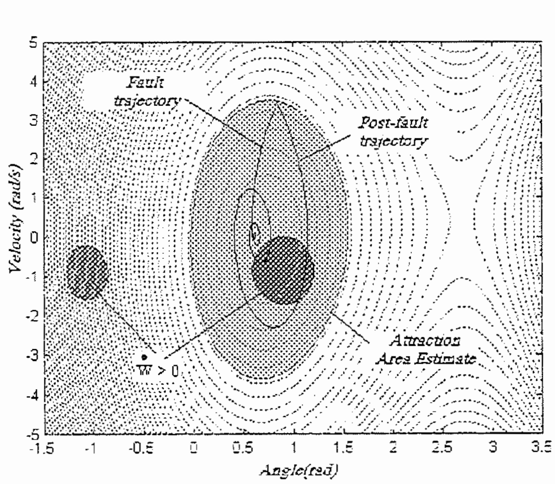

Figure 5: Level Curves of W. E'_q is fixed and equal to 1.3

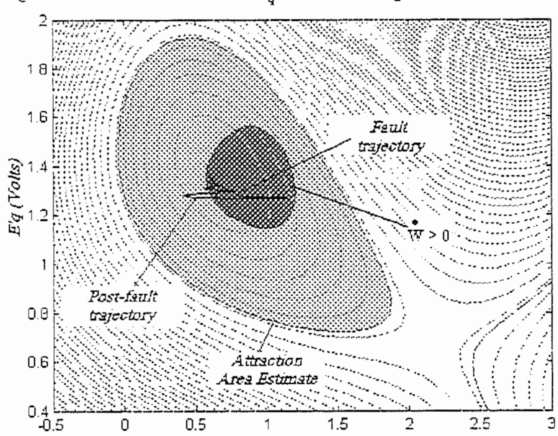

Figure 6: Level Curves of W. ω is fixed and equal to -0.8

4 Extended Lyapunov Functions for Multimachine Systems

4.1 Two-machine versus infinite bus system

Before considering the general multi-machine case, let us firstly consider the two-machine versus infinite bus system of Figure 7.

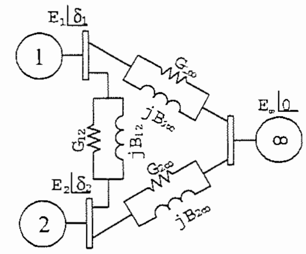

Figure 7: Two-machine versus infinite bus system

The following differential equations:

$$\begin{cases} \dot{\delta}_1 & = \omega_1 \\ M_1\dot{\omega}_1 & = P_1 - C_1 \sin\delta_1 - D_1 \cos\delta_1 - \\ & \quad -C_{12}\sin(\delta_1 - \delta_2) - D_{12}\cos(\delta_1 - \delta_2) - T_1\omega_1 \\ \dot{\delta}_2 & = \omega_2 \\ M_2\dot{\omega}_2 & = P_2 - C_2\sin\delta_2 - D_2\cos\delta_2 - \\ & \quad -C_{12}\sin(\delta_2 - \delta_1) - D_{12}\cos(\delta_2 - \delta_1) - T_2\omega_2 \end{cases} \quad (17)$$

describe the dynamical behavior of this system. When the transfer conductances are neglected in the model ($D_{12} = 0$), there exists a general Lyapunov Function in the usual sense which can be used to study the stability of this system. This function can be easily found by a traditional integration process and is given by:

$$\begin{aligned} V_{FL}(\delta_1, \omega_1, \delta_2, \omega_2) = \quad & M_1\tfrac{\omega_1^2}{2} - P_1\delta_1 - C_1\cos\delta_1 + D_1\sin\delta_1 + \\ & M_2\tfrac{\omega_2^2}{2} - P_2\delta_2 - C_2\cos\delta_2 + D_2\sin\delta_2 - \\ & -C_{12}\cos(\delta_1 - \delta_2) + \alpha \end{aligned} \quad (18)$$

where α is an arbitrary constant.

However when $D_{12} \neq 0$ the integration process yields a path dependent integral and it is impossible to prove that its derivative , along the trajectories, is semi-negative definite. As consequence the original invariance principle cannot be used to study the stability regions of these systems.

