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ON THE REPRESENTATION DIMENSION OF SOME CLASSES OF ALGEBRAS

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ABSTRACT. We show that the representation dimension of the following classes of algebras is at most 3: (a) Artin algebras A such that the functor $\mathrm{Hom}_A(D(A), -)$ has finite length (or dually, $\mathrm{Hom}_A(-, A)$ has finite length). These algebras coincide with the right (left) glued algebras, as introduced in [1]; and (b) Trivial extensions of iterated tilted algebras.

The concept of representation dimension of an artin algebra A, denoted by $\mathrm{rep.dim}A$, was introduced by M. Auslander [3] in the early 70's in an attempt to, paraphrasing him, *give a reasonable way of measuring how far A is from being representation-finite* (see Section 1 below for the appropriate definitions).

For some time, this notion stayed apart from the main lines of investigation in the representation theory of algebras. Recently, its interest was revived by some works of Xi [11, 12] but mainly by the connection, which we shall recall in the sequel, with the finitistic and Nakayama conjectures given by Igusa and Todorov [6] (see also [9]).

It has been shown by Auslander that an algebra A is representation-finite if and only if $\mathrm{rep.dim}A = 2$. On the other hand, despite shown in [9] that $\mathrm{rep.dim}A$ is always finite, so far there is no example of algebras with representation dimension greater than 3. It follows from [6] that if A has representation dimension at most 3, then its finitistic dimension is finite. It is clear, then, the interest in deepening the study of the representation dimension. For a discussion on the connection between the finitistic and the Nakayama conjecture we refer the reader to [8].

The purpose of the present work is to calculate the representation dimension for some classes of algebras. We will show, for instance, that it is at most three for the following algebras:

(a) Artin algebras A such that the functor $\mathrm{Hom}_A(D(A), -)$ has finite length (or dually, $\mathrm{Hom}_A(-, A)$ has finite length). These algebras

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coincide with the right (left) glued algebras, as introduced in [1]. The class of right glued algebras includes all the hereditary algebras as well as all tilted algebras with complete slices in their preinjective components.

(b) Trivial extensions of iterated tilted algebras.

For that, we shall prove a criterion which appears implicitely in the works of Auslander [3] and Xi [11]. Also, as a consequence of this criterion, we get a better insight of the relations between the representation dimension of an algebra which is a one-point extension $B[M]$ of an algebra B, and the representation dimension of B itself. This extends some results of [11].

This paper is organized as follows. In the first section, after recalling some preliminary notions needed along the work, we state and prove the above mentioned criterion. Sections 2 and 3 deal with the calculation of the representation dimension of the algebras mentioned in (a) and (b) above while in Section 4, we show some results concerning one-point extension algebras.

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1. Preliminaries

1.1. Throughtout this paper, all our algebras are artin algebras. For an algebra A, we denote by $\mathrm{mod}A$ its category of finitely generated left A-modules and by $\mathrm{ind}A$ a full subcategory of $\mathrm{mod}A$ having as objects a full set of representatives of the isomorphism classes of the indecomposable A-modules. Also, given $M \in \mathrm{mod}A$, we denote by $\mathrm{add}M$ the full subcategory of $\mathrm{mod}A$ having as objects the direct sums of indecomposable summands of M. We denote by $\mathrm{pd}_A M$ (or $\mathrm{id}_A M$) the projective dimension (or injective dimension, respectively) of M. Finally, we denote by $\mathrm{gl.dim}A$ the global dimension of A, that is, the suppremum of the projective dimensions of modules in $\mathrm{mod}A$.

We recall that an A-module M is a *generator* (or a *cogenerator*) for $\mathrm{mod}A$ provided for each $X \in \mathrm{mod}A$, there exists an epimorphism $M' \longrightarrow X$ (or a monomorphism $X \longrightarrow M'$) with $M' \in \mathrm{add}M$.

For unexplained notions and facts needed on $\mathrm{mod}A$ we refer the reader to [4].

1.2. The notion of representation-dimension of an algebra was introduced in [3] by Auslander. We refer to this work for the original

definition. For us, it will be more convenient to use the following characterization, also proven in [3].

DEFINITION. The *representation-dimension* of an artin algebra is the number rep.dim A
$= \inf \{gl.dim(\mathrm{End}_A M) :$ M is a generator-cogenerator of mod$A\}$.

