# Mass ordering sum rule for the neutrino disappearance channels in T2K, NOvA, and JUNO

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(Received 17 April 2024; revised 19 August 2024; accepted 15 January 2025; published 31 January 2025)

We revisit, reformulate, and extend a method for determining the neutrino mass ordering by using precision measurements of the atmospheric $\Delta m^2$ s in both electron and muon neutrino disappearance channels, first proposed by the authors in 2005 [H. Nunokawa *et al.*, Phys. Rev. D **72**, 013009 (2005)]. The mass ordering is a very important outstanding question for our understanding of the elusive neutrino and determination of the mass ordering has consequences to particle physics, nuclear physics, and cosmology. The JUNO reactor experiment will start data taking this year, and the precision of the atmospheric $\Delta m^2$ s from electron antineutrino measurements will improve by a factor of 3 from Daya Bay's 2.4% to 0.8% within a year. This measurement, when combined with the atmospheric $\Delta m^2$'s measurements from T2K and NOvA for muon neutrino disappearance, will contribute substantially to the $\Delta\chi^2$ between the two remaining neutrino mass orderings. In this paper we derive for the first time a mass ordering sum rule that can be used to address the possibility that JUNO's atmospheric $\Delta m^2$'s measurement, when combined with other experiments in particular T2K and NOvA, can determine the neutrino mass ordering at the $3\sigma$ confidence level within one year of operation. For a confidence level of $5\sigma$ in a single experiment, we will have to wait until the middle of the next decade when the DUNE experiment is operating.

DOI: 10.1103/PhysRevD.111.013008

## I. INTRODUCTION

We have known for more than a quarter of a century that neutrinos are massive [1] but we still do not know whether the neutrino with the least amount of $\nu_e$, usually labeled $\nu_3$, is at the top or bottom of the neutrino mass spectrum. This is the neutrino mass ordering (MO) question and is essential for understanding the mass pattern in the neutrino sector. As the role of neutrinos in particle physics, nuclear physics, astrophysics, and cosmology cannot be understated, this pattern will have tangible consequences in these fields.

The SNO experiment [2] determined that the MO of the other two neutrino mass eigenstates was such that the neutrino with the most $\nu_e$, usually labeled $\nu_1$, was lighter than the other mass state, $\nu_2$, which has a smaller $\nu_e$ fraction than $\nu_1$ but a larger $\nu_e$ fraction than $\nu_3$. Thus, the remaining possible MO for the neutrino mass states is either $m_1 < m_2 < m_3$ which is known as the normal ordering (NO) or $m_3 < m_1 < m_2$ which is known as the inverted ordering (IO), see Fig. 1. The mass squared splitting between $\nu_2$ and $\nu_1$ was measured with good precision by the KamLAND experiment to be [3]

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx +7.5 \times 10^{-5}\ \mathrm{eV}^2, \tag{1}$$

whereas the magnitude of the mass-squared splitting between $\nu_3$ and $\nu_1$ has been determined by a number of experiments to be 30 times larger, i.e.,

$$\Delta m_{31}^2 \equiv m_3^2 - m_1^2 \approx \pm 2.5 \times 10^{-3}\ \mathrm{eV}^2, \tag{2}$$

where the ambiguity in the sign comes from the undetermined MO [4].

There exist numerous ways to determine the MO and hence the above sign in the literature [6–19]. Nevertheless, the use of matter effects in neutrino oscillations has been guiding most of the experimental efforts. The long-baseline accelerator neutrino experiments NOvA [20] and T2K [21] as well as the atmospheric neutrinos experiments Super-Kamiokande [22], Ice-Cube [23], and KM3NeT/ORCA [24], operate in a regime where neutrino oscillations are mostly driven by the higher mass-squared splitting and matter effects are significant in the $\nu_\mu \to \nu_e$ (and antineutrino) appearance

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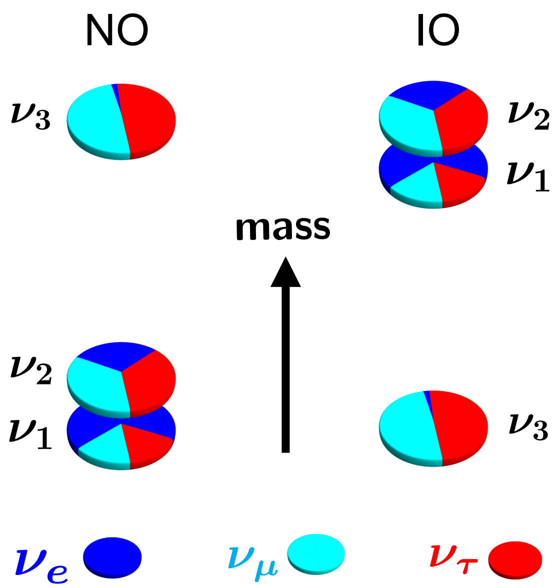


FIG. 1. The normal and inverted mass orderings for the three neutrino mass states, from [5]. Dark blue is the $\nu_e$ fraction, cyan the $\nu_\mu$ fraction, and red the $\nu_\tau$ fraction. If the mass state with the least fraction of $\nu_e$, labeled $\nu_3$, is at the top of the spectrum, this is called the normal ordering (labeled NO), whereas if it is at the bottom of the spectrum it is called the inverted ordering (IO). SNO [2] determined the MO of the other two mass states using solar neutrinos. The set $(\nu_1, \nu_2)$, usually called the solar pair, has the state with most $\nu_e$, labeled $\nu_1$, below the other member of the pair, labeled $\nu_2$.