In order to solve this problem a new function is proposed and the extension of the Invariance Principle is used to study the stability of this system. It will be shown that this new function is a Lyapunov Function in a wider sense, that is, in the sense of the extension of the Invariance Principle if the transfer conductance D_{12} is small enough. With that in mind consider the following function:

$$\begin{aligned} & W(\delta_1, \omega_1, \delta_2, \omega_2) = M_1\tfrac{\omega_1^2}{2} - P_1\delta_1 - C_1\cos\delta_1 + D_1\sin\delta_1 - \\ & -\beta_1\omega_1\left[P_1 - C_1\sin\delta_1 - D_1\cos\delta_1 - C_{12}\sin(\delta_1 - \delta_2) - \right. \\ & \left. D_{12}\cos(\delta_1 - \delta_2)\right] + M_2\tfrac{\omega_2^2}{2} - P_2\delta_2 - C_2\cos\delta_2 + D_2\sin\delta_2 - \\ & -\beta_2\omega_2\left[P_2 - C_2\sin\delta_2 - D_2\cos\delta_2 - C_{12}\sin(\delta_2 - \delta_1) - \right. \\ & \left. D_{12}\cos(\delta_2 - \delta_1)\right] - C_{12}\cos(\delta_1 - \delta_2) + \alpha \end{aligned} \quad (19)$$

where β_1 and β_2 are parameters to be determined and α is an arbitrary constant.

Calculating the derivative of this function along the system orbits one finds:

$$\begin{aligned} \dot{W} = & -\{T_1 + \beta_1[-C_1\cos\delta_1 + D_1\sin\delta_1 - C_{12}\cos(\delta_1 - \delta_2) \\ & +D_{12}\sin(\delta_1 - \delta_2)]\}\,\omega_1^2 + \tfrac{\beta_1 T_1}{M_1}\omega_1\left[P_1 - C_1\sin\delta_1 - D_1\cos\delta_1\right. \\ & \left. -C_{12}\sin(\delta_1 - \delta_2) - D_{12}\cos(\delta_1 - \delta_2)\right] - \tfrac{\beta_1}{M_1}\left[P_1 - C_1\sin\delta_1\right. \\ & \left. -D_1\cos\delta_1 - C_{12}\sin(\delta_1 - \delta_2) - D_{12}\cos(\delta_1 - \delta_2)\right]^2 \\ & -D_{12}\cos(\delta_1 - \delta_2)\omega_1 - \beta_1\omega_1\omega_2\left[C_{12}\cos(\delta_1 - \delta_2)\right. \\ & \left. -D_{12}\sin(\delta_1 - \delta_2)\right] \\ & \quad -\{T_2 + \beta_2[-C_2\cos\delta_2 + D_2\sin\delta_2 - C_{12}\cos(\delta_2 - \delta_1) \\ & +D_{12}\sin(\delta_2 - \delta_1)]\}\,\omega_2^2 + \tfrac{\beta_2 T_2}{M_2}\omega_2\left[P_2 - C_2\sin\delta_2 - D_2\cos\delta_2\right. \\ & \left. -C_{12}\sin(\delta_2 - \delta_1) - D_{12}\cos(\delta_2 - \delta_1)\right] - \tfrac{\beta_2}{M_2}\left[P_2 - C_2\sin\delta_2\right. \\ & \left. -D_2\cos\delta_2 - C_{12}\sin(\delta_2 - \delta_1) - D_{12}\cos(\delta_2 - \delta_1)\right]^2 \\ & -D_{12}\cos(\delta_2 - \delta_1)\omega_2 - \beta_2\omega_1\omega_2\left[C_{12}\cos(\delta_2 - \delta_1)\right. \\ & \left. -D_{12}\sin(\delta_2 - \delta_1)\right] \end{aligned} \quad (20)$$

Choosing $\beta = \beta_1 = \beta_2$ one finds:

$$-\dot{W} = \begin{bmatrix} P_{l1}(\delta_1,\delta_2) \\ \omega_1 \\ P_{l2}(\delta_1,\delta_2) \\ \omega_2 \end{bmatrix}^T A \begin{bmatrix} P_{l1}(\delta_1,\delta_2) \\ \omega_1 \\ P_{l2}(\delta_1,\delta_2) \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}^T B \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + D_{12}\cos(\delta_1-\delta_2)(\omega_1+\omega_2) \quad (21)$$

where

$$A = \left[\begin{array}{c|c} A_{11} & 0 \\ \hline 0 & A_{22} \end{array}\right],$$

$$B = \left[\begin{array}{c|c} \frac{T_1}{2} & \beta C_{12}cos(\delta_1-\delta_2) \\ \hline \beta C_{12}cos(\delta_1-\delta_2) & \frac{T_2}{2} \end{array}\right],$$

