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Performance Assessments of an Advanced Control System in an Iron Ore Industrial Grinding Circuit

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Abstract

Optimizing beneficiation processes in the iron ore industry is essential to meet increasing demand while dealing with declining ore grade. Since grinding has a major impact on product quality and production costs, the implementation of Advanced Process Control (APC) emerges as an effective strategy to enhance efficiency and operational stability. This study quantifies the operational improvements achieved after the implementation of an APC system in the ball grinding circuit of the Mineração Usiminas industrial processing plant. The assessment was based on an ON/OFF test conducted over 67 days, during which operational data were collected for periods with the APC system enabled and disabled, supported by statistical tests and a literature review. The results show significant improvements in both stability and throughput under the unconstrained operating scenario. The standard deviation of the hydrocyclone feed pulp density setpoint decreased by 63%, while the circuit throughput increased from 541 to 571 tph. Moreover, the specific energy consumption was reduced by more than 5% in the same scenario. These findings demonstrate the tangible benefits of implementing Advanced Process Control in industrial grinding operations.

Keywords: comminution; advanced process control; iron ore; grinding and process optimization

1. Introduction

Natural resources play a fundamental role in sustaining industrial production chains on a global scale. Despite the challenges posed by climate, economic and social aspects, their exploitation remains indispensable [1,2]. In recent years, an increasing number of studies have been dedicated to a more sustainable and technically efficient exploration [3–5]. The growing global demand for raw materials contrasts with the gradual exhaustion of high-quality deposits, which makes process optimization an increasingly relevant guideline, both in the present and in future projections [2,6–9].

The grinding process represents a significant portion of the energy consumption of an industrial mineral processing plant and, consequently, of the processing cost [10,11]. The efficient operation of the grinding circuit directly influences both the overall metal

recovery and the quality of the product obtained at the mill [12–14]. Although the relationship between overall recovery and product quality is typically inversely proportional, Advanced Process Control strategies can improve these competing objectives while simultaneously improving the use of electrical energy [15–18]. Numerous studies, such as the work of [19–25], have confirmed that the implementation of Advanced Process Control (APC) has resulted in significant improvements in the operation and performance of the grinding process.

Beyond the technological advances in process optimization, mining plants have also sought to incorporate broader socioeconomic and sustainability competencies, which align closely with the principles of Mining 4.0. This paradigm integrates artificial intelligence (AI), the Internet of Things (IoT), and smart sensors to improve efficiency, safety, and sustainability while optimizing operations and reducing environmental impact [12,26,27]. Although challenges remain in terms of equipment integration and social sustainability, the trend points toward intelligent, interconnected, and adaptive mining systems—a path that leads to the future concept of Mining 5.0 [28]. Although there are numerous laboratory studies, industrial validations of APC systems remain rare.

The grinding circuit of Mineração Usiminas (MUSA) operates with a basic PID (Proportional–Integral–Derivative) control strategy, whose main function is to control the level of the sump to avoid overflow. This approach prioritizes operational safety but disregards the control of critical process variables, such as hydrocyclone feed pressure and pulp density.

This work aims to evaluate the performance of an advanced control strategy applied to an industrial grinding circuit focusing on stabilizing key operational variables, increasing circuit throughput, and reducing the specific energy consumption. It is hypothesized that this strategy enhances overall circuit efficiency, product quality, and operational safety, while minimizing pulp losses—iron ore and water—thus promoting more sustainable resource use and reducing environmental impact.

2. Grinding

Mineral processing plants generally includes comminution, which comprises crushing and grinding operations. The comminution stage aims to reduce the size of the particles, for either liberating the selected minerals or adapting the particle size distribution to the requirements of subsequent processes [29]. On an industrial scale, grinding is predominantly carried out in tubular mills, which can be classified according to the type of grinding medium used: autogenous grinding (AG), in which the ore itself acts as a grinding medium; semi-autogenous grinding (SAG), which combines the use of ore with a fraction of steel grinding media; and rod or ball mills, in which fragmentation is promoted exclusively by metallic grinding bodies with a cylindrical or spherical shape, respectively [30].

Grinding is one of the most energy-intensive steps in mineral processing, accounting for about 30% to 50% of a plant's total energy [1,31–33]. Therefore, increasing the efficiency of this operation is crucial both for reducing operating costs and for mitigating the environmental impacts associated with power generation. In recent decades, driven by the advances of Industry 4.0, efforts have intensified in the development and application of automation and advanced control techniques aimed at increasing grinding efficiency.

In the 1970s and 1980s, control systems applied to comminution were mostly based on relatively simple loops using PID controllers [34]. The advances in computer systems and the popularization of techniques such as fuzzy logic, artificial intelligence and neural networks enhanced the developments of more robust specialist systems, capable of dealing with the dynamic complexity of grinding circuits [35,36].

The integration of advanced control into the grinding circuit aims to optimize variables such as circuit throughput, pulp dilution and hydrocyclone feed pressure. Chen et al. [35] highlighted the importance of controlling particle size distribution and the degree of particle liberation, which are key factors to maximize metal recovery in subsequent processing stages. Wang et al. [36] reinforce that efficient grinding control is decisive for improving energy efficiency and concentrate quality. Zhou et al. [37], in a comprehensive review on control methods in grinding, highlighted the relevance of hybrid approaches that combine historical data and process knowledge to ensure robust performance in industrial environments.

The integration of Industry 4.0 and Industrial Internet of Things (IIoT) technologies in mineral processing was discussed by Jämsä-Jounela [31], indicating a trend towards smarter, autonomous, and energy-efficient automation systems. Yamashita et al. [38] investigated multi-objective tuning techniques of Model Predictive Controllers (MPC) applied to the specific challenges of grinding circuits, such as slow dynamics and long dead times. Apelt and Thornihill [19] demonstrated that the application of well-tuned MPC can generate production increases of 1%–2% per year, representing approximately USD 1 million annually in copper beneficiation plants.

2.1. Grinding at Mineração Usiminas

Mineração Usiminas (MUSA) was created in 2010 as a joint venture between Usiminas and Sumitomo Corporation for producing iron ore concentrate for the Brazilian domestic market, as well as international markets [39]. Located in Itatiaiuçu, state of Minas Gerais, MUSA is currently the fifth largest iron ore producer in Brazil [40]. It includes four industrial mineral processing plants, i.e., ITM West, ITM Samambaia, ITM Flotation and ITM East.

