Electromechanical Energy Conversion Through Active Learning

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Contents

Preface

Why should the subject of *electromechanical energy conversion* be studied in the 21st century? What is the appeal of this discipline, based on 19th century concepts, that might attract engineering students who were born in this century?

The relevance of this discipline is justified by strong concerns around environmentally sensitive issues. In the past, the subject of electromechanical energy conversion was driven by finding solutions to the control of motion in industrial activities. Although this task is still important, another concern has been presented to the engineers of this century.

The turnaround caused by the impact of fossil fuels on global warming triggered disruption to human mobility, reversing the idea that the automobile was a final solution to it becoming a problem to be solved.

Ambitious projects that were believed to be economically unviable decades ago became the best alternatives for minimizing the environmental impact caused by human mobility.

In this scenario, electromechanical energy conversion became again the center of a new technological advance. The advent of both new materials and the technological improvements on control theory has led to the development of high-performance electromechanical devices, both electrical motors and generators, as well as electromechanical actuators that are lighter, more efficient and accurate.

Not only the electromechanical energy conversion needs, but also techniques for learning this discipline have been changed in this century. The content has improved due to new devices that have appeared and students have acquired competence in actively searching for knowledge.

The main objective of this book is to guide instructors and students on this new technological path, encouraging the use of active learning and providing the right content to support a modern curricular structure for engineering schools.

The initiation of the design of electromechanical devices and their applications is integrated with the content, with the objective of leading the student to practice 'hands-on', that is, facing the challenges that do not have a closed solution, but that require creativity and determination to find a solution that is economically viable.

The proposed projects should involve some kind of competition to challenge the students, not individual competition but those involving team competition. This kind of activity is driven for practicing *teamwork* where the student is still in the undergraduate program.

The authors have been applying with success a *flipped classroom* for more than five years when teaching electromechanical energy conversion. The authors suggest the use of *project based learning* techniques to be discussed with the students. Competitions, that reward the team who won the race, promote an increase in both the teacher/student relationship and classroom efficiency.

The final goal can be achieved if the instructor guides the students to give them the competences and motivation to play with the basics of the design of electromechanical devices involving electromechanical energy conversion.

Acknowledgments

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Jose Cardoso (University of São Paulo—USP) was born in Marilia, SP Brazil, in 1949. He received his BS, MS and PhD degree in electrical engineering from Escola Politécnica da USP in 1974, 1979 and 1986, respectively. Since 1999, he has been a Professor with the Electrical Energy and Automation Engineering Department, Escola Politécnica of USP. He is the author of four books, more than 70 articles, and has advised more than 40 theses/dissertations. His research interests include numerical methods for electromagnetics, theoretical electromagnetics, electrical machines, electromechanical biomedical application, electrical railway research and engineering education. He is the president of SBMAG—Brazilian Electromagnetics Society and former Dean of Escola Politécnica da USP. He is the founder of the LMAG—the Electromagnetic Applied Laboratory. Cardoso was a recipient of the 2013 Emeritus Engineer of the Year in São Paulo State. He was awarded Doctor Honoris Causa by the Grenoble-INP Institute of Engineering in 2019.

Mauricio Barbosa de Camargo Salles

Mauricio Salles (University of São Paulo—USP) was born in Guarulhos, SP Brazil, in 1975. He has been Assistant Professor with the Electrical Energy and Automation Engineering Department, Escola Politécnica of the University of Sao Paulo, since 2010. In 2004, he obtained the MSc degree from the State University of Campinas (UNICAMP), Campinas/SP, Brazil. From 2006 to 2008, he was part of the research team of the Institute of Electrical Machines at the RWTH Aachen University. In 2009, he received the Doctorate degree from the University of Sao Paulo (USP). Between 2014 and 2015, he was Visiting Scholar at Harvard John A Paulson School of Engineering and Applied Sciences. In 2018, he was an Invited Professor at Ecole Centrale de Lille in France. His main interests are on distributed generation, power system dynamics, control and stability, renewable energy, energy storage and electricity markets. He is one of the founders of the Laboratory of Advanced Electric Grids—LGrid.

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Mauricio Costa was born in Sao Paulo, SP Brazil, in 1973. He has been Director at Electromagnetics Technology since 2008, where he works on consulting projects involving electromagnetic field simulation. In 1998, he obtained the MSc degree from the Escola Politécnica of the University of Sao Paulo (POLI-USP), Brazil. In 2001, he received his Doctorate degree from the Grenoble Institute of Engineering (Grenoble INP), France. From 2005 to 2008, he worked at IPT (Institute for Technological Research of Sao Paulo) as the leader of Electromagnetic Compatibility team at Electrical and Optical Equipment Laboratory. His main interests are finite element and other numerical methods, electromagnetic devices simulation and optimization, software development and power systems.

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Chapter 1

The magnetic circuit

1.1 Some magnetic properties of the material

Associated with electron displacement, we can explain several magnetic properties of the material. The circular movement of the electron could be imagined as an elementary current loop that produces an elementary magnetic flux density. In the same way, we can imagine that the spin movement of the electron is a source of an elementary magnetic flux density as well.

For simplicity, it is possible to associate a net elementary flux density produced by the movement of all electric charges of the atom or molecule of the material (figure 1.1).

Normally, the net elementary flux density is null in most materials.

Although some atoms or molecules present a non-null elementary flux density field, the final result of the vectorial summation of all elementary flux density fields of the material is also null when the material was never magnetized.

The composition effects of several substances in the same molecule or the arrangements of several atoms in a crystal could be classified based on its reaction to the effect of a given magnetic field.

Diamagnetic materials: (i) These become weakly magnetized in the opposite direction of the magnetic field intensity in which they are placed; (ii) the flux density in diamagnetic materials varies linearly with the magnetic field intensity.

Examples of diamagnetic materials: Cu, Ag, Zn, Bi, Au, water, etc (figure 1.2).

Paramagnetic materials: (i) These become weakly magnetized in the same direction as the magnetic field intensity in which they are placed; (ii) the flux density in paramagnetic materials varies linearly with the magnetic field intensity (figure 1.3).

Examples of paramagnetic materials: Pt, Mg, Al, Cr, etc.

Ferromagnetic materials: (i) These become strongly magnetized in the same direction as the magnetic field intensity in which they are placed; (ii) the flux density in

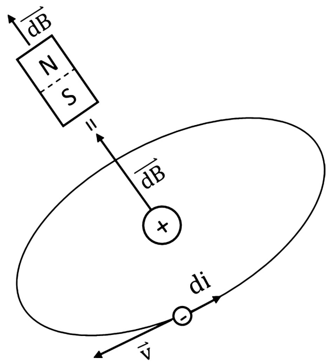

Figure 1.1. Elementary magnetic flux density produced by an atom. No elementary magnetic flux density is produced by most atoms in Nature.

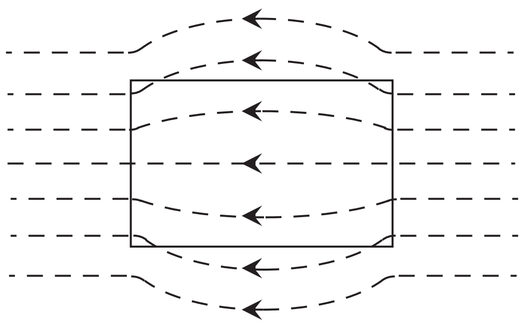

Figure 1.2. Slab of diamagnetic material embedded in a uniform flux density field. Observe that the diamagnetic material spreads the magnetic flux density line from inside to outside of the slab.

