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Units of Integral Group Rings of Frobenius Groups.

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Abstract

Let G be a finite Frobenius group with Frobenius kernel N and a Frobenius complement X. We prove that if $u \in \mathcal{U}_1\mathbb{Z}G$ is a torsion unit then the order of u divides either $|N|$ or $|X|$. As a consequence we prove that Zassenhaus' Conjecture holds in some cases and that Problem 8 of [12] has a positive answer for finite groups that are subgroups of the multiplicative group of a division ring and for a large family of Frobenius groups. Moreover, we prove that normalized group basis in the integral group ring of a Frobenius group are Frobenius groups.

1 Introduction

Let $\mathcal{U}(\mathbb{Z}G)$ denote the group of units of the integral group ring of a finite group G, and set $\mathcal{U}_1(\mathbb{Z}G) = \{u \in \mathcal{U}(\mathbb{Z}G) \mid \varepsilon(u) = 1\}$, where $\varepsilon : \mathbb{Z}G \to \mathbb{Z}$ denotes the augmentation map. A well-known theorem of G. Higman (see [12, Theorem 20.9]) implies that, if G is abelian, then any element of finite order of $\mathcal{U}_1(\mathbb{Z}G)$ is trivial; i.e., it belongs to G. In the non-commutative

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setting H.J. Zassenhaus has formulated a conjecture that extends the result above: if $x \in \mathcal{U}_1(\mathbb{Z}G)$ is an element of finite order then there exists a unit $u \in \mathbf{Q}G$ such that $u^{-1}xu \in G$. When such a unit exists, we say that x is *rationally conjugate* to an element of G. This conjecture is denoted **(ZC1)** in [12] and we shall keep this notation.

This conjecture has been confirmed for several classes of groups (see [5], [9], [10], [12], [14] , [15]) and no counterexamples to **(ZC1)** are known. A weaker version of this conjecture is stated as Resaerch Problem 8 in [12] and it is as follows.

Problem 8. *Let G be a group and let $u \in \mathcal{U}_1\mathbb{Z}G$ be a torsion unit. Then there exists an element $g \in G$ such that $o(u) = o(g)$.*

For example, it is known that this version of the conjecture holds for metabelian groups ([12, Lemma 37.14]) though it is still undecided whether **(ZC1)** holds in this case.

Let G be a finite Frobenius group with Frobenius kernel N and Frobenius complement X. It is well-known that the order of every element of G divides either $|N|$ or $|X|$ and, in a way, this property characterizes finite Frobenius groups (see [8, Proposition 17.2]). We shall show that something similar holds for units in the integral group ring of G; namely, we shall prove that if G is a finite Frobenius group as above, then for every torsion unit $u \in \mathcal{U}_1\mathbb{Z}G$, we have that the order of u divides either $|N|$ or $|X|$.

This result will allow us to show that normalized group basis in the integral group ring of a Frobenius group are themselves Frobenius groups and will be useful in showing that Problem 8 has a positive answer for a large family of Frobenius groups. Also, we prove that **(ZC1)** holds for some of these groups.

2 Main result

We shall first prove our main result.

Theorem 2.1 *Let G be a Frobenius group with Frobenius kernel N and a Frobenius complement X. If $u \in \mathcal{U}_1\mathbb{Z}G$ is a unit of finite order, then $o(u)$ divides either $|N|$ or $|X|$.*

Proof. Set $n = |N|$ and $m = |X|$. Suppose, by way of contradiction, that there exists a torsion unit $u \in \mathcal{U}_1\mathbb{Z}G$ that is not as in our statement; i.e., such that $(n, o(u))$ and $(m, o(u))$ are both different from 1. Then, we may also assume that there exists a unit $u = \sum u(g)g \in \mathcal{U}_1\mathbb{Z}G$ such that

$o(u) = pq$ where p and q are prime divisors of n and m respectively and, taking adequate powers, we can write $u = vw$ where $o(v) = p$ and $o(w) = q$.