1.3. The first aim is to show a criterion for the calculation of the representation dimension of an algebra. From now on, A will denote an artin algebra, and let $\mathcal{C}$ be a full subcategory of mod$A$. We recall that a map $f : C \longrightarrow M$ is called a right $\mathcal{C}$-approximation of the A-module M if C is in $\mathcal{C}$ and the sequence $(-, C) \longrightarrow (-, M) \longrightarrow 0$ is exact in $\mathcal{C}$. Moreover, we will say that an exact sequence

$$0 \longrightarrow C_r \xrightarrow{f_r} \cdots \xrightarrow{f_2} M_1 \xrightarrow{f_1} M \longrightarrow 0$$

is a *$\mathcal{C}$-approximation resolution of M* if C_i is in $\mathcal{C}$ for all i and the sequence

$$0 \longrightarrow (-, C_r) \xrightarrow{f_r} \cdots \xrightarrow{f_2} (-, C_1) \xrightarrow{f_1} (-, M) \longrightarrow 0$$

is exact in $\mathcal{C}$. We say that r is the *length of the resolution.*

DEFINITION. An A-module $\bar{X}$ is said to have the *r-resolution property* if each A-module M has an *add*$\bar{X}$-approximation resolution of length r.

REMARKS. (a) The condition of the above definition can be replaced by a similar one holding for indecomposable modules M.

(b) Clearly, the modules in *add*$\bar{X}$ always have an *add*$\bar{X}$-approximation resolution of length 1.

(c) Given a module M in a subcategory $\mathcal{C}$, one can, dually, define a *left $\mathcal{C}$-approximation of M*, and a *$\mathcal{C}$-approximation coresolution of M*. Also, one can look at the notion of *r-coresolution property.*

1.4. EXAMPLES. (a) Let A be a representation-finite algebra, and let $M_1, M_2, \cdots, M_s$ be a set of all isoclasses of indecomposable A-modules. Clearly, $\bar{X} = M_1 \oplus \cdots \oplus M_s$ has the 1-resolution property, since add$\bar{X}$ = mod$A$. It is also not difficult to see that if an algebra A has a module M satisfying the 1-resolution property, then A is representation-finite, and add$M$ = mod$A$.

(b) Let H be a non-semisimple hereditary algebra and let $\bar{X} = H \oplus \mathrm{D}(H)$. We show that such $\bar{X}$ has the 2-resolution property. Let M be an indecomposable H-module not in add $\bar{X}$, and consider the minimal projective resolution $0 \longrightarrow P_1 \longrightarrow P_0 \longrightarrow M \longrightarrow 0$ of M. Clearly,

$\mathrm{Hom}_H(H \oplus \mathrm{D}(H), M) = \mathrm{Hom}_H(H, M)$ because M is not in add$\bar{X}$ and therefore is not injective, so the sequence

$$0 \longrightarrow (H \oplus \mathrm{D}(H), P_1) \longrightarrow (H \oplus \mathrm{D}(H), P_0) \longrightarrow (H \oplus \mathrm{D}(H), M) \longrightarrow 0$$

is exact and we are done.

1.5. We will see in the sequel that if a module $\bar{X}$ satisfies the r-resolution property then gl.dim $(End_A \bar{X}) \leq r + 1$. If, moreover, $\bar{X}$ is a generator-cogenerator of mod$A$, then rep.dim$A \leq r + 1$. This result has been used by Auslander in [3] and by Xi in [11], in order to give bounds for the representation dimension of some classes of algebras.

In the following theorem, which is the main result of this section, we will prove that the converse statement also holds.

THEOREM. *Let A be an artin algebra. The following statements are equivalent for a positive integer r:*

(a) *rep.dim $A \leq r + 1$;*
(b) *there exists a generator-cogenerator of mod$A$ satisfying the r-resolution property.*

Though, as we mentioned above, the implication (b) $\Rightarrow$ (a) has been implicitely proven in [3] and [11], for the convenience of the reader we shall provide here a complete proof of this result. For that, it is convenient to recall some facts.

1.6. For a module Y, denote by $\mathcal{F}_Y$ the category of all coherent functors F: $(\mathrm{add}Y)^{op} \longrightarrow \mathcal{A}b$, where $\mathcal{A}b$ denotes the category of abelian groups. Recall that a functor F: $(\mathrm{add}Y)^{op} \longrightarrow \mathcal{A}b$ is called *coherent* provided there is a morphism $Y_1 \longrightarrow Y_2$ in add$Y$ such that the sequence

$$(-, Y_1) \longrightarrow (-, Y_2) \longrightarrow F \longrightarrow 0$$

is exact in add$Y$. Here we denote by $(-, C)$ the restriction of the functor $\mathrm{Hom}_A(-, C)$: mod$A \longrightarrow \mathcal{A}b$ to add$Y$. It follows from [3] (Proposition, Ch.III, p.104) that the categories $\mathcal{F}_Y$ and mod$(\mathrm{End}_A Y)$ are equivalent and so, in particular, gl.dim$(\mathrm{End}_A Y) =$ gl.dim $(\mathcal{F}_Y)$.