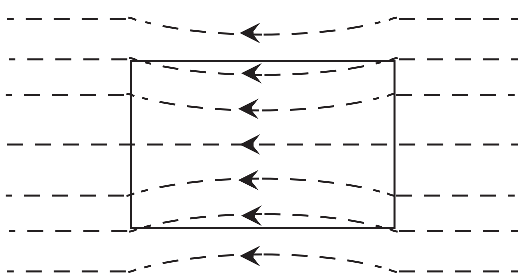

Figure 1.3. Slab of paramagnetic material embedded in a uniform flux density field. Observe that the paramagnetic material attracts the magnetic flux density line from outside to inside of the slab.

ferromagnetic materials varies non-linearly with the magnetic field intensity. For a small range of magnetic field intensity, ferromagnetic materials present linear variation behavior. (iii) Ferromagnetic materials present saturation (non-linearity), hysteresis (behavior dependent of the sense of flux density) and retentivity (property of a permanent magnet). (iv) The magnetic properties of ferromagnetic materials are very sensitive to both thermal treatment and mechanical stress (figure 1.4).

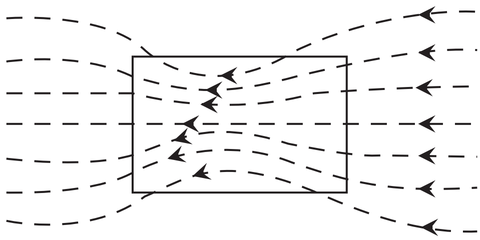

Figure 1.4. Slab of ferromagnetic material embedded in a uniform flux density field. Observe that the ferromagnetic material strongly attracts the magnetic flux density line from the outside to the inside of the slab.

Examples of ferromagnetic materials: Fe, Co, Ni, etc.

The property associated with the magnetic behavior of materials is called magnetic permeability, symbolized by the Greek letter μ (mu).

For engineering, the vacuum has a smaller magnetic permeability the value of which, in the International System (IS), is $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}.

Indeed, the magnetic permeability of diamagnetic materials is smaller than the vacuum, but the difference between them is negligible. The magnetic permeability of some diamagnetic materials is: $\mu_{\text{Cu}} = 0.999\,99 \times \mu_0$; $\mu_{\text{Ag}} = 0.999\,97 \times \mu_0$; $\mu_{\text{Au}} = 0.999\,96 \times \mu_0$.

The magnetic permeability of paramagnetic materials is a little bit bigger than the vacuum, but also the difference between them is negligible. The magnetic permeability of some paramagnetic materials is: $\mu_{\text{Al}} = 1.000\,008 \times \mu_0$; $\mu_{\text{W}} = 1.000\,002 \times \mu_0$.

Hence, for all engineering effects the permeability of both diamagnetic and paramagnetic materials, including the air, is $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}.

From electromagnetism, the following relation:

$$\mu_r = \frac{\mu}{\mu_0}$$

is defined as the relative magnetic permeability (non-dimensional). For both diamagnetic and paramagnetic materials, the relative magnetic permeability is (virtually) unitary.

Most materials used in the construction of electrical engineering apparatus are made by ferromagnetic materials due to their high value of magnetic permeability. From electromagnetism, the magnetic permeability of ferromagnetic materials is non-linear, i.e., the magnetic permeability is dependent on the flux density or $\mu = \mu(B)$. As the magnetic permeability of ferromagnetic materials is not constant, the magnetization characteristics curve is not linear either. Both curves of the magnetic permeability and the magnetization characteristics of ferromagnetic materials are, respectively, represented in figures 1.5 and 1.6.

1.2 The basis for the linear magnetic circuit

Let us suppose a rectangular cross-section toroid with N closely spaced turns fed by a DC current source, as shown in figure 1.7.

Due to both the symmetry and the right-hand rule, the magnetic flux density presents a circular path in the clockwise sense.

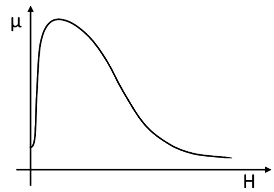

Figure 1.5. Magnetic permeability curve $\mu \times H$.

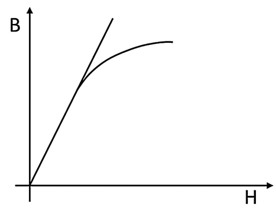

Figure 1.6. Magnetization characteristics curve $B \times H$. For small range of flux density, the $B \times H$ curve is straightforward.

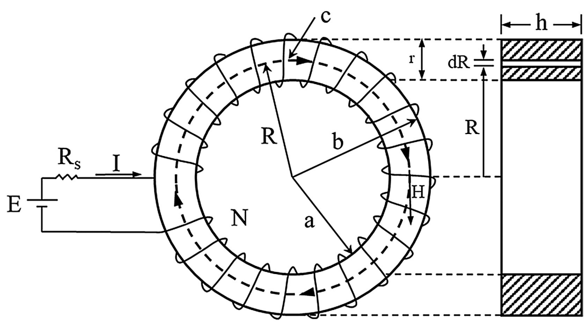

Figure 1.7. A toroid with *N* closely spaced turns.

Ampere's law establishes that:

$$\oint_C \boldsymbol{H} \cdot \boldsymbol{dl} = I_t \tag{1.1}$$

where:

$\boldsymbol{H}$: magnetic field intensity vector (A m^{-1})

I_t: the total amount of linkage current by the closed path C (A)

Let us suppose that the circle of radius R is oriented in the clockwise sense as shown in figure 1.8.

Once the intensity of $\boldsymbol{H}$ is constant along the path C, Ampere's law can be written as:

$$H \oint_C \boldsymbol{dl} = NI \tag{1.2}$$

where NI is the total amount of linkage current by the circle R.

So, we can write:

$$H \times 2\pi R = NI \tag{1.3}$$

and the magnetic field intensity inside the toroid is:

$$H = \frac{NI}{2\pi R}\ (\text{A m}^{-1}) \tag{1.4}$$

Outside the toroid, the magnetic field intensity $\boldsymbol{H}$ is null because there is no linkage current with the path.

The relationship of $\boldsymbol{B}$ and $\boldsymbol{H}$ is the constitutive relation:

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{1.5}$$

The magnetic flux density inside the toroid is given by:

$$B = \frac{\mu NI}{2\pi R}\ (\text{Wb m}^{-2}) \tag{1.6}$$

Figure 1.9 shows the curve $B = B(R)$. If we suppose that $a \gg r$, there is no big variation of B inside the toroid. Under this condition, we can admit without any loss of accuracy that B is constant in a cross-section of the toroid and obtain its value at the mean radius R_m as follows:

$$B = \frac{\mu NI}{l_m}(\text{Wb m}^{-2}) \tag{1.7}$$

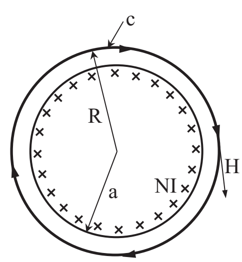

Figure 1.8. The linkage current by the circle of radius R.

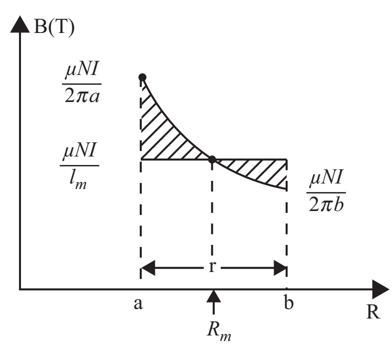

Figure 1.9. B variation inside the toroid. For $a \gg r$ we can admit $B = B(R_m)$ in the cross-section.

with:

$l_m = 2\pi R_m$: The mean length of magnetic path inside the toroid.

In all electromagnetic designs, the condition $a \gg r$ is very common. In a transformer design, the bigger dimension of the core cross-section is smaller than a tenth of the main dimension of the transformer.

It is easy to evaluate the magnetic flux in the toroid cross-section if we suppose B constant inside it.

The magnetic flux is given by:

$$\varnothing = \int_S \boldsymbol{B} \cdot \boldsymbol{dS} (\text{Wb}) \tag{1.8}$$

For $\boldsymbol{B}$ constant inside the toroid and aligned with $\boldsymbol{dS}$, expression (1.8) can be rewritten as:

$$\varnothing = BS \ (\text{Wb}) \tag{1.9}$$

Substituting B by its value expressed in (1.7), the magnetic flux $\varnothing$ is given by:

$$\varnothing = \frac{\mu NIS}{l_m} = \frac{NI}{\frac{1}{\mu}\frac{l_m}{S}} (\text{Wb}) \tag{1.10}$$

At this point, it is convenient to compare this result with one extract from the solution of a simple electric circuit like the one shown in figure 1.10.