Let $\Psi : \mathbb{Z}G \longrightarrow \mathbb{Z}X$ denote the homomorphism of $\mathbb{Z}$-algebras induced by the natural homomorphism $G \to X$. If $\Psi(v) \neq 1$ then $o(\Psi(v)) = p$ and [12, Lemma 37.3] would imply that $p \mid m$, so we must have that $\Psi(v) = 1$. Since w has order q, it follows from [12, Lemma 7.3] that $supp(w)$ contains an element g of order q such that $\widetilde{w}(g) = \sum_{h\sim g} u(h) \neq 0$. Then, $\Psi(w) \neq 1$ and we have that $\Psi(u) = \Psi(v)\Psi(w) = \Psi(w) \neq 1$.

Consequently, $\Psi(u) \neq 1$ is a unit of finite order in $\mathbb{Z}X$ and it follows by a theorem of Berman-Higman [12, Proposition 1.4] that

$$\sum_{g\in N} u(g) = 0. \tag{1}$$

Since p and q are different primes, it follows that $q \not\equiv 0 \pmod{p}$ so we can find a positive integer t such that $q^t \equiv 1 \pmod{p}$. As v and w commute, we have that $u^{q^t} = v$.

Using [12, Lemma 7.1] we can compute:

$$u^{q^t} \equiv \sum (u(g)g)^{q^t} \ \ (mod\ ([\mathbb{Z}G, \mathbb{Z}G] + q\mathbb{Z}G))$$

and, factoring, we can write this expression in the form

$$u^{q^t} \equiv \sum u'(g)g^{q^t} \ \ (mod\ ([\mathbb{Z}G, \mathbb{Z}G] + q\mathbb{Z}G)),$$

where the sum runs over different elements of G. Notice that, since u^{q^t} is a unit of finite order, we have that the coefficient of 1 in this expression is 0. Given an element $g \in G$, if $g^{q^t} \in N$ we have that $o(g^{q^t})$ is a divisor of $|N|$ and, since $(n, m) = 1$ it follows that $o(g) \mid n$ so $g \in N$. Thus, $g^{q^t} \in N$ if and only if $g \in N$. Also, [12, Lemma 7.2] shows that if $\alpha \in [\mathbb{Z}G, \mathbb{Z}G]$ then $\widetilde{\alpha}(g) = 0$ for all $g \in G$. Hence, we have that:

$$1 = \Psi(v) = \Psi(u^{q^t}) \equiv \sum_{g\in N} u'(g) \ \ (mod\ q).$$

Since each coefficient $u'(g)$ is a sum of q^tth powers of coefficients of u and, for every integer a we have that $a^{q^t} \equiv a \ (mod\ q)$, the relation above shows that:

$$1 \equiv \sum_{g\in N} u(g) \ \ (mod\ q).$$

This contradicts (1), proving our statement. □

As an immediate consequence, using [12, Theorem 41.12] and a theorem of Weiss [12, 40.4] we have the following.

Corollary 2.2 *Let G be a finite Frobenius group of order $p^n q^m$, where p, q are rational primes. Then ZC1 holds for G.*

We now prove a particular case of the conjecture of Zassenhaus.

Lemma 2.3 *Let G be a Frobenius group with Frobenius kernel N and Frobenius complement X and let $u \in \mathcal{U}_1\mathbb{Z}G$ be a torsion unit of prime order. Then u is rationally conjugate to an element in G.*

Proof. Let $o(u) = p$, a prime rational integer.Then, p divides either $|N|$ or $|X|$. Assume first that $n \mid |N|$. Since N is nilpotent, its Sylow p-subgroup P is a direct factor of N and we can write G in the form $P \rtimes H$ where p does not divide $|H|$. Then, [12, Theorem 41.12] shows that u is rationally conjugate to a group element.

On the other hand, assume now that p is a divisor of $|X|$. It was shown in [3, Theorem 6.1] that if either $p \neq 2$ or $p = 2$ and G cannot be mapped homomorphically to S_5 then every finite p-subgroup of $\mathcal{U}_1\mathbb{Z}G$ is rationally conjugate to a subgroup of G. Thus, we need only to consider the case when $p = 2$ and G can be mapped onto S_5. In this latter case, G is not solvable and we know, from Zassenhaus' theorem [8, p. 204] that X contains a subgroup X_0 such that $[X : X_0] \leq 2$ and $X_0 = SL(2,5) \times M$, where M is a metacyclic group whose Sylow subgroups are cyclic and $|M|$ is prime to $2, 3$ and 5. Hence $|M|$ is relatively prime to its index in X and by Schur-Zassenhaus Theorem [11, 9.1.2], there exists a subgroup H such that $|H| = [X : M]$ and we can write $X = M \rtimes H$. Also, notice that, if $X = SL(2,5) \times M$, then G cannot be mapped onto S_5, so we must have that $[X : X_0] = 2$. Hence, we have that $|H| = 2|SL(2,5)| = 240$.