The ITM Flotation process flow sheet is shown in Figure 1. The plant feed comprises the coarse tailings from the West and Samambaia ITM. The grinding circuit product is directed to desliming, followed by flotation, thickening and filtering. The flotation circuit operates with two parallel and independent lines, consisting of flotation cells and columns. Both tailings and concentrate are thickened and filtered before stacking.
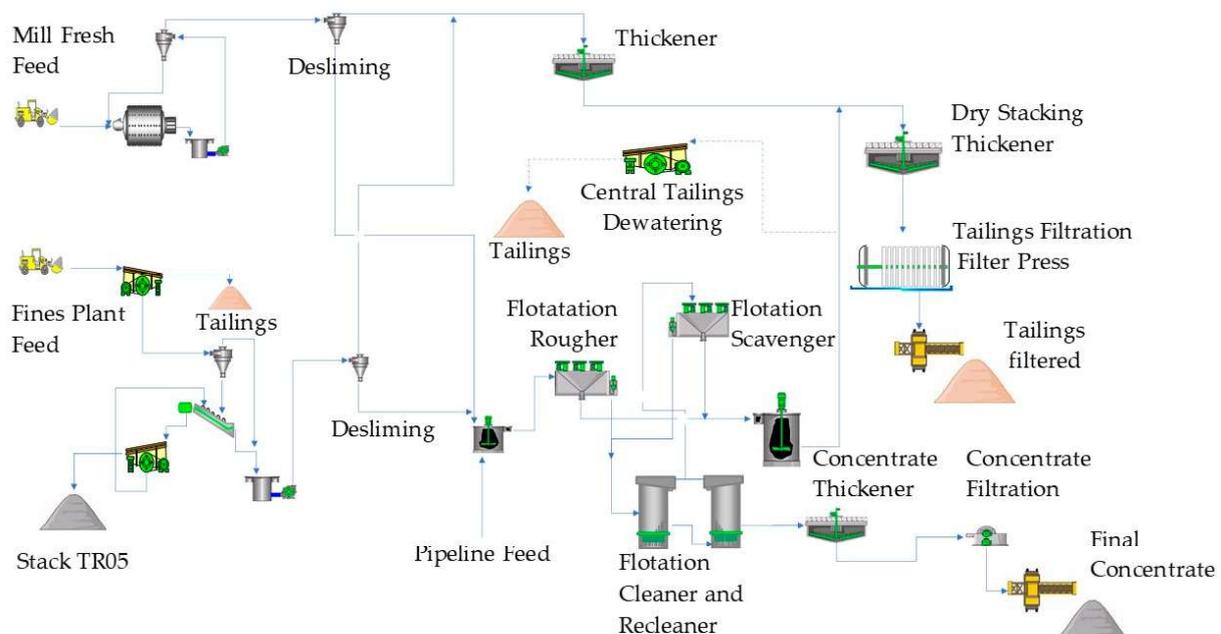

Figure 1. Simplified flow sheet of ITM Flotation at Usiminas processing plant.

Table 1 presents the physical and chemical characteristics of the grinding circuit feed measured throughout the test period. The operational Bond Work Index (WI) was calculated for the feed material.

Table 1. Physical and chemical characteristics of the feed material during the test period.

Main Characteristics of the Feed Material			
Chemical		Physical	
Fe Grade	SiO$_2$ Grade	Density of Solids	WI
45.2%	31.0%	3.9 g/cm^3	14.6 kWh/t

The grinding circuit consists of a single CITIC ball mill 5.8 m in diameter and 12.4 m in length, equipped with a 7.1 MW electric motor, operating at a rotational speed of 13.4 rpm, which corresponds to 75% of the respective critical speed at a 31% ball charge, the latter comprising steel balls with 2.5-inch-diameter top size. The mill operates in a closed direct configuration with hydrocyclones. The circuit nominal throughput is 540 t/h of solids.

The fresh feed is conveyed to the ball mill feed, whose discharge flows onto a sump (CX-01), where water is added for achieving a solids concentration in the range of 60%–65%. The pulp is pumped (BP-01 or BP-02) to a 26-inch-diameter hydrocyclone nest, all equipped with an 8-inch vortex and 4.5-inch apex. The hydrocyclone nest combined underflow gravitates back to the ball mill feed, therefore consisting in the circulating load of the closed-circuit configuration. The circuit product consisting of the combined hydrocyclone nest overflow is directed to a dedicated sump (CX-02), from which it is pumped to the desliming circuit. In terms of size distribution, the circuit product specification is 92% passing 0.150 mm, as well as a P$_{80}$ (80% passing) of 0.106 mm.

The circuit monitoring is carried out by automatic instruments for measuring pulp density, pulp flowrate and pressure of the hydrocyclone feed, as well as ball mill speed. Sampling is conducted by automatic samplers in the overflow of the hydrocyclones, together with manual sampling of the mill fresh feed. Laboratory analyses include determination of Fe, SiO$_2$, P, Mn, Al$_2$O$_3$, PPC, TiO$_2$, MgO and CaO grades, in addition to particle size distributions. Specifically, Fe and other elemental grades are determined using X-ray fluorescence (XRF) analysis, after the samples are homogenized, quartered, and pulverized. A flux is added to produce fused pellets, which are subsequently analyzed by XRF to obtain the chemical composition. Meanwhile, the particle size distribution is determined as part of the laboratory procedure using an automatic suspended sieve shaker.

2.2. Circuit Operation

Before the implementation of the APC system, a few abnormal operating conditions were observed. A relatively frequent condition was the combination of very high circulating load (520%) together with a very high bypass in the hydrocyclones (72.5%), the latter referred to as the amount of fines returning to the ball mill, instead of being directed to the grinding circuit product. The very high inefficiency in classification and very high flowrates around the hydrocyclone nest restricted the circuit throughput.

The absence of adequate pulp density control in CX-01 was also observed. The addition of water in CX-01 was regulated on a sump-level basis, therefore disregarding the pulp density. As the sump level was set to 80% by water addition, it conflicted with the pump (BP-01 and 02) rotation speed, which was set for maintaining the same sump (CX-01) level at 20%. As a result, the pumps operated continuously at full speed, without reaching the proper pressures and densities in feeding the hydrocyclones.

The operational conditions described above were identified as the main factors limiting grinding circuit performance, and their adjustment is expected to increase overall throughput while ensuring consistent product quality.

3. Materials and Methods

The adaptative control approach was selected as the method for coping with the instability and challenging operating conditions occurring at the MUSA's industrial grinding circuit. Accordingly, an APC system was developed and implemented, targeting a consistent higher overall performance of the plant. This section describes the strategy adopted for the application of the APC, the main parameters monitored, the architecture of the control system and the methods used to evaluate the performance of the grinding process.

3.1. BALL MILL ACE^{TM}

Ball Mill ACE (ANDRITZ Control Expert, ANDRITZ, Graz, Austria) is an advanced control and optimization solution for the ball grinding circuits, focused on reducing process variability and adjusting the optimal operating points through layers of supervision and optimization. The ACE advanced control strategy is structured in two hierarchical levels. The first level aims to stabilize the process with Model Predictive Controllers (MPC) with the patented BrainWave$^{®}$ technology (ANDRITZ, Vancouver, BC, Canada). The second level, composed of supervision and optimization logics implemented in the IDEAS™ (ANDRITZ, Atlanta, GA, USA) and Metris All-in-one Platform™ (ANDRITZ, Porto Alegre, Brazil) platforms, was designed to optimize the process setpoints and manage the constraints and auxiliary controls necessary to maximize the performance of the grinding circuit.