The electric current in this simple circuit is:

$$I = \frac{E}{\frac{1}{\sigma}\frac{l}{S}} = \frac{E}{R} (A) \tag{1.11}$$

If we compare both expressions (1.10) and (1.11), we can easily identify the analogy between their values that can be summarized in table 1.1:

Following this analogy, we describe the toroid magnetic circuit by the analog electric circuit of figure 1.11.

Only to justify the approach of considering the flux density B constant in the cross-section, let us evaluate the exact value of ϕ applying (1.8). As both flux density and elemental surface vector are aligned, equation (1.8) can be expressed in its scalar form as:

$$\varnothing_{\text{exact}} = \int_S B \cdot dS (\text{Wb}) \tag{1.12}$$

Substituting B for its value obtained in (1.6) and considering that $dS = h\, dR$ extracted from the figure 1.7, we have:

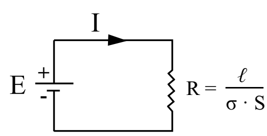

Figure 1.10. Electric circuit.

Table 1.1. Analogy electric circuit × magnetic circuit.

Electric circuit	Magnetic circuit
E or V: Electromotive force (emf) (V)	$\mathcal{F} = NI$: Magnetomotive force (mmf) (A)
σ: Electric conductivity (S m^{-1})	μ: Magnetic permeability (H m^{-1})
I: Electric current (A)	$\varnothing$: Magnetic flux (Wb)
$R = \frac{l}{\sigma S}$: Electrical resistance (Ω)	$\mathcal{R} = \frac{l}{\mu S}$: Magnetic reluctance (A Wb^{-1})
$G = \frac{1}{R}$: Electrical conductance (S)	$\mathcal{P} = \frac{1}{\mathcal{R}}$: Magnetic permeance (Wb A^{-1})
$J = \frac{I}{S}$: Current density (A m^{-2})	$B = \frac{\phi}{S}$: Flux density (Wb m^{-2})
E: Electric field (V m^{-1})	H: Magnetic field intensity (A m^{-1})

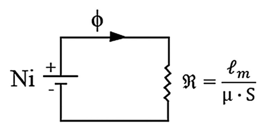

Figure 1.11. Magnetic circuit.

$$\varnothing_{\text{exact}} = \frac{\mu N I h}{2\pi} \int_a^b \frac{dR}{R} = \frac{\mu N I h}{2\pi} \ln \frac{b}{a} \text{ (Wb)} \tag{1.13}$$

Considering that the toroid has the following dimensions:

$$a = 25 \text{ cm}$$
$$b = 30 \text{ cm}$$
$$h = 10 \text{ cm}$$

we have the requirements established for (1.7). Comparing its value with the one extract from (1.9), we obtain the relation:

$$\frac{\varnothing}{\varnothing_{\text{exact}}} = 0.997$$

whose error is less than 0.4%!

1.3 The real world

The toroid assemblage is not a simple task. There are several difficulties forbidding the use of this kind of assemblage in electromechanical devices.

Even with the complex geometry of all electrical apparatus, their studies become easier since they are made with high permeability ferromagnetic materials. This kind of material has the capacity of concentrate the flux density that makes it easy to apply Ampere's law as we will see later.

The general aspect of a magnetic structure is the one shown in figure 1.12. Despite using a classical toroid assemblage, the magnetic structure is normally assembled as a polygon with a uniform cross-section on each leg.

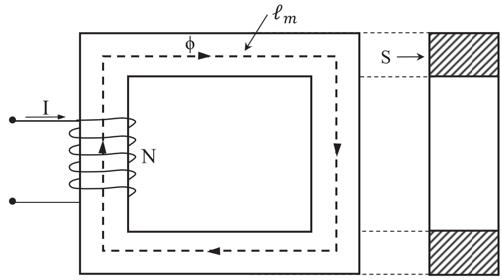

Figure 1.12. Magnetic structure—each leg has a uniform cross-section. The flux density is constant on each leg of the structure.

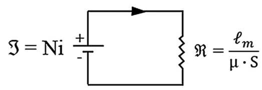

Figure 1.13. Analog electric circuit of magnetic structure: l_m Mean length of magnetic path. S Cross-sectional area: $\mathcal{R}$ Equivalent reluctance of magnetic structure.

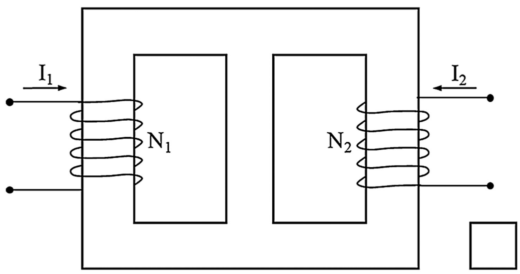

Figure 1.14. Magnetic structure with more than two legs. Two sources of mmf.

For each leg, we evaluate its reluctance by the expression:

$$\mathcal{R}_i = \frac{l_i}{\mu_i S_i} \tag{1.14}$$

Using this procedure for this example, the final analog circuit (figure 1.13) for it is where the equivalent reluctance for this magnetic structure is:

$$\mathcal{R} = \sum_{i=1}^{4} \mathcal{R}_i \tag{1.15}$$

If we have a more complex magnetic structure, a more complex analog circuit should be assembled. Figures 1.14–1.17 show some examples of more complex magnetic structures and their analog circuit.

All types of analysis of an electric circuit are applied to the analysis of the analog electric circuit of a magnetic structure.

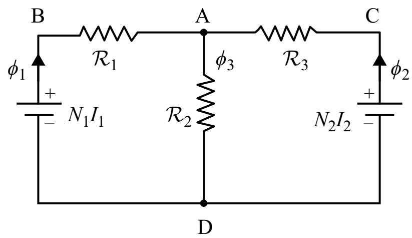

Figure 1.15. Analog electric circuit of figure 1.14. The right-hand rule gives the sense of mmf.

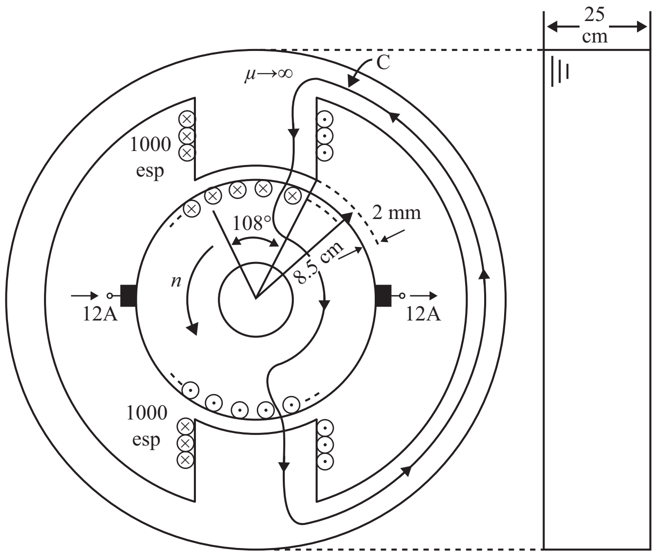

Figure 1.16. Magnetic structure of a DC machine. Two sources of mmf.

