RT-MAT 2007 - 14

A POLYNOMIAL ALGORITHM FOR DECIDING 3-SAT

Maria Angela Weiss

Novembro 2007

Esta é uma publicação preliminar ("preprint").

A Polynomial Algorithm for Deciding **3-sat**

Techical Report

Departamento de Matemática
Universidade de São Paulo, São Paulo - Brasil

M A Weiss

Abstract

We present here a **KE**–tableaux based method for solving **3-sat**. We show that this method is polynomial. We solve, in this way, a question alternative to the classic question whether **P** = **NP**.

1 Introduction

Our goals here are to establish a polynomial algorithm for writing and solving polynomially a macro for a tableau for **3-sat**.

The class of **sat** problems was shown to be **NP–complete**. The pioneer work shown of Cook, in [1] and Cook and Reckhow in [2]. The focus is to establish lower bounds in complexity, as the pioneer work of Meyer and Sotckmeyer and Meyer [5] and [4]. The literature in this area is rich of very nice surveys, like [3] and [6].

Our approach on the problem of solving **3-sat**is using the approach of tableaux. Tableaux branches can grow exponentially, but we propose a macro based on a bound over the number of clauses of a given **3-sat** formula and this approach can be give answers in some open questions.

2 Basic Definitions

This Section is divided into two parts. The algorithms we have developed are a "brand new feature". Definitions can be, even in this introductory section, a mandatory reading for the sake of the full comprehension. Definitions 2.1 to 2.11 belong to the class of "keystone definitions" for being the base of our algorithm. The other definitions can get more of the reader focus accordingly to the needs.

A **3-sat** formula Ψ is a conjunctions of a number, say n, of formulas L_n, where each L_i is a disjunction of three literals. We write

$$\Psi \equiv (l_1^1 \vee l_1^2 \vee l_1^3) \wedge \cdots \wedge (l_n^1 \vee l_n^2 \vee l_n^3) \equiv C_1 \wedge \cdots \wedge C_n$$

Any subformula of Ψ of the form $C_k \equiv l_k^1 \vee l_k^2 \vee l_k^3$, $1 \leq k \leq n$ is called a *conjunct* of Ψ

Definition 2.1 *The set of literals of a formula Ψ is denoted by* $\mathsf{Letter}(\Psi)$.

A pair of a literal and its negation is called a conjugated pair.

Notation 2.2 *Consider the* **3-sat** *formula*

$$\Psi \equiv (p_1 \vee q_{11} \vee q_{12}) \wedge (p_1 \vee q_{21} \vee q_{22}) \wedge \cdots \wedge (p_1 \vee q_{2t} \vee q_{2t})$$

The factorization *of* Ψ

$$p_1 \vee \big((q_{11} \vee q_{12}) \wedge (q_{21} \vee q_{22}) \wedge \cdots \wedge (q_{2t} \vee q_{2t})\big) \qquad (1)$$

is denoted $p_1 \vee S_{p_1}$.

Definition 2.3 *Given a* **3-sat** *formula* $\Psi \equiv \psi_1 \wedge \cdots \wedge \psi_n$, *a* partition *for a formula* Ψ *is a rewritten of* Ψ *as (the equivalent) formula*

$$(p_1 \vee S_{p_1}) \wedge (\neg p_1 \vee S_{\neg p_1}) \wedge \cdots \wedge (p_k \vee S_{p_k}) \wedge (\neg p_k \vee S_{\neg p_k}) \wedge S_3 \qquad (2)$$

where for all l_1, l_2 *in* $\{p_1, \neg p_1, \ldots, p_k, \neg p_k\}$, $l_1 \cap \mathsf{Letter}(S_{l_2}) = \emptyset$.

Schematically, we have:

$$\begin{array}{cccc}
p_1 & \boxed{S_{p_1}} & \neg p_1 & \boxed{S_{\neg p_1}} \\
p_2 & \boxed{S_{p_2}} & \neg p_2 & \boxed{S_{\neg p_2}} \\
 & \vdots & & \vdots \\
p_k & \boxed{S_{p_k}} & \neg p_k & \boxed{S_{\neg p_k}} \\
 & & \boxed{S_3^1} &
\end{array}$$

We can keep on making partitions of the above formula S_3^1. The set of atoms in S_3^1 is contained in the union of the atoms of each S_l and not necessarily S_3^1 is a **2-sat** formula or empty, as the below formula

$$\Psi = (a \vee b \vee c) \wedge (a \vee b \vee d) \wedge (a \vee c \vee d) \wedge (b \vee c \vee d)$$

If Ψ already is of the form

$$(p_{01} \vee S_{p_{01}}) \wedge (\neg p_{01} \vee S_{\neg p_{01}}) \wedge \cdots \wedge (p_{0k} \vee S_{p_{0k}}) \wedge (\neg p_{0k} \vee S_{\neg p_{0k}}) \wedge S_3^1$$

Successively, do the partitions:

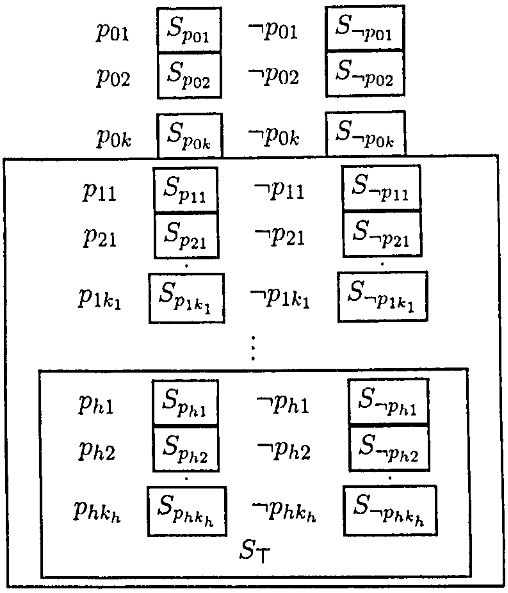
$$S_3^1 \equiv (p_{11} \vee S_{p_{11}}) \wedge (\neg p_{11} \vee S_{\neg p_{11}}) \wedge \cdots \wedge (p_{1k_1} \vee S_{p_{1k_1}}) \wedge (\neg p_{1k_1} \vee S_{\neg p_{1k_1}}) \wedge S_3^2$$

$$S_3^2 \equiv (p_{21} \vee S_{p_{21}}) \wedge (\neg p_{21} \vee S_{\neg p_{21}}) \wedge \cdots \wedge (p_{2k_2} \vee S_{p_{2k_2}}) \wedge (\neg p_{2k_2} \vee S_{\neg p_{2k_2}}) \wedge S_3^3$$

$\ldots$

$$S_3^h \equiv (p_{h1} \vee S_{p_{h1}}) \wedge (\neg p_{h1} \vee S_{\neg p_{h1}}) \wedge .. \wedge (p_{hk_h} \vee S_{p_{hk_h}}) \wedge (\neg p_{hk_h} \vee S_{\neg p_{hk_h}}) \wedge S_\top$$

where $S_\top$ is either a **2 − sat** formula or the formula $\top$. Finally, obtain the below partition:



Definition 2.4 *The set* $\mathbb{L}abel = \{p_{ij} | (1 \leq i \leq h) \wedge (1 \leq j \leq k_i\}$ *is called the set of* labels *and each literal in* $\mathbb{L}abel$ *is a label.*

We distinguish the set of labels according to its depth. A literal l_{ij} *belongs to the level* i*.*

Notice that if a literal l belongs to the level 0, then it cannot belong to the set of vertices, if a literal l belongs to the level j, then it cannot be an vertices of a level $j' > j$.