$$A_{11} = \left[\begin{array}{c|c} \frac{\beta}{M_1} & -\frac{\beta T_1}{2M_1} \\ \hline -\frac{\beta T_1}{2M_1} & \begin{array}{c} \frac{T_1}{2} + \beta\,[-C_1\cos\delta_1 + D_1\sin\delta_1 \\ -C_{12}\cos(\delta_1-\delta_2) + D_{12}\sin(\delta_1-\delta_2)] \end{array} \end{array}\right]$$

and

$$A_{22} = \left[\begin{array}{c|c} \frac{\beta}{M_2} & -\frac{\beta T_2}{2M_2} \\ \hline -\frac{\beta T_2}{2M_2} & \begin{array}{c} \frac{T_2}{2} + \beta\,[-C_2\cos\delta_2 + D_2\sin\delta_2 \\ -C_{12}\cos(\delta_2-\delta_1) + D_{12}\sin(\delta_2-\delta_1)] \end{array} \end{array}\right]$$

Similarly to the previous case, the derivative of W is composed by quadratic terms plus the term $D_{12}\cos(\delta_1 - \delta_2)(\omega_1+\omega_2)$. Parameter β can be chosen in order to make the quadratic term positive definite. Applying Silvester's Criteria one can easily find that this is certainly guaranteed if

$$\beta^2 < \frac{T_1 T_2}{4C_{12}^2},$$

$$0 < \beta < \frac{T_1}{2\left(\frac{T_1^2}{4M} + C_1 + D_1 + C_{12} + D_{12}\right)}$$

and

$$0 < \beta < \frac{T_2}{2\left(\frac{T_2^2}{4M} + C_2 + D_2 + C_{12} + D_{12}\right)}$$

In this way, only the term $D_{12}\cos(\delta_1-\delta_2)(\omega_1+\omega_2)$ will be responsible for generating regions where the derivative of W is positive.

Once β has been chosen, a real number L must be found such that the conditions of Theorem 2.2 are satisfied. In the next example, these conditions are numerically checked.

Example 4.1 *Consider the system of Figure 7 with* $P_1 = 1.25$, $P_2 = 1.5$, $C_1 = 1.7$, $C_2 = 2.0$, $D_1 = D_2 = 0.1$, $C_{12} = 0.5$, $D_{12} = 0.05$, $T_1 = T_2 = 0.1$ *and* $M_1 = M_2 = 0.05$. *The level curves of* W *are depicted in Figure 8 for* $\alpha = 5.1337$ *and* $\beta = 0.0093$. *These curves were drawn in the plane* $\omega_1 = \omega_2 = -0.4$. *The region where the derivative of* W *is positive is composed by two small bounded sets. One of them is close to the stable equilibrium point and corresponds to the set* C *in Theorem 2.2. The maximum value of* W *in* $\bar{C}$ *defines the set* $\bar{\Omega}_l$ *which is an attractor estimate, i.e. all the solutions starting into the stability region will enter in this attractor estimate in a finite time. In this example, one finds numerically* $l = 0.0417$. *To estimate the stability region or attraction area of the attractor we must choose the largest number* L *of Theorem 2.2 such that the conditions of Theorem 2.2 are satisfied. In practice, we must guarantee that* $\bar{\Omega}_L$ *is bounded and does not intercept the region close to the unstable equilibrium point where the derivative is positive. In this example* $L = 0.8667$ *is found numerically. Figure 8 illustrates the attractor estimate projection and the stability region estimate projection on the plane* $\omega_1 = \omega_2 = -0.4$.

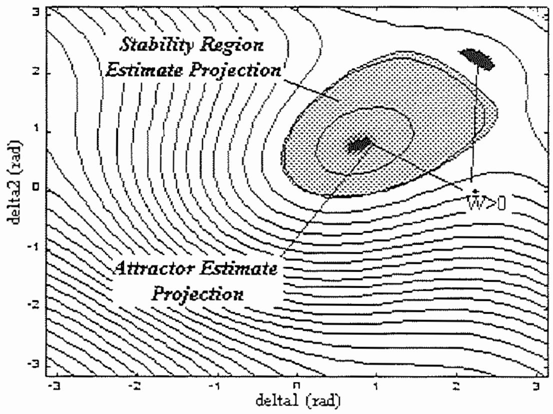

Figure 8: *Level curves of W*

4.2 Multimachine-systems

Consider a system composed by n machines where the n^{th}-machine is an infinite bus. One can show, similarly to the case of two-machines, that the following function