From the nodes law, we have:

$$\sum_n \varnothing_i = 0 \text{ at node } i$$

From the voltage law, we can write:

$$\sum_{i=1}^{n} (\mathcal{F}_i - \mathcal{R}_i \phi_i) = 0 \text{ at the mesh } i$$

Applying these laws at the analog circuit of figure 1.15, we have:

$$\phi_1 + \phi_2 - \phi_3 = 0 \text{ at the node A}$$
$$N_1 I_1 - \mathcal{R}_1 \phi_1 - \mathcal{R}_2 \phi_3 = 0 \text{ at mesh BAD}$$
$$N_2 I_2 - \mathcal{R}_3 \phi_2 - \mathcal{R}_2 \phi_3 = 0 \text{ at the mesh ACD}$$
$$N_1 I_1 - \mathcal{R}_1 \phi_1 + \mathcal{R}_3 \phi_2 - N_2 I_2 = 0 \text{ at the mesh ABCD}$$

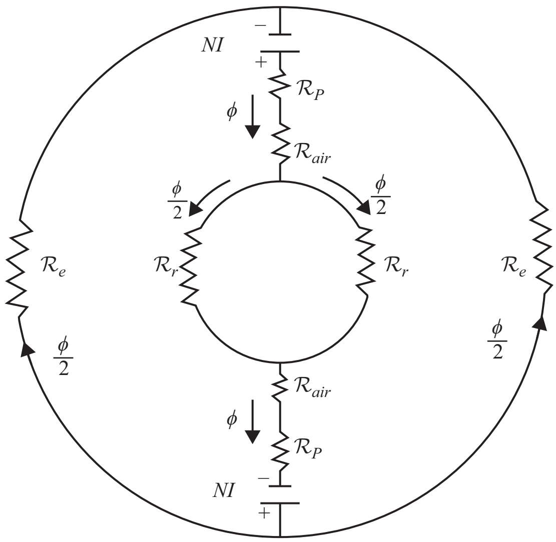

Figure 1.17. Analog electric circuit of a DC machine. The right-hand rule gives the sense of mmf.

For the analog electric circuit of figure 1.17, the mesh law applied on one side of the circuit gives:

$$-NI + (\mathcal{R}_p + \mathcal{R}_{ar})\phi + \mathcal{R}_r\frac{\phi}{2} + (\mathcal{R}_p + \mathcal{R}_{ar})\phi - NI + \mathcal{R}_e\frac{\phi}{2} = 0$$

or else:

$$2NI - 2(\mathcal{R}_p + \mathcal{R}_{ar})\phi - \mathcal{R}_r\frac{\phi}{2} - \mathcal{R}_e\frac{\phi}{2} = 0$$

Due to the symmetry only, this equation is enough to evaluate the magnetic performance of the magnetic circuit.

1.4 Air-gap in a magnetic structure

The presence of an air-gap in a magnetic structure is very common. There are two possibilities: intentional or unintentional. In the first case, the air-gap is necessary for separate moving parts from the static ones, as we can observe in electrical machines. In the DC machine of figure 1.16, the stator (static part) is separated from the rotor (moving part) by a small air-gap.

The second case is the unintentional air-gap. This kind of air-gap appears during the assemblage of closed magnetic structures, like the one used in transformers, when a natural, very small, air-gap arises in the junction of the steel strips, as shown in figure 1.18.

As the unintentional air-gap is normally less than a tenth of millimeter, it is often neglected in the magnetic circuit solution, but the intentional air-gap should be considered because it has a very important role in the magnetic circuit's performance.

Two undesirable effects appear around the air-gap. The first one is the fringing effect on the flux lines that makes the crossing surface of the magnetic flux bigger.

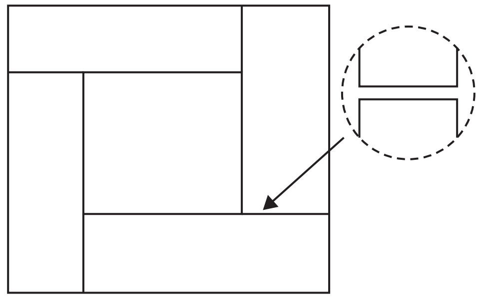

Figure 1.18. Unintentional air-gap. This air-gap is normally neglected in the magnetic circuit solution.

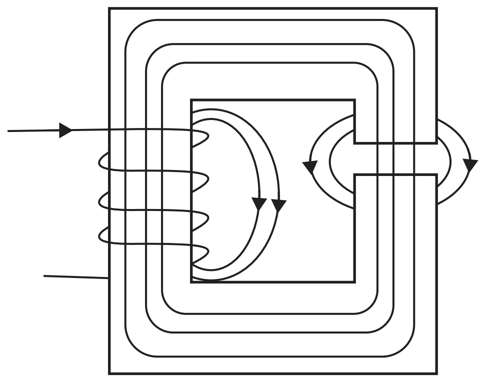

Figure 1.19. Leakage and fringing effects in the air-gap.

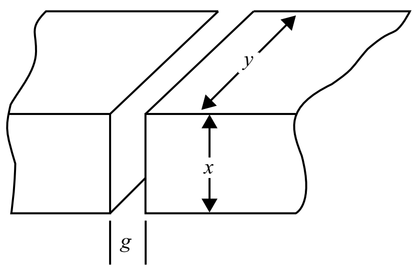

Figure 1.20. The fringing effect evaluation $g \ll x, y$.

The air-gap also increases the leakage flux, which means that some lines of the total flux produced by the coil are not self-closed by the main magnetic circuit.

Figure 1.19 shows both effects. The fringing effect could be considered in the magnetic circuit analysis by using an empirical formula that works well for small air-gaps. On the other hand, the evaluation of the leakage flux is not a simple task and we will not discuss it in this book.

The fringing effects are considered when correcting the crossing surface of the magnetic flux using the empirical formula (figure 1.20):

$$S_g = (x + g)(y + g) \tag{1.16}$$

when the surface of both sides has the same dimension. If the dimensions are different, we must use the following equation:

$$S_g = (x + 2g)(y + 2g) \tag{1.17}$$

where x and y are the dimensions of the smaller face bounding the air-gap.

Example 1.1

The magnetic structure of figure 1.21 was built with cast steel. As the final flux density is low, the relative magnetic permeability may be taken as 1000.

Determine: (a) The reluctance of all legs of the structure; (b) the electric current necessary for establishing a magnetic flux of 3.2×10^{-4} Wb in the air-gap.

Note: All dimensions are in millimeters. The fringing effect should be considered and the leakage flux neglected. $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}.

Solution

The analog electric circuit of the magnetic structure (figure 1.22) is:

Reluctance of lateral legs

$$\mathcal{R}_1 = \frac{l_1}{\mu S_1}$$

where:

$$\mu = 1000\mu_0 = 4\pi \times 10^{-4} \text{ H m}^{-1}$$
$$l_1 = 2(10 + 60 + 10) + (10 + 80 + 10) = 260 \text{ mm}$$
$$l_1 = 0.26 \text{ m}$$
$$S_1 = 20 \times 20 = 400 \text{ mm}^2$$
$$S_1 = 400 \times 10^{-6} \text{ m}^2$$
$$\mathcal{R}_1 = \frac{0.26}{4\pi \times 10^{-4} \times 400 \times 10^{-6}} = 5 \times 10^5 \text{ A Wb}^{-1}$$

Reluctance of central leg

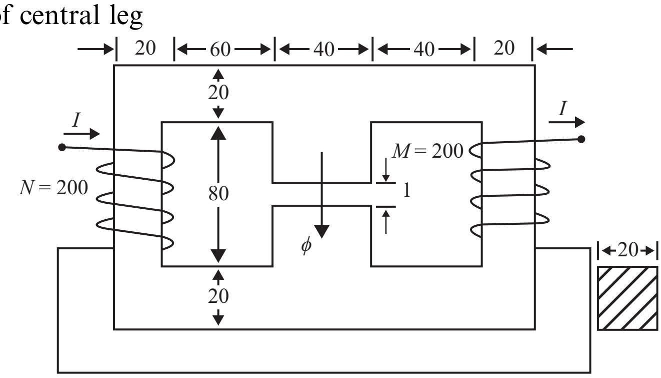

Figure 1.21. Magnetic structure of example 1.4.1.