If an A-module M has an add$Y$-approximation resolution of length s, then $\mathrm{pd}(-, M) \leq s - 1$. If, moreover, Y is a generator of mod$A$, then the converse holds. In fact, if $\mathrm{pd}(-, M) \leq s - 1$, let

$$0 \longrightarrow (-, Y_{s-1}) \longrightarrow \cdots \longrightarrow (-, Y_0) \longrightarrow (-, M) \longrightarrow 0$$

with $Y_i \in$ add$Y$, be a sequence which is exact in add$Y$. Since, by hypothesis, A is in add$Y$, by evaluating the above sequence at A, we infer that there exists an exact sequence

$$0 \longrightarrow Y_{s-1} \longrightarrow \cdots \longrightarrow Y_0 \longrightarrow M \longrightarrow 0$$

inducing the above one, proving then that M has an add$Y$-approximation resolution of length s, as desired.

1.7. *Proof of Theorem 1.5.*

(b) $\Rightarrow$ (a) Let $\bar{X}$ be a module satisfying the r-resolution property. We will show that gl.dim$\mathcal{F}_{\bar{X}} \leq r+1$, leading to the required result. Let F be a functor in $\mathcal{F}_{\bar{X}}$. By definition, there exists a morphism $X'' \xrightarrow{f} X'$ in add$\bar{X}$ such that

$$(*) \qquad (-, X'') \longrightarrow (-, X') \longrightarrow F \longrightarrow 0$$

is exact in add$\bar{X}$. Denote $M = \mathrm{Ker} f$. Now, since $\bar{X}$ has the r-resolution property, there exists an exact sequence

$$0 \longrightarrow X_r \longrightarrow \cdots \longrightarrow X_1 \longrightarrow M \longrightarrow 0$$

with $X_i \in \mathrm{add}\bar{X}$ such that the induced sequence

$$(**) \qquad 0 \longrightarrow (-, X_r) \longrightarrow \cdots \longrightarrow (-, X_1) \longrightarrow (-, M) \longrightarrow 0$$

is exact in add$\bar{X}$. Glueing together $(*)$ and $(**)$ we end up with a sequence

$$0 \to (-, X_r) \to \cdots \to (-, X_1) \to (-, X'') \to (-, X') \to F \to 0$$

which is exact in add$\bar{X}$, showing that $\mathrm{pd}(F) \leq r+1$. Therefore, gl.dim$\mathcal{F}_{\bar{X}} \leq r+1$ and rep.dim$A \leq r+1$, as required. This proves the implication (b) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b) Suppose rep.dim$A = s \leq r+1$. Then there exists a module $\bar{X}$ such that $A \oplus \mathrm{D}(A)$ is in add $\bar{X}$ and gl.dim $(\mathrm{End}_A \bar{X}) = s$. By the above remarks, gl.dim$\mathcal{F}_{\bar{X}} = s$. We claim that $\bar{X}$ has the $(s-1)$-resolution property. In fact, let $M \in \mathrm{mod}A$, not in add$\bar{X}$, and consider a minimal injective presentation $0 \longrightarrow M \longrightarrow I_0 \xrightarrow{f_0} I_1$ of M. Hence, for $F = \mathrm{Coker}(-, f_0)$, we have that

$$(*) \qquad 0 \longrightarrow (-, M) \longrightarrow (-, I_0) \longrightarrow (-, I_1) \longrightarrow F \longrightarrow 0$$

is exact. Since $\bar{X}$ is a cogenerator of mod$A$ we get that I_0, I_1 are in add $\bar{X}$, thus $F \in \mathcal{F}_{\bar{X}}$. Now, M is not in add$\bar{X}$, so $(-, M)$ is not projective. Since gl.dim $\mathcal{F}_{\bar{X}} = s$ we then infer that

$$\mathrm{pd}\ (-, M) = \mathrm{pd}\ (F) - 2 \leq s - 2$$

As observed before the proof of the theorem, this implies that M has a right add$\bar{X}$-approximation of length smaller than $s \leq r+1$. This ends the proof of the theorem. □

1.8. COROLLARY. *Let A be an artin algebra. Then $rep.dim A = r+1$ if and only if there exists a generator-cogenerator of $modA$ satisfying the r-resolution property but there is none satisfying the s-resolution property for $s < r$.*

1.9. COROLLARY. *Let A be a representation-infinite algebra. Then $rep.dim A = 3$ if and only if there exists a generator-cogenerator of $modA$ satisfying the 2-resolution property.*

2. GLUED ALGEBRAS

2.1. We will prove in this section that the algebras such that the length of $\mathrm{Hom}_A(D(A), -)$ is finite (or dually the length of $\mathrm{Hom}_A(-, A)$) is finite) have representation dimension at most three. These algebras were studied by Assem-Coelho, who introduced in [1] the right (left) glued algebras, which coincide with them. We refer the reader to this work for their original definition. We will prove the result for right glued algebras, the corresponding result for left glued algebras follows by duality. We shall use here a characterization whose proof can be found in [1, 2]. Given $X, Y \in \mathrm{ind}A$, we say that X is a *predecessor of* Y or that Y is a *successor of* X provided there exists a sequence $X = X_0 \longrightarrow X_1 \longrightarrow \cdots X_t = Y$ of non-zero morphisms between indecomposable modules. For a given algebra A, define the subcategory

$$\mathcal{L}_A = \{X \in \mathrm{ind}A : \text{ for each predecessor } Y \text{ of } X,\ \mathrm{pd}_A Y \leq 1\}$$