BrainWave$^{®}$ controllers are predictive based on models implemented by state–space matrices derived from Laguerre's orthogonal basis functions [41]. In particular, BrainWave$^{®}$ includes a characteristic algorithm dedicated to multivariate processing, with long time constants, resulting in integrative responses, therefore surpassing the traditional approaches of expert systems or other controllers.

The IDEAS™ and Metris All-in-one Platform™ platforms are used for field signal processing and logic programming to manage the grinding system, interacting directly with the various BrainWave$^{®}$ controllers and other process variables. The platforms generate setpoints and coordinate the activation of MPCs, interlocking logics, and communication diagnosis, among other functions.

The ANDRITZ BrainWave$^{®}$ MPC, IDEAS™ Supervisory Control Logic, and ANDRITZ Metris All-in-one Platform™ Digital Solutions software, which make up the ACE system, were run on a dedicated server, provided by ANDRITZ, and integrated into the existing automation and process control network of the MUSA industrial grinding circuit through OLE Process Control (OPC) communication.

The control strategy was organized into three layers, highlighted in Figure 2 by colored blocks. The first layer, represented in gray, corresponds to the management of operational constraints, whose function is to preserve the integrity of the equipment and ensure the safe operation of the process. Included in the first layer is the control of the level of sump 01 (CX-01), the level of sump 02 (CX-02), the electric current of the BP01/02 pump motors, and the hydrocyclone feed pressure.
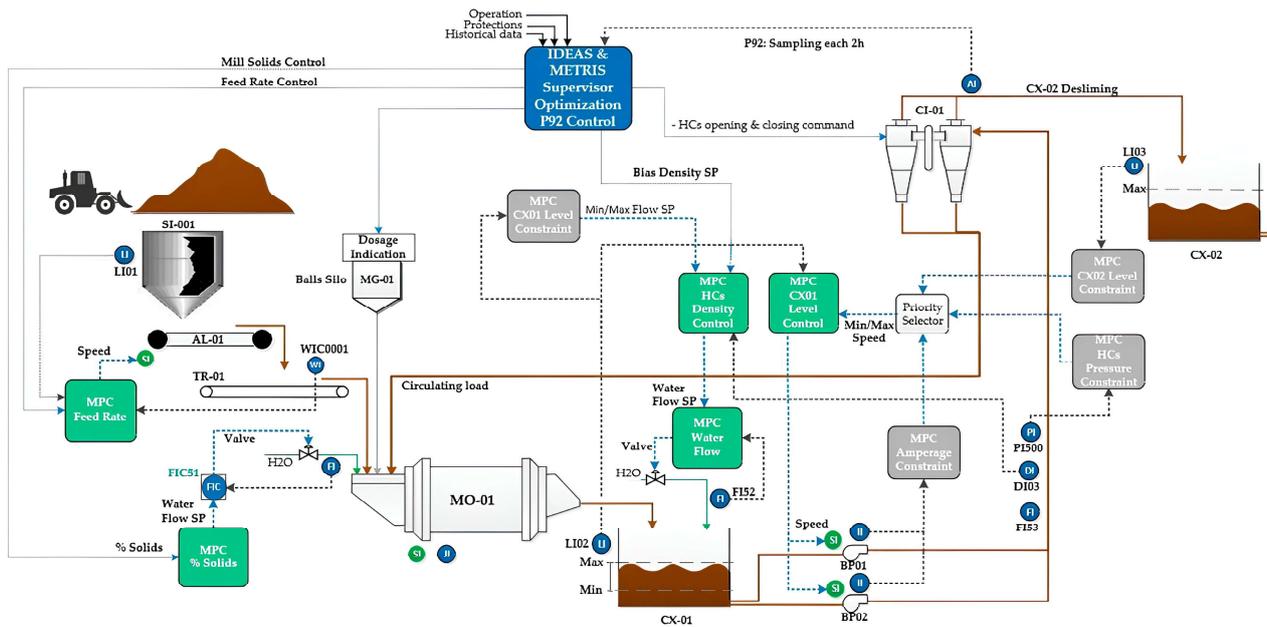
Figure 2. Control strategy for BALL MILL ACE—MUSA.

The second layer, represented in green in Figure 2, covers the main controllers, dedicated to stabilizing the critical variables of the grinding and classification process. In this second layer, the following variables are controlled: the mill fresh feed throughput, the concentration of solids to the ball mill, the level of CX-01 and the hydrocyclone feed pulp density.

The third layer, indicated in blue in Figure 2, is dedicated to process supervision and optimization. It includes three modules that interact with the main controllers to enhance performance, maintain stability, and support safe and efficient operation:

i.   The first is a fuzzy granulometry control module, designed to regulate the hydrocyclone overflow particle size, measured through laboratory analyses performed approximately every two hours. Because of this low-frequency measurement, fuzzy logic is used to interpret process behavior based on the granulometry error (difference between the target and measured value) and its rate of change. The controller then computes a bias that adjusts the hydrocyclone density setpoint—controlled by an MPC (HC density control)—gradually stabilizing the product size.

ii.   The second module focuses on throughput maximization under safe and stable conditions. It continuously assesses the circuit stability using key variables such as feed silo level, mill power, circulating load, sump CX-01 level, hydrocyclone density, and product granulometry. When all parameters remain within predefined limits for a minimum period of two hours, the module increases the MPC feed rate setpoint. After each adjustment, the stability assessment cycle restarts, allowing further incremental increases as long as operating conditions remain suitable. On the other hand, mill power draw, hydrocyclone feed density, and BP01/02 pump motor current are continuously monitored, and if any exceed their safety limits or indicate instability, the module automatically reduces the feed rate setpoint to maintain equipment integrity and circuit stability.

iii. The third module supports grinding media management, maintaining mill power within the optimal operating range. It estimates the required media replenishment based on the historical media consumption per unit of energy and accumulated operating hours since the last replenishment. The module also evaluates recent mill

power trends and adjusts the calculated media dosage when power deviates from the optimal range.

The multilayer APC strategy is designed for coordinated and efficient action on the process, ensuring operational safety, stability of critical variables, and continuous search for better performance. Constraint control acts directly on operational variables that must remain within strict limits. The main layer controls the process variables at the setpoints, while the supervision and optimization layers promote strategic adjustments aimed at maximizing circuit throughput and improving energy efficiency. The configuration of these layers was carried out in collaboration with the plant's process, automation, and operation teams. Table 2 presents a summary of the loops involved in each layer, respective manipulated and controlled variables, and corresponding operational objectives, including the controllers of the BALL MILL ACE.

Table 2. Summary of controlling levels of the APC installed at the MUSA grinding circuit.