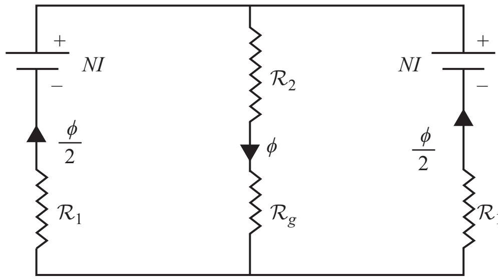

Figure 1.22. Analog circuit of magnetic structure. The magnetic circuit is symmetric.

$$\mathcal{R}_2 = \frac{l_2}{\mu S_2}$$
$$\mu = 1000\mu_0 = 4\pi \times 10^{-4}\ \text{H m}^{-1}$$
$$l_2 = 10 + 80 + 10 = 100\ \text{mm}$$
$$l_2 = 0.1\ \text{m}$$
$$S_2 = 40 \times 20 = 800\ \text{mm}^2$$
$$S_2 = 800 \times 10^{-6}\ \text{m}^2$$
$$\mathcal{R}_2 = \frac{0.1}{4\pi \times 10^{-4} \times 800 \times 10^{-6}} = 10^5\ \text{A Wb}^{-1}$$

Reluctance of air-gap

$$\mathcal{R}_g = \frac{g}{\mu_0 S_g}$$
$$\mu_0 = 4\pi \times 10^{-7}\ \text{H m}^{-1}$$
$$g = 10^{-3}\ \text{m}$$
$$S_g = (40 + 1) \times (20 + 1) \times 10^{-6} = 861 \times 10^{-6}\ \text{m}^2$$
$$\mathcal{R}_g = \frac{10^{-3}}{4\pi \times 10^{-7} \times 861 \times 10^{-6}} = 9.2 \times 10^5\ \text{A Wb}^{-1}$$

As the magnetic structure is symmetric, just one mesh is enough to solve the magnetic circuit. For that,

$$-NI + (\mathcal{R}_2 + \mathcal{R}_g)\phi + \mathcal{R}_1\frac{\phi}{2} = 0$$

or else,

$$-200I + (1 + 9, 2) \times 10^5 \times 3, 2 \times 10^{-4} + 5 \times 10^5 \times 1, 6 \times 10^{-4} = 0$$

that results in

$$I \cong 2\text{A}$$

1.5 Axisymmetric geometry

Until now, we have learned how to evaluate the performance of a magnetic circuit with a planar symmetry. In this case, the flux density distribution inside the structure can be represented by a bi-dimensional (2D) symmetry, i.e., the flux density distribution is the same at any section parallel to the plane of the paper (x, y plane). No field variation is observed in the z-direction.

Although this kind of symmetry is the most common, we find several kinds of electromagnetic devices with axisymmetric geometry (or cylindrical geometry). In this kind of symmetry, the flux density distribution remains the same in all planes that contain the axis of rotation.

Example 1.2

Figure 1.23 shows a cylindrical reactor built with an ideal magnetic material ($\mu \rightarrow \infty$). The magnetic flux in the air-gap is 25×10^{-4} Wb. Determine the current in the 100 turns coil for establishing the magnetic flux required. Both the fringing effects and leakage magnetic flux are neglected.

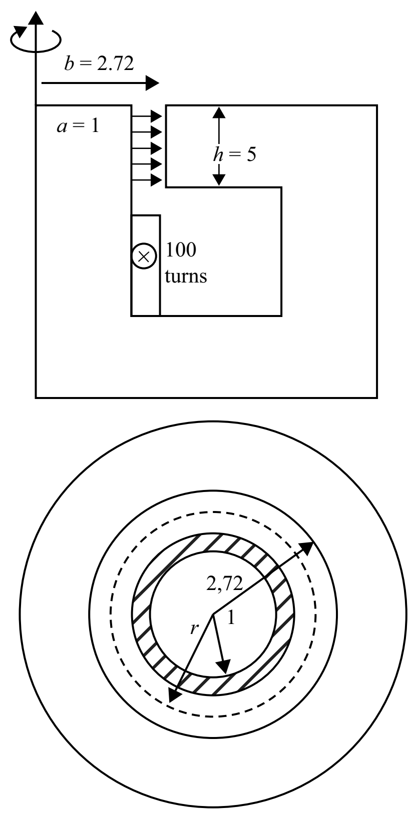

Figure 1.23. Figure of example 1.5.1. Dimensions in centimeters.

Solution
The magnetic circuit of this electromagnetic device is composed of two reluctances. One of them is due to the air-gap ($\mathcal{R}_g$) and the other one is due to the magnetic material ($\mathcal{R}_m$), as shown in figure 1.24.

As the magnetic material has a very high magnetic permeability, the magnetic reluctance of the structure ($\mathcal{R}_m$) is considered null in this example.

To evaluate the air-gap reluctance ($\mathcal{R}_g$), we first need to evaluate the elemental reluctance ($d\mathcal{R}_g$) based on the small ring shown in figure 1.25 located at a generic position $a < r < b$.

The expression for evaluating the elemental reluctance $d\mathcal{R}_g$ is such that:

$$d\mathcal{R}_g = \frac{dr}{\mu_0 \times 2\pi rh}$$

By integration, we get:

$$\mathcal{R}_g = \frac{1}{2\pi\mu_0 h}\int_a^b \frac{dr}{r}$$

That results in:

$$\mathcal{R}_g = \frac{\ln\frac{b}{a}}{2\pi\mu_0 h}$$

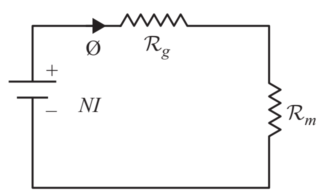

Figure 1.24. Analog electric circuit. $\mathcal{R}_g$: air-gap reluctance; $\mathcal{R}_m$: the magnetic material reluctance. ∅: magnetic radial flux.

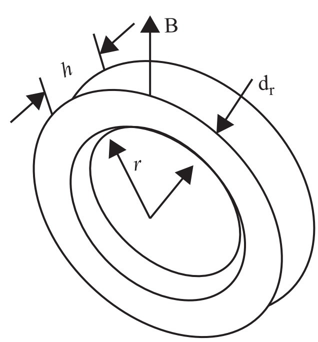

Figure 1.25. Elemental reluctance. This ring is located at a generic position $a < r < b$.

Once $a = 1$ cm and $b = 2.72$ cm, substituting for its value, the final result is:

$$\mathcal{R}_g = 0.253 \times 10^5 \text{ A Wb}^{-1}$$

Based on the analog circuit of figure 1.24, we have:

$$NI = \mathcal{R}_g \varnothing$$

so,

$$100I = 0.253 \times 10^5 \times 25 \times 10^{-4}$$

Resulting in:

$$I = 0.63 \text{ A}$$

1.6 The permanent magnet

Some materials have the property of retaining a residual magnetic flux density even without any electrical current excitation.

When submitted to a high excitation, some of the atoms of this kind of material remain oriented in the direction of impressed mmf even after the mmf is kept down. This is due to the properties of the crystalline structure of the material that gives enough cohesion between groups of atoms to keep these crystals oriented to give a resultant magnetic flux density that is non-null.

The technology of permanent magnets (PM) grew considerably in the late 20th century with the development of the rare-earth permanent magnet. This kind of PM has a very high energy product (the $B \times H$ product) that makes viable the development of several kinds of devices, not only for special applications like high efficiency electrical motors and others, but also for common applications in electrical appliances.

The PM in the magnetic circuit is analogous to a 'battery' in the electric circuit. The battery releases an emf between its plates and a PM releases a mmf inside the material. Figure 1.26 shows the analogy between the battery and PM.

The magnetization characteristics of a PM extracted from its hysteresis curve has the aspect shown in figure 1.27.

B_r is the residual magnetic flux density obtained from the orientation of atoms of the crystals after both a strong and sudden excitation is applied, and H_c is the coercive magnetic field that represents the intensity of the magnetic field intensity necessary for the PM demagnetization.

In the magnetic circuit of figure 1.28, the shaded area is a PM. A linear magnetic material of magnetic permeability μ of mean length l and uniform cross-section S is excited by this PM.