THEOREM. [1, 2] *The following statements are equivalent for an artin algebra A:*

(a) *A is a right glued algebra.*
(b) *the length of $Hom_A(D(A), -)$ is finite.*
(c) *all but finitely many indecomposable A-modules have projective dimension at most one.*
(d) *$\mathcal{L}_A$ is cofinite in $indA$.*

Clearly, the class of right glued algebras includes all the representation-finite ones. Not so immediate, but it also includes all the tilted algebras with complete slices in a preinjective component. Further examples can be found in [1].

2.2. Our main result of this section is the following.

THEOREM. *Let A be a representation-infinite right glued algebra. Then $rep.dim A = 3$.*

Proof. By (1.7), it is enough to exhibit a generator cogenerator of mod$A$ satisfying the 2-resolution property. Since A is right glued, the set $\mathcal{X}_1 = \text{ind}A \setminus \mathcal{L}_A$ is finite. Also, by [2](1.5), the set

$$\mathcal{X}_2 = \{Y \in \mathcal{L}_A : Y \text{ is a successor of an injective in ind}A\}$$

is finite. So, the set

$$\mathcal{X} = \mathcal{X}_1 \cup \{P : P \text{ is a projective in ind}A\} \cup \mathcal{X}_2$$

is finite, say $\mathcal{X} = \{X_1, \cdots, X_s\}$. Write $\bar{X} = X_1 \oplus \cdots \oplus X_s$. Clearly, such module is a generator-cogenerator of mod$A$, and we claim that it satisfies the 2-resolution property. Let now M be an indecomposable A-module not in add$(\bar{X})$. Then $M \in \mathcal{L}_A$, $\text{Hom}_A(\mathcal{X}_1, M) = 0$ because $\mathcal{L}_A$ is closed under predecessors, and $\text{Hom}_A(\mathcal{X}_2, M) = 0$ because $\mathcal{X}_2$ is closed under successors. Moreover, since $M \in \mathcal{L}_A$ and is not projective, $\text{pd}_A M = 1$.

Let now $0 \longrightarrow P_1 \xrightarrow{f_1} P_0 \xrightarrow{f_0} M \longrightarrow 0$ be the minimal projective resolution of M in mod$A$. We will prove that this sequence is an add$\bar{X}$-approximation resolution of M. This amounts to prove that $(X, P_0) \longrightarrow (X, M) \longrightarrow 0$ is exact for each indecomposable $X \in \text{add}\bar{X}$. This clearly holds if X is projective, and it also holds if X is not projective, since then $\text{Hom}_A(X, M) = 0$, as observed above. Thus the proof of the theorem is complete. □

2.3. The following result is the dual of Theorem 2.2. We leave the details of the proof for the reader.

THEOREM. *Let A be a representation-infinite left glued algebra. Then $rep.dim A = 3$.*

2.4. The above results imply Theorem 5.1 of [11]. Also, for reference, we mention the following corollary which improves Corolllary 5.2 of [11].

COROLLARY. *Let A be a tilted algebra. If A has a complete slice in either a postprojective component or in a preinjective component then, $rep.dim A = 3$.*

3. TRIVIAL EXTENSIONS OF ITERATED TILTED ALGEBRAS

3.1. Along this section, H will denote a hereditary algebra. We will prove here that the representation-dimension of the trivial extension $T(H)$ is at most 3. As a consequence, we will have the same bound for the representation dimension of the trivial extensions of iterated tilted algebras, using results by Happel [7] and by Xi in [12]. We shall first

recall some background on this construction. For further details, we refer the reader to [5] and [10].

The *trivial extension of an algebra* A is the algebra $T(A) = A \ltimes \mathrm{D}(A)$ defined as follows. As a vector space, $T(A) = A \oplus D(A)$, and the product is defined by $(a,f)(b,g) = (ab, ag + fb)$, for $a, b \in A, f, g \in D(A)$. The algebra $T(A)$ is selfinjective.