Control Layer	Controlled Variable	Objective	Manipulated Variable	Controller
Restriction	CX-01 level	Avoid overflow and emptying of CX-01	Water addition to CX-01	MPC BrainWave®
	CX-02 level	Avoid overflow of CX-02	Rotating speed of BP01 or BP02	MPC BrainWave®
	Hydrocyclone feed pressure	Ensure pressure within stipulated operating interval	Rotating speed of BP01 or BP02	MPC BrainWave®
	BP-01 and 02 electric current	Avoid damage to the electric motors	Rotating speed of BP01 or BP02	MPC BrainWave®
Main	Circuit throughput	Keep the throughput steady	Feeder speed	MPC BrainWave®
	Solids concentration in the mill	Keep the concentration in the setpoint	Water addition to the mill	Cascade of the MPC BrainWave® and PID
	Feed pulp density to hydrocyclones	Keep density steady	Water flowrate and water addition valve to CX-01	Cascade of MPC BrainWave® Hydrocyclone feed density and CX-01 water addition flowrate
	CX-01 level	Keep the level within the stipulated operating interval	Rotating speed of BP01 or BP02	MPC BrainWave®
Supervision and Optimization	Product size distribution	Adjust the hydrocyclone cutting size	Bias of the density setpoint	Fuzzy Metris All-in-one Platform™
	Circuit throughput	Maximize throughput	Circuit feed setpoint	Expert IDEAS and Fuzzy Metris All-in-one Platform™
	Mill power draw	Keep the power draw within the stipulated operating interval	Amount of grinding media added to the mill	Expert IDEAS and Fuzzy Metris All-in-one Platform™

3.2. Performance Testing

To evaluate the efficiency of the APC system with the BALL MILL ACE strategy in improving the operational stability and circuit throughput of the grinding process, a performance test was conducted based on the ON/OFF comparative method. The

experiment was carried out over 67 days, consisting in weekly alternations between periods with the APC system enabled (ON) and disabled (OFF).

The test covered the period from 28 October 2024 to 5 January 2025, starting with the APC system OFF (28 October to 3 November) and then ON (4 November to 10 November). This weekly ON/OFF alternation continued until the end of the test campaign. A total of 13 days were excluded from the analysis due to scheduled maintenance shutdowns, off-spec feed conditions, and other factors that could affect the reliability of the comparison.

This alternating strategy was chosen to minimize the effects of natural variations in ore feed characteristics, ensuring comparable operating conditions between the two control scenarios. The test design, with five effective weeks under each configuration (ON and OFF), followed predefined operational assumptions to ensure both regimes operated within equivalent process ranges and to isolate the multivariable control effect on the main circuit variables.

The analysis included the following four main variables: (i) circuit throughput, (ii) hydrocyclone feed pulp density, (iii) product particle size distribution and (iv) size-specific energy consumption. The circuit throughput and hydrocyclone feed pulp density were obtained directly from the plant's instrumentation system. The product size distribution was derived from laboratory testing. For this analysis, a sub-sample of the hydrocyclone overflow was collected every 15 min to compose a composite sample every two hours, resulting in 12 composite samples per day, each consisting of eight increments. As these are routine samples involving a large sample volume, duplicate or triplicate analyses were not performed. Size-specific energy consumption was calculated based on these measurements using Equation (1).

Data were extracted using the PI DataLink tool, with a sampling time interval of 20 s. After collecting, the data were statistically treated for removing outliers and were later classified into the ON and OFF groups, as described in Section 3.2.

The specific energy consumption was calculated based on the ratio between the mill power draw (kW) and the throughput (t/h) passing at 0.150 mm generated by the grinding, as expressed by Equation (1):

$$\text{Size specific energy consumption} = \frac{\text{Mill power draw}}{\text{Net product throughput passing at 0.15 mm}} \quad (1)$$

The specific energy consumption was selected for assessing the energy efficiency of the grinding process.

To assess the statistical significance of the differences between the ON and OFF scenarios of the APC system, a statistical inference analysis was conducted based on the sample distributions of the selected performance variables. Initially, the Shapiro–Wilk test was applied to verify the adherence of the data to the normal distribution, since this test is recognized for its sensitivity in small to moderate samples [42]. The assumed null hypothesis was that the data follows a normal distribution. However, in all the variables evaluated, the results indicated rejection of the null hypothesis (p-value < 0.05), showing that the analyzed samples did not present sufficient evidence to guarantee statistical normality.

In view of this result, non-parametric statistical methods were adopted, which are more robust when the distribution of data does not meet the assumptions of normality. To evaluate possible differences in the median circuit throughput and specific energy consumption, the Mann–Whitney U test [43] was applied, suitable for comparisons between two independent samples. In the case of the analysis of variability, for comparing the standard deviation of the pulp density setpoint and the particle size of the product, Levene's test [44] was used, which allows the evaluation of the homogeneity of variances between groups.

Statistical comparisons were performed between the APC OFF scenario and three APC ON operational scenarios, the latter further detailed in the next section. Thus, for each performance variable, three sets of independent statistical tests were conducted, always comparing an ON scenario with the OFF scenario. In all analyses, the level of significance adopted was 5% ($\alpha = 0.05$), and the *p*-values obtained guided the interpretation of the effects of the APC on the process parameters evaluated.

In addition to the experimental design with alternation between periods with the ON and OFF system, the performance test was conducted under controlled operating conditions, for ensuring comparison between the analyzed scenarios. These conditions were (i) circuit throughput equal to or greater than 450 tons per hour (tph), (ii) mill power draw higher than 5500 kW, (iii) hydrocyclone feed pressure ranging between 0.7 kgf/cm^2 and 1.5 kgf/cm^2, (iv) hydrocyclone feed pulp density between 1.70 g/cm^3 and 1.90 g/cm^3, and (v) simultaneous operation of five hydrocyclone units. Strict compliance with these conditions was essential to isolate the effect of the advanced control system and ensure the statistical validity of the obtained results.

3.3. Data Collection and Analysis

This section describes the methodology used in data collection and the analysis of collected data. This step is essential to ensure the reliability of the results, eliminating external influences that may compromise the interpretation of the data and ensuring that the comparison between the scenarios with APC ON and OFF is accurate and representative of the operational reality. After the analysis, a total of 13 days were excluded from the database, essentially due to scheduled maintenance or ore feed being out of specification.

Two filtering methods were applied on the database as follows:

- The first filter, referred to as Base Filter, was applied to select the data from steady periods under nominal operating conditions. These periods were defined by a throughput of at least 450 tph sustained for 10 min and a hydrocyclone feed pulp density of at least 1.7 g/cm^3 sustained for 30 min.
- The second filter, referred to as 3STD, was essentially a steadiness criterion, adopted as the variation of three standard deviations around the mean to eliminate outliers, without compromising the representativeness of stable periods. The sampling interval adopted was 20 s per collection point.

Table 3 shows the summary of data corresponding to APC ON and OFF periods, including the results of applied filtering criteria.

Table 3. Summary of data for the OFF and ON periods.

