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WAVES AND INSTABILITIES IN PLASMA

# Drift and Geodesic Effects on the Ion Sound Eigenmode in Tokamak Plasmas[1]

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**Abstract**—A kinetic treatment of geodesic acoustic modes (GAMs), taking into account ion parallel dynamics, drift and the second poloidal harmonic effects is presented. It is shown that first and second harmonics of the ion sound modes, which have respectively positive and negative radial dispersion, can be coupled due to the geodesic and drift effects. This coupling results in the drift geodesic ion sound eigenmode with a frequency below the standard GAM continuum frequency. Such eigenmode may be able to explain the split modes observed in some experiments.

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## 1. INTRODUCTION

Geodesic acoustic modes (GAMs) supported by plasma compressibility have first been theoretically predicted [1] using the MHD approach. The GAMs involve electric field fluctuations, which are mainly electrostatic with $M = 0$, $\pm 1$ poloidal and $N = 0$ toroidal mode numbers. These modes have been observed in numerous experiments and numerical simulations, (see, e.g., [2, 3]). In two-fluid theory, the mode frequency becomes $\omega_{\mathrm{GAM}}^2 \approx (7T_i/2 + 2T_e)\big/R_0^2 m_i$ and is further modified by kinetic [4] and drift kinetic effects [5–11]. For finite $N$ and $M$, coupling with Alfvén continuum with $N \approx M/q$, the GAM continuum creates a combined Alfvén–geodesic continuum. In vicinity of its maximum/minimum, but out of the continuum where their dissipation is small, discrete Alfvén eigenmodes (AEs) or beta induced Alfvén eigenmode (BAE) [2, 5, 12, 13] exist. Such modes may be driven by neutral beam (NB) or ion cyclotron resonance (ICR) heating. Various modes of these types have been observed in experiments and confirmed by numerical calculations of energetic particle effect [14, 15]. That effect was proposed to use for channeling of BAE energy to heat electrons [16]. Typically, BAE oscillations are much easier detected by magnetic probes due to large magnetic field component, contrary to geodesic acoustic modes (BAAEs) with $N \approx M/q$ [9], which involves acoustic sidebands $\omega_{s,N} - \omega_{s,N-1} = \omega_s$ where $\omega_s = \sqrt{T_e/m_i}\big/qR_0$, that was detected due to strong NB drive despite of their Landau dissipation for $T_e \sim 1.3T_i$. Fluctuations in the GAM frequency range have been observed in tokamaks with strong NB and/or ICR heating [2, 9, 12–14, 17], as well in tokamaks with ohmic and ECR heating [18–22]. Strong evidence exist that experimentally observed frequencies follow the electron temperature [19] even in presence of plasma turbulence [23] suggesting the important role of geodesic and ion sound effects. Many of these observations are generically referred as geodesic oscillations. The experimental values of geodesic oscillations in the inner plasma regions bulk have the frequency [18–23] below the theoretically predicted GAMs continuum.

Earlier kinetic study of GAMs was mainly done in a "continuum limit" assuming $\partial\tilde{f}/\partial r = 0$ for the perturbed distribution function in the drift kinetic equation. In this approximation, GAMs involve the $M = 0$ component of the potential and $M = 1$ components of density and potential. More advanced theoretical calculations show that the GAMs also involve second poloidal harmonic fluctuations, with $M = 2$ poloidal and $N = 0$ toroidal mode numbers, and $\partial\tilde{f}/\partial r \neq 0$, due to the finite orbit width (FOW) effect [7–11]. The second harmonic effect creates the radial eigenmode structure and substantially changes the eigenmode frequency [7–10], as well the wave dissipation [11]. The basic geodesic perturbations naturally involve the

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$M = 1$ harmonic of the ion sound motion. In the next order, including the second order poloidal harmonics, GAMs become coupled to the second poloidal harmonics of the ion-sound mode $\omega = 2\omega_s$.

The nature of the experimentally observed geodesic eigenmodes is still not clear. It is also possible that several different mechanisms might responsible for their dispersion and manifest themselves in different conditions. Here, we present a kinetic treatment of the electrostatic GAMs to study the formation of the ion-sound eigenmodes due to coupling of the diamagnetic drift and the geodesic FOW effects. Thus, we extend earlier studied second harmonic effect [10] that is directly responsible the formation of new ion-sound eigenmodes in tokamak plasmas. We show here that the dispersion effects due to averaged geodesic curvature and drift effects result in the eigenmode based on the second harmonic of the ion sound.