Let $\mathcal{A}$ be an additive category and $F : \mathcal{A} \longrightarrow \mathcal{A}$ be an additive functor. The *trivial extension* $\mathcal{A} \ltimes F$ *of* $\mathcal{A}$ *by* F, defined in [5], Section 1, is the category whose objects are the maps $\alpha : F(A) \longrightarrow A$ such that the composition $\alpha \cdot F(\alpha) = 0$. For objects $\alpha : F(A) \longrightarrow A$ and $\beta : F(B) \longrightarrow B$ in $\mathcal{A} \ltimes F$, a morphism $f : \alpha \longrightarrow \beta$ is a morphism $f : A \longrightarrow B$ such that $\beta F(f) = f\alpha$. When A is an artin algebra and $F = \mathrm{D}(A) \otimes_A -$: mod$A \longrightarrow$ mod$A$, then mod$A \ltimes F$ is equivalent to mod $T(A)$. For an A-module X and a morphism $f : D(A) \otimes_A X \longrightarrow X$, the $T(A)$-module structure is defined on X by $(a,g) \cdot x = ax + f(g \otimes x)$, for $x \in X$, $a \in A$ and $g \in D(A)$ ([5], p.19).

In the case we are primarily interested, that is, the trivial extension of the hereditary algebra H, the modules in mod$T(H)$ can be seen as triples (X_1, X_2, f) with $X_1, X_2 \in$ mod$H$, and f: $\mathrm{D}(H) \otimes X_1 \longrightarrow X_2$ a surjective H-map. A morphism from (X_1, X_2, f) to (Y_1, Y_2, g) is a triple $(\alpha_{11}, \alpha_{22}, \alpha_{21})$ of morphisms in mod$H$, $\alpha_{11} : X_1 \longrightarrow Y_1$, $\alpha_{22} : X_2 \longrightarrow Y_2$ and $\alpha_{21} : X_1 \longrightarrow Y_2$ such that $\alpha_{22} f = g(1 \otimes \alpha_{11})$. This description of the $T(H)$-modules was given by Tachikawa in [10]. To see that the morphisms are appropiately defined we use the equivalence between mod$T(H)$ and mod$A \ltimes F$, considering the triples as elements in mod$A \ltimes F$ in the following way.

The element in mod$A \ltimes F$ corresponding to the triple (X_1, X_2, f) is the map

$$\left(\begin{smallmatrix} 0 & 0 \\ f & 0 \end{smallmatrix}\right) : D(H) \otimes (X_1 \oplus X_2) \simeq D(H) \otimes X_1 \oplus D(H) \otimes X_2 \longrightarrow X_1 \oplus X_2$$

Then a morphism from (X_1, X_2, f) to (Y_1, Y_2, g) corresponds to a map $\left(\begin{smallmatrix} 0 & 0 \\ f & 0 \end{smallmatrix}\right) \longrightarrow \left(\begin{smallmatrix} 0 & 0 \\ g & 0 \end{smallmatrix}\right)$ in mod$A \ltimes F$, that is, a map $\alpha = \left(\begin{smallmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{smallmatrix}\right) : X_1 \oplus X_2 \longrightarrow Y_1 \oplus Y_2$ such that

$$\left(\begin{smallmatrix} 0 & 0 \\ g & 0 \end{smallmatrix}\right)\left(id_{D(H)} \otimes \left(\begin{smallmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & 0 \\ f & 0 \end{smallmatrix}\right)$$

This is satisfied precisely when $\alpha_{12} = 0$ and the triple $(\alpha_{11}, \alpha_{22}, \alpha_{21})$ satisfies the above stated condition.

Since H is hereditary then $\mathrm{D}(H) \otimes X$, being a homomorphic image of $D(H)^n$ for some n, is injective. Thus, $\mathrm{D}(H)X$ is also injective, and $X \simeq \mathrm{D}(H)X \oplus X/D(H)X$ in mod$H$. Observe that the triple associated to the $T(H)$-module X is $(X/D(H)X, D(H)X, f)$, where f: $\mathrm{D}(H) \otimes X/D(H)X \longrightarrow D(H)X$ is the multiplication map.

From now on, we will write the adjoint functors $\mathrm{D}(H) \otimes_H -$ and $\mathrm{Hom}_H(\mathrm{D}(H), -)$ by F and G, respectively, and by $\epsilon : FG \longrightarrow Id$ and $\eta : Id \longrightarrow GF$ the adjuntion morphisms.

Following [10], to a given indecomposable H-module X, two indecomposable $T(H)$-modules can be assigned as follows. The $T(H)$-module $(X, 0, 0)$, called module of the *first type* and which we shall also denote by X. On the other hand, we consider a fixed minimal injective coresolution $0 \longrightarrow X \longrightarrow I_0(X) \xrightarrow{f} I_1(X) \longrightarrow 0$ of X, and assign to X the indecomposable $T(H)$-module $\tilde{X} = (G(I_0(X)), I_1(X), f\epsilon_{I_0(X)})$. We say that $\tilde{X}$ is a module of the *second type*. It follows from [10] that an indecomposable nonprojective $T(H)$-module can be identified either with an H-module or with a module of the second type.