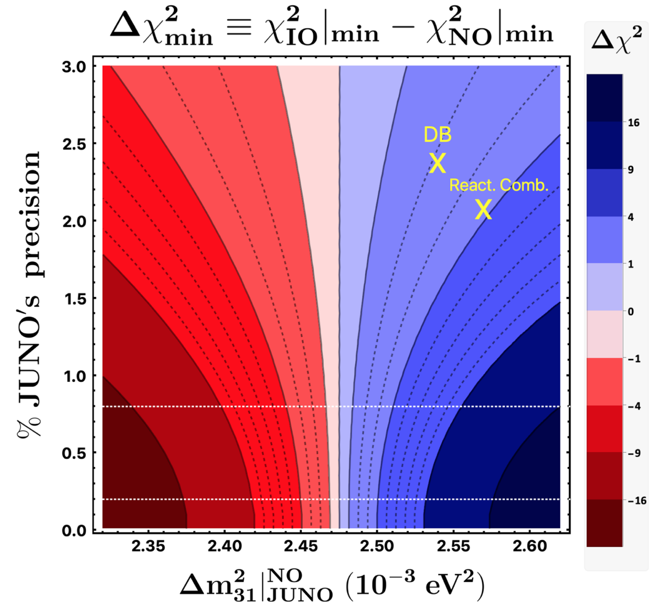


FIG. 2. Isocontours of $\Delta\chi^2_{\rm min} \equiv \chi^2_{\rm IO}|_{\rm min} - \chi^2_{\rm NO}|_{\rm min}$ in the plane of JUNO's $(\Delta m^2_{31}|^{\rm NO} \otimes \sigma)$ fit to the far detector spectrum assuming normal ordering (NO). The precision is expressed as a percent of $\Delta m^2_{31}|^{\rm NO}$. The blue region favors NO whereas the red region favors IO. The current values from the Daya Bay (DB) experiment and from DB/RENO combined according to NuFIT (React. Comb.) are also shown. The white dashed lines mark the precision achievable by JUNO after 100 days (0.8%) and six years (0.2%).

probabilities. They are responsible for the current status of our understanding of the MO question. At present T2K/NOvA, individually, have a slight preference for NO, while the combined fit flips this preference to IO [25,26], although the preference is weak. Ice-Cube has no preference for either. On the other hand, Super-Kamiokande data seems to favor NO by 92.3% CL even though their Monte Carlo simulation indicates they should not be able to discriminate the ordering better than ~80% CL; see [22]. It is the statistical weight of their data, corresponding to an exposure of 364.8 kiloton-years, which makes the final global data fit prefer NO [26]. So, at the current time, we do not have $3\sigma$ or more preference for one ordering over the other.

In this article we revisit, reformulate, and extend one way proposed by the authors of this paper [6] which requires precision measurements of $|\Delta m^2_{31}|$ or $|\Delta m^2_{32}|$ by both $\nu_e/\bar{\nu}_e$ and $\nu_\mu/\bar{\nu}_\mu$ disappearance experiments. The reason for writing this paper now is that within the next year the JUNO experiment [27,28] is expected to improve the precision of these measurements for $\bar{\nu}_e$ disappearance by a factor of approximately 3, i.e., from Daya Bay's (DB) 2.4% to better than 0.8%, a very significant improvement.

There are a number of papers, such as [29–31], which use the method first presented in [6] to address the MO question. Here, we revisit the combination of long baseline experiments and reactor experiments by simplifying and extend all of these analyses. First, we derive a MO sum rule, Eq. (7), for the electron and muon neutrino disappearance channels. Then we use NuFIT's combined T2K/NOvA analysis to generate a contour plot of the $\Delta\chi^2$ between the MOs in the plane of JUNO's measurement of $\Delta m^2_{31}$[NO] and its precision, Fig. 2. This plot allows the reader to immediately estimate the $\Delta\chi^2$ between the two MOs as soon as JUNO presents their $\Delta m^2_{31}|^{\rm NO}$ measurement with its corresponding precision. Neither the sum rule nor the contour plot has appeared elsewhere.

In [6], with more details available in [32], we have shown that the effective atmospheric $\Delta m^2$ $(\Delta m^2_{\rm atm})$ for $\nu_e$ and $\bar{\nu}_e$ disappearance at a baseline divided by neutrino energy $(L/E)$ of 0.5 km/GeV, in vacuum, is given by [33]

$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{12}. \quad (3)$$

This is the only $\Delta m^2$ measured by DB [34] or RENO [35] without additional information from other experiments. The final precision obtained by DB was 2.4%, see [36]:

$$|\Delta m^2_{ee}|_{\rm DB} = 2.519 \pm 0.060 \times 10^{-3}\ {\rm eV}^2. \quad (4)$$

Since the DB experiment is insensitive to the neutrino MO, the magnitude of $\Delta m^2_{ee}$ is the same for both orderings and the sign is undetermined. Matter effects on the magnitude of $\Delta m^2_{ee}$ are smaller than one-tenth of one percent level, see [37], and are therefore much smaller than the measurement uncertainties.