## 2. BASIC EQUATIONS

To calculate GAM spectra, the quasi-toroidal coordinates $(r, \vartheta, \zeta)$ are employed within the large aspect ratio tokamak approximation, $R_0 \gg r$, where circular magnetic surfaces ($R = R_0 + r\cos\vartheta,\ z = r\cos\vartheta$) are formed by the toroidal and poloidal magnetic field components, $B_\zeta = B_0 R_0/R$, $B_\vartheta = rB_\zeta/qR_0$, and $B_\vartheta \ll B_\zeta$. Using the eikonal approach, the perturbed distribution with $N = 0$ for plasma species ($\alpha$) is taken in the form

$$f = (-e_\alpha F_\alpha \varphi/T_\alpha + g_\alpha)\exp\left(\mathrm{i}\int\limits_{k_r-\mathrm{real}}^{r} k_r dr - \mathrm{i}\omega t\right). \quad (1)$$

That leads to the standard kinetic equation with magnetic and diamagnetic drift terms

$$\begin{aligned}&(\Omega_\alpha - V_{r\alpha}k_r\sin\vartheta)qg_\alpha + \mathrm{i}w\frac{\partial g_\alpha}{\partial\vartheta}\\ &= \frac{e_\alpha}{T_\alpha}qF_\alpha J_0^2(uk_r\rho_\alpha)\left(\Omega_\alpha - \mathrm{i}\hat{\Omega}_{*\alpha}\frac{\partial}{\partial\vartheta}\right)\varphi,\end{aligned} \quad (2)$$

where $\varphi$ is the electrostatic potential; $V_{r\alpha} = -(2w^2 + u^2)\rho_\alpha/2$ is the normalized magnetic drift velocity; $\rho_\alpha = v_{T\alpha}/\omega_{c\alpha}$ is the Larmor radius; $F_\alpha$ is the Maxwellian distribution function; $\hat{\Omega}_{*\alpha} = \Omega_{*\alpha} \times \left[2 + \eta(w^2 + u^2 - 3)\right]/2$; $\Omega_{*\alpha} = v_{T\alpha}/\omega_{c\alpha}\varepsilon d_r$ is dimensionless diamagnetic drift frequency; $\varepsilon = r/R_0$; $\omega_{c\alpha} = e_\alpha B/m_\alpha c$ and $\Omega_\alpha = \omega R_0/v_{T\alpha}$ are the cyclotron and normalized wave frequencies, respectively; $\partial n_0/\partial r = -n_0/d_r$ is the density gradient; $\eta_\alpha = \partial\ln T_\alpha/\partial\ln n_0$; $w = v_\parallel/v_{Ti}$ and $u = v_\perp/v_{Ti}$ are velocities normalized to the thermal speeds, $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$, where the electron and ion temperatures are allowed to be different, $T_e = t_e T_i$, with $t_e$ being an independent parameter; and $J_0(x)$ is the zero order Bessel function. The perturbations can be expressed as a sum of several poloidal harmonics ($M = 0,\ \pm1,\ \pm2$) in the form $\varphi = \varphi_0 + \varphi_s\sin\vartheta + \varphi_c\cos\vartheta + \varphi_{2s}\sin2\vartheta + \varphi_{2c}\cos2\vartheta$ [9–11]. The electrons are considered in the adiabatic approach, $w \to \infty$, which is valid for the large circulation frequency and the wave dissipation in electrons is extremely small due to the large bounce frequency $\sqrt{\varepsilon}v_{Te}/qR \gg \omega$ [24]. This simplifies the perturbed electron density equation to the form

$$\begin{aligned}\tilde{n}_e = (en_0/T_e)(&\varphi_s\sin\vartheta + \varphi_c\cos\vartheta\\ &+ \varphi_{2s}\sin2\vartheta + \varphi_{2c}\cos2\vartheta).\end{aligned} \quad (3)$$