We extend the above notation to arbitrary H-modules $X = X_1 \oplus \cdots \oplus X_n$, with X_i in $\mathrm{ind}A$ by writing $\tilde{X} = \tilde{X_1} \oplus \cdots \oplus \tilde{X_n}$

3.2. Given $X, Y \in \mathrm{ind}H$, there is naturally a morphism

$$\psi\colon \mathrm{Hom}_{T(H)}(\tilde{X}, \tilde{Y}) \longrightarrow \mathrm{Hom}_H(X, Y)$$

defined by the commutative diagram

$$(*) \qquad \begin{array}{ccccccccc} 0 & \longrightarrow & X & \longrightarrow & I_0(X) & \xrightarrow{f} & I_1(X) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \psi(\alpha)} & & \downarrow & & \downarrow{\scriptstyle \alpha_1} & & \\ 0 & \longrightarrow & Y & \longrightarrow & I_0(Y) & \xrightarrow{g} & I_1(Y) & \longrightarrow & 0 \end{array}$$

for $\alpha = (\alpha_1, \alpha_2, \alpha_{21}) : \tilde{X} \longrightarrow \tilde{Y}$, where the middle vertical map is $\epsilon_{I_0(Y)} F(\alpha_0) \epsilon^{-1}_{I_0(X)}$.

Then ψ is surjective, functorial in X and Y, and induces an isomorphism

$$\underline{\psi} : \underline{\mathrm{Hom}}_{T(H)}(\tilde{X}, \tilde{Y}) \longrightarrow \mathrm{Hom}_H(X, Y)$$

as follows from the following lemma. Recall that

$$\underline{\mathrm{Hom}}(X, Y) = \mathrm{Hom}(X, Y)/P(X, Y),$$

where $P(X, Y)$ denotes the set of morphims from X to Y which factor through a projective module.

LEMMA. *Let $\alpha = (\alpha_1, \alpha_2, \alpha_{21}) : (X_1, X_2, f) \longrightarrow (Y_1, Y_2, g)$ be a morphism in $\mathrm{mod}T(H)$ and asume that there exists $\rho : X_2 \longrightarrow F(Y_1)$ such that $F(\alpha_1) = \rho f$. Then:*

(a) $(0, 0, \alpha_{21})$ *factors through the projective module* $(G(Y_2), Y_2, \epsilon_{Y_2})$.

(b) $(\alpha_1, \alpha_2, 0)$ *factors through* $(Y_1, F(Y_1), id)$.

(c) *If Y_1 is a projective H-module then α factors through a projective module.*

Proof. (a) Observe that $(0,0,\alpha_{21})$ is the composition of

$$(0,0,\alpha_{21}) : (X_1, X_2, f) \longrightarrow (G(Y_2), Y_2, \epsilon_{Y_2})$$

and

$$(0, id, 0) : (G(Y_2), Y_2, \epsilon_{Y_2}). \longrightarrow (Y_1, Y_2, g).$$

(b) Since $F(\alpha_1) = \rho f$, using that f is surjective we get that $\alpha_2 = g\rho$. Then we can write $(\alpha_1, \alpha_2, 0) = (id, g, 0)((\alpha_1, \rho, 0)$. (c) It follows from (a) and (b) observing that $(Y_1, F(Y_1), id)$ is projective in mod$T(H)$ when Y_1 is projective in mod$H$. □

3.3. The main result of this section is the following.

THEOREM. *Let H be a hereditary algebra. Then rep.dim $T(H) \leq 3$.*

Proof. Let $\bar{X} = H \oplus D(H) \oplus T(H) \oplus \tilde{H}$ be in mod$T(H)$. We shall prove that the generator-cogenerator module $\bar{X}$ satisfies the 2-resolution property. Let $N' \in$ ind$T(H)$.

Case 1. $N = N'$ is a module of the first type (that is, $N' \in$ mod$H$). As seen in (1.4)(b), $H \oplus$ D(H) satisfies the 2-resolution property in mod$H$, and so, there exists an exact sequence

$$0 \longrightarrow Y_2 \longrightarrow Y_1 \xrightarrow{g} N \longrightarrow 0$$

such that

$$0 \longrightarrow_H (-, Y_2) \longrightarrow_H (-, Y_1) \longrightarrow_H (-, N) \longrightarrow 0$$

is exact in add$(H \oplus$ D$(H))$. Clearly, also

$$(*) \qquad 0 \longrightarrow_{T(H)} (-, Y_2) \longrightarrow_{T(H)} (-, Y_1) \longrightarrow_{T(H)} (-, N) \longrightarrow 0$$