Let us evaluate the magnetic flux created by the PM inside both the PM and the magnetic material.

The analog electric circuit of this magnetic circuit is shown in figure 1.29.

Solving the electric circuit of figure 1.29, we get:

$$\mathcal{F} = \mathcal{R} \varnothing$$

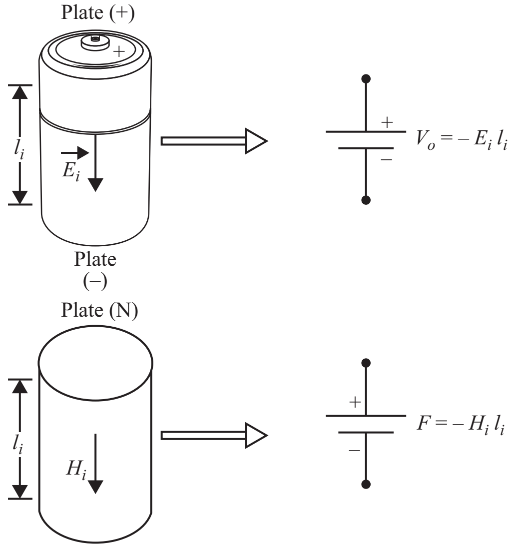

Figure 1.26. Analogy between a battery and a PM. The electric field inside the battery is in the opposite sense to the emf. The magnetic field intensity inside the material is in the opposite sense to the mmf.

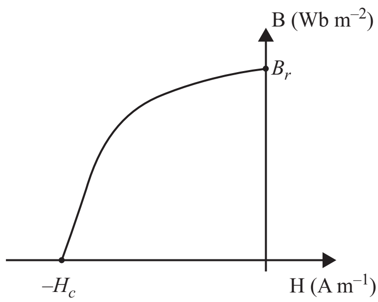

Figure 1.27. Magnetization characteristics of PM. B_r: residual flux density Wb m^{-2}. H_c: coercive magnetic field A m^{-1}.

or,

$$-H_i l_i = \mathcal{R} B_i S$$

that results in:

$$B_i = -\frac{\mathcal{P} l_i}{S} H_i \tag{1.18}$$

where:

$$\mathcal{P} = \frac{1}{\mathcal{R}} [\text{Wb A}^{-1}] \tag{1.19}$$

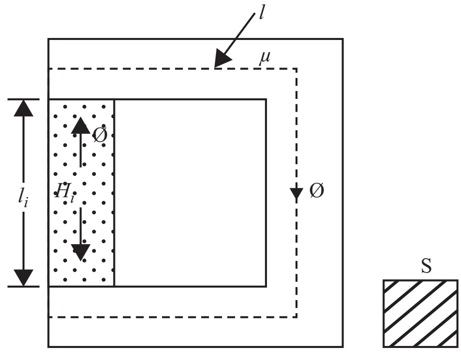

Figure 1.28. Magnetic circuit + permanent magnet. The shaded area is a PM with length l_i.

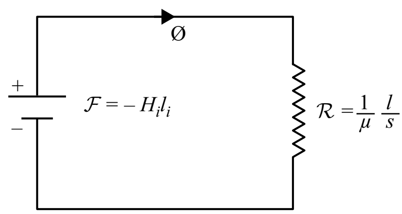

Figure 1.29. The analog electric circuit of magnetic circuit. $\mathcal{F} = -H_i l_i$: The mmf created by a PM. The magnetic flux is in the opposite sense of the magnetic field intensity at the PM.

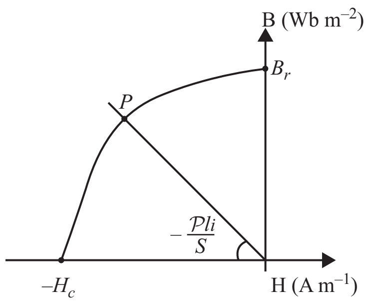

Figure 1.30. Operation point of a PM.

is the permeance of the magnetic circuit.

The relation (B_i, H_i) for the magnetic circuit is a straight line of angular coefficient $-\frac{\mathcal{P}l_i}{S}$. The operation point of the PM is the intersection of the straight line of the magnetic circuit with the magnetization characteristics of PM represented by the point P in figure 1.30.

1.7 The inductance

We can associate with each coil a parameter called 'self-inductance' or simply 'inductance' that characterizes its capacity for producing a magnetic flux.

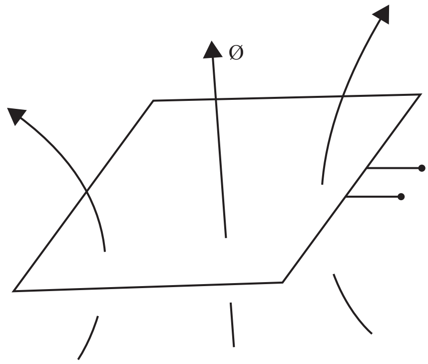

Figure 1.31. Magnetic flux crossing one single turn coil.

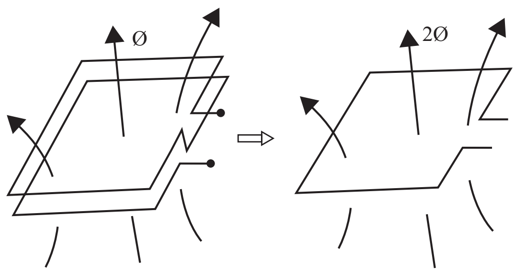

Figure 1.32. Two turns coil crossing by magnetic flux ∅. Equivalent to a single coil crossing by magnetic flux 2∅.

Let us consider ∅ the magnetic flux that crosses one turn coil as shown in figure 1.31.

It is convenient to consider this single turn as a planned electric circuit.

Let us consider now the same magnetic flux crossing a two turns coil as shown in figure 1.32. As the electric circuit has two plans (one for each turn), the total amount of magnetic flux that crosses the circuit is 2∅, i.e., the magnetic flux crossing a two turns coil is equivalent to double the magnetic flux crossing one single turn coil.

So, if we have N turns coil crossed by the same magnetic flux, as shown in figure 1.33, the total amount of magnetic flux crossing the coil is $N\varnothing$.

The product $N\varnothing$ is called 'concatenate magnetic flux' with the coil, and it is represented by the Greek letter λ:

$$\lambda = N\varnothing \tag{1.20}$$

The magnetic flux crossing the coil can be created by an external source or created by the coil itself when it is carrying an electric current. A linear magnetic circuit under this condition shows a direct proportionality between λ and the electric current i. Therefore, we can express the relation of both values as:

$$\lambda = Li \tag{1.21}$$

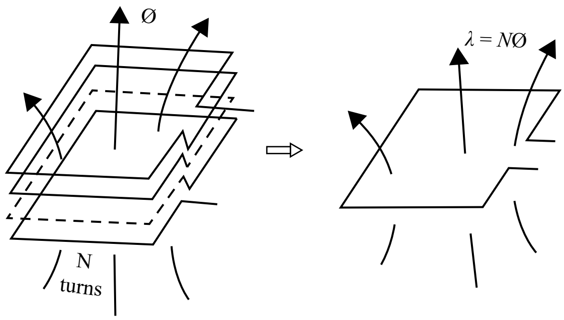

Figure 1.33. *N* turns coil crossing by magnetic flux ∅. Equivalent to single coil crossing by magnetic flux *N*∅.

The constant L is called 'self-inductance' of the coil. It is possible to evaluate such a parameter in simple magnetic circuits, remembering that,

$$Ni = \mathcal{R}\varnothing \tag{1.22}$$

From equations (1.20)–(1.22), we can extract the 'inductance' making:

$$Ni = \mathcal{R}\frac{Li}{N}$$

That results in:

$$L = \frac{N^2}{\mathcal{R}} \tag{1.23}$$

1.8 Summary

The concept of magnetic circuits introduced in this chapter is the first step for a complete understanding of the theory of electromechanical energy conversion. Almost all electromagnetic devices are built with magnetic material that can be split by a prism with both a uniform cross-section and uniform magnetic permeability.