$$W = \sum_{i=1}^{n-1}\Big\{ M_i\frac{\omega_i^2}{2} - P_i\delta_i - C_i\cos\delta_i + D_i\sin\delta_i - $$
$$-\beta_i\omega_i\,[P_i - C_i\sin\delta_i - D_i\cos\delta_i -$$
$$-\sum_{\substack{j=1 \\ j\neq i}}^{n-1} C_{ij}\sin(\delta_i-\delta_j) - \sum_{\substack{j=1 \\ j\neq i}}^{n-1} D_{ij}\cos(\delta_i-\delta_j)]$$
$$-\sum_{j=i+1}^{n-1} C_{ij}\cos(\delta_1-\delta_2)\Big\} + \alpha \quad (22)$$

where α is an arbitrary constant, is a Lyapunov function in the sense of the extended Invariance Principle if the transfer conductances are small enough. Choosing $\beta_i = \beta$, $i = 1,\ldots,n$, then the derivative of W along the orbits is given by

$$-\dot{W} = \begin{bmatrix} P_{l1}(\delta) \\ \omega_1 \\ \vdots \\ P_{ln-1}(\delta) \\ \omega_{n-1} \end{bmatrix}^T A \begin{bmatrix} P_{l1}(\delta) \\ \omega_1 \\ \vdots \\ P_{ln-1}(\delta) \\ \omega_{n-1} \end{bmatrix} + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} \begin{bmatrix} \omega_i \\ \omega_j \end{bmatrix}^T B_{ij} \begin{bmatrix} \omega_i \\ \omega_j \end{bmatrix} + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} D_{ij}\cos(\delta_i-\delta_j)(\omega_i+\omega_j)$$

where A is a block diagonal matrix. Each block of A is given by

$$A_{ii} = \left[\begin{array}{c|c} \frac{\beta}{M_i} & -\frac{\beta T_i}{2M_i} \\ \hline -\frac{\beta T_i}{2M_i} & \begin{array}{c} \frac{T_i}{n-1} + \beta(-C_{in}cos(\delta_i-\delta_n) + D_{in}sin(\delta_i-\delta_n) - \\ \sum_{j\neq i}^{n-1} C_{ij}cos(\delta_i-\delta_j) + \sum_{j\neq i}^{n-1} D_{ij}sin(\delta_i-\delta_j)) \end{array} \end{array}\right]$$

and

$$B_{ij} = \left[\begin{array}{c|c} \frac{T_i}{n-1} & C_{ij}cos(\delta_i-\delta_j) \\ \hline C_{ij}cos(\delta_i-\delta_j) & \frac{T_j}{n-1} \end{array}\right]$$

The quadratic terms will be positive definite if

$$\beta^2 < \frac{T_i T_j}{(n-1)^2 C_{ij}^2}, \quad i \neq j; \; i, j = 1, \ldots, n-1$$

and

$$0 < \beta < \frac{T_i}{(n-1)\left(\frac{T_i^2}{4M} + \sum_{j \neq i}^{n} C_{ij} \sum_{j \neq i}^{n} + D_{ij}\right)}, \quad i = 1, \ldots, n-1$$

In this way, only the term $D_{12} \cos(\delta_1 - \delta_2)(\omega_1 + \omega_2)$ will be responsible for generating regions where the derivative of W is positive.

Therefore, the same ideas presented in the previous cases can be applied to multi-machine systems. Obviously there are some difficulties to be overcome in order to make the idea suitable for applications in large systems. First of all, it is more difficult to check the conditions required by Theorem 2.2 than checking the conditions required by the original invariance principle. In both theorems it is necessary to guarantee the boundedness of the set Ω_L. From the point of view of the classical invariance principle, it is enough to take L as the potential energy of the unstable equilibrium point which has the lowest energy between all unstable equilibrium points around the stable equilibrium point of interest. The experience has shown that this choice guarantees the boundedness of the set Ω_L and guarantees that the stable equilibrium point of interest is the unique invariant set contained into Ω_L. From the point of view of the extended Invariance Principle, the boundedness of Ω_L is also required and indeed it is necessary to assure that the set C containing the stable equilibrium point of interest is strictly contained into Ω_L and that the set Ω_L does not intersect any other set where the derivative of the Lyapunov Function is positive.