Since H is a subgroup of X it is a Frobenius complement and [8, Theorem 18.1 (iii)] shows that H contains a unique element a of order 2, which is central.

Let $\overline{u}$ denote the image of u in $\mathbb{Z}(X/M) \cong \mathbb{Z}H$. Then $o(\overline{u}) = 2$ and we know, from [12, Lemma 7.1] that $a \in supp(\overline{u})$. Since a is central, it follows that $a = \overline{u}$. Hence, [2, Lemma 2.1] shows that u is rationally conjugate to a group element. □

As an applications of the results of this section, we prove the following.

Theorem 2.4 *Let G be a finite Frobenius group and let H be a normalized group basis of $\mathbb{Z}G$. Then H is a Frobenius group.*

Proof. Let G be a Frobenius group with Frobenius kernel N and Frobenius complement X and let H be a normalized group basis: i.e., a subgroup

$H \subset \mathcal{U}_1\mathbb{Z}G$ such that $|H| = |G|$. Set $H_0 = H \cap (1 + \Delta(G,N))$ and let $h \in H_0$ be an element of prime order p. We claim that $p \mid |N|$. In fact, write $h = 1 + \tau$ with $\tau \in D(G,N))$. According to Lemma 2.3, there exists an element $\alpha \in \mathbf{Q}G$ and $g \in G$ such that $g = \alpha^{-1}h\alpha = 1 + \alpha{-}1\tau\alpha$, where $\alpha{-}1\tau\alpha \in \Delta_{\mathbf{Q}}(G,N) \cap \mathbb{Z}G \subset \Delta(G,N)$. Hence, $\alpha^{-1}h\alpha \in (1{+}\Delta(G,N)) \cap G = N$.

The argument above shows that every prime dividig $|H_0|$ is a divisor of $|N|$. Since $|H_0|$ divides $|G| = |N||X|$ and $(|N|,|X|) = 1$, it follows that $|H_0| \mid |N|$.

Notice that H/H_0 can be included in $\mathbb{Z}(G/N) \cong \mathbb{Z}X$, so $[H : H_0] \mid |X|$. Consequently, $(|H_0|,[H : H_0]) = 1$ and by the Schur-Zassenhaus Theorem [11, 9.1.2] we can write $H = H_0 \rtimes X_0$, with $(|H_0|,|X_0|) = 1$. Since $|H| = |G|$, it follows that $|H_0| = |N|$ and $|X_0| = |X|$. It follows from Theorem 2.1 that if $h \in H$ then either $o(k) \mid |H_0|$ or $o(h) \mid |X_0|$. Hence, [8, Theorem 37.17] shows that H is a Frobenius groups. □

3 Problem 8

As a consequence of the results in the previous section, we shall show that problem 8 has a positive answer for a large family of Frobenius groups. We begin with two easy lemmas.

Lemma 3.1 *Let H be a subgroup of a group G. If* **(ZC1)** *holds in $\mathbb{Z}G$, then problem 8 has a positive answer for $\mathbb{Z}H$.*

Proof. Let $\alpha \in \mathbb{Z}H$ be an element of finite order. Since **(ZC1)** holds in $\mathbb{Z}G$ we have that α is conjugate, in $\mathbf{Q}G$, to an element $g \in G$. It follows that $\tilde{\alpha}(g) = 1$ and $\tilde{\alpha}(g') = 0$ if $g' \not\sim g$.