is exact in add$(H \oplus$ D$(H))$ (as $T(H)$-modules). Since $T(H)$ is projective, then $(*)$ is also exact in add$(T(H))$. The proof will be complete in this case once we prove that $\mathrm{Hom}_{T(H)}(\tilde{H}, -)$ preserves the exactness of $0 \longrightarrow Y_2 \longrightarrow Y_1 \xrightarrow{g} N \longrightarrow 0$. So, let P be an indecomposable projective H-module. Then $\tilde{P} = (Q, I_1(P), h\,\epsilon_{I_0(P)})$, where

$$0 \longrightarrow P \longrightarrow I_0(P) \xrightarrow{h} I_1(P) \longrightarrow 0$$

is a minimal injective resolution, and $Q = G(I_0(P))$. Observe that a map $f\colon \tilde{P} \longrightarrow N$ is given by $(\alpha_0, 0, 0)$, where $\alpha_0\colon Q \longrightarrow N$. Since Q is projective, there exists $\beta\colon Q \longrightarrow Y_1$ such that $g\beta = \alpha_0$, and

$(\beta,0,0)\colon \tilde{P} \longrightarrow Y_1$ lifts $(g,0,0)$, as desired.

Case 2. N' is not a module of the first type (i.e. $N' \notin \mathrm{mod}H$). So $N' = \tilde{N}$, for some $N \in \mathrm{ind}H$. As above, consider an exact sequence $(*)\ 0 \longrightarrow Y_2 \longrightarrow Y_1 \longrightarrow N \longrightarrow 0$ with $Y_1, Y_2 \in \mathrm{add}\ (H \oplus \mathrm{D}(H))$, which remains exact under $\mathrm{Hom}_H(H \oplus \mathrm{D}(H), -)$. Let I_0, I_1 be injective modules so that

$$
(*) \qquad
\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & Y_2 & \longrightarrow & Y_1 & \longrightarrow & N & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & I_0(Y_2) & \longrightarrow & I_0 & \longrightarrow & I_0(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & I_1(Y_2) & \longrightarrow & I_1 & \longrightarrow & I_1(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
$$

is exact and commutes. Observe that $I_0 = I_0(Y_1) \oplus I$, $I_1 = I_1(Y_1) \oplus I$, where I is an injective module and $f\colon I_0 \longrightarrow I_1$ is $f = \left(\begin{smallmatrix} f_0 & 0 \\ 0 & \sigma \end{smallmatrix}\right)$ in the above decomposition with σ being an isomorphism. Clearly, we get a sequence

$$(\tilde{*}) \qquad 0 \longrightarrow \tilde{Y}_2 \longrightarrow \tilde{Y}_1 \oplus Q \longrightarrow \tilde{N} \longrightarrow 0$$

with $Q = (G(I), I, \epsilon_I)$, which is projective in $\mathrm{mod}T(H)$. To prove that this sequence is exact we observe two facts:

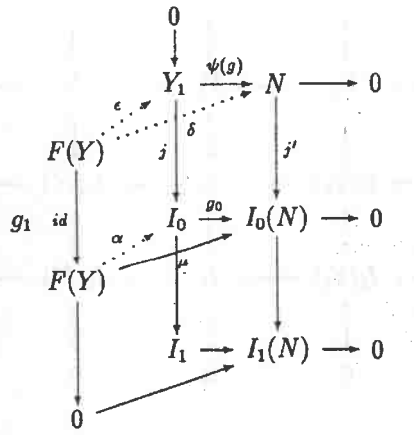
(a) The exact sequence $0 \longrightarrow I_0(Y_2) \longrightarrow I_0 \longrightarrow I_0(N) \longrightarrow 0$ splits, and therefore it remains exact under G.

(b) All the maps in $(\tilde{*})$ are of the form $(\alpha_1, \alpha_2, 0)$, with α_i in $\mathrm{mod}H$.

Then, using that a short exact sequence is exact in $\mathcal{A} \ltimes F$ when the corresponding sequence in $\mathcal{A}$ is exact ([5], Corollary 1.2) we obtain that $(\tilde{*})$ is exact. Also, each of $\tilde{Y}_1, \tilde{Y}_2$ belongs to $\mathrm{add}(\tilde{H} \oplus \widetilde{D(H)})$, and therefore to $\mathrm{add}\bar{X}$, because $\widetilde{D(H)} \simeq (H,0,0)$. We shall now prove that

$\mathrm{Hom}_{T(H)}(\tilde{X}, -)$ keeps $(\tilde{*})$ exact.