Similarly, for $\nu_\mu$ (and $\bar{\nu}_\mu$) disappearance, in vacuum, $\Delta m^2_{\mu\mu}$ is given by

$$\Delta m^2_{\mu\mu} \approx \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{12} + \sin\theta_{13}\cos\delta \Delta m^2_{21}, \tag{5}$$

where $\delta$ is the $CP$ phase and we have set $\sin 2\theta_{12} \tan\theta_{23} \approx 1$ for simplicity in the coefficient of the $\cos\delta$ term. For the current long baseline experiments such as T2K/NOvA, even though matter effects are significant for the appearance channels, it was recently shown by Denton and Parke, in [38], that for the $\nu_\mu$ disappearance channels matter effects are at the tenths of one percent level. Therefore the $\nu_\mu$ disappearance channels are effectively in vacuum as there is a cancellation between the matter effects in $\nu_\mu \to \nu_e$ and in $\nu_\mu \to \nu_\tau$ for the long baseline experiments T2K/NOvA. As a result, these disappearance channels are also insensitive to the neutrino MO. Similar to DB and RENO, $|\Delta m^2_{\mu\mu}|$ is the only $\Delta m^2$ that is measurable in T2K/NOvA without information from other experiments.

For NO $\Delta m^2_{31} > \Delta m^2_{32}$ whereas for IO $|\Delta m^2_{31}| < |\Delta m^2_{32}|$, therefore, since $\sin^2\theta_{12} \approx 0.3$ we can determine the MO by comparing the magnitude of $|\Delta m^2_{ee}|$ to $|\Delta m^2_{\mu\mu}|$:

$$|\Delta m^2_{ee}| > (<) |\Delta m^2_{\mu\mu}| \quad \text{for NO(IO)}, \tag{6}$$

although the size of the difference between $|\Delta m^2_{ee}|$ and $|\Delta m^2_{\mu\mu}|$ is at the couple of percent level and therefore precise measurements are required. Fortunately, we are about to enter an era where very precise measurements will be made for $\bar{\nu}_e$ disappearance by the JUNO experiment.

The above observations can be converted into a"MO sum rule for the neutrino disappearance channels":

$$(\Delta m^2_{31}|^{\rm NO}_{\mu\,\rm disp} - \Delta m^2_{31}|^{\rm NO}_{e\,\rm disp}) + (|\Delta m^2_{32}|^{\rm IO}_{e\,\rm disp} - |\Delta m^2_{32}|^{\rm IO}_{\mu\,\rm disp})$$
$$= (2\cos 2\theta_{12} - 2\sin\theta_{13}\widehat{\cos\delta})\Delta m^2_{21}, \tag{7}$$

where the rhs can also be written as $(2.4 - 0.9\widehat{\cos\delta})\% \times |\Delta m^2_{\rm atm}|$. The subscript "$\mu$ disp" means the results from $\nu_\mu$ disappearance measurements in T2K/NOvA, whereas "e disp" means the result from $\bar{\nu}_e$ disappearance experiments such as DB and RENO. The symbol $\widehat{\cos\delta}$ is the average $\cos\delta$ for the NO and IO fits. If one changes which $\Delta m^2$ one uses for both experiments for a given MO, the rhs of this sum rule is unchanged. For a detailed derivation of this sum rule, see Appendix A.

From this sum rule it is clear that if NO is nature's choice then the first term will be zero, within measurement uncertainties, and if IO is nature's choice the second term will be zero but in both scenarios the sum of the two must add up to the rhs, independent of the MO. Consequently, if NO is nature's ordering, the measurements of the $\Delta m^2_{32}$ s assuming IO will not align between $\nu_e$ and $\nu_\mu$ disappearance within ≈2.4% and similar for the IO ordering. The measurement uncertainties of the experiments DB, T2K/NOvA are now small enough that this method is already contributing to the global fits on the neutrino MO. This can be perceived in the latest NuFIT figure on the synergies for the $\Delta m^2_{3\ell}$ s, see [39]; one can observe a preference for NO although the precision of the current measurements is not sufficient for a $3\sigma$ determination of the neutrino MO.

The question of immediate current interest is how will the precision measurements of the $\Delta m^2_{3i}$ s by JUNO affect the determination of the MO as JUNO measurements are expected to have an uncertainty smaller than one-third of DBs. This measurement is expected to come very quickly after JUNO turns on, most likely in the first year of operation.

## II. REACTOR MEASUREMENT ($\bar{\nu}_e$ DISAPPEARANCE)

JUNO is a medium baseline (~50 km) high precision reactor antineutrino oscillation experiment aiming to determine the neutrino MO by a careful measurement of the $\bar{\nu}_e$ energy spectrum using an idea first proposed in [13] and further investigated in [40,41]. It was shown in [42] that medium baseline reactor experiments can, in principle, determine the ordering by precisely measuring the effective combination $\Delta m^2_{ee}$ and the sign (+/− for NO/IO) of a phase, $\Phi_\odot$, responsible for the advancement or retardation of the atmospheric oscillations. This phase is given by

$$\Phi_\odot \equiv \arctan(\cos 2\theta_{12} \tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12},$$

where $\Delta_{21} = \Delta m^2_{21} L/4E$. It is derived in Appendix B, Eq. (B3). This is a very challenging measurement due to various systematic effects (energy resolution, nonlinear detector response, etc.) that have to be tamed and understood to a very high level. The JUNO collaboration claimed, in their 2015 paper [27], that it will take six years to determine the MO at $3\sigma$, although more recent papers, see, e.g., [43], have questioned that claim [44]. JUNO's recent update [28] does not contain an update on their expected MO sensitivity.