A line with a single literal in it, say, l is equivalent to $l \vee \bot$.

We refute or do not refute a given formula Ψ under the hypothesis that any given maximal consistent set of labels $\mathrm{L} = \{l_i, \dots, l_k\}$ forms a branch of a labeled tableau for Ψ in which each node is a literal of the maximal consistent set labeled as false, F.

Observe that if there is a literal r so that $r \in \mathsf{Letter}(\Psi)$ and $\neg r \notin \mathsf{Letter}(\Psi)$, then Ψ is false iff the formula

$$\Psi' \equiv \{C_1 | 1 \leq i \leq n \wedge r \notin \mathsf{Letter}(C_i)\}$$

is false since a valuation $r = \top$ trivializes our search.

From now on, we fix a partitioned **3-sat** formula Ψ

Definition 2.5 *A* maximal set *is a maximal consistent subset of* $\mathbb{L}abel$.

Definition 2.6 *The* the mirror tableau for Ψ *is the tableau whose branches are of the form*

$F \quad l_{01}$
$\vdots$
$F \quad l_{0k_0}$
$\vdots$
$F \quad l_{hk_h}$
$T \quad S_{l_{01}}$
$\vdots$
$T \quad S_{l_{0k_0}}$
$\vdots$
$T \quad S_{l_{hk_h}}$

where the nodes labeled as false range over maximal sets.

We show in this paper that there is a quick way to select this branches of a **KE**-tableaux and decide if Φ is valid or no. The search over the generated branches starts on looking for contradictions over groups of conjunctions of clauses S_{ij}.

Definition 2.7 *Define the cylindrical digraph* $\mathbb{C}lndr$*-graph,* $(L_{0,1}, \Rightarrow, :::)$*:*

- $L_{0,1} = L \times \{0,1\} \cup \{\bot\}$, *where L is the set of literals that belong to some S_{ij};*

- *If $a \vee b$ is a clause of $S_{l_1}, \ldots, S_{l_k}$, then $a_0 \Rightarrow b_1$ and $b_0 \Rightarrow a_1$ are vertices and both have as label the string $l_1 \ldots l_k$ of literals in $\mathbb{L}$abel;*
- *If a is an isolated line of $\{S_{l_1}, \ldots, S_{l_s}\}$, then $a_0 \Rightarrow \bot$ and $\bot \Rightarrow a_1$ are edges in $\mathbb{C}$lndr-graph both with label given by the string $l_1 \ldots l_s$*
- *For all conjugated, a and $\neg a$ in $\mathsf{Letter}(\Psi)$, $a_1 ::: \neg a_0$ and $\neg a_1 ::: a_0$.*

Definition 2.8 *Given two strings of labels, l_1 and l_2, we say that both strings are equivalent iff there is a set of labels $\{r_1, \ldots, r_n\}$ so that both l_1 and l_2 are a concatenation of* all *elements of the given set.*

From now on, labels are written as its quotient by the above defined equivalence class.

Definition 2.9 *Given a subdigraph $\mathtt{B}$ of the cylindrical digraph and two vertices a_0 and b_1,*

1. *An alternated sequence from a_0 to b_1 is a linear subdigraph of the form*

$$a_0 \Rightarrow c_1^1 ::: \neg c_0^1 \ldots c_1^k ::: \neg c_0^k \Rightarrow b_1$$

2. *An interval between a_0 and b_1 is the subdigraph of $\mathtt{B}$ that contains all alternated sequences from a_0 to b_1. Denote this interval by $[a_0, b_1]$;*
3. *A* maximal linear sequence *is an alternated sequence*

$$a_0 \overset{l_1}{\Rightarrow} b_1^1 ::: \neg b_0^1 \overset{l_2}{\Rightarrow} b_1^2 ::: \neg b_0^2 \ldots b_1^2 ::: \neg b_0^m \overset{l_m}{\Rightarrow} c_1$$

   maximal for the property: $\forall 1 \leq i \leq m(\neg b_0^i \overset{l}{\Rightarrow} d_1) \rightarrow (b_1^{i+1} = d_1)$;
4. *Given an interval $[a_0, b_1]$ a* sequence in the interval $[a_0, b_1]$ *is any alternated sequence with starting point a_0 and ending b_1.*

Lemma 2.10 *The number of lines of a partitioned formula Ψ is bounded by the* square *of its set of atoms.*

Proof: The number of literals Ψ, $|\mathsf{Letter}(\Psi)|$, establishes the number maximum of edges in $\Leftrightarrow$. This bound, given by $|\mathsf{Letter}(\Psi)|^2$, is the maximum of edges one can draw in a polygon with $|\mathsf{Letter}(\Psi)|$ arrows and it is given by the sum of an arithmetic progression

$$(|\mathsf{Letter}(\Psi)|) + (|\mathsf{Letter}(\Psi)| - 1) + \cdots + 1$$

□

Now, consider operations over subdigraphs of the cylindrical digraph. Two subdigraphs $G_1 = (V_1, \Rightarrow, :::)$ and $G_2 = (V_2, \Rightarrow, :::)$ can be turned into two *disjoint* digraphs by, say, by taking the natural bijective mapping of V_i onto $V_i \times i$, $i \in \{1, 2\}$ and a set of edges, $\Rightarrow$ and $:::$ inherited from the similar edges of G_i. For shortness, we write vertex of each G_i by using its first coordinate. In other words, a vertex $(a_j.i)$, $i \in \{1,2\}$, $j \in \{0,1\}$ is written as a_j. Edges have no especial distinction on its notation. The context of writing the disjoint subdigraphs is very clear.

Definition 2.11 *Let $G_1 = (V_1, \Rightarrow, :::)$ and $G_2 = (V_2, \Rightarrow, :::)$ be two disjoint digraphs. Let $a_1, a_0 \in V_1$ and $b_1, \neg a_0 \in V_2$ and l a string of labels.*

1. *Define the disjoint join of the two digraphs by a_1 and $\neg a_0$ as the disjoint union of the digraphs G_1 and G_2 plus a new edge $a_1 ::: \neg a_0$.*

   Denote the join of G_1 and G_2 by $G_1 a_1 ::: \neg a_0 G_2$.

2. *Define the disjoint join with label l of G_1 and G_2 by $a_0 \in V_1$ and $b_1 \in V_2$ as the disjoint union of G_1 and G_2 with all the edges inherited form each digraph and the addition of the labeled edge $a_0 \overset{l}{\Rightarrow} b_1$.*

   Denote the join of G_1 and G_2 by $a_0 \in V_1$ and $b_1 \in V_2$ with label l by $G_1 a_0 \overset{l}{\Rightarrow} b_1 G_2$.