For each prism, it is required to evaluate its reluctance given by:

$$\mathcal{R}_i = \frac{l_i}{\mu_i S_i} \tag{1.24}$$

In the next step we compose the analog electric circuit with all reluctance at the same topology as the magnetic circuit.

In the analog electric circuit, all currents are associated with the magnetic flux in each branch and all drop voltages are equivalent to the drop magnetomotive force in the component (figure 1.34).

When the magnetic circuit has a permanent magnet, the operation point is given by the intersection of the straight line of the magnetic circuit with angular coefficient $-\frac{\mathcal{P}l_i}{S}$ with the magnetization characteristics of a PM.

The inductance of an N turns coil embedded in a linear magnetic circuit of equivalent reluctance $\mathcal{R}_{eq}$ is given by:

$$L = \frac{N^2}{\mathcal{R}_{eq}} \tag{1.25}$$

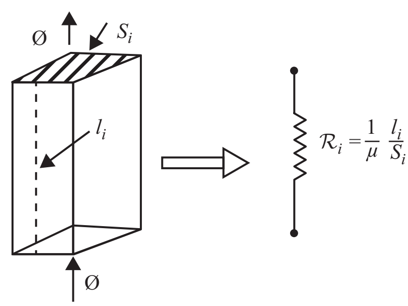

Figure 1.34. Equivalence magnetic circuit × analog electric circuit. The approximations should be done at the corners and at the circle arcs. l_i: the mean length of magnetic path of prism i. S_i: cross-sectional area.

Problems

1.1 A circular cross-section toroid (figure 1.35) is made with linear magnetic material of relative permeability = 5000. The toroid dimensions are: $a = 50$ cm; $b = 60$ cm and $N = 500$ turns. The coil carries a 2A direct current. Determine:
   (a) The flux density distribution for $a \leqslant r \leqslant b$.
   (b) Which value of B can be considered constant in the toroid cross-section?
   (c) The magnetic flux in the cross-section using the value of flux density obtained in (b).
   (d) The reluctance of the magnetic circuit.
   (e) The self-inductance of the coil.

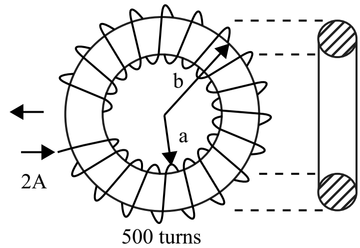

Figure 1.35. Problem 1.1. Circular cross section toroid.

Answers: (b) $B_{cte} = 1.82$ Wb m^{-2}; (c) $\phi = 14.3 \cdot 10^{-3}$ Wb ; $R = 70\,028$ A Wb^{-1}; $L = 3.57$ H.

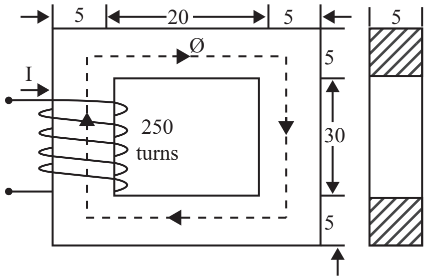

Figure 1.36. Problem 1.2. Sheet steel reactor.

1.2 A reactor shown in figure 1.36 was built of sheet steel. The maximum value of flux density inside the reactor is less than 0.4 Wb m^{-2}, to make this possible consider it as a linear magnetic material of relative magnetic permeability of 5500 ($\mu_0 = 4\pi \times 10^{-7}$ H). The coil has 250 turns and all dimensions are in cm.

Determine:

(a) The DC electric current for establishing a magnetic flux $\varnothing$ of 7.5×10^{-4} Wb in the core.

(b) The self-inductance of the coil.

Answers: (a) $I = 208$ mA ; (b) 900 mH.

1.3 In the reactor of figure 1.36 an air-gap of 1 mm is inserted in the right leg of the core. Neglecting the leakage flux but considering the fringing effect, determine:

(a) The DC electric current for establishing a magnetic flux $\varnothing$ of 7.5×10^{-4} Wb in the core.

(b) The self-inductance of the coil;

Answers: (a) $I = 1.16$ A ; (b) L = 161 mH.

1.4 The symmetric magnetic structure of figure 1.37 is built with two different magnetic materials. The first one has the relative magnetic permeability of 4000 (left side) and the other one 6000 (right side). All dimensions are in cm.

Determine:

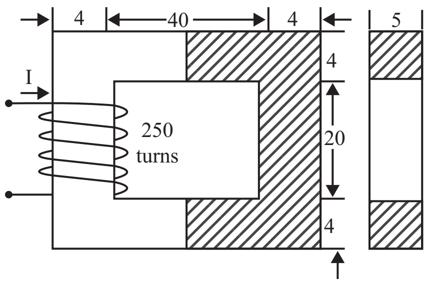

Figure 1.37. Problem 1.4.

(a) The DC electric current in the 250 turns coil for establishing 10^{-3} Wb in the core.
(b) The percentage distribution of mmf in each part of the structure.
(c) The self-inductance of the coil.

Answers: (a) I = 451 mA ; (b) 60% (left side) and 40% (right side) ; (c) L = 554 mH.

1.5 The magnetic structure of figure 1.38 is made by transformer steel and is operating at a low range of flux density. In this condition, the material can be considered linear with a relative magnetic permeability of 15 000. All dimensions are in centimeters and all leakage flux is neglected.

The flux density inside the coil is 0.4 Wb m^{-2}.

Determine:

(a) The electric current in the coil for establishing this flux density.
(b) The magnetic flux and flux density in all legs of the structure.
(c) The inductance of the coil

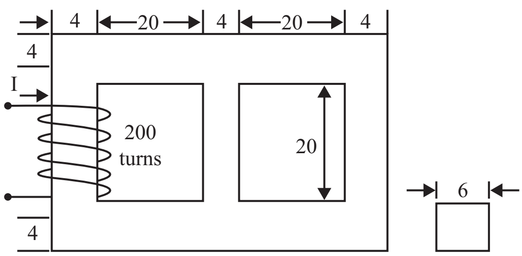

Figure 1.38. Magnetic structure for problem 1.5.

Answers: (a) $I = 95$ mA; (b) $\phi_1 = 9.6 \cdot 10^{-4}$ Wb; $B_1 = 0.4$ T; $\phi_1 = 7.2 \cdot 10^{-4}$ Wb; $B_1 = 0.3$ Wb m^{-2}. $\Phi_3 = 2.4 \cdot 10^{-4}$ Wb; $B_3 = 0.1$ Wb m^{-2}; (c) $L = 2$H.

1.6 The magnetic structure of figure 1.39 is built with a linear magnetic material of relative magnetic permeability 10 000. The magnetic flux in the central leg of the core is 1.0 Wb m^{-2}. The right leg has a 100 turns coil and

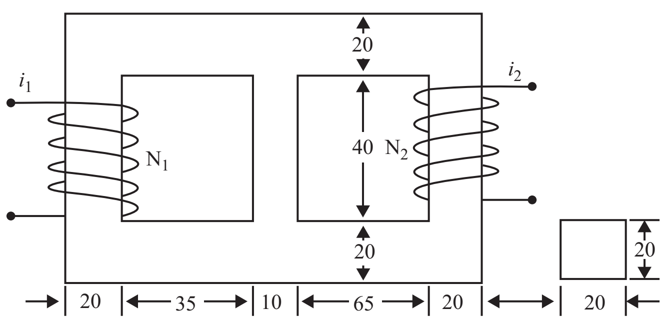

Figure 1.39. Magnetic structure for problem 1.6. The mmf in the left leg ($\mathcal{F}_1 = N_1 i_1$) is 100 A.

the *mmf* in the left leg is ($\mathcal{F}_1 = N_1 i_1$) 100 A. All dimensions are in centimeter and all leakage magnetic flux should be neglected.

Determine:

(a) The DC electric current in the 100 turns coil.

(b) The flux density in all legs of the structure.