Criterion	Number of Data	
	Period OFF	Period ON
Not filtered	11,219,208	11,399,024
Only Base Filter	9,563,736	10,782,863
Both Base Filter and 3STD Filter	8,971,053	10,326,316
Percent Base Filter and 3STD Filter	79.96%	90.59%

The two datasets that resulted from both Base Filter and 3STD Filter shown in Table 3 represented 28 full days of operation under the conditions of the APC ON and OFF control systems. They summed up more than 19 million data points. The analysis of the process variables indicated trends that were associated with operational constraints on the stability of the control system. A critical constraint was the level of CX-01. Accordingly, in the APC ON scenario, the system imposed an automatic restriction when the CX-01 level exceeded 85% to prevent sump overflow. However, this limitation directly affected the

hydrocyclone feed pulp density control, resulting in severe limitations to maintain it in the set point, which in turn impacted the hydrocyclones' operation and consequently increased the circulating load.

Under abnormal operating conditions—such as high pump motor current, elevated CX-02 level, or excessive hydrocyclone pressure—protective constraints are activated, automatically reducing the pump speed to mitigate these issues. This action typically causes the CX-01 level to rise, while the density control may continue to add water, further contributing to the level increase. When the CX-01 level reaches its configured upper limit, the system restricts additional water flow to prevent overflow, prioritizing level control over density control. As a result, the density control becomes subordinated to the sump level constraint, compromising its performance and, ultimately, the efficiency of the grinding circuit classification. Another important source of operational instabilities in the same part of the circuit was the electric current of BP-01 and BP-02, as it limited the pump speed and therefore the respective pulp flowrate, for modulating the CX-01 pulp level.

Based on the above-described situations, the APC ON was divided into three operational scenarios, as follows:

- DB_Geral: Dataset of the entire 28-day period.
- DBCX1_OK: Dataset of the 9 days in which the level restriction of CX-01 acted on less than 35% of the daily operational time.
- DBCX1_NOK: Dataset of the 19 days in which the level restriction of CX-01 acted on more than 35% of the daily operational time.

4. Results and Discussion

This chapter describes the results obtained from the comparisons between the APC OFF and APC ON selected periods, the latter including three operational scenarios, as discussed in the previous chapter. The four sections include individual analysis of the following variables: (i) grinding circuit throughput, (ii) hydrocyclone feed pulp density, (iii) grinding size and (iv) size-specific energy consumption.

4.1. Circuit Throughput

The strategy of the APC BALL MILL ACE includes a module for maximizing circuit throughput. The adopted criterion is based on the stability of critical process variables. Accordingly, in situations where the selected parameters remain steady within the adopted operating limits, the system promotes gradual increases in throughput. Conversely, persisting operational instability and restrictions result in automatic reductions in throughput.

Figure 3 shows the effect of the APC implementation on circuit throughput. The upper chart (green) shows the process behavior under APC OFF, where the feed rate presents higher variability and less consistent tracking of the setpoint. In contrast, the lower chart represents the APC ON (blue) condition, where the throughput distribution is concentrated at higher values, with clearer alignment to the feed rate setpoint. The shaded areas indicate days with reduced influence from the CX01 level constraint—DBCX1_OK—allowing a more direct comparison of the controller's effectiveness.
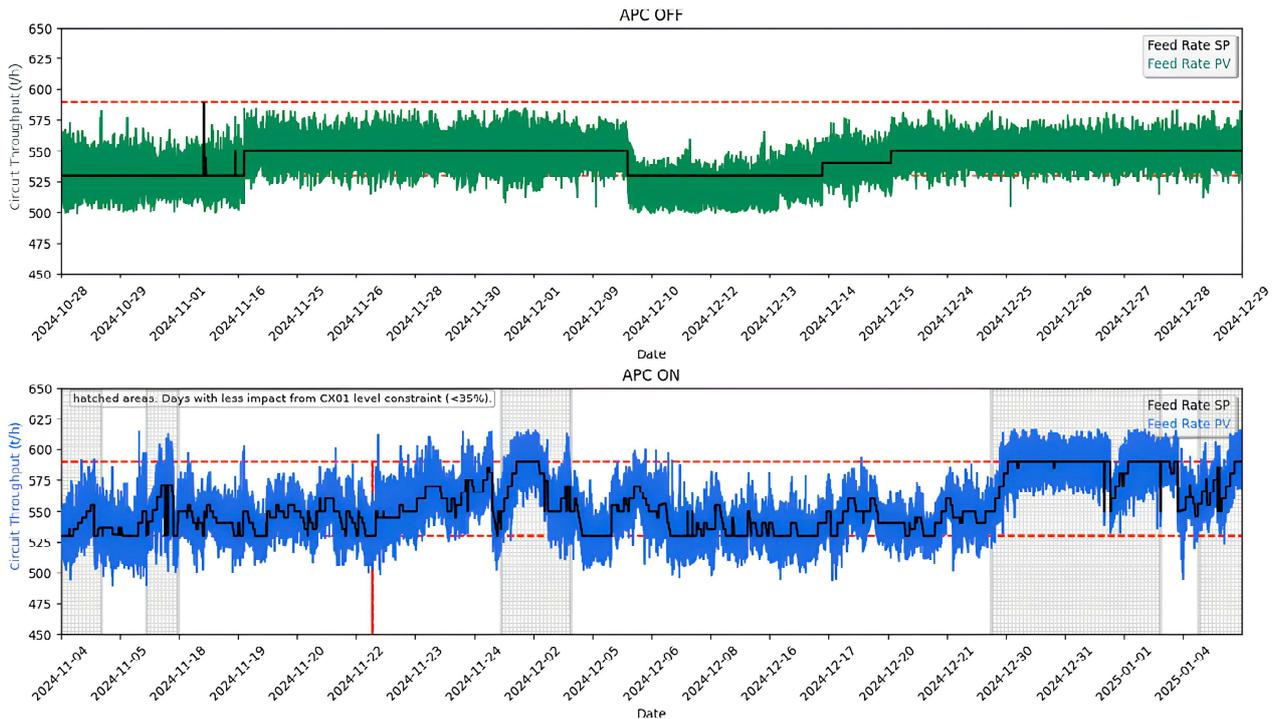
Figure 3. Variations in circuit throughput during the selected periods of APC OFF and APC ON—DB_Geral.

Table 4 summarizes the average circuit throughput values in tons per hour (t/h) under APC OFF and APC ON conditions. To provide a more consistent basis for comparison, the APC ON periods were further divided according to the influence of the sump level constraint in CX-01. Specifically, the subset DBCX1_OK corresponds to periods with reduced impact of this constraint, while the DBCX1_NOK subset represents the remaining periods. This approach allows for a clearer distinction of the APC performance under different operating conditions and highlights the average throughput achieved in each scenario.

Table 4. Average circuit throughput (tph) for APC OFF and APC ON conditions, showing percentage variation relative to APC OFF.