Definition 2.12 *Given two strings of labels l and m, the operation $l \setminus m$ is the string of labels formed by all labels in l that do not belong to m.*

Definition 2.13 *Given a subdigraph $\mathcal{SB}$ of a cylindrical digraph,*

1. *If m is a string of labels, define the subdigraph $\mathcal{SB} \setminus m$ as the subdigraph of $\mathcal{SB}$ whose vertices and edges are inherited by the vertices and edges of $\mathcal{SB}$ and moreover, for any arrow with label l, $a_0 \overset{l}{\Rightarrow} b_1$, we have either,*

   $$a_0 \overset{l \setminus m}{\Rightarrow} b_1 \text{ if } l \setminus m \neq \emptyset$$

   or the edge erased, if $l \subseteq m$;

2. *If $a^1_{i_1}, a^2_{i_2}, \ldots, a^k_{i_k}$ are vertex for $i_j \in \{0,1\}$, $1 \leq j \leq k$, denote by $\mathcal{SB}VL(a^1_{i_1}, a^2_{i_2}, \ldots, a^k_{i_k})$ to the digraph generated by $\mathcal{SB}$ whose set of vertices is the set of vertices of $\mathcal{SB}$ less the vertices $\{a^1_{i_1}, a^2_{i_2}, \ldots, a^k_{i_k}\}$.*

3 Algorithm I - Building the Macro Tableau

Given a formula Ψ as an input, the output is either *Ψ is non-valid* or *Ψ is valid.* From now on, we suppose Ψ is *always* written in any of its partitioned form.

We introduce an algorithm that is based on **KE**-tableau methods. We work backwards with regard to the usual tableaux operations and, after fixing one set of labels, $\mathbb{L}abel$, we apply Bivalence Rule, we develop a quick search for the set of closed branches. If it can be done, we obtain a closed **KE**-tableau for Ψ.

We start our algorithm of writing a macro for the search for closed branches in the mirror tableau. The basic idea is to look for contradictions over alternated paths in the cylindrical digraph.

Definition 3.1 *Given an set of strings of labels,*

$$\{l_1, l_2, \ldots, l_k\}$$

its associated linear macrobranch is an array of the form

$$\begin{array}{ll} F & l_1 \\ F & l_2 \\ \vdots & \\ F & l_k \end{array}$$

where $F\ l_1$ encodes the choice of a literal r_1 in the string l_1 and its labeling as F. Similarly, $F\ l_2$, a choice of a literal r_2 in the string l_2 labeled as F. Successfully, $F\ l_k$, a choice of a literal r_k in l_k, labeled F.

A choice of consistent literals $r_i \in l_i$, $1 \leq i \leq k$, defines the possible branch

$$\begin{array}{ll} F & r_1 \\ F & r_2 \\ \vdots & \\ F & r_k \end{array}$$

A possible branch is a subbranch of the mirror tableaux. We write a macrotableau where we write all the subbranches in the mirror tableau that are closed.

Definition 3.2 *Given an alternated sequence*

$$a_0 \overset{l_1}{\Rightarrow} b_1^1 ::: \neg b_0^1 \overset{l_2}{\Rightarrow} b_1^2 ::: \neg b_0^2 \ldots b_1^k ::: \neg b_0^k \overset{l_k}{\Rightarrow} c_1$$

consider the set of strings of labels associated to each edge $r_0 \overset{l_i}{\Rightarrow} s_1$ in the alternated sequence, $\{l_1, l_2, \ldots, l_k\}$.

A linear macrobranch generated by the given sequence is the linear macrobranch associated to $\{l_1, l_2, \ldots, l_k\}$.

Notice that a possible branch is a subbranch of the mirror tableau.

Definition 3.3 *A linear macrobranch is a macrobranch. Inductively, we built macrobranches by using the operations* $\bigwedge$ *and* $\bigvee$, *as we define here.*

Given two macrobranches B_1 *and* B_2,

1. *The conjunction of* B_1 *and* B_2, *denoted* $B_1 \bigwedge B_2$ *is the macrobranch*
   B_1
   B_2
   obtained by the join of the macrobranches B_1 *and* B_2 *is a macrobranch and its meaning is the use of* $\bigwedge$*-rule over all possibles branches of* B_1 *with all possible branches of* B_2.

   A possible branch in $B_1 \bigwedge B_2$ *is the choice of a branch* l_1 *in* B_1, *a branch* l_2 *in* B_2 *to form the branch* $\begin{matrix} l_1 \\ l_2 \end{matrix}$

2. *The disjunction of* B_1 *and* B_2, *denoted by* $B_1 \bigvee B_2$ *is the macrobranch*

   $$B_1 \qquad B_2$$

   and its meaning is the use of $\bigvee$*-rule over all possible branches of* B_1 *and* B_2.

Definition 3.4 *Consider the linear macrobranches* $\{B_1, \ldots, B_n\}$, $n \geq 1$.

1. $\bigwedge_{1 \leq j \leq n} B_j$ *is the macrobranch* $B_1 \bigwedge \cdots \bigwedge B_n$. *An analogous notation is given to* $\bigvee_{1 \leq j \leq n} B_j$.

   $B_1 = \bigwedge_{i=1} B_i = \bigvee_{i=1} B_i$. *Parenthesis commmutates in both* $\bigwedge$ *and* $\bigvee$ *and are omitted when its meaning is clear.*

   Recursively, a macrobranch is obtained from operations $\bigvee$ *and* $\bigwedge$ *over macrobranches. We call each of its macrobranches we use to make the building up of a more complex macrobranch a* submacrobranch. *With abuse of notation we use the notation* $\in$ *to denote the binary relation of being a submacrobranch of a given macrobranch;*

2. *Define the partial order over the linear submacrobranches in a macrobranch* B *as the transitive closure of*

   $$\forall B_i \in \{B_1, \ldots, B_n\} \forall B_j \in \{B_{n+1}, \ldots, B_m\}$$
   $$(((\textstyle\bigvee_{1 \leq j \leq n} B_i) \bigwedge (\bigvee_{n+1 \leq j \leq m} B_j) \in B) \rightarrow B_k \leq B_l)$$

3. *Let $\{B_1, B_2, \dots, B_s\}$ be linear macrobranches so that $C = B_1 < B_2 < \dots < B_s$ is a maximal chain of a macrobranch B. A possible branch over C is a choice of a possible branch over $\bigwedge_{1\leq i\leq s} B_i$.*

   A possible branch in B is a possible branch in some chain of B;

4. *Given a possible branch*
   $F\ l_1$
   $\vdots$
   $F\ l_1$
   each $F\ \ l_i$ is called a (labeled) node.

Macrotableaux are a way to write a tableau from the hypothesis we have nodes of the form $F(r_1 \wedge r_2 \wedge \dots \wedge r_n)$ in which r_i ranges over a given string of labels. So, all the properties of tableaux are translated and rewritten in here. The association of macrobranches to intervals in a cylindrical digraph is our way to make shortcuts on solving **3-sat**.

We translate intervals in a cylindrical digraph to macrobranch. A maximal sequence

$$a_0 \overset{l_1}{\Rightarrow} b_1^1 ::: \neg b_0^1 \overset{l_2}{\Rightarrow} b_1^2 ::: \neg b_0^2 \dots b_1^2 ::: \neg b_0^m \overset{l_m}{\Rightarrow} c_1$$

has a forking in at least two paths, say the maximal sequences for $1 \leq i \leq n$ and $n \geq 2$,

$$\neg c_0 \overset{l_{1i}}{\Rightarrow} d_1^{1i} ::: \neg d_0^{1i} \overset{l_{2i}}{\Rightarrow} d_1^{2i} ::: \neg d_0^{2i} \dots d_1^{2i} ::: \neg d_0^{mi} \overset{l_{mi}}{\Rightarrow} e_1^i$$

If the below sequences are associated, respectively, to the macrobranch B and B_i for $1 \leq i \leq n$, then we have the macrobranches $B \bigwedge (\bigvee_{1\leq i\leq n} B_i)$:

$$\begin{array}{ccc} & B & \\ \swarrow & & \searrow \\ B_1 & \cdots\cdots & B_n \end{array}$$

In general, we associate macrobranches to intervals.