A general solution of Eq. (2) for ions is found using fluid limit $\omega R_0/v_{Ti} \gg V_{r\alpha}k_r,\ 1/q$ and $k_r^2\rho_\alpha^2 \ll 1$. That allows calculations of the harmonics of the perturbed distributions for the ion species $g = g_0 + g_s\sin\vartheta + g_c\cos\vartheta + g_{2s}\sin2\vartheta + g_{2c}\cos2\vartheta$. Then, equations for harmonics of the ion density oscillations are obtained by integration of respective perturbed distributions in Eq. (1) in velocity space. Those equations are simplified, omitting $\alpha$ index due to only one ion plasma specie, and using the expansion over the Larmor radius up to fourth order ignoring terms of the order $\kappa^4/q^2$, $\kappa^4\Omega_*^2$, and $\kappa^2/q^4$, which are assumed to be small where the parameters $\kappa^2 = k_r^2\rho^2$ and $\Omega_*^2$ are assumed to have the same order. Next, using quasineutrality conditions, we get the set of equations

$$\begin{aligned}\tilde{n}_0 = \frac{e_i n_0}{2T_i}\Bigg\{&\left[\left(\frac{7}{\Omega^2} - 2\right)\kappa^2 + \left(\frac{3}{2} - \frac{13}{\Omega^2} + \frac{747}{4\Omega^4}\right)\kappa^4\right]\varphi_0\\ &- 2\mathrm{i}(1+\eta)\kappa\frac{\Omega_*}{\Omega^2}\varphi_c + 7\mathrm{i}(1+2\eta)\kappa^2\frac{\Omega_*}{\Omega^3}\varphi_{2s}\\ &-\left[2 + \left(\frac{27}{\Omega^2} - 3\right)\kappa^2\right]\frac{\kappa}{\Omega}\varphi_s\\ &+\left[\left(13 - \frac{747}{4\Omega^2}\right)\kappa^2 - 7\right]\frac{\kappa^2}{2\Omega^2}\varphi_{2c}\Bigg\},\end{aligned} \quad (4a)$$

$$\begin{aligned}\sum_{ei}\tilde{n}_s = \frac{e_i n_0}{T_i}\Bigg\{&\left[\left(3\Omega^2 + \frac{5}{q^2} - 27\right)\kappa^2 - 2\Omega^2 - \frac{4}{q^2}\right]\\ &\times\frac{\kappa}{\Omega^3}\varphi_0 + \left[\left(\frac{21}{4} - \Omega^2 - \frac{1}{q^2}\right)\kappa^2 + \frac{1}{q^2} - \frac{\Omega^2}{t_e}\right]\frac{\varphi_s}{\Omega^2}\\ &- \mathrm{i}\left(\Omega^2 + \frac{(1+\eta)}{q^2}\right)\frac{\Omega_*}{\Omega^3}\varphi_c - 2\mathrm{i}(1+\eta)\frac{\Omega_*}{\Omega^2}\kappa\varphi_{2s}\\ &+\left[1 + \frac{14}{q^2\Omega^2} + \left(\frac{27}{\Omega^2} - 3\right)\frac{\kappa^2}{2}\right]\frac{\kappa}{\Omega}\varphi_{2c}\Bigg\},\end{aligned} \quad (4b)$$

$$\sum_{ei}\tilde{n}_c = \frac{e_i n_0}{T_i}\left\{\left[\frac{1}{q^2}-\frac{\Omega^2}{t_e}+\left(\frac{7}{4}-\Omega^2-\frac{1}{q^2}\right)\kappa^2\right]\frac{\varphi_c}{\Omega^2}\right.$$
$$-\,\mathrm{i}\left(\Omega^2+\frac{(1+\eta)}{q^2}\right)\frac{\Omega_*}{\Omega^3}\varphi_s-2\mathrm{i}(1+\eta)\frac{\Omega_*}{\Omega^2}\kappa\varphi_{2c} \quad (4c)$$
$$\left.-\left[1+\frac{14}{q^2\Omega^2}+\left(\frac{27}{\Omega^2}-3\right)\frac{\kappa^2}{2}\right]\frac{\kappa}{\Omega}\varphi_{2s}\right\},$$

$$\sum_{ei}\tilde{n}_{2s} = \frac{e_i n_0}{T_i}\left\{\left[\frac{4}{q^2}-\frac{\Omega^2}{t_e}+\kappa^2\left(\frac{7}{4}-\Omega^2-\frac{4}{q^2}\right)\right]\frac{\varphi_{2s}}{\Omega^2}\right.$$
$$+\,\mathrm{i}\,(1+\eta)\,\kappa\frac{\Omega_*}{\Omega^2}\varphi_s+2\mathrm{i}\left(1-(1+\eta)\kappa^2\right)\frac{\Omega_*}{\Omega}\varphi_{2c} \quad (4d)$$
$$\left.-\left[1+\frac{14}{\Omega^2 q^2}+\left(9-\frac{3}{q^2}\right)\frac{\kappa^2}{2\Omega^2}\right]\frac{\kappa}{\Omega}\varphi_c\right\},$$