On the other hand, JUNO is expected to reach, after a few months of operation, and much sooner than they can start to be sensitive to the MO, unprecedented subpercent precision on the determination of $|\Delta m^2_{\rm atm}|$, where $|\Delta m^2_{\rm atm}|$ could be any one of $|\Delta m^2_{ee}|$, or $|\Delta m^2_{31}|$, or $|\Delta m^2_{32}|$ depending on the experiment's analysis choice for both MOs but all are related to one another. JUNO claims that after 100 days of data taking they will be able to determine $|\Delta m^2_{\rm atm}|$ at

0.8% precision and will continue to improve ultimately reaching 0.2% precision; see [28]. In contrast, the expected $\Delta\chi^2$ between the NO and IO in JUNO's fits is expected to grow quite slowly, at no more than 1.5 units per year.

It was shown in [43], and reviewed Appendix B, that due to the phase advance (NO) or retardation (IO) of the atmospheric oscillations, the best fits to the spectrum at the far JUNO (JU) detector will give a $|\Delta m^2_{\rm ee}|$ for IO which is 0.7% larger than $|\Delta m^2_{\rm ee}|$ for NO, i.e.,

$$|\Delta m^2_{\rm ee}|^{\rm IO}_{\rm JU} = \Delta m^2_{\rm ee}|^{\rm NO}_{\rm JU} + 1.8\times 10^{-5}\ {\rm eV}^2. \qquad (8)$$

Note this shift is a fraction of $\Delta m^2_{21}$ and comes from the fact that in IO the atmospheric oscillations are retarded with respect to NO and therefore $\Delta m^2_{\rm ee}$ for IO is slightly larger than for NO to compensate for this difference in the phase shift. We have used that $0.007\times|\Delta m^2_{\rm ee}|\approx 0.018\times 10^{-3}\ {\rm eV}^2$ and the value of $\Delta m^2_{21}$ is given by Eq. (1). Using Eq. (3) for each MO we can relate this result for any of the other possible $|\Delta m^2_{\rm atm}|$ as in Appendix A.

This modifies the MO sum rule for the neutrino disappearance channels, given in Eq. (7), by increasing the rhs by 0.7%, so that for T2K/NOvA (LBL) and JUNO (JU) we have the following sum rule:

$$(\Delta m^2_{31}|^{\rm NO}_{\rm LBL} - \Delta m^2_{31}|^{\rm NO}_{\rm JU}) + (|\Delta m^2_{32}|^{\rm IO}_{\rm JU} - |\Delta m^2_{32}|^{\rm IO}_{\rm LBL}) \approx (3.1 - 0.9\widehat{\cos\delta})\%|\Delta m^2_{\rm atm}|. \qquad (9)$$

With enough precision on the measurements of the $\Delta m^2_{\rm atm}$ s between $\nu_e$ and $\nu_\mu$ disappearance, the $\Delta\chi^2$ between the two MO fits can contribute significantly to the determination of the MO.

We can describe for JUNO the $\chi^2$ fit to data to determine $|\Delta m^2_{\rm atm}|$, as the parabola which will depend on the assumed best fit value $\Delta m^2_{\rm atm}|^{\rm NO}$ or $|\Delta m^2_{\rm atm}|^{\rm IO}$ and $\sigma_{\rm JU}$, the precision of the measurement, which does not depend on the ordering

$$\chi^2_{\rm KO}(\Delta m^2, \sigma_{\rm JU})_{\rm JU} = \left(\frac{\Delta m^2 - \Delta m^2_{\rm atm}|^{\rm KO}}{\sigma_{\rm JU}}\right)^2, \qquad (10)$$

where KO = NO or IO. Note that the subscript "atm" can be any one of (ee, 31, 32) and they do not need to be the same for NO and IO. Different experiments and different global fit groups may make different choices. Here, we will use the choice used by the NuFIT Collaboration, which is $\Delta m^2_{31}$ for NO and $\Delta m^2_{32}$ for IO and the relationship that for the JUNO experiment

$$|\Delta m^2_{32}|^{\rm IO}_{\rm JU} = \Delta m^2_{31}|^{\rm NO}_{\rm JU} + 4.7\times 10^{-5}\ {\rm eV}^2. \qquad (11)$$

The physics conclusions will be independent of the (31, 32) arbitrary choices.

## III. LONG-BASELINE ACCELERATOR MEASUREMENT ($\nu_\mu/\bar{\nu}_\mu$ DISAPPEARANCE)

T2K is a long-baseline (~295 km) accelerator neutrino oscillation experiment in Japan that has collected a total of $1.97\times 10^{21}$ and $1.63\times 10^{21}$ protons on target in neutrino and antineutrino modes, respectively. Similarly, NOvA is a long-baseline (~810 km) accelerator neutrino oscillation experiment in the U.S. that also has collected $1.36\times 10^{21}$ and $1.25\times 10^{21}$ protons on target of data in neutrino and antineutrino modes, respectively. Both experiments operate as a $\nu_\mu\to\nu_\mu/\bar{\nu}_\mu\to\bar{\nu}_\mu$ disappearance experiment as well as a $\nu_\mu\to\nu_e/\bar{\nu}_\mu\to\bar{\nu}_e$ appearance experiment. Their disappearance measurements have no sensitivity to the MO but are responsible for the precise determination of $|\Delta m^2_{32}|$ (or equivalently $|\Delta m^2_{31}|$). T2K results given in [45] are $\Delta m^2_{32}|^{\rm NO} = (2.49\pm 0.05)\times 10^{-3}\ {\rm eV}^2$ and $|\Delta m^2_{31}|^{\rm IO} = (2.46\pm 0.05)\times 10^{-3}\ {\rm eV}^2$; note that uncertainty is ~2%. NOvA's results are given by $\Delta m^2_{32}|^{\rm NO} = (2.39\pm 0.06)\times 10^{-3}\ {\rm eV}^2$ and $|\Delta m^2_{32}|^{\rm IO} = (2.44\pm 0.06)\times 10^{-3}\ {\rm eV}^2$; see [20]. Given the consistency of T2K/NOvA disappearance measurements, they can be combined, as in [26], for both a NO or IO fit. Using the update of these fits, given in [39], we have