Definition 3.5 *Given an interval $[a_0, b_1]$, the macrobranch associated to $[a_0, b_1]$ is defined by the macrobranch B so that*

- *For all maximal sequence M in $[a_0, b_1]$, the linear macrobranch B_M generated by M is a member of B;*

- *If a maximal sequence M has the forking with t maximal sequences whose associated linear macrobranches are M_1, ... M_t. Then the macrobranch built by the $\bigvee$-rule,*

$$M \bigwedge (\bigvee_{1 \leq j \leq t} M_j)$$

is a submacrobranch of B.

Definition 3.6 *Given a macrobranch B, define its* transposed, *B^t as*

1. *The transposed of a node $F\ r_1 r_2 \ldots r_n$ in a linear macrobranch is the linear macrobranch l^t*
   $F\ \neg r_1$
   $F\ \neg r_2$
   $\vdots$
   $F\ \neg r_n$

2. *The transposed a linear macrobranch with n nodes l_1,...l_n is $\bigvee_{1 \leq i \leq n} l_i^t$;*

3. *$(B_1 \bigvee B_2)^t = B^t \bigwedge B_2^t$ and $(B_1 \bigwedge B_2)^t = B_1^t \bigvee B_2^t$.*

Notice that if B is a linear macrobranch whose nodes are the labeled strings $F\ l_i$, $1 \leq i \leq n$, for some number n, its transposed is

$$\bigvee_{1 \leq i \leq n} F\ (l_i)^t$$

where if l_i is the string $r_{i1} \ldots r_{in_i}$, then its transposed is
$F \neg r_{i1}$
$\vdots$
$F \neg r_{in_i}$

Definition 3.7 *Given a chain $C_1, \ldots, C_n$ in a macrotableau B, its transposed, $\bigvee_{1 \leq i \leq n} C_i^t$ is called an* antichain.

Definition 3.8 *A possible branch of B*
$F\ r_1$
$\vdots$
$F\ r_n$
is closed iff

$F\ r_1$
$\vdots$
$F\ r_n$
$T\ S_{r_1}$
$\vdots$
$T\ S_{r_n}$
is a closed subbranch of the mirror tableau.

A macrobranch is closed iff any of its possible branches are closed.

Definition 3.9 *Any literal a so that the interval $[a_0, a_1]$ is a non-empty digraph is called a* necessarily true *literal.*

The set of necessarily true literal is denoted by Nec.

A necessarily true literal a ought be labeled as *true* regarding to any of its linear macrobranches generated by alternated sequences in the interval $[a_0, a_1]$,

$$a_0 \overset{l_1}{\Rightarrow} b_1^1 ::: \neg b_0^1 \ldots \neg b_0^n \overset{l_n}{\Rightarrow} a_1$$

If otherwise, labeling a with *false* will result in a contradiction. Indeed, if the sequence is $a_0 \overset{l}{\Rightarrow} \perp \overset{l}{\Rightarrow} a_1$, a is an isolated branch in any label r in the string l. So the branches of the mirror tableau for Ψ with the nodes $F\ \ r$ and $F\ \ a$ are closed. If the vertex $\perp$ does not belong to the alternated sequence, we have that the same reasoning holds for branches of a tableau for Ψ with $F\ \ r_1, \ldots, F\ \ r_n$ and $F\ \ a$, for any r_i in the string of labels l_i, $1 \leq i \leq n$. Indeed, a label $F\ \ a$ implies $T\ \ b^1$, $F\ \ \neg b^1$ and so on until we obtain $F\ \ \neg b^n$ and, therefore, $T\ \ a$, a contradiction, whereas $T\ \ a$ does not leads to a contradiction over the sequence at all.

We keep this reasoning as a building block for writing and searching for contradictions over macrobranches.

Definition 3.10 *A literal $r \in \mathbb{L}abel \cap (\cup Letter(S_{ij}))$ is called* necessarily false literal. *The set of necessarily false literals is denoted by* $NecFls$.

A necessarily false literal r ought be labeled as $F\ \ r$.

Consider the three non-empty intervals,

$$[\neg a_0, \neg c_1] \quad [r_0, s_1] \quad [\neg a_0, r_1]$$

where $a, c \in$ Nec and $r, s \in NecFls$.

The literals a, c and r, s are labeled, respectively, as **True** and **False** over their macrobranches, respectively, $B_{\neg a_0 \neg c_1}$, $B_{r_0 s_1}$ and $B_{\neg a_0 r_1}$.

In this way, if the macrobranches associated to $[a_0, a_1]$ and $[c_0, c_1]$ are, respectively $B_{a_0a_1}$, $B_{c_0c_1}$. We will show, in Proposition 3.13, that three macrobranches

$$\begin{matrix} B_{a_0a_1} & & \\ B_{\neg a_0 \neg c_1} & B_{r_0 s_1} & \begin{matrix} B_{a_0a_1} \\ B_{\neg a_0 r_1} \end{matrix} \\ B_{c_0c_1} & & \end{matrix}$$

are closed, or, to be more precise, the above defined macroberanches are closed.

Definition 3.11 *Given a cylindrical digraph, its set of* **closed digraphs**, $\mathcal{CSD}$ *is defined as*

1. *For all* $a \in \mathsf{Nec} \cap \mathsf{Nec}Fls$, *the digraph,*

$$([a_0, a_1] \setminus (\neg a))a_0 \overset{a}{\Rightarrow} \perp\{\perp\}$$

where $\{\perp\}$ *is the digraph with only one vertex, the vertex* $\perp$;

2. *For all* $a, b \in \mathsf{Nec}$, *if* $[\neg a_0, \neg b_1]\mathsf{V}L(a_0, a_1, \neg a_1) \neq \emptyset$, *consider*

$$([a_0, a_1]a_1 ::: \neg a_0([\neg a_0, \neg b_1]\mathsf{V}L(a_0, a_1, \neg a_1))\neg b_1 ::: b_0[b_0, b_1]) \setminus (ab)$$

3. *For all* $a \in \mathsf{Nec}$, $b \in \mathsf{Nec}Fls$, *if* $[\neg a_0, b_1]\mathsf{V}L(a_0, a_1, \neg a_1) \neq \emptyset$, *consider*

$$([a_0, a_1]a_1 ::: \neg a_0([\neg a_0, b_1]\mathsf{V}L(a_0, a_1, \neg a_1)) \setminus (a\neg b)$$

4. *For all* $a, b \in \mathsf{Nec}Fls$, *if* $[a_0, b_1] \neq \emptyset$, *consider*

$$[a_0, b_1] \setminus (\neg a \neg b)$$

Definition 3.12 *Given a set of closed digraphs,* $\mathcal{CSD}$, *its associated set of* possible closed macrobranches, $\mathcal{A}$, *is the set of macrobranches generated by each digraph in* $\mathcal{CSD}$.