$$\sum_{ei}\tilde{n}_{2c} = \frac{e_i n_0}{T_i}\left\{\left[1+\frac{14}{\Omega^2 q^2}+\left(\frac{27}{\Omega^2}-3\right)\frac{\kappa^2}{2}\right]\frac{\kappa}{\Omega}\varphi_s\right.$$
$$+\,\mathrm{i}\,(1+\eta)\,\kappa\frac{\Omega_*}{\Omega^2}\varphi_c\text{-}2\mathrm{i}\left(1-(1+\eta)\kappa^2\right)\frac{\Omega_*}{\Omega}\varphi_{2s}$$
$$+\left[\frac{4}{q^2}-\frac{\Omega^2}{t_e}+\kappa^2\left(\frac{7}{4}-\Omega^2-\frac{4}{q^2}\right)\right]\frac{\varphi_{2c}}{\Omega^2} \quad (4e)$$
$$\left.-\left(7+13\kappa^2\right)\frac{\kappa^2}{2\Omega^2}\varphi_0\right\}.$$

We begin the evaluation of the dispersion equation by finding the second harmonic perturbations of the potential from Eqs. (4d), (4e)

$$\varphi_{2c} = \frac{t_e}{\hat{Q}_2}\left\{\mathrm{i}\left[(1+\eta)\,P_2+2t_e\Omega^2\right]\Omega\Omega_*\kappa\varphi_c\right.$$
$$-\frac{7}{2}\kappa^2 P_2\varphi_0+\left[\left(1+\frac{14}{\Omega^2 q^2}\right)\kappa P_2+2t_e\Omega\Omega_*^2\kappa \quad (5a)\right.$$
$$\left.\left.+\left(\frac{27}{\Omega^2}-3\right)\frac{\kappa^3}{2}P_2\right]\varphi_s\right\},$$

$$\varphi_{2s} \approx \frac{t_e}{\hat{Q}_2}\left\{\mathrm{i}\left[2t_e\left(\Omega^2+\frac{14}{q^2}\right)+P_2\right](1+\eta)\right.$$
$$\times\,\Omega_*\kappa\varphi_s-\left[t_e\,(1+\eta)^2\Omega_*^2+2P_2\right]\Omega\kappa\varphi_c \quad (5b)$$
$$\left.-7\,(1+\eta)\,\Omega_* P_2\kappa^2\varphi_0\right\},$$

where $\hat{Q}_2 = P_2^2-4t_e^2\Omega^2(1+\eta)^2\,\Omega_*^2$, $P_2 = \Omega^2-4\Omega_S^2+k_r^2\rho^2 D_2$, $D_2 = t_e\Omega^2+4\Omega_S^2-\frac{7}{4}t_e$, $\kappa^2 = k_r^2\rho^2$, $\Omega_s^2 = t_e/q^2 = \omega_{is}^2R_0^2/v_{Ti}^2$ is the normalized ion-sound frequency.

Next, substituting the second harmonics (5a) and (5b) into Eqs. (4a)–(4c), we find equations for the $\varphi_{c,s}$ amplitudes of the potential via the $\varphi_0$ amplitude. Now, a general dispersion equation may be found after substitution of the $\varphi_{c,s}$ expressions into Eq. (4a). Finally, expanding this expression into series up to the order of $k_r^2\rho^2$, we get the equation

$$\left[\left(\frac{7}{2\Omega^2}-1\right)\hat{Q}_{1s}+2t_e\right]\left(\hat{Q}_2+k_r^2\rho^2 DP_2\right)\hat{Q}_{1c}$$
$$+\,t_e^2\left(\Omega^2-\frac{3}{2}+2\eta\right)\Omega_*^2\hat{Q}_2 = 0, \quad (6)$$

where $\hat{Q}_{1c} = \hat{Q}_{1s}+7t_ek_r^2\rho^2/2$, $\hat{Q}_{1s} = \Omega^2-\Omega_S^2+k_r^2\rho^2D_1$,

$$D_1 = t_e\Omega^2+\Omega_S^2-21t_e/4$$
$$-\left(\Omega^2+28\Omega_S^2\right)\left(\Omega^2-4\Omega_S^2\right)t_e/\hat{Q}_2, \quad (7a)$$

and the dispersion coefficient $D$ defined by the FOW effect is

$$D = t_e\left(\frac{54}{\Omega^2}-6\right)\left(\Omega^2-4\Omega_S^2\right)+7t_e^2$$
$$+\left[\frac{49t_e}{8\Omega^2}+\left(\frac{3}{4}-\frac{13}{2\Omega^2}+\frac{747}{8\Omega^4}\right)\left(\Omega^2-4\Omega_S^2\right)\right] \quad (7b)$$
$$\times\left(\Omega^2-\Omega_S^2\right).$$