Since $supp(\alpha) \subset H$, this shows that there exists an element $h \in H$ which is conjugate to g in G. Consequently, $o(h) = o(g) = o(\alpha)$, as desired. □

Lemma 3.2 *Let $G = G_1 \times G_2$ be a direct product of two groups such that $(|G_1|,|G_2|) = 1$. If problem 8 has a positive answer in $\mathbb{Z}G_i$, $i = 1,2$, then it also has a positive answer in $\mathbb{Z}G$.*

Proof. Let $\alpha \in \mathbb{Z}G$ be an element of finite order. Then, we can write $\alpha = \alpha_1\alpha_2$, where $o(a_i) \mid |G_i|$, $i = 1,2$, $o(\alpha) = o(\alpha_1)o(\alpha_2)$ and thus $(o(\alpha_1),|G_2|) = 1$. So, using coprime reduction [2, Lemma 2.], we have that α_1 is congruent, in $\mathbf{Q}G$, to an element $\beta_1 \in \mathbb{Z}H_1$. As we are assuming that problem 8 has a positive answer in $\mathbb{Z}H_1$, it follows that there exists an element $g_1 \in G_1$ such that $o(\beta_1) = o(g_1)$ and hence $o(\alpha_1) = o(g_1)$. A similar argument shows that we can find $g_2 \in G_2$ such that $o(\alpha_2) = o(g_2)$.

Since g_1 and g_2 commute, we see that $o(g_1g_2) = o(g_1)o(g_2) = 0(\alpha)$. □

Now, we turn to Frobenius groups. Since the Sylow subgroups of a Frobenius complement are either cyclic, with the possible exception of the Sylow 2-subgroups, which are either cyclic or generalized quaternion [8, Theorem 18.1], we begin with the following, which is actually an easy consequence of known results. Notice that Frobenius complements of odd order are included in the class considered.

Theorem 3.3 *Let G be a group whose Sylow subgroups are all cyclic. Then,* **(ZC3)** *holds in* $\mathbb{Z}G$.

Proof. If all the Sylow subgroups of a group G are cyclic, it follows from a theorem of Hölder, Burnside and Zassenhaus [11, 10.1.10] that G has a presentation

$$G = \langle a, b \mid a^m = b^n = 1,\ b^{-1}ab = a^r\rangle,$$

where $r^n \equiv \ (mod\ m)$, m is odd, $0 \leq r \leq m$ and m and $n(r-1)$ are coprime.

Since G is thus split metacyclic, it follows from a result of Polcino Milies, Ritter and Sehgal [12, Theorem 42.1] that **(ZC3)** holds for G. □

I.N. Herstein raised, in 1953, the question of classifying finite groups of the multiplicative group of a division ring D and showed that, if $char(D) = p > 0$, then these are cyclic p'-groups and problem 8 clearly has a positive answer in this case. The case when $char(D) = 0$ was solved by S. Amitsur in 1955, so we shall refere to finite groups that can be realized in this way as *Amitsur groups.* Amitsur groups are known to be Frobenius complements (see [13, 2.1.2]). The classification theorem is as follows.

Theorem 3.4 ([1]) *Let G be a finite group. Then G is a subgroup of a division ring of characteristic* 0 *if and only if G is isomorphic to one of the following groups.*

(i) A subgroup of a division ring all of whose Sylow subgroups are cyclic.

(ii) The binary octahedral group of order 48.

(iii) A group of the form $C_m \rtimes Q_{2^n}$ *where* C_m *is a cyclic group of odd order* m, $Q_{2^n} = \langle a, b \mid a^{2^{n-2}} = b^2,\ b^4 = 1,\ a^b = a^{-1}\rangle$ *denotes a quaternion group of order* 2^n, *a centralizes* C_m *and b inverts elements of* C_m.

(iv) A group of the form $Q \times M$, *where* Q *is the quaternion group of order* 8, M *is a group of odd order, all of whose Sylow subgroups are cyclic and* 2 *has odd multiplicative order modulo* $|M|$.

(v) *A group of the form $SL(2,3) \times M$, where M is a group of order coprime to 6, all of whose Sylow subgroups are cyclic and 2 has odd multiplicative order modulo $|M|$.*

(vi) *The binary icosahedral group $SL(2,5)$.*

We are now ready to prove that problem 8 has a positive answer for this family of groups.

Theorem 3.5 *Let G be an Amitsur group. If $\alpha \in U_1(\mathbb{Z}G)$ is a unit of finite order, then there exists an element $g \in G$ such that $o(\alpha) = o(g)$.*

Proof. We shall study separately each of the possible cases.

(*i*) is an immediate consequence of Theorem 3.3 and (*ii*) follows from the fact that **(ZC3)** holds for the binary octahedral group [2].