$$\chi^2_{\rm KO}(\Delta m^2_{31})_{\rm LBL} = \left(\frac{\Delta m^2_{31} - \Delta m^2_{31}|^{\rm KO}}{\sigma^{\rm KO}_{\rm LBL}}\right)^2 \qquad (12)$$

with KO = NO, $(\Delta m^2_{31}|^{\rm NO}, \sigma^{\rm NO}_{\rm LBL}) = (2.516, 0.031)\times 10^{-3}\ {\rm eV}^2$ and for KO = IO, $(|\Delta m^2_{32}|^{\rm IO}, \sigma^{\rm IO}_{\rm LBL}) = (2.485, 0.031)\times 10^{-3}\ {\rm eV}^2$. Both the best fit point and the uncertainty are used to construct these $\Delta\chi^2$ that will be implored in the combined LBL plus JUNO fit, Eq. (13). The uncertainties on $\cos\delta$ are included in uncertainties on $\Delta m^2_{3i}$ for NO/IO. Since the uncertainty for NO and IO are the same, $\sigma^{\rm IO}_{\rm LBL} = \sigma^{\rm NO}_{\rm LBL}$, we can drop the MO on this symbol. Also, it is important that this combined uncertainty is $0.031/2.5\sim 1.2\%$, much less than the rhs of Eqs. (7) and (9). T2K/NOvA's results are not expected to change significantly in the next few years due to the large statistics already collected by these experiments.

## IV. COMBINING JUNO WITH T2K AND NOVA

Given the expectation from the sum rule and the LBL measurement uncertainties, we expect that with $\lesssim 1\%$ precision from JUNO, one can exclude one of the MOs at $3\sigma$ confidence level from a joint fit.

We combine the results of T2K/NOvA and JUNO by just adding the $\chi^2$ for the combined LBL results to that of JUNO, using the best fit for NO and the measurement precision of JUNO as variables as follows: using for KO = NO or IO

$$\chi^2_{\rm KO}(\Delta m^2_{31}, \sigma_{\rm JU}) = \chi^2_{\rm KO}(\Delta m^2_{31}, \sigma_{\rm JU})_{\rm JU} + \chi^2_{\rm KO}(\Delta m^2_{31})_{\rm LBL},$$

where for JUNO we use the relationship given in Eq. (11) for the best fit values. Then the difference in the $\Delta\chi^2$ is given by

$$\Delta\chi^2(\Delta m^2_{31}|^{\rm NO}_{\rm JU}, \sigma_{\rm JU})_{\rm min} = \chi^2_{\rm IO}|_{\rm min} - \chi^2_{\rm NO}|_{\rm min}. \quad (13)$$

Now everything is determined except for the best fit value $\Delta m^2_{31}|^{\rm NO}$ and precision of this measurement $\sigma_{\rm JU}$ by the JUNO experiment. In Fig. 2 we have plotted the $\Delta\chi^2_{\rm min}$ defined as the minimum for IO minus the minimum for NO as a function of JUNO's $\Delta m^2_{31}|^{\rm NO}$ and the fractional precision of the measurement by JUNO. On this fan shaped figure we have shown for reference DB's result as well as the one from the combined fit of DB and RENO data given by the NuFIT group. Clearly as JUNO's precision on the measurement improves as one moves down this plot which increases the $\Delta\chi^2$, however the central value could also move right (left) thereby increasing (decreasing) the $\Delta\chi^2$. This plot assumes that the disappearance $\Delta m^2_{\rm atm}$ results from T2K/NOvA will not significantly change from what they are now. So, as soon as JUNO presents results on the $|\Delta m^2_{\rm atm}|$ fit to the spectrum at the far detector, one can read off from this plot the contribution that $\nu_\mu/\bar{\nu}_\mu$ and $\bar{\nu}_e$ disappearance measurements from T2K/NOvA and JUNO make to the $\Delta\chi^2$ for the MO determination. If JUNO has the same central value as NuFIT (React. Comb.) but with a precision below 1%, then the contribution from these disappearance measurements will be greater than nine units of $\Delta\chi^2$, i.e., greater than $3\sigma$.

## V. CONCLUSIONS

We reformulate and extend an idea we have had on how to determine the neutrino MO, almost 20 years ago, using only neutrino disappearance data in vacuum at $L/E \sim 500$ km/GeV. While T2K/NOvA are performed at this $L/E$, JUNO's $L/E$ is 30 times larger at 15 km/MeV, requiring a significant extension to the original scheme.

In fact, three new concepts are required before the original idea of [6] can be applied to a combined fit of T2K/NOvA and JUNO: first, the reformulation in terms of a sum rule, second, the modification of the sum rule to account for the fact that JUNO operates at a different $L/E$ than T2K/NOvA, and third, that matter effects in muon neutrino disappearance are negligible for T2K/NOvA.