The macrotableau for Ψ *is* $\bigvee\{B | B \in \mathcal{A}\}$.

Proposition 3.13 *If* B *is a macrobranch in* $\mathcal{A}$, *then* B *is closed.*

Proof: Case 1) $a \in \mathsf{Nec}Fls \cap \mathsf{Nec}$. For any sequence in the interval $[a_0, a_1]$,

$$a_0 \overset{s_1}{\Rightarrow} d_1^1 ::: \neg d_0^1 \ldots \neg d_0^m \overset{s_n}{\Rightarrow} a_1$$

associated to a linear macrobranch Lin, for any possible branch in lin, with nodes in the set $\{r_1, \ldots, r_n\}$, we have the branch for a tableau

$T\ \Psi$
$F\ r_1$
$\vdots$
$F\ r_n$
$T\ a \vee d^1$
$T\ \neg d^1 \vee d^2$
$\vdots$
$T\ \neg d^n \vee a$

that can be developed, by the use of Bivalence rule, as

$$
\begin{array}{ccc}
 & T\ \Psi & \\
 & F\ r_1 & \\
 & \vdots & \\
T\ a \ \text{---} & F\ r_n & \text{---}\ T\ \neg a \\
\textit{open} & & T\ d^1 \\
 & & T\ d^2 \\
 & & \vdots \\
 & & T\ a \\
 & & \textit{closed}
\end{array}
$$

Suppose $\neg a \notin \{r_1, \ldots, r_n\}$. Add the node $F\ a$, we obtain the closed branch

$$
\begin{array}{ccc}
 & T\ \Psi & \\
 & F\ r_1 & \\
 & \vdots & \\
 & F\ r_n & \\
T\ a \ \text{---} & F\ a & \text{---}\ T\ \neg a \\
\textit{closed} & & T\ d^1 \\
 & & T\ d^2 \\
 & & \vdots \\
 & & T\ a \\
 & & \textit{closed}
\end{array}
$$

Hence, the subdigraph

$$([a_0, a_1] \setminus (\neg a))a_0 \overset{a}{\Rightarrow} \bot\{\bot\}$$

has its generated macrotableau closed.

Case 2): The literals a and b are necessarily true and case 1), at least one of them is necessarily false, does not happen.

The intervals $[a_0, a_1]$ and $[b_0, b_1]$ generate two macrobranches, B_a and B_b that close if we add to them, respectively, the nodes $F\ a$ and $F\ b$.

Let $B_{\neg a \neg b}$ be the macrobranch generated by the interval $[\neg a_0, \neg b_1]$. If we add the the macrobranch $B_{\neg a \neg b}$ the two nodes $F \neg a$ and $F \neg b$, then the macrobranch closes.

In conclusion, the macrobranch $B_a \bigwedge B_b \bigwedge B_{\neg a \neg b}$ is closed.

The hypothesis of a or b being necessarily false was ruled out. So, we conclude that the macrotableau generated by

$$[a_0, a_1]a_1 ::: \neg a_0([\neg a_0, \neg b_1]\mathrm{V}L(a_0, a_1, \neg a_1))\neg b_1 ::: b_0[b_0, b_1]$$

closes under all possible nodes $F\ a$ or $F\ b$ or ($F \neg a$ and $F \neg b$).

Case 3) If a is necessarily true, the macrobranch generated by $[a_0, a_1]$ closes with the addition of the node $F\ a$.

The macrobranch generated by $[\neg a_0, b_1]$ closes added to the nodes $F \neg a$ and Tb. Hence, if b is necessarily false, the macrobranch generated by $[a_0, a_1]a_1 ::: \neg a_0([\neg a_0, b_1]\mathrm{V}L(a_0, a_1, \neg a_1)$ closes under the nodes $F\ a$ or $T\ b$ or ($T\ a$ and $T\ b$). We cannot suppose $\{a \neg b\} \subseteq \mathsf{Nec}Fls$. So, the macrobranch generated by

$$([a_0, a_1]a_1 ::: \neg a_0([\neg a_0, b_1]\mathrm{V}L(a_0, a_1, \neg a_1)) \setminus (a \neg b)$$

is closed.

Case 4) Consider the interval $[a_0, b_1]$. Any alternated sequence in this interval generates a linear macrobranch so that the node $F\ a$ forces $T\ b$, a contradiction if we assume that a and b are nodes of the mirror tableau.

Discharge the labels $\neg a$ and $\neg b$, for we deal with the hypothesis that $F\ a$ and $F\ b$ are nodes in the mirror tableau, which is the tableau obtained by using Branching Rule.

Conclude that macrobranch generated by

$$[a_0, b_1] \setminus (\neg a \neg b)$$

is closed. □

Proposition 3.14 *The set $\mathcal{CSD}$ has its size bounded by a polynomial.*

Proof: Observe that one can easily write in polynomial time and size the set of non-empty intervals $[a_0, b_1]$, for $(a, b) \in \mathsf{Letter}(\Psi) \times \mathsf{Letter}(\Psi)$. □

The reciprocal of Theorem 3.13 is shown next.

Theorem 3.15 *For any closed branch Br in the mirror tableau there is a macrobranch B so that a possible branch l of B is a subbranch of Br.*

Proof: Let Br be a branch for the mirror tableau for $T\ \Psi$ so that none of its subbranches is a possible branch in some macrotableau in $\mathcal{A}$.

$F\ l_1$
$\vdots$
$F\ l_t$
$T\ S_{l_1}$
$\vdots$
$T\ S_{l_t}$

that is, for no subdibraph S in $\mathcal{CSD}$ (see Definition 3.11), there is a possible branch generated by S that is a subbranch of Br.

Our search is restricted to CIL, the digraph given by the cylindrical digraph without labels ranging over the literals complementary to each l_i, $1 \leq i \leq t$. $\mathbb{C}lndr \setminus (\neg l_1, \ldots, \neg l_t)$. On making such a restriction, we avoid writing nodes of the fom $F\ p$ for any $p \in \{\neg l_1, \ldots, \neg l_t\}$.

Consider the set NT of the necessarily true literals in CIL and NF the necessarily false literals among the set $\{l_1, \ldots, l_t\}$. By hypothesis, $NT \cap NF = \emptyset$, for otherwise, Case 1 of Definition 3.11 holds.

If NT in non-empty, for any $a \in NT$, extend Br by adding all the branches of the form

$F\ \neg a$
$T\ \neg a \vee r_1$
$T\ \neg r_1 \vee r_2$ where $\neg a \vee r_1$ and for all $1 \leq i \leq j$, $\neg r_1 \vee r_2$ belong to some
$\vdots$
$T\ \neg r_{j_1 - 1} \vee r_j$
S_i.

The above branch is equivalent to the branch

$F\ \neg a$
$T\ r_1$
$T\ r_2$
$\vdots$
$T\ r_j$

and by our hypothesis, cases 2 and 3 of Definition 3.11 were ruled out, so, $\{r_1, r_2, \ldots, r_j\} \cap NF = \emptyset$ (Case 3 of the same Definition does not happen). Also, $\{\neg r_1 . \neg r_2, \ldots, \neg r_j\} \cap NT = \emptyset$ (Case 2, does not happen). Therefore, we conclude that subbranches of the above form remain open.