Scenario	Circuit Throughput (tph)		
	Average		Observation
	(tph)	(%)	
APC OFF	541.2	Reference	
APC ON—DB_Geral	553.7	+2.3	APC ON vs. APC OFF
APC ON—DBCX1_OK	571.7	+5.6	APC ON (Hatched areas) vs. APC OFF
APC ON—DBCX1_NOK	545.1	+0.7	APC ON (No hatched areas) vs. APC OFF

With the implementation of the APC, the circuit throughput showed a clear improvement across all evaluated scenarios. The most expressive result occurred under stable operating conditions (DBCX1_OK), with an average increase of about 5.6%, confirming the system's ability to optimize feed rate under stable operating conditions. In addition, the DB_Geral period showed a moderate improvement, while the DBCX1_NOK period presented only marginal variation, indicating that APC performance is strongly dependent on process stability.

The observed behavior indicates that the APC flow maximization module was effective in stable operational contexts, as evidenced in the set DBCX1_OK. The upper limitation

imposed on the module (590 tph), defined by operational criteria, may have constrained additional increments in the DB_Geral and DBCX1_OK scenarios. In the DBCX1_NOK set, the high incidence of the level restriction of CX-01 compromised the performance of the module, resulting in marginal gain.

4.2. Hydrocyclone Feed Pulp Density

Controlling the pulp density in the hydrocyclone feed is a critical factor in ensuring the efficiency of the classification process, which in turn modulates the circulating load and ultimately both grinding size and circuit throughput.

Figure 4 presents the error histograms based on hydrocyclone feed pulp density setpoint, comparing the three scenarios of APC ON in blue (DB_Geral, DBCX1_OK and DBCX1_NOK) with the APC OFF in green. In all cases, solid lines represent normalized histograms, while the bars depict probability density estimates for each condition. The three tables summarize the standard deviation (STD) values and the relative percentage variation (%STD) between the selected conditions.
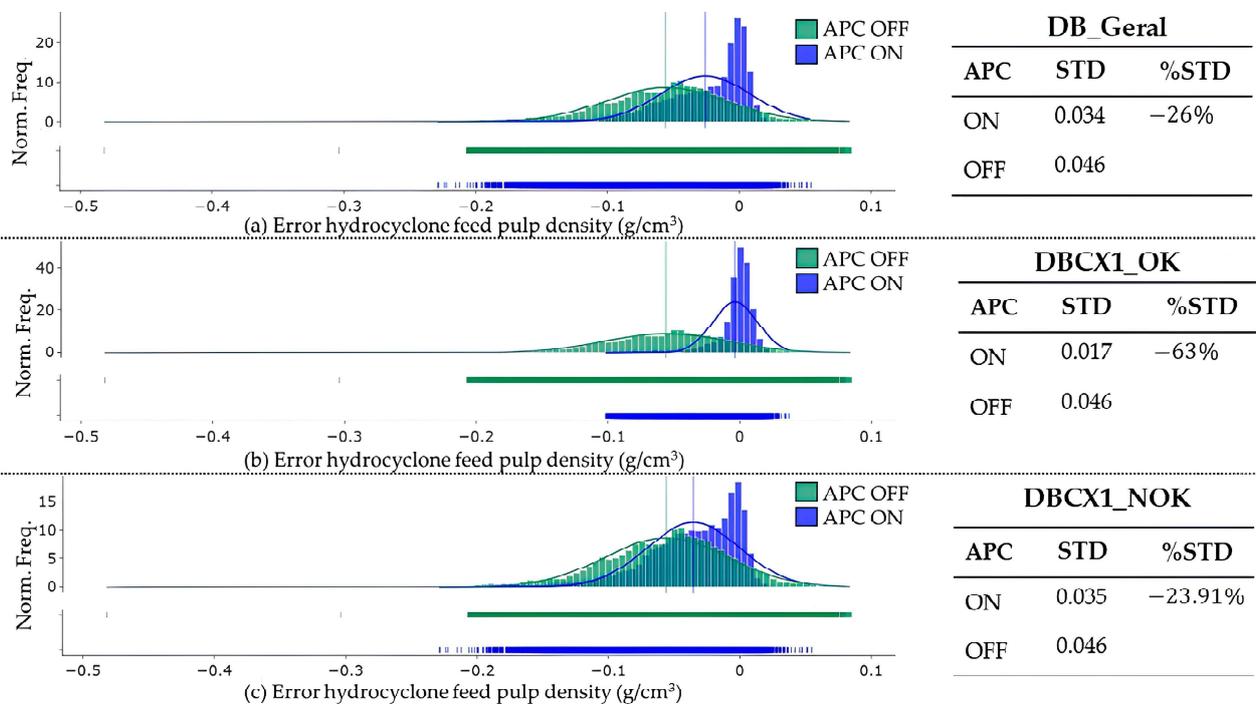
DB_Geral		
APC	**STD**	**%STD**
ON	0.034	−26%
OFF	0.046	

DBCX1_OK		
APC	**STD**	**%STD**
ON	0.017	−63%
OFF	0.046	

DBCX1_NOK		
APC	**STD**	**%STD**
ON	0.035	−23.91%
OFF	0.046	

(a) Error hydrocyclone feed pulp density (g/cm³)
(b) Error hydrocyclone feed pulp density (g/cm³)
(c) Error hydrocyclone feed pulp density (g/cm³)

Figure 4. Normalized frequency distribution of the error in hydrocyclone feed pulp density.

The application of the APC resulted in a consistent reduction in the variability of the hydrocyclone feed pulp density in all the evaluated scenarios. The most significant improvement occurred under stable conditions (DBCX1_OK), as shown in Figure 4b, with variability decreasing by about 63%, confirming the controller's ability to maintain closer adherence to the setpoint. Moderate reductions were also observed in DB_Geral (Figure 4a) and DBCX1_NOK (Figure 4c), demonstrating that the system enhances process stability even under less favorable operating conditions.

4.3. Grinding Size

The grinding size, here represented by the percent retained at 0.150 mm, is directly influenced by the control of density and pressure in the hydrocyclone pulp feed. The APC uses a control strategy based on a fuzzy controller, which dynamically adjusts the density setpoint according to the deviation observed in retention at 0.150 mm. Accordingly, if the

value of retention at 0.150 mm is higher than the stipulated setpoint, the controller reduces the pulp density setpoint by increasing the water flowrate to CX-01, therefore diluting the feed pulp, which in turn increases the pulp level in the sump. To stabilize the sump level, the controller increases the pump rotation, which in turn results in increasing the hydrocyclone feed pulp pressure. Both pulp feed density reduction and pressure increase result in a finer hydrocyclone overflow size distribution, therefore reducing the retention at 0.150 mm.