This is auspicious today in light of the current precision on $|\Delta m^2_{\rm atm}|$ (31 for NO and 32 for IO) achieved by long-baseline experiments (~2% individually and ~1.2% combined) and of the imminent few per mil determination of $|\Delta m^2_{\rm atm}|$ by JUNO. This kind of unprecedented accuracy allows one to discuss a MO sum rule for the neutrino disappearance channels [see Eq. (9)] which may be used to determine the MO in the near future solely using data from these disappearance experiments. To show this with more clarity, we combined in a $\chi^2$ function the present results from T2K/NOvA $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance measurements using both the best fit value and the uncertainty with that expected from $\bar{\nu}_e$ disappearance at JUNO, as a function of the assumed best fit value ($\Delta m^2_{31}|^{\rm NO}_{\rm JU}$) and fractional accuracy of the JUNO measurement ($\sigma_{\rm JU}$). In Fig. 2, one can see the main result, the values of $\Delta\chi^2_{\rm min} = \chi^2_{\rm IO}|_{\rm min} - \chi^2_{\rm NO}|_{\rm min}$ in the plane of JUNO's $(\Delta m^2_{31}|^{\rm NO} \otimes \sigma)$. This is a very useful figure because as soon as JUNO presents its first result on their $|\Delta m^2_{\rm atm}|$ measurement, one can read from it if NO is preferred, and if so, how much it is preferred in terms of how many units of $\Delta\chi^2$. In this manner it is conceivable, if JUNO measures $|\Delta m^2_{\rm atm}|$ close to the one given by combining DB/RENO data, that NO could be soon (in a year or so) determined, by the combined (T2K/NOvA and JUNO) disappearance measurements alone, to better than $3\sigma$, i.e., a confidence level of 99.73%.

The determination of the neutrino MO is crucial to establish the neutrino mass pattern with important implications. In particle physics it will guide model building, in cosmology it will help the understanding of large scale structure formation and affect the possibility of direct detection of relic neutrinos, and in nuclear physics it will impact neutrinoless double $\beta$ and $\beta$ decay experiments. By using the ideas and the proposal given in this manuscript, this might happen within the next year.

## ACKNOWLEDGMENTS

S. J. P. would like to thank Peter Denton for many useful discussions on matter effects for $\nu_\mu$ disappearance. S. J. P. acknowledges support by the United States Department of Energy under Grant Contract No. DE-AC02-07CH11359. R. Z. F. is partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under Contract No. 2019/04837-9, and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). We would both like to thank our longtime collaborator, Hiroshi Nunokawa, for many enlightening discussions on this topic over many years. This project has also received support from the European Union's Horizon 2020 research and innovation program under the Marie Sklodowska-Curie Grant Agreement No. 860881-HIDDeN as well as under the Marie Sklodowska-Curie Staff Exchange Grant Agreement No. 101086085—ASYMMETRY.

## DATA AVAILABILITY

No data were created or analyzed in this study.

## APPENDIX A: DERIVATION OF MASS ORDERING SUM RULE

Here we derive the "mass ordering sum rule for neutrino disappearance channels" from the observations of [6]. Daya Bay and Reno measure the same $|\Delta m^2_{ee}|$ for both mass orderings. Therefore from Eq. (3) we have the following:

TABLE I. This table relates the different $\Delta m^2_{\rm atm}$ s for the best fits of the JUNO experiment, such that each entry gives the difference between $\Delta(\Delta m^2) \equiv |\Delta m^2_{ij}|^{\rm IO} - \Delta m^2_{kl}|^{\rm NO}$ in units of $10^{-5}$ eV$^2$ where (ij) and (kl) are (ee, 31, 32). A value of 2.5 in this table represents a 1.0% difference. The difference between $|\Delta m^2_{32}|^{\rm IO}$ and $\Delta m^2_{32}|^{\rm NO}$ is ~5%, much larger than the measurement error expected in JUNO, whereas the difference between $|\Delta m^2_{31}|^{\rm IO}$ and $\Delta m^2_{31}|^{\rm NO}$ is ~1%.

| $\Delta(\Delta m^2)/10^{-5}$ eV$^2$ | NO ee | NO 31 | NO 32 |
|---|---|---|---|
| IO ee | 1.8 | −0.5 | 6.9 |
| IO 31 | −0.5 | −2.7 | 4.7 |
| IO 32 | 6.9 | 4.7 | 12.1 |

$$\begin{aligned} \Delta m^2_{31}|^{\rm NO}_{e\,\rm disp} &= |\Delta m^2_{ee}| + \sin^2\theta_{12}\Delta m^2_{21} \\ |\Delta m^2_{32}|^{\rm IO}_{e\,\rm disp} &= |\Delta m^2_{ee}| + \cos^2\theta_{12}\Delta m^2_{21} \\ |\Delta m^2_{32}|^{\rm IO}_{e\,\rm isp} - \Delta m^2_{31}|^{\rm NO}_{e\,\rm disp} &= \cos 2\theta_{12}\Delta m^2_{21}. \end{aligned} \tag{A1}$$

Daya Bay's reported measurements given in [36] of $\Delta m^2_{32}|^{\rm NO}_{\rm DB} = 2.466 \pm 0.060 \times 10^{-3}$ eV$^2$ and $|\Delta m^2_{32}|^{\rm IO}_{\rm DB} = 2.571 \pm 0.060 \times 10^{-3}$ eV$^2$ satisfy Eq. (A1), after adding $\Delta m^2_{21}$ to $\Delta m^2_{32}|^{\rm NO}$ to obtain $\Delta m^2_{31}|^{\rm NO}$. The agreement is much, much smaller than the quoted uncertainty, demonstrating that these two results are connected via $|\Delta m^2_{ee}|$.