If NF is non-empty, for any $b \in NF$, consider branches of the form

$T\ b \vee r_1$
$T\ \neg r_1 \vee r_2$
$\vdots$
$T\ \neg r_{j_1 - 1} \vee r_{j_1}$

Take the branches maximal in the sense that no new node of the form $T\ \neg r_j \vee r_{j+1}$ can be added to it

As $b \in NF$, the node $F\ b$ belongs to Br. So, the above branch is equivalent to

$T\ r_1$
$T\ r_2$
$\vdots$
$T\ r_{j_1}$

As cases 3 and 4 of Definition 3.11 were ruled out, none of the literals in the above branch are in NF or a conjugated pair of a literal in NT and therefore no literal labeled as T and F occurs. The branch is left open.

It remains to analyze the branches with no literals in NF or NT in it. Consider maximal branches of the form

$T\ \neg s_1 \vee s_2$
$T\ \neg s_2 \vee s_3$
$\vdots$
$T\ \neg s_{j-1} \vee s_j$

The above branch is developed, using Branching Rule as

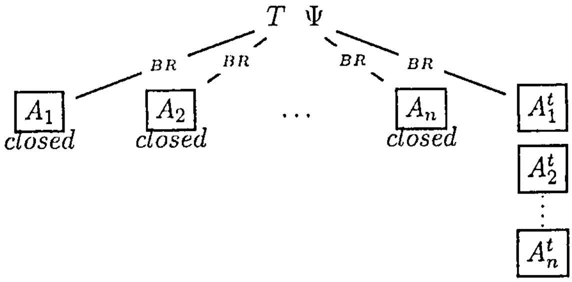
$$T\ \neg s_1 \qquad \begin{array}{l} T\ s_1 \\ T\ s_2 \\ \vdots \\ T\ s_{j_1} \end{array}$$

The second branch remains open. Indeed, no $\neg s_i$ belongs to NT and no s_i belongs to NF. □

4 Algorithm II - Polynomial Resolution

Now, we deal with a decision procedure in order to decide if a given set of possible closed branches indeed entails the mirror tableau, and therefore, the tableau closes and the given **3-sat** formula Ψ is not valid or the tableau does not close, so Ψ is valid. That is, we develop the tools to polynomially decide if for any branch of the mirror tableau, there is a possible branch in $\mathcal{A}$ that is a subbranch of it (so Ψ is non-valid) or not (Ψ is valid).

Our general strategy for solving the tableaux is the following: Given a set of possible closed branches $\mathcal{A}$ for a formula Ψ, then we solve the tableau for Ψ by Bivalence rule over a set $A \in \mathcal{A}$, which we recall, entails all closed branches. In a schema,



The last branch $(\bigvee_{1\leq i\leq n} A_i)^t$, was built up due to a branching rule, generalized over all possible branches. All other possible branches in $\mathcal{A}$ are *closed* and we describe here a procedure in order to decide whether any possible branch in any macrobranch of $\mathcal{A}$ represents a closed tableau, that is, any branch of the mirror tableau contains a possible branch of some A_i. Call resolution to the process of deciding whether branches in the mirror tableau are a possible line in $\mathcal{A}$ or not.

Recall that the set of all nodes in a possible branch is a consistent set of literlas in $\mathbb{L}abel$.

Definition 4.1 *A macrobranch B is inconsistent iff there is no possible line in B.*

In a rephrasing, a macrobranch B is inconsistent iff for any attempt of writing a possible branch we can find two nodes with complementary literals in it.

Proposition 4.2 *Let $\mathcal{A} = \bigvee_{1\leq i\leq n} A_i$ be a set of possible closed branches. A branch Br is a possible branch of $\bigwedge_{1\leq i\leq n} A_i^t$ iff Br it is not a possible branch in any macrotableau $A \in \mathcal{A}$.*

Proof: Let C be the set of all chains in $\bigwedge_{1\leq i\leq n} A_i^t$. For all $\{C_1, \ldots, C_m\} \in C$ we have that $\{C_1^t, \ldots, C_m^t\}$ is an antichain in $\bigvee A_i$. That is, for all possible branch in $\bigvee_{1\leq i\leq n} A_i$, there is a node whose literals in its the string of labels are complementary of the literals in the possible branch.

So, for all $A \in \mathcal{A}$ and any chain $\{B_1, \ldots, B_p\}$ in A has non empty intersection with $\{C_1^t, \ldots, C_m^t\}$. Suppose that the linear branch $\bigwedge_{1\leq i\leq m} C_i$ is of the form

$$l_1$$
$$\vdots$$
$$l_m$$

then for all s, there is a node in B_s substring of $\neg l_1 \vdots \neg l_m$. ◻

In other words, $\bigwedge A_i^t$ is consistent iff there is a branch of the mirror tableau that is not a possible branch in $\mathcal{A}$. So, the tableau generated by $\mathcal{A}$ closes iff there is no possible branch in $\bigwedge A_i^t$.

Definition 4.3 *Given two strings of labels l_1, l_2,*

$$l_1 = r_{11}r_{21}\dots r_{n_1 1}$$
$$l_2 = r_{12}r_{22}\dots r_{n_2 2}$$

we say that l_1 and l_2 are compatible if no two pairs of conjugated letters belong to $\{r_{11}, r_{21}, \dots r_{n_1}, r_{12}, r_{22}, \dots, r_{n_2 2}\}$

Notice that a string of the form $a\neg ab_1 \dots b_n$ is not compatible with any other string.

Recall that lines belong to macrobranches and macrobranches are numbered and ordered according to a partial order in $\mathcal{A}$.

A line l belong to the set $A_1, A_2, \dots, A_l$ and its position is $j_1, j_2, \dots, j_l$.

Definition 4.4 *We say that the set of possible closed branches $\mathcal{A}$* represents the mirror tableau *iff for any maximal consistent set $\{l_1, \dots, l_n\}$, there is a possible line in $\mathcal{A}$ that is a subbranch of the branch of the mirror tableau whose nodes are $F\ l_i$, for all $1 \le i \le n$.*

If $\mathcal{A}$ represents the mirror tableau, then the mirror tableau for Ψ (that is any tableau for Ψ) is closed.

Corollary 4.5 *Given a set $\mathcal{A} = \{B_1, \dots, B_n\}$, the following assertions are equivalent:*

1. *$\mathcal{A}$ represents the mirror tableau;*
2. *$\bigwedge_{1 \le i \le n} B_i^t$ is inconsistent;*
3. *For all antichain $Ant = \{C_1, \dots, C_m\}$ in $\mathcal{A}$, for all $1 \le i \le m$ in Ant and strings of labels l_{ik_i} in C_i, there is a pair (i, j) so that the strings l_{ik_i} and l_{jk_j} are incompatible.*

Proof: Equivalences of 1 and 2 follows from Proposition 4.2.

2 iff 3 follows from $\bigwedge_{1 \le i \le n} B_i^t$ is inconsistent iff for any chain $\{C_1, \dots, C_p\}$ there is no consistent line in $\bigwedge_{1 \le i \le p} C_i$. That is for all possible lines l_{ik_i} in each C_i, there is a pair (i, j) so that a node in l_{ik_i} and a node inl_{jk_j} form a conjugated pair.