For $k_r^2 = 0$, Eq. (6) describes two types of continuum modes: GAM and ion-sound harmonic modes, which are coupled via the drift term [5]. In the small drift frequency limit $\Omega_*^2\to 0$, GAM and ion-sound harmonic modes are decoupled and Eq. (6) splits in the ion-sound mode equation of the $\cos\vartheta$ density oscillations $\hat{Q}_{1c} = 0$ and the GAM equation

$$\left[\left(\frac{7}{2\Omega^2}-1\right)\left(\Omega^2-\Omega_S^2\right)+2t_e\right]\left[\left(\Omega^2-4\Omega_S^2\right)^2\right.$$
$$\left.-\,4t_e^2(1+\eta)^2\Omega^2\Omega_*^2\right]+k_r^2\rho^2D_{\mathrm{GAM}}\left(\Omega^2-4\Omega_S^2\right) = 0, \quad (8)$$

where

$$D_{\mathrm{GAM}} = \left[\frac{49t_e}{8\Omega^2}+\left(\frac{3}{4}-\frac{13}{2\Omega^2}+\frac{747}{8\Omega^4}\right)\left(\Omega^2-4\Omega_S^2\right)\right]$$
$$\times\left(\Omega^2-\Omega_S^2\right)+t_e\left(\frac{167}{4\Omega^2}-\frac{5}{2}\right)\left(\Omega^2-4\Omega_S^2\right)$$
$$+\left(1-\frac{7}{2\Omega^2}\right)\left[t_e\left(t_e\Omega^2+28\Omega_S^2\right)-\left(\Omega^2-4\Omega_S^2\right)\right. \quad (9)$$
$$\left.\times\left(t_e\Omega^2+\Omega_S^2-\frac{7}{4}t_e\right)\right]+7t_e^2+\left(t_e\Omega^2+4\Omega_S^2-\frac{7}{4}t_e\right)$$
$$\times\left[\left(\frac{7}{2\Omega^2}-1\right)\left(\Omega^2-\Omega_S^2\right)+2t_e\right].$$

Using the limit $\Omega^2 \gg \Omega_S^2$ in Eq. (9), we get the GAM dispersion equation

$$\Omega^2 = 7/2 + 2t_e + k_r^2\rho^2\left(\left(\frac{3}{4} - 2t_e\right)\Omega_{\text{GAM}}^2 + 3t_e^2 + 8t_e - \frac{13}{2} + \frac{285t_e + 747}{8\Omega_{\text{GAM}}^2}\right), \quad (10)$$

that is equivalent to the dispersion obtained in [7, 8] combining it with the continuum frequency $\Omega_{\text{GAM}}^2 = 7/2 + 2t_e$. The GAM propagates in positive direction $\partial\Omega/\partial k_r > 0$ from the continuum and changes direction for $t_e > 5.46$. It means that GAM propagation is impossible in the internal region of the GAM continuum for typical tokamak discharges. Ignoring correction of the order $1/q^2 \ll 1$ on the GAM continuum frequency, we find that there are two additional traveling modes, $\hat{Q}_2 + k_r^2\rho^2 DP_2 \cong 0$, at the second harmonic continuum position $\Omega^2 \approx 4\Omega_S^2 \ll \Omega_{\text{GAM}}^2$, where

$$\Omega^2 - \left(1 \pm 2t_e\frac{|(1+\eta)\Omega_*|}{\Omega}\right)4\Omega_S^2 - k_r^2\rho^2\left[\frac{931}{128} + \frac{7}{8}t_e + 6\Omega_S^4 - \Omega_S^2\left(5t_e + \frac{151}{8}\right)\right] = 0. \quad (11)$$

The mode may propagate below the drift second harmonic continuum, when it has negative dispersion $\partial\Omega/\partial k_r < 0$, approximately, for $t_e > 0.35 + 0.26q^2$.