Figure 5 presents the distributions of percent retained at 0.150 mm according to three scenarios of APC ON in blue (DB_Geral, DBCX1_OK and DBCX1_NOK), as compared with the APC OFF in green. Solid lines correspond to normalized frequency distributions, while the bars indicate probability density estimates. The tables summarize the standard deviation (STD) values and the relative percentage variation (%STD) between the selected conditions.
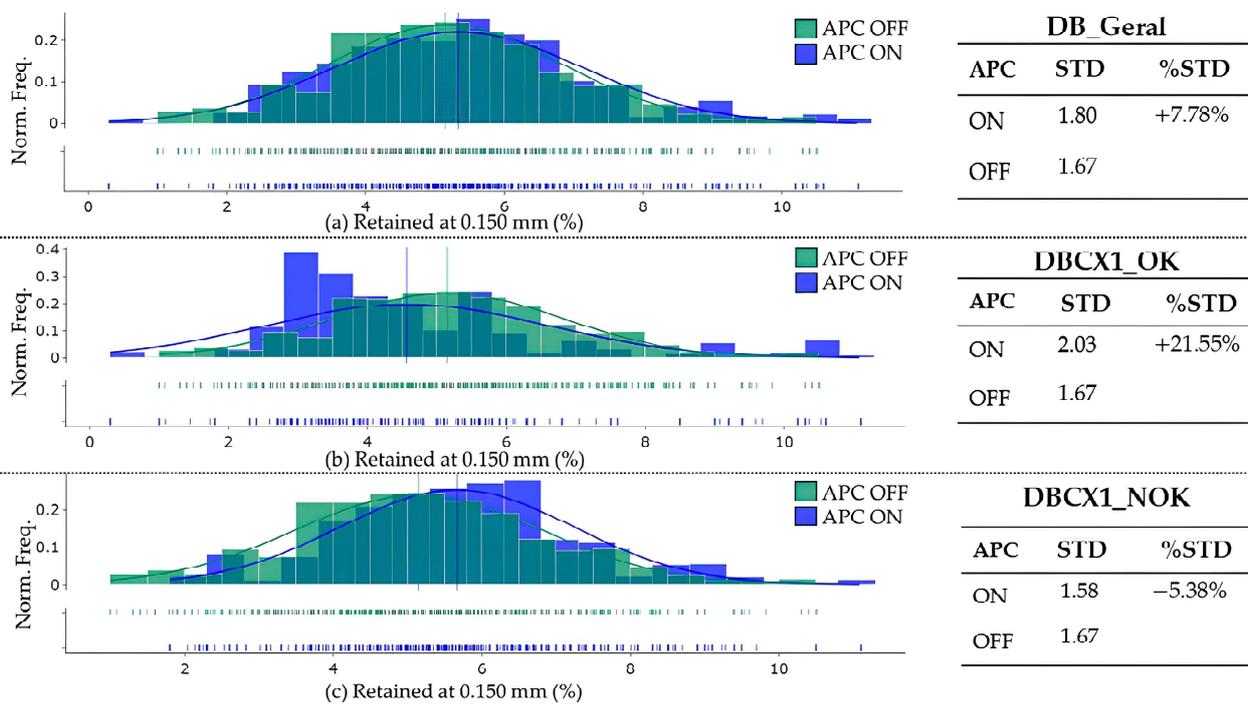

DB_Geral

APC	STD	%STD
ON	1.80	+7.78%
OFF	1.67	

DBCX1_OK

APC	STD	%STD
ON	2.03	+21.55%
OFF	1.67	

DBCX1_NOK

APC	STD	%STD
ON	1.58	−5.38%
OFF	1.67	

Figure 5. Normalized frequency histograms of the percentage retained at 0.150 mm under APC OFF (green) and APC ON (blue) conditions.

As illustrated in Figure 5a,b, which correspond respectively to scenarios DB_Geral and DBCX1_OK, the implementation of the APC resulted in slightly higher variability in grinding size, with the latter showing an increase of approximately 20%. It is observed that, despite the active pulp density control and a more stable average hydrocyclone feed pressure, there was an increase in the standard deviation in both cases. Similarly, a small improvement was observed in DBCX1_NOK (Figure 5c).

The analysis of such scenarios indicates that as the level control of CX-01 imposes restrictions when the level reaches 85% or higher, it limits the density controller to maintain the variable at the respective setpoint, therefore affecting the range of granulometry reduction. This problem does not occur in the APC OFF period, as in such a situation there is no pulp density control, meaning that both the water flowrate added to the CX-01 and the pump speed operate as a function solely of the sump level.

4.4. Size-Specific Energy Consumption

Figure 6 presents the distributions of size-specific energy consumption (kWh/t) according to three scenarios of APC ON in blue (DB_Geral, DBCX1_OK and DBCX1_NOK), as compared with the APC OFF in green. Solid lines correspond to normalized frequency distributions, while the bars indicate probability density estimates. The tables summarize the average (Avg) values for the corresponding conditions, as well as the variation in the average towards the APC ON cases, as compared with the APC OFF conditions.
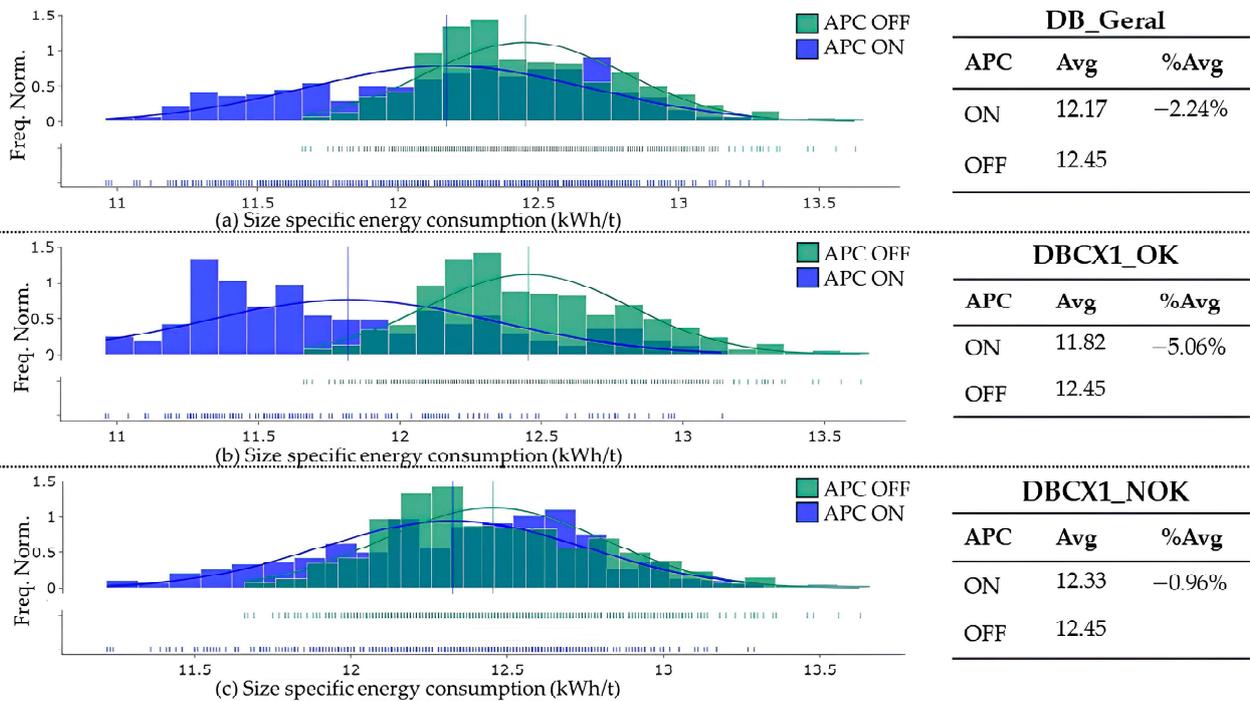
Figure 6. Normalized frequency histograms of size-specific energy consumption (kWh/t) under APC OFF (green) and APC ON (blue) conditions.