Equivalently, for all antichain $\{C_1^t, \dots, C_p^t\}$ and strings l_{ik_i} in each C_i^t, there is a pair (i, j) so that the strings l_{ik_i} and l_{jk_j} are non-compatible. ◻

Theorem 4.6 *There is a polynomial algorithm for deciding a set $\mathcal{A}$,*

Proof: Let $\{B_1, B_2, \ldots, B_m\}$ be the set of all linear macrobranches in $\mathcal{A}$. Suppose that the label F was dropped in each node and we consider only strings of literals in each node. For all $1 \leq i \leq m$, let $l_{i1}, \ldots, l_{in_i}$ be an enumeration of each node in B_i.

For all l_{ik} and $l_{i'k'}$, if

1. l_{ik} and $l_{i'k'}$ are compatible or
2. l_{ik} and $l_{i'k'}$ are incompatible and the are comparable linear macrobranches in, say, the macrobranches $A_1, \ldots, A_p \in \mathcal{A}$;

then we write all sequences of the form

$$l_{1k_1}, l_{2k_2}, \ldots, l_{mk_m}$$

two-by-two compatible or in comparable positions in the macrobranches $A_1, \ldots, A_q \in \mathcal{A}$, we choose B_i the minimum position where macrobranches are comparable and B_2, ..., B_s where the linear macrobranch is not comparable.

Notice that this is a antichain and $\bigwedge_{1k_1 \leq sk_s \leq mk_m} l^t_{sk_s}$ is a possible branch of $\bigwedge\{A^t | A \in \mathcal{A}\}$. ◻

5 Conclusion

We described in this paper two algorithms for solving **3-sat**. One of them describe how to obtain macrotableau for **3-sat** formulas. The second algorithm deal with the resolution of such macrotableaux.

Regarding to the first algorithm, we have to write subdigraphs of the cylindrical digraph. An straightforward observation is the following Theorem:

Theorem 5.1 *Writing the intervals for building the set $\mathcal{A}$ is polynomial in space and time.*

The algorithm of solving the macrotableau involves deciding if strings of labels do have or do not have a non compatible associated string.

Searching for non compatible strings of labels has order 2 regarding to the set of strings and, therefore, order 4 regarding to the set of literals in Ψ. So, we have the following Theorem:

Theorem 5.2 *Solving and deciding a given macrotableau is polynomial in time and space.*

6 Examples

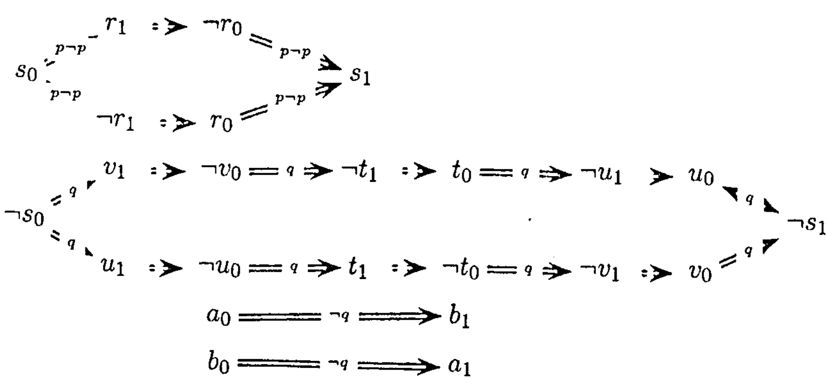
Example 6.1 *Consider the formula*

$$\Psi \equiv (p \vee r \vee s) \wedge (\neg p \vee r \vee s) \wedge (p \vee \neg r \vee s) \wedge (\neg p \vee \neg r \vee s) \wedge$$
$$(q \vee \neg r \vee u) \wedge (q \vee \neg r \vee v) \wedge (q \vee \neg u \vee t) \wedge (q \vee \neg v \vee t)$$
$$(\neg q \vee a \vee b)$$

and the following partition:

p	$r \vee s$ $\neg r \vee s$	$\neg p$	$r \vee s$ $\neg r \vee s$
q	$\neg s \vee u$ $\neg s \vee v$ $\neg u \vee t$ $\neg v \vee \neg t$	$\neg q$	$a \vee b$

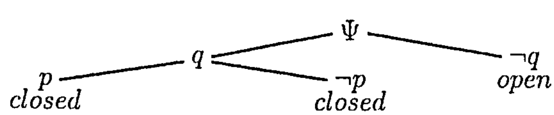
The cylindrical digraph is given by:



A simple computation show us that the set of conjugated literals $\{s, \neg s\}$ is the set of necessarily true literals. We skip the straightforward computation of the set of necessarily true and $\mathcal{A}$. The set $\mathcal{A}$ is given by

$$\frac{p, \neg p}{q}$$

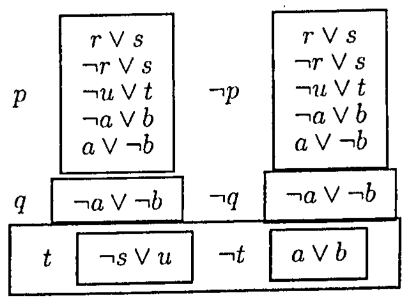
We have that the below set is not complete.
The tableau for Ψ, shown below, is open.



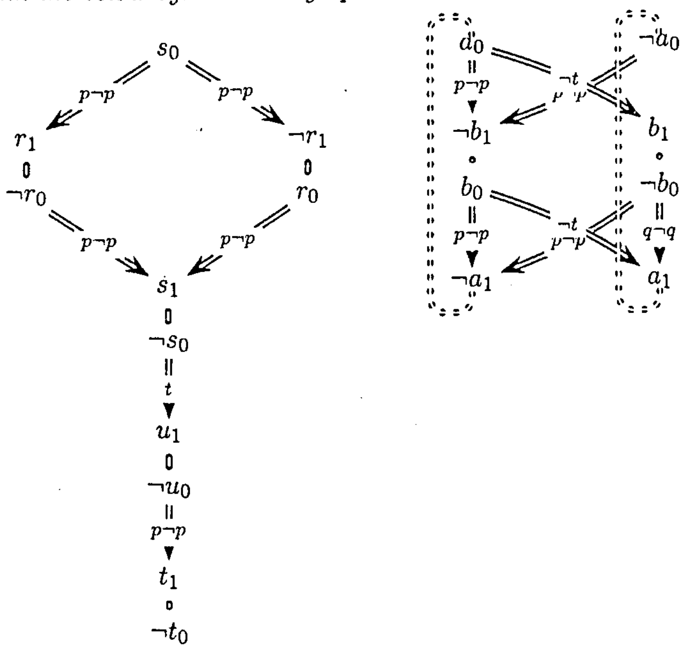
Example 6.2 *Consider the formula*

$$(p \vee r \vee s) \wedge (\neg p \vee r \vee s) \wedge (p \vee \neg r \vee s) \wedge (\neg p \vee \neg r \vee s) \wedge$$
$$(p \vee \neg u \vee t) \wedge (\neg p \vee \neg u \vee t) \wedge (p \vee \neg a \vee b) \wedge (\neg p \vee \neg a \vee b) \wedge$$
$$(p \vee a \vee \neg b) \wedge (\neg p \vee a \vee \neg b) \wedge (q \vee \neg a \vee \neg b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge$$
$$(\neg s \vee t \vee u) \wedge (\neg t \vee a \vee b)$$