The other condition $\hat{Q}_{1c} = 0$ for the $\cos\vartheta$-density oscillation leads to the equation

$$\left(\Omega^2 - \Omega_S^2\right)\left[\left(\left(\Omega^2 - 4\Omega_S^2\right)^2 - 4\Omega^2(1+\eta)^2\Omega_*^2\right)\right] + D_{is}k_r^2\rho^2\left(\Omega^2 - 4\Omega_S^2\right) = 0, \quad (12)$$

where

$$D_{is} = \left[\left((t_e - 2)\Omega^2 + 3\Omega_S^2 - \frac{7}{4}t_e\right)\left(\Omega^2 - 4\Omega_S^2\right) - t_e(t_e\Omega^2 + 28\Omega_S^2) + 2\left(t_e\Omega^2 + 4\Omega_S^2 - \frac{7}{4}t_e\right) \times \left(\Omega^2 - \Omega_S^2\right)\right].$$

Typically, as $q^2 \gg 1$, $\Omega^2 \le 4\Omega_S^2 \ll \Omega_{\text{GAM}}^2$, the dispersion coefficient $D_{is} < 0$ is negative.

It means that the ion sound mode may propagate in the regions where $\Omega^2 < \Omega_S^2$, as well, in the region $\Omega^2 > 4\Omega_S^2\left(1 \pm 2t_e|\Omega_*|/\Omega\right)$. In the last case, the mode may interact with the second harmonic modes shown in Eq. (11), which dispersion may be negative $\partial\Omega/\partial k_r < 0$. This means that the region of existence for the both modes is overlapped in the interval of the radius defined by the second harmonic roots of the ion-sound continua $4\Omega_S^2\left(1 - 2t_e|\Omega_*|/\Omega\right) < \Omega^2 < 4\Omega_S^2\left(1 + 2t_e|\Omega_*|/\Omega\right)$. These modes may be coupled by the drift term in Eq. (6) that defines real region of some ion sound eigenmode formation.

Generally, taking into account simplifications discussed above and small drift corrections $\Omega_*^2 \ll \Omega_S^2 < \Omega^2 \ll \Omega_{\text{GAM}}^2$, Eq. (6) is modified to the form

$$\left\{\Omega_{\text{GAM}}^2\left[\left(\Omega^2 - 4\Omega_S^2\right)^2 - 4t_e^2(1+\eta)^2\Omega^2\Omega_*^2\right] + k_r^2\rho^2\left(\Omega^2 - 4\Omega_S^2\right)D_{\text{GAM}}\right\}\left\{\left(\Omega^2 - \Omega_S^2\right) \times\left[\left(\Omega^2 - 4\Omega_S^2\right)^2 - 4t_e^2(1+\eta)^2\Omega^2\Omega_*^2\right] + k_r^2\rho^2\left(\Omega^2 - 4\Omega_S^2\right)D_{is}\right\} + t_e^2\left(\Omega^2 - \frac{3}{2} + 2\eta\right) \times \Omega_*^2\left[\left(\Omega^2 - 4\Omega_S^2\right)^2 - 4t_e^2(1+\eta)^2\Omega^2\Omega_*^2\right]^2 = 0. \quad (13)$$

Now, we focus on the analysis of the second harmonic resonance region $r_{-2} < r < r_{+2}$ at $\Omega^2 = 4\Omega_{0S}^2(1+\delta)$, $\delta = (r - r_0)/d_{qT}$, $1/d_{qT} = -d\ln\left(T_e/q^2\right)/dr$, which is limited by the reflection points $|r_0 - r_{\pm 2}| = t_e d_{qT}|\Omega_*|/\Omega_S|_{r=r0}$. In this region, we find two solutions

$$\delta_{1,2} \approx \pm t_e\frac{|(1+\eta)\Omega_*|}{2\Omega_{0S}} - \frac{k_r^2\rho^2}{2}\left(\frac{D_{\text{GAM}}}{3\Omega_{0S}^2} + \frac{D_{is}}{\Omega_{\text{GAM}}^2}\right)_{r=r0}. \quad (14)$$