Answers: (a) $i_2 = 635$ mA; (b) $B_1 = 0.41$ T; $B_2 = 1$ T; $B_3 = 0.09$ T.

1.7 The electromagnet of figure 1.40 is made of sheet steel magnetized at a low range of flux density. In this condition the sheet steel could be considered a linear magnetic structure of 5000 relative magnetic permeability. The mmf developed by the coil is 1500 A. All dimensions are in centimeters, the leakage magnetic flux is neglected, and the fringing effect should be considered.

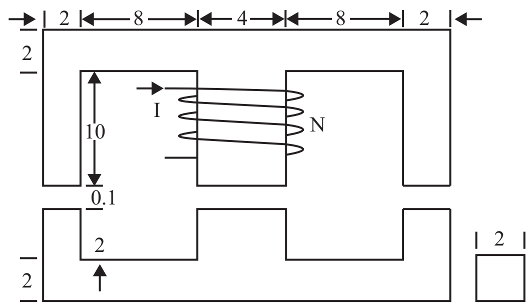

Figure 1.40. Electromagnet of sheet steel. All dimensions in centimeters.

It will be demonstrated later that in each air-gap a magnetic force per unit area of boundary is:

$$\frac{F}{S} = \frac{1}{2}\frac{B^2}{\mu_0} \text{N m}^{-2}$$

Determine:

(a) The magnetic flux in each air-gap.

(b) The magnetic force in each air-gap and the total amount of magnetic force in the electromagnet.

Answers: (a) $\phi_1 = \phi_3 = 3.9 \cdot 10^{-4}$ Wb; $\phi_2 = 7.8 \cdot 10^{-4}$ Wb.

(b) $F_1 = F_3 = 151.3$ N; $F_2 = 302.6$ N.

1.8 The DC electric machine shown in figure 1.41 is built with an ideal magnetic material ($\mu \to \infty$). Determine the electrical current in the 1000 turns/pole field coil for establishing a flux density of 0.6 Wb m^{-2} in the air-gap. No electric current is carried by the armature coil.

Answer: $I = 9.5$ A.

1.9 A kind of covered spool is made of an ideal magnetic material ($\mu \to \infty$), as shown in figure 1.42. This magnetic structure is excited by a 200 turns coil that produces a magnetic flux 25×10^{-4} Wb in the air-gap. Determine the respective current value to produce the required magnetic flux. No fringing effect or leakage magnetic flux should be considered in the solution. All dimensions are in centimeters.

Answer: 1.0 A.

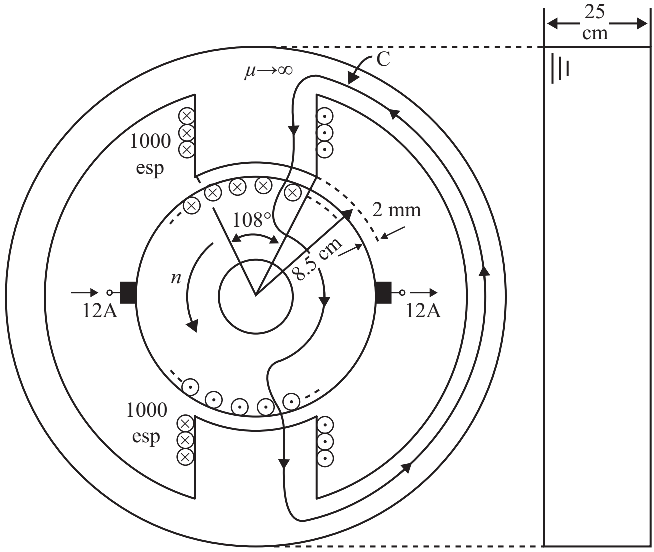

Figure 1.41. DC electrical machine magnetic circuit.

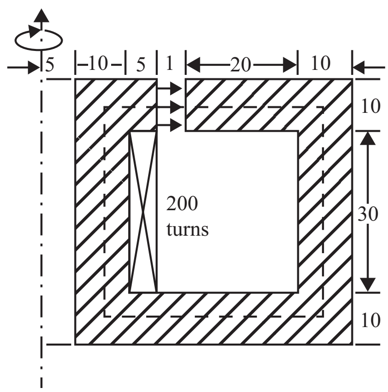

Figure 1.42. Covered spool—cylindrical symmetry.

1.10 The magnetic device shown in figure 1.43 is made of a soft magnetic material that can be considered ideal ($\mu \to \infty$) and a permanent magnet rigid enough that we can consider its magnetic characteristics linear such that $B_r = 0.5$ T and $H_c = 500\ 000$ A m^{-1} (figure 1.44).

The length of the PM is 10 mm in the direction of magnetization and a cross-section area is 10 cm^2. The cross-section of the air-gap area is also 10 cm^2. Fringing of flux around the air-gaps may be ignored. Determine for both $l_g = 2$ and $l_g = 5$ mm (figure 1.43).

Compute the magnetic flux density in both the PM and the air-gap.

Answer: 0.27 T.

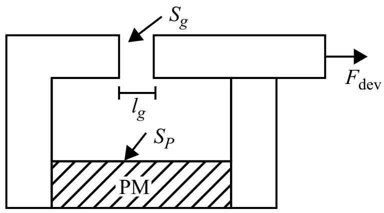

Figure 1.43. Magnetic device of example 5.4. $S_g = 10\ \text{cm}^2$ - $S_p = 10\ \text{cm}^2$. $l_g = 2-5$ mm; $l_p = 102$ mm. $B_r = 0.5$ T; $H_c = 500\ 000\ \text{A m}^{-1}$

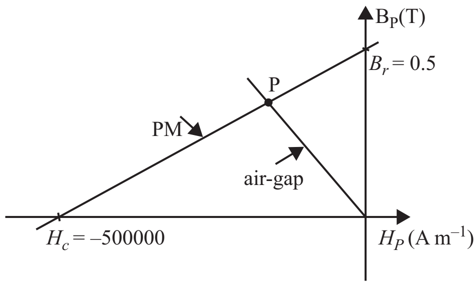

Figure 1.44. PM and air-gap characteristics.

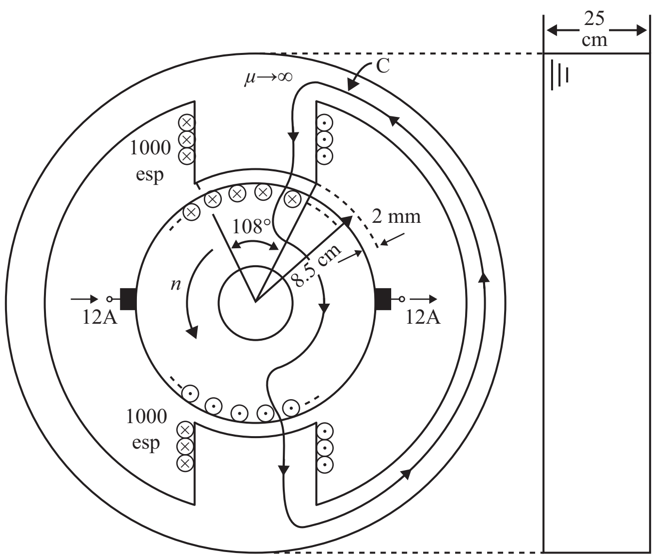

Figure 1.45. Draft for project 1.

Project 1

Design the field coil of the DC electrical machine shown in figure 1.45 considering the stator made with magnetic material of 5000 relative magnetic permeability, the rotor made with magnetic material of 12 000 relative magnetic permeability.

The leakage magnetic flux is neglected, and the fringing effect should be considered.

The field coil is fed by a 220 VDC source. As the duty work of this machine is very high, the current density in the field coil should not exceed 3 A mm^{-2}.

Further reading

[1] Gourishankar V 1966 *Electromechanical Energy Conversion* (Scranton: International Textbook Company)

[2] Skilling H H 1962 *Electromechanics: A First Course in Electromechanical Energy Conversion* (New York: Wiley)

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