Partitioned as



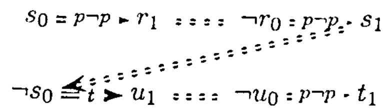
and obtain the below cylindrical digraph:



We have $\mathsf{Nec} = \{s, a, \neg a, b, \neg b\}$. *the closed digraph*

$$([s_0, s_1]s_1 ::: \neg s_0([\neg s_0, t_1] \vee L(s_0, s_1, \neg s_1)) \setminus (s \neg t)$$

after reduction over $[s_0, s_1]$ *is given by*



As s *is necessarily true, the vertices* u_1, t_1 *are labeled as* **true** *and the literal* t *belongs to* NecFls. *In other words,* F t *implies* T t. *So, the below chart belongs to* $\mathcal{A}$*:*

$$\frac{p, \neg p}{\frac{p, \neg p}{\frac{t}{p, \neg p}}}$$

The below line says "under true t*, we have false* t*". After reduction, we have the macrobranch:* $\dfrac{t}{p, \neg p}$

The other macrobranch in $\mathcal{A}$ *was built on analyzing the necessarily true literals* $a, \neg a$ *and* $b, \neg b$*:*

$$\frac{p, \neg p}{\frac{q, \neg q}{\frac{p, \neg p}{\neg t}}}$$

the below macrobranch is reduced to: $\dfrac{p, \neg p}{\dfrac{q, \neg q}{\neg t}}$

Both macrobranches have non compatible lines. Lines $p, \neg p$ *and* $q, \neg q$ *are non compatible in itself, so, in any antichain. Lines* t *and* $\neg t$ *are non compatible lines and belong to distinct macroarrays.*

References

[1] Cook, S.A. The complexity of theorem proving procedures. *Proc 3rd STOC*, pages 151–158, 1971.

[2] Cook, S.A. Reckhow A.R. The relative efficiency of propositional proof systems. *J. of Symbolic Logic*, 44(1):36–50, 1979.

[3] Stockmeyer L. J. Classifying the computational complexity problems. *The Journal of Symbolic Logic*, 52 No 1:1–43, 1987.

[4] A.R. Meyer. weak monadic second-ordet theory of successor is not elementary recursive. In *Lecture Notes in Mathematics.*

[5] Meyer, A.R. Stockmeyer L. J. The equivalence problem for regular expressions with squaring requires exponential space. *Proceedings of the 13th IEEE Symposium on Switching and Automata Theory*, pages 125–129, 1972.

[6] P. Pudlak. Satisfiability - algorithms and logic. In *Mathematical Foundations of Computer Science*, pages 129–141. Springer, 1998.

TRABALHOS DO DEPARTAMENTO DE MATEMÁTICA

TÍTULOS PUBLICADOS

2006-01 MIRANDA, J. C. S. Adaptive maximum probability estimation of multidimensional Poisson processes intensity function. 9p.

2006-02 MIRANDA, J. C. S. Some inequalities and asymptotics for a weighted alternate binomial sum. 7p.

2006-03 BELITSKII, G., BERSHADSKY, M., SERGEICHUK, V. and ZHARKO, N. Classification of 2 x 2 x 2 matrices; criterion of positivity. 16p.

2006-04 FUTORNY, V. and SERGEICHUK, V. Classification of sesquilinear forms whit the first argument on a subspace or a factor space. 25p.

2006-05 DOKUCHAEV, M. A., KIRICHENKO, V. V., NOVIKOV, B.V. and PETRAVCHUK, A. P. On incidence modulo ideal rings. 41p.

2006-06 DOKUCHAEV, M., DEL RÍO, Á. and SIMÓN, J. J. Globalizations of partial actions on non unital rings. 10p.

2006-07 DOKUCHAEV, M., FERRERO, M. and PAQUES, A. Partial actions and Galois theory. 20p.

2006-08 KIRICHENKO, V. V. On semiperfect rings of injective dimension one. 25p.

2006-09 SHESTAKOV, I. and ZHUKAVETS. N. The free alternative superalgebra on one old generator. 30p.

2006-10 MIRANDA, J. C. S. Probability density functions of the wavelet coefficients of a wavelet multidimensional poisson intensity estimator. 7p.

2006-11 ALEXANDRINO, M. M and GORODSKI, C. Singular Riemannian foliations with sections, transnormal maps and basic forms. 17p.

2006-12 FERNANDEZ, J. C. G. On right-nilalgebras of index 4. 21p.

2006-13 FUTORNY, V. and OVSIENKO, S. Galois Algebras I: Structure Theory. 52p.

2006-14 FUTORNY, V. and OVSIENKO, S. Galois Algebras II: Representation Theory. 36p.

2006-15 DRUCK, I. O. Frações: uma análise de dificuldades conceituais. 16p.

2006-16 FUTORNY, V, HORN, R. A. and SERGEICHUK V. V. Tridiagonal canonical matrices of bilinear or sesquilinear forms and of pairs of symmetric, skew-symmetric, or Hermitian forms. 21p.

2006-17 MIRANDA, J. C. S. Infinite horizon non ruin probability for a non homogeneous risk process with time-varying premium and interest rates. 4p.

2007-01 GREBENEV, V. N., GRISHKOV, A. N. and OBERLACK, M. Lie algebra methods in Statistical Theory of Turbulence. 29p.

2007-02 FUTORNY, V. and SERGEICHUK, V. Change of the congruence canonical form of 2 x 2 and 3 x 3 matrices under perturbations. 18p.

2007-03 KASHUBA, I. and PATERA, J. Discrete and continuous exponential transforms of simple Lie groups of rank two. 24p.

2007-04 FUTORNY, V. and SERGEICHUK, V. Miniversal deformations of matrices of bilinear forms. 34p.

2007-05 GONÇALVES, D. L., HAYAT, C., MELLO, M.H.P.L. and ZIESCHANG, H. Spin-structures of bundles on surfaces and the fundamental group. 22p.

2007-06 GOODAIRE, E. G. and MILIES, C.P. Polynomial and group identities in alternative loop algebras.7p.

2007-07 GOODAIRE, E. G. and MILIES, C.P. Group identities on symmetric units in alternative loop algebras. 8p.

2007-08 ALEXANDRINO, M. M. Singular holonomy of singular Riemannian foliations with sections. 16p.

2007-09 ASPERTI, A. C. and VALÉRIO, B. C. Ruled Weingarten surfaces in a 3-dimensional space form. 6p.

2007-10 ASPERTI, A. C. and VILHENA, J. A. M. Spacelike surfaces in $\mathbb{L}^4$ with prescribed Gauss map and nonzero mean curvature. 22p.

2007-11 GOODAIRE, E. G. and MILIES, C. P. Involutions of RA loops. 13p.

2007-12 FERRAZ, R. A., MILIES, C.P., DEL RIO, Á. and SIMÓN, J. J. Group codes from subgroups. 9p.

2007-13 ALEXANDRINO, M. M. and TÖBEN, D. Equifocality of a singular Riemannian foliation. 10p.

2007-14 WEISS. M. A. A polynomial algorithm for deciding 3-sat. 23p.

Nota: Os títulos publicados nos Relatórios Técnicos dos anos de 1980 a 2005 estão à disposição no Departamento de Matemática do IME-USP.
Cidade Universitária "Armando de Salles Oliveira"
Rua do Matão, 1010 - Cidade Universitária