Using the above expressions, for each MO, we can obtain any of the $\Delta m^2_{\rm atm}$ s used in the literature for the best fit of the JUNO experiment. In Table I we provide how they relate to each other numerically.

Similarly for the disappearance channels in T2K and NOvA, both experiments measure the same $|\Delta m^2_{\mu\mu}|$ for both mass orderings. Matter effects are negligible in this channel for both T2K and NOvA. In fact, it was recently shown by Denton and Parke in [38], that for the $\nu_\mu$ disappearance channels matter effects are at the tenths of one percent level for these experiments. This observation is very important for the derivations in this paper, as even though matter effects were known to be small for muon neutrino disappearance nobody had quantitatively answered the question, "How small is small ?"

Therefore from Eq. (5) we have the following:

$$\begin{aligned} \Delta m^2_{31}|^{\rm NO}_{\mu\,\rm disp} &= |\Delta m^2_{\mu\mu}| + (\cos^2\theta_{12} - \sin\theta_{13}\cos\delta^{\rm NO})\Delta m^2_{21} \\ |\Delta m^2_{32}|^{\rm IO}_{\mu\,\rm disp} &= |\Delta m^2_{\mu\mu}| + (\sin^2\theta_{12} + \sin\theta_{13}\cos\delta^{\rm IO})\Delta m^2_{21} \\ \Delta m^2_{31}|^{\rm NO}_{\mu\,\rm disp} - |\Delta m^2_{32}|^{\rm IO}_{\mu\,\rm disp} &= (\cos 2\theta_{12} - 2\sin\theta_{13}\widehat{\cos\delta})\Delta m^2_{21}, \end{aligned} \tag{A2}$$

where $\widehat{\cos\delta} \equiv \frac{1}{2}(\cos\delta^{\rm NO} + \cos\delta^{\rm IO})$. T2K provides enough significant figures such that again the identity Eq. (A2) can be checked. T2K results given in Table 13 of [45] are reported as $\Delta m^2_{32}|^{\rm NO}_{\rm T2K} = 2.494 \pm 0.054 \times 10^{-3}$ eV$^2$ and $|\Delta m^2_{31}|^{\rm IO}_{\rm T2K} = 2.463 \pm 0.049 \times 10^{-3}$ eV$^2$ again after correcting for the $31 \leftrightarrow 32$ for both mass ordering, we get excellent agreement when using $\widehat{\cos\delta} \approx 0$. The agreement is much, much smaller than the quoted uncertainty, demonstrating that these two results are connected via $|\Delta m^2_{\mu\mu}|$. NOvA, unfortunately, does not provide enough significant figures for their measurements to demonstrate the $|\Delta m^2_{\mu\mu}|$ connection as convincingly as T2K, but their measurements are in agreement with (A2). Also, by comparing the magnitude of $|\Delta m^2_{\rm ee}|$ and $|\Delta m^2_{\mu\mu}|$, we see from Eq. (6) that there is a hierarchy depending on the MO, although the size of the difference between $|\Delta m^2_{\rm ee}|$ and $|\Delta m^2_{\mu\mu}|$ is at the couple of percent level and therefore precise measurements are required. If T2K and NOvA reported the $|\Delta m^2_{\mu\mu}|$ fits to their disappearance only data, this comparison could be made more directly without any information needed on $\cos\delta$.

Now, adding and rearranging Eqs. (A1) and (A2), we have the mass ordering sum rule for neutrino disappearance channels given in Eq. (7):

$$\begin{aligned} &(\Delta m^2_{31}|^{\rm NO}_{\mu\,\rm disp} - \Delta m^2_{31}|^{\rm NO}_{e\,\rm disp}) + (|\Delta m^2_{32}|^{\rm IO}_{e\,\rm disp} - |\Delta m^2_{32}|^{\rm IO}_{\mu\,\rm disp}) \\ &= (2\cos 2\theta_{12} - 2\sin\theta_{13}\widehat{\cos\delta})\Delta m^2_{21} \\ &\approx (2.4 - 0.9\widehat{\cos\delta})\%|\Delta m^2_{\rm atm}|, \end{aligned} \tag{A3}$$

where for the last line we have used $\Delta m^2_{21}/|\Delta m^2_{\rm atm}| = 0.03$, $\sin^2\theta_{12} = 0.3$, and $\sin\theta_{13} = 0.15$. Note the interchange between $\mu$ disp and e disp when going from NO to IO. Also, this sum is invariant if we replace $\Delta m^2_{31}$ with $\Delta m^2_{32}$ for NO or/and $\Delta m^2_{32}$ with $\Delta m^2_{31}$ for IO. We use the above choice because that is the choice made by NuFIT, but our physics conclusions are independent of this choice. The best fit values of $\delta$ and their uncertainties are included in the extraction of the allowed regions for $|\Delta m^2_{31}|^{\rm NO}$ and $|\Delta m^2_{32}|^{\rm IO}$ in the NuFIT combined fit of T2K and NOvA. In passing it is worth noting that the NuFIT results given in [39] suggest that $\widehat{\cos\delta} \le 0$.