5 Conclusions

In this paper, the extension of the Invariance Principle was successfully applied to support theoretically the proposal of new functions which are Lyapunov Functions in a wider sense (their derivative can assume positive values in some bounded regions) for power system models which do not have Lyapunov Functions in the usual sense. These functions were shown to be suitable for transient stability studies and estimates of the critical clearing time were obtained using a solid theoretical background without approximations or conjectures. Further studies are necessary to reduce the conservativeness of the obtained estimates when the one-axis model is employed.

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REFERENCES

[1] L. F. C. Alberto, N. G. Bretas, "Application of Melnikov's Method for Computing Heteroclinic Orbits in a Classical SMIB Power System Model", IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications, vol. 47, n. 7, Jul 2000

[2] L. F. C. Alberto, N. G. Bretas, "Synchronism versus Stability in Power Systems", International Journal of Electrical Energy & Power Systems, v.21, n.4, 1999, pp.261-267

[3] L. F. C. Alberto, N. G. Bretas, "Required Damping to Assure Multi-Swing Transient Stability: The SMIB Case", International Journal of Electrical Energy & Power Systems, v.22, n.3, 2000, pp. 179-185

[4] L. F. C. Alberto, F. H. J. R. Silva, N. G. Bretas, "Direct Methods for Transient Stability Analysis in Power Systems: State of Art and Future Perspectives", Proc. IEEE Porto Power Tech Conference 2001, Portugal, September 2001.

[5] T. Athay, R. Podmore, S. Virmani, "A Practical Method for Direct Analysis of Transient Stability", IEEE Trans., vol. PAS-98, n. 2, March/April 1979, pp. 573-584

[6] P. D. Aylett, "The Energy-Integral Criterion of Transient Stability Limits of Power Systems", Proc. IEE, vol. 105C, 1958, pp.527-536

[7] H. D. Chiang, F. F. Wu, P. P. Varaya, "A BCU Method for Direct Analysis of Power System Transient Stability ", IEEE Trans. on Power Systems, August 1994, pp. 1194-1208

[8] H. D. Chiang, F. F. Wu, P. P. Varaya, "Foundations of the Potential Energy Boundary Surface Method for Power System Transient Stability Analysis ", IEEE Transactions on Circuits and Systems, vol. 35, no. 6, June 1988.

[9] H. D. Chiang, "Study of the existence of energy functions for power systems with losses", IEEE Trans. on Circuits and Systems, vol. CAS-36, pp. 1423-1429, 1989.

[10] H. G. Kwatny, L. Y. Bahar, A. K. Pasrija, "Energy-like Lyapunov Functions for Power System Stability Analysis", IEEE Trans. on Circuits and Systems, vol. CAS-32, n. 11, Nov 1985, pp. 1140-1149

[11] J. P. LaSalle, "The extent of asymptotic stability", Proceedings of the National Academy of Sciences, Vol. 46, no.3, pp. 363-365, March(1960).

[12] J. P. LaSalle, "Some extensions of Liapunov's second method", IRE Trans. on Circuit Theory, CT-7 pp. 520-527 (1960).

[13] J. P. LaSalle, "Asymptotic stability criteria", Proceedings of Symposia in Applied Mathematics, Vol. 13, Hydrodynamic Instability, AMS, pp. 299-307 (1962).

[14] J. P. LaSalle, "Stability theory for difference equations. Studies in ordinary differencial equation", Studies in ordinary differencial equations, Stud. in Math., Vol. 14, Math. Assoc. of America, Washington, D.C.(Reviewer G.R.Sell), pp.1-31 (1977).

[15] G. Ledwich, E. Palmer, "Energy Function for Power Systems with Transmission Losses", IEEE Trans. on Power Systems, v. 12, n. 2, 1997

[16] M.A. Pai, "Power System Stability - Analysis by the Direct Method of Lyapunov ", North-Holland Publishing Company, Amsterdam-New York-Oxford, 1981

[17] H. M. Rodrigues, L. F. C. Alberto, N. G. Bretas, "On the Invariance Principle. Generalizations and Applications to Synchronism", IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications, vol. 47, n. 5, May 2000

[18] H. M. Rodrigues, L. F. C. Alberto, N. G. Bretas, "Uniform Invariance Principle and Synchronization. Robustness with Respect to Parameter Variation", Journal of Differential Equations, vol. 169, n.1, Part 3, Jan 2001, pp.228-254