In typical tokamak discharges, the value $3\Omega_{0S}^2 D_{\text{GAM}} + \Omega_{\text{GAM}}^2 D_{is} < 0$ is negative for the temperature ratio $t_e < 0.83q^4/(1 + q^2 - 0.11q^4)$, because we have mode propagation region at $r_{-2} < r < 0$ for first mode, $0 < r < r_{+2}$ for the other one and $r = r_0$ is a mode coupling point. Further, to calculate the respective mode structure, we use of the geometric optics property (quantization rule) for the radial wave vector

$$\int_{r_{-2}}^{r_0} k_r dr + \int_{r_0}^{r_{+2}} k_r dr = \frac{2\sqrt{2}d_{qT}}{3\rho\sqrt{A}}\left(t_e\frac{|(1+\eta)\Omega_*|}{\Omega_{0S}}\right)^{3/2} = p\pi, \quad (15)$$

where $k_r$ is defined by Eq. (14), $A = -\left(D_{\rm GAM}/3\Omega_{0S}^2 + D_{is}/\Omega_G^2\right)_{r=r0}$, $r_{\pm 2}$ are the points of the drift second harmonic ion sound continua and $p = 1, 2, 3\ldots$. It means that we have the mode traveling region $4\omega_S^2\left(1 - t_e\left|(1+\eta)\,\omega_*\right|/\omega\right)_{r=r2} < \omega^2 < 4\omega_S^2 \times \left(1 + t_e\left|(1+\eta)\,\omega_*\right|/\omega\right)_{r=r2}$ where drift geodesic ion sound eigenmode (DGISE) may be formed for some fixed frequency.

## 3. DISCUSSION AND CONCLUSIONS

A full numerical analysis of the ion-sound eigenmodes for exact T-10 parameters is under progress and will be reported elsewhere. However, simple estimates show that the DGISE resonant modes may exist between first and second ion sound harmonics $\omega^2 = \omega_S^2(r_1)_{\kappa=0} = 4\omega_S^2(r_{-2})_{\kappa=0}$. From Eq. (12), one finds that the $A$ parameter is positive for the temperature ratio $T_e/T_i > 0.83q^4/(1 + q^2 - 0.11q^4)$,which is required for the eigenmode existence. Moreover, we estimate a possibility of the DGISE formation at the second harmonic continuum, $\omega^2 \approx 4\omega_S^2(r)_{\kappa=0}$, for the T-10 parameters [19]: $r \approx 20$ cm, $q \approx 2$, $\eta = 2.3$, $t_e = T_e/T_i \approx 3$, $\rho \approx 0.13$ cm, $d_{qT} \approx 4.1$ cm, and $\left|\Omega_*\right| t_e/\Omega_{0S} \approx 0.33$, we have $A \approx 9$, that gives $p \approx 1$ from Eq. (15) and $f_{2s} \approx 16$ kHz, which is very near to those observed in experiments.

In this paper, we do not investigate the detailed excitation mechanism for the studied eigenmodes. It has been recently shown [24, 26] that the geodesic type modes may be driven by the electron current with the characteristic growth rate $\gamma \propto t_e\omega_* V_0/R\omega_{\rm GAM}$, where $V_0$ is the electron current speed and $\omega_*$ is the drift frequency. Here, we propose that is the origin of the DGISE instability. For T-10 experiments [18, 19], the electron current velocity is 4.5 times larger than the ion-sound one over most part of the radius $0 < r < r_2$, where the eigenmode may propagate. Our estimates show that is sufficient to overcome the ion Landau damping [10, 11] for the low $p$-eigenmodes.

Contrary to the AE/BAE oscillations [5, 8, 10–16], which have typical Alfvénic character, the DGISE oscillations are electrostatic, related to the geodesic drift effects, and may propagate radially due to the FOW effect. It should be noted that the modes found in 2D numerical calculations in [25, Fig. 4] appears to be similar to DGISE that is described by Eq. (15).

In summary, using kinetic theory, we have shown that the finite orbit width effect related to magnetic and diamagnetic drifts couple the first and the second ion-sound harmonics, forming a new type of the geodesic eigenmodes, named drift geodesic ion-sound eigenmode with frequency of the order of the core ion-sound frequency and substantially below the standard GAM frequency. Two types of the eigenmodes may be formed: second harmonic drift geodesic ion sound mode and combined first and second harmonics eigenmode. The eigenmode is radially localized in the middle region between the local resonances of the main and second harmonic of the ion-sound modes. Due to radial dispersion the mode frequency stays substantially below from maximum of the 2D harmonic ion-sound continuum. It is suggested that the developed theory may explain the geodesic type modes in the frequency band $f = 18$–$24$ kHz observed in T-10 experiments.

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