Because of the phase advance (NO) or retardation (IO) of the atmospheric oscillations, JUNO does not measure exactly the same $|\Delta m^2_{ee}|$ for both mass orderings, see [43], in fact

$$\begin{aligned} |\Delta m^2_{ee}|^{\rm IO} &\approx 1.007 \times \Delta m^2_{ee}|^{\rm NO} \\ &= \Delta m^2_{ee}|^{\rm NO} + 1.8 \times 10^{-5}\ {\rm eV}^2. \end{aligned} \tag{A4}$$

This changes Eq. (A1) and also the mass ordering sum rule for neutrino disappearance channels to what is given in Eq. (9) by adding 0.7% times $|\Delta m^2_{\rm atm}|$ to the rhs:

$$\begin{aligned} &(\Delta m^2_{31}|^{\rm NO}_{\rm LBL} - \Delta m^2_{31}|^{\rm NO}_{\rm JU}) + (|\Delta m^2_{32}|^{\rm IO}_{\rm JU} - |\Delta m^2_{32}|^{\rm IO}_{\rm LBL}) \\ &\approx (3.1 - 0.9\widehat{\cos\delta})\%|\Delta m^2_{\rm atm}|. \end{aligned} \tag{A5}$$

Here the label e disp ($\mu$ disp) has been replaced with the label "JU" ("LBL") as this sum rule is specific for T2K, NOvA (LBL), and JUNO (JU). The additional 0.7% has an impact on Fig. 2 because of the precision of the JUNO measurements on $\Delta m^2_{\rm atm}$ s.

If nature's choice for the mass ordering is NO, then

$$(\Delta m^2_{31}|^{\rm NO}_{\rm LBL} - \Delta m^2_{31}|^{\rm NO}_{\rm JU}) \approx 0$$

and

$$(|\Delta m^2_{32}|^{\rm IO}_{\rm JU} - |\Delta m^2_{32}|^{\rm IO}_{\rm LBL}) \approx (3.1 - 0.9\widehat{\cos\delta})\%|\Delta m^2_{\rm atm}| \quad \text{(A6)}$$

whereas for IO

$$(\Delta m^2_{31}|^{\rm NO}_{\rm LBL} - \Delta m^2_{31}|^{\rm NO}_{\rm JU}) \approx (3.1 - 0.9\widehat{\cos\delta})\%|\Delta m^2_{\rm atm}|$$

and

$$(|\Delta m^2_{32}|^{\rm IO}_{\rm JU} - |\Delta m^2_{32}|^{\rm IO}_{\rm LBL}) \approx 0, \quad \text{(A7)}$$

where "≈" should be interpreted to mean "within measurement uncertainties." The current measurement uncertainty for the combined T2K and NOvA measurement is ∼1.2%. JUNO's measurement uncertainty will reach 0.8% within one year of data taking and reach 0.2% within six years. If T2K and NOvA can improve their measurement uncertainties on the $\Delta m^2_{\rm atm}$ s even modestly, in the future, this can impact the $\Delta\chi^2$ between the two mass orderings substantially.

The uncertainty on the solar parameters can be inferred from Eqs. (7) and (A3). A 5% uncertainty on $\cos 2\theta_{12}\Delta m^2_{21}$ corresponds to a 5% uncertainty on the rhs of Eq. (7). Thus, the 2.4% could have been written as $(2.4 \pm 0.12)\%$ to show these uncertainties. But these uncertainties are negligible when compared with other uncertainties. Similar for the sum rule used for T2K, NOvA, and JUNO, Eqs. (9) and (A5), the 3.1% could have been written as $(3.1 \pm 0.12)\%$ to include the solar parameter uncertainties.

The sum rule is not used to obtain Fig. 2 but it gives us a guide to the expected size of the misalignment in the $\Delta m^2$ for the MO that is not nature's choice. In Fig. 2 we fix the solar parameters at their best fit values determined by NuFIT [26] $\Delta m^2_{21} = 7.41 \times 10^{-5}$ eV$^2$ and $\sin^2\theta_{12} = 0.303$. Small changes in these parameters when measured by JUNO will not affect our results as the experimental uncertainty on $\Delta m^2_{\rm atm}$ from the LBL experiments will dominate.

## APPENDIX B: REVIEW OF ELECTRON NEUTRINO DISAPPEARANCE PROBABILITY

We start from the usual expression for the $\bar{\nu}_e$ disappearance probability in vacuum ($\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$),

$$P_{\bar{\nu}_e\to\bar{\nu}_e} = 1 - \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21} - \sin^2 2\theta_{13}[\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}]. \quad \text{(B1)}$$

Using the methods from Refs. [32,42], it is simple to show without approximation that

$$P_{\bar{\nu}_e\to\bar{\nu}_e} = 1 - \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13} \times \left(1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\cos\left[2|\Delta_{ee}| \pm \Phi_\odot\right]\right) \quad \text{(B2)}$$

where $\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$,

$$\text{and} \quad \Phi_\odot \equiv \arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}, \quad \text{(B3)}$$

The $\sqrt{(\cdots)}$ gives the amplitude modulation of the $\theta_{13}$ oscillations and the $\pm\Phi_\odot$ is the phase advance and phase retardation of these oscillations for the NO and IO, respectively. Note the difference between NO and IO only appears in the sign in front of the phase $\Phi_\odot$ and only depends on the solar parameters. This phase advancement/retardation means that, for JUNO, the spectrum for IO can be *best matched* to the spectrum for NO when

$$|\Delta m^2_{\rm ee}|^{\rm IO}_{\rm JU} = 1.007 \times \Delta m^2_{\rm ee}|^{\rm NO}_{\rm JU},$$

as shown in Fig. 2 of [43], i.e., the best fit points for $\Delta m^2_{\rm ee}$ for NO and IO are related by this equation.

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