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Analysis of non-linear plates by the boundary element method applied to concrete slabs

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BEM formulations have already been proposed to deal with an enormous number of complex engineering problems, many of them exhibiting non-linear behaviour. Analysis of plate bending, written within the context of classical Kirchhoff's hypothesis, is one problem where BEM formulations have been successfully applied [1, 2, 3, 4]. Elastoplastic plate bending analysis has first appeared in the work written by Moshaiov and Vorus [5] who have implemented a particular incremental scheme based on the initial moment technique. Chueri and Venturini [6] have studied the problem as well, proposing a formulation to be applied to reinforced concrete slabs. In this context, it is important to introduce BEM formulations capable of dealing with plate that exhibits variable thickness or stiffness in general, as has appeared in recent works [7, 8]. Those formulations could be properly modified to give the incremental procedure to deal with non-linear problems. Although no fundamental solution can be derived regarding this kind of problem, some procedures can be followed to introduce the possibility of varying the stiffness using the classical fundamental solution accomplished by a convenient treatment of the remaining domain integral. The present paper will discuss the non-linear formulation of Kirchhoff plates in bending to model reinforced concrete elements. Integral equations of displacements, bending and twisting moments and shear forces will be derived. The Betti's reciprocal work theorem has been taken to derive those integral representations, starting by assuming either the structural element with varying thickness or an initial moments field applied over the domain to model the material behaviour. A simple and particular plastic criterion appropriate to model reinforced concrete slabs, written in terms of internal moments and curvatures, will be discussed. In what follows, the formulation is extended to incorporate layered models, where the initial moment fields are evaluated by numerical integrations along the plate thickness. The plastic model assumed to govern the concrete behaviour, given in terms of stresses, is enforced at Gauss points taken along the thickness. The steel model is independently verified at the actual reinforced positions. Two different schemes to model concrete slabs are discussed, First, a very simple procedure is adopted assuming the neutral axis to be at the plate middle surface. Then, the model is improved leading to the second alternative, now including a scheme to define properly the neutral axis position enforcing no in plane stress resultant. The algorithm proposed is appropriate to deal with simple case of linear plate with varying thickness and with more complex plates where different reinforcements are adopted for the two directions leading to the orthotropic case of plates.

After implementing those models some examples are taken to illustrate the accuracy of the proposed formulations.

References

[1] BEZINE, G.P.; Boundary integral formulation for plate flexure with arbitrary boundary conditions, *Mech. Res. Comm.*, 1978, 5(4), 197-206.

[2] STERN, M.A.; A general boundary integral formulation for the numerical solution of plate bending problems, *Int. J. Solids Struct.*, 1979, **15**, 769-782.

[3] HARTMANN, F.; ZOTEMANTEL, R.; The direct boundary element method in plate bending, *Int. J. Num. Meth. Engrg.*, 1986, **23**(11), 2049-2069.

[4] VENTURINI, W.S., PAIVA, J.B. Boundary elements for plate bending analysis, Engineering Analysis, **11**, 1-8, 1993.

[5] MOSHAIOV, A., VORUS, W.S. Elasto-plastic plate bending analysis by a boundary element method with internal initial plastic moments, *Int. J. Solids Struct.*, **22**, 1213-1229,1986.

[6] CHUEIRI, L.H.M. AND VENTURINI, W.V., Elastoplastic BEM to model concrete slabs. In *Boundary Elements XVII*, eds. Brebbia, C. A. et al, Computational Mechanics Publications, Southampton, 1995.

[7] Sapountzakis, E.J., Katsikadelis, J.T., Boundary element solution of plates with variable thickness. *J. Eng. Mech.*, **117**(6), 1241-1256, 1991.

[8] Nerantzaki, M.S., Analysis of plates with variable thickness by the analog equation method. *Boundary Elements XVII,* eds. Brebbia, C. A. et al, Computational Mechanics Publications, Southampton, 1995