Figure 6 shows that the APC contributed to reducing specific energy consumption under all evaluated scenarios. The most notable improvement occurred in DBCX1_OK (Figure 6b), with an average decrease of about 5%, highlighting the system's effectiveness under stable operating conditions. Smaller gains were observed in DB_Geral (Figure 6a) and DBCX1_NOK (Figure 6c), reflecting the impact of process variability on energy efficiency. Overall, these results reinforce that the benefits of advanced control are maximized when process stability is maintained, particularly through effective sump level management.

The implementation of APC enhanced grinding circuit performance by stabilizing key operational variables and enabling operation closer to optimal setpoints. By continuously adjusting the feed rate according to ore characteristics and process stability, the system reduces unnecessary fluctuations in mill load, resulting in higher throughput with lower specific energy consumption. In this regard, reduced density variability contributes to improved energy efficiency, as less energy is spent on overgrinding or correcting transient instabilities.

Moreover, when the specific energy consumption was analyzed as a function of the feed rate ranges, it was found that the grinding circuit operated for longer periods at higher throughput levels under APC ON scenarios. This behavior demonstrates that the control system promoted a more stable and productive operating regime, enabling higher feed rates to be sustained with lower specific energy consumption. These findings further confirm the energy efficiency improvement provided by the APC implementation.

5. Summary

Table 5 summarizes the results obtained in the selected parameters as a function of the selected scenarios, the latter including periods with APC OFF and the three scenarios with APC ON (DB_Geral, DBCX1_OK, and DBCX1_NOK). The averaged values were adopted for the circuit throughput and the size-specific energy consumption, whereas the standard deviations were considered for the hydrocyclone pulp feed density and the grinding size. The averaged values were selected because the respective absolute values are key factors to assess the performance of the APC system. Conversely, the variation in hydrocyclone pulp feed density and the grinding size represented the circuit operational stability. Percentage changes ($\Delta\%$) were calculated based on the reference condition, APC OFF.

Table 5. Summary of the operational performance results under the different control conditions analyzed.

Parameter	APC OFF	APC ON						
		DB_Geral	$\Delta\%$	p-Value	DB_CX1_OK	$\Delta\%$	DB_CX1_NOK	$\Delta\%$
Circuit throughput (t/h)	541.2	553.7	+2.3	0.0447	571.7	+5.6	545.1	+0.7
Hydrocyclones feed pulp density (g/cm^3)	0.046	0.034	−26	0	0.017	−63	0.035	−24
Retained at 0.150 mm (%)	1.67	1.80	+7.78	1.11×10^{-76}	2.03	+20.83	1.58	−5.38
Size-specific energy consumption (kWh/t)	12.45	12.17	−2.24	2.24×10^{-16}	11.82	−5.06	12.33	−0.96

The statistical analyses were performed for all datasets (DB_Geral, DBCX1_OK, and DBCX1_NOK). Nonetheless, only the p-values of the complete dataset (DB_Geral) are reported in Table 5, as it best represents the overall circuit performance under APC operation. The results of the subsets were omitted to avoid redundancy, given that DBCX1_OK consistently showed superior results under unrestricted conditions, while DBCX1_NOK corresponds to constrained periods that do not reflect the typical APC performance.

Overall, the results confirm that the APC system effectively improves the grinding circuit's stability and efficiency, particularly when the CX-01 level restriction is not active. Nonetheless, a limitation of the current APC implementation is that its optimal performance is achieved primarily under stable operating conditions, without restrictions on CX-01. During periods of higher process variability or frequent activation of CX-01 level restrictions, the system's ability to maximize throughput and minimize energy consumption is reduced. Future work will focus on strategies to mitigate these limitations, including the potential integration of an adaptive control layer or additional sump sensors, aiming to reduce or eliminate the operational constraints associated with CX-01 and enhance the overall robustness of the APC system.

6. Conclusions

A comprehensive testing campaign was carried out at Mineração Usiminas's industrial grinding circuit, resulting in a consistent database to assess the performance associated with the implemented Advanced Process Control (APC), in this case the BALL MILL ACE. The dataset selected according to the stipulated filter represented 28 operating days, comprising eight parameters obtained every 20 s by the system, summing up to more than 19 million individual records.

As the level of sump CX-01 installed in the pumping system for the hydrocyclone feed limited the control of hydrocyclone pulp feed density and pressure, the dataset was divided into the following three selected scenarios for periods where the APC was on: DB_Geral, DBCX1_OK and DBCX1_NOK. The scenario where the APC was off (APC OFF) was also included in the comparisons.

The results indicate that APC improves operational stability and circuit throughput, particularly when the CX-01 level does not significantly restrict the system. Stabilization of the combined hydrocyclone feed pulp density and pressure contributed to higher throughput and reduced energy consumption.

Although differences in grinding size were relatively small and within operational limits, the APC contributed to overall process stability. The analysis showed that the effectiveness of the density control is limited when the CX-01 level reaches high values, which can affect grinding size. In general, however, APC operation led to improved control of hydrocyclone feed conditions and resulted in better energy efficiency compared with periods when the system was off.

Overall, the study confirms that APC implementation has a positive impact on industrial grinding circuits by enhancing stability, efficiency, and energy performance, while highlighting the operational conditions that can limit its full potential.

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Data Availability Statement: The data are contained within the article.

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Abbreviations

The following abbreviations are used in this manuscript:

ACE	ANDRITZ Control Expert
AI	Artificial Intelligence
AG	Autogenous Grinding
APC	Advanced Process Control
IIoT	Industrial Internet of Things
IoT	Internet of Things
ITM	Industrial Mineral Processing Plants
MPC	Model Predictive Control (or Controller, depending on context)
MUSA	Mineração Usiminas
ON/OFF	Enabled/Disabled mode
PID	Proportional–Integral–Derivative (control strategy)
P_{80}	80% of the material passing a specific size (0.106 mm in this study)
SAG	Semi-Autogenous Grinding
3STD	Three Standard Deviations (steadiness criterion)

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