To prove our statement in case (*iii*), consider $\alpha \in U_1(\mathbb{Z}G)$ of finite order, which can be written in the form $\alpha = \alpha_1\alpha_2$, where α_1 is of odd order and α_2 has order a power of 2. We claim that $\alpha_1 \in 1 + \Delta(G, C_m)$.

In fact, consider the natural projection $\mathbb{Z}G \to \mathbb{Z}(G/C_m)$ and denote by $\overline{\alpha_1}$ the image of α_1 under this mapping. Then $o(\overline{\alpha_1}) \mid a(\alpha_1)$ and is thus odd. As G/C_m is of order a power of 2, it follows that $\overline{\alpha_1} = 1$ so $\alpha_1 \in 1 + \Delta(G, C_m)$, as claimed.

Now a result of Luthar and Trama [7, Corollary 1.2] shows that α_1 is rationally conjugate to an element $g_1 \in G$, so $o(\alpha_1) = o(g_1)$.

On the other hand, as $(o(\alpha_2), m) = 1$ and C_m is, in particular, nilpotent we can apply [12, Theorem 37.17] and it follows that α_2 is rationally conjugate to an element $\beta_2 \in \mathbb{Z}Q$. As Q_{2^n} is also nilpotent, a theorem of A. Weiss [12, Theorem 40.4] shows that β_2 is rationally conjugate to an element $g_2 \in Q_{2^n}$. Since $o(g_2) \leq 2^{n-1}$, we can find an element $a_2 \in \langle a \rangle$ such that $o(a_2) = o(\beta_2) = o(\alpha_2)$.

As $g_1 \in C_m$, we have that g_1 and a_2 commute, so $o(g_1a_2) = o(g_1)o(a_2) = o(\alpha)$, as desired.

To prove our statement in case (*iv*) we observe that problem 8 has a positive solution Q and also for M, because of Theorem 3.3. Hence, Lemma 3.2 shows that it has a positive solution in this case.

The proof in case (*v*) is similar, as $SL(2,3)$ is a subgroup of the binary octahedral group, so problem 8 also has a positive solution in $\mathbb{Z}(SL2,3))$ by Lemma 3.1 and the argument proceeds as above.

Finally, it was shown in [3] that **(ZC3)** holds for $SL(2,5)$, so the proof is complete. □

Finally, we return to Frobenius groups.

Theorem 3.6 *Let G be a Frobenius group with Frobenius kernel N and Frobenius complement X. If X is an Amitsur group and $\alpha \in U_1(\mathbb{Z}G)$ is a unit of finite order, then there exists an element $g \in G$ such that $o(\alpha) = o(g)$.*

Proof. Set $n = o(g)$. In view of Theorem 2.1, we have that n divides either $|N|$ or $|X|$. Assume first that n is a divisor of $|N|$. Then, we can write u as a product $u = u_1 u_2 \ldots u_t$ where each element u_i has order a power of a prime p_i which is a divisor of $|N|$, $1 \leq i \leq t$.

Let P_i denote the Sylow p_i-subgoup of N, $1 \leq i \leq t$. since N is nilpotent, $P_i \triangleleft N$ so also $P_i \triangleleft G$ and we can write G in the form $G = P_i \rtimes X_i$ for some subgroup X_i of G and $u_i \in P_i$. Then, by [12, Theorem 41.12] it follows that there exists an element $g_i \in G$ such that u_i is rationally conjugate to g_i, $1 \leq i \leq t$. Since N is nilpotent, the elements g_i, $1 \leq i \leq t$ commute pairwise, so if we set $g = \prod_{i=1}^{n} g_i$ we have that $o(g) = \prod_{i=1}^{n} o(g_i) = o(u)$.

Assume now that n divides $|X|$. In this case, using [12, Theorem 37.17] we see that u is rationally conjugate to an element $u_1 \in \mathcal{U}_1\mathbb{Z}X$. In view of Theorem 3.5, there exists an element $x \in X$ such that $o(x) = o(u_1) = o(\alpha)$ and the result follows. □.

Theorem 3.7 *Let G be a Frobenius group of odd order. If $\alpha \in U_1(\mathbb{Z}G)$ is a unit of finite order, then there exists an element $g \in G$ such that $o(\alpha) = o(g)$.*

Proof. Since all Sylow subgroups of a Frobenius complement of odd order are cyclic, the result follows, as above, from Theorems 2.1 and 3.3. □

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