Let Y in add$\bar{X}$. We proved in Lemma 3.2 that maps of the form $(0,0,\theta_{21}) : Y \longrightarrow \tilde{N}$ factor through a projective module, and therefore they can be lifted to $\tilde{Y}_1 \oplus Q \longrightarrow \tilde{N}$. Thus it is enough to prove that maps of the form $(\theta_0, \theta_1, 0) : Y \longrightarrow \tilde{N}$ can be lifted to $\tilde{Y}_1 \oplus Q \longrightarrow \tilde{N}$. Consider first $Y \in \mathrm{add}\,(H \oplus \mathrm{D}(H))$ and let $\theta = (\theta_0, 0, 0) : Y \longrightarrow \tilde{N}$ (so, $\epsilon_{I_0(N)} F(\theta_0) : F(Y) \longrightarrow I_0(N)$). We then have the following diagram, where $\mu = \epsilon_{I_0(N)} F(\theta_0)$:



First observe that $\mu = \epsilon_{I_0(N)} F(\theta_0)$ lifts to δ: $F(Y) \longrightarrow N$. Since $_H(D(H), -)$ keeps the sequence $(*)$ exact, and $F(Y)$ is injective (see (3.1)), we infer that δ lifts through $\psi(g)$: $Y_1 \longrightarrow N$. So $j'\delta = \epsilon_{I_0(N)} F(\theta_0)$ and $\delta = \psi(g)\epsilon$, for some $\delta : F(Y) \longrightarrow N$ and $\epsilon : F(Y) \longrightarrow Y_1$. Let $\alpha = j\epsilon$. Then $g_0\alpha = g_0(j\epsilon) = j'\psi(g)\epsilon = j'\delta = \epsilon_{I_0(N)} F(\theta_0)$. Let $\alpha_0 = G(\alpha)\eta_Y$. Then $(\alpha_0, 0, 0)$: $Y \longrightarrow (G(I_0), I_1, f\,\epsilon_{I_0}) = \tilde{Y}_1 \oplus Q$ lifts $(\theta_0, 0, 0)$, as required.

It remains to show that $\mathrm{Hom}_{T(H)}(\tilde{P}, -)$ keeps $(\tilde{*})$ exact for each projective H-module P. We have a commutative diagram

$$(*) \qquad \begin{array}{ccccc} {}_{T(H)}(\tilde{P}, \tilde{Y}_1) & \longrightarrow & {}_{T(H)}(\tilde{P}, \tilde{N}) & \longrightarrow & 0 \\ \downarrow & & \downarrow & & \\ {}_H(P, Y_1) & \longrightarrow & {}_H(P, N) & \longrightarrow & 0 \end{array}$$

where the lower sequence is exact and the vertical arrows are the isomorphisms defined in (3.2). Then we have that the upper sequence is exact. Consequently, $(\tilde{P}, \underline{g})$: ${}_{T(H)}(\tilde{P}, \tilde{Y}_1 \oplus Q) \longrightarrow {}_{T(H)}(\tilde{P}, \tilde{N})$ is

surjective. It follows then that

$$(\tilde{P}, g) : {}_{T(H)}(\tilde{P}, \tilde{Y}_1 \oplus Q) \longrightarrow {}_{T(H)}(\tilde{P}, \tilde{N})$$

is also surjective, because $g : \tilde{Y}_1 \oplus Q \longrightarrow \tilde{N}$ is an epimorphism. This ends the proof of the theorem. □

3.4. COROLLARY. *Let A be an iterated tilted algebra. Then rep.dim $T(A) \leq 3$.*

Proof. It follows from [7] that such an A is derived equivalent to a hereditary algebra H. So, by [12], rep.dim$T(A)$ = rep.dim$T(H)$ and the result follows using our theorem above. □

4. ONE-POINT EXTENSION ALGEBRAS

4.1. In this section we compare the representation dimension of an artin algebra B and the representation dimension of a one point extension of B, under appropiate hypothesis. More precisely, we will prove the following proposition, extending results proven in [11] for one point extensions of finite dimensional algebras by simple injective modules. We refer the reader to [4] for an account on the one-point extension construction. We also observe that a dual version of this result holds for one-point coextensions. We leave to the reader the details of the corresponding proof.

PROPOSITION. *Let B be an artin algebra, D a division ring, M a $B-D$ bimodule and $A = B[M]$ the one point extension of B by M. Assume that the set of successors of M in ind$B$ is finite. Then:*

(a) rep.dim $B \leq$ rep.dim$A$;

(b) *If* ind$B$ *is cofinite in* ind$A$ *then* rep.dim B = rep.dimA.

Proof. (a) Let $\{Z_1, \cdots, Z_t\}$ be the set of successors of M in ind$B$. We will consider the A-modules as triples (D^n, X, f), with X in mod$B$ and $f : M \otimes D^n \longrightarrow X$ a B-morphism, in the usual way (See, e.g., [4], III,2). We start by observing that any A-module K can be written in the form $K = (D^n, Z, g) \oplus (0, N, 0)$ with $Z \in add(Z_1 \oplus \cdots \oplus Z_t)$. In fact, let $K = (D^n, K_1, h)$, and let N' be an indecomposable summand of K_1 which is not a successor of M. Then ${}_B(M, N') = 0$ and therefore $(0, N', 0)$ is a summand of K.

Let rep.dim A $= r + 1$ and let $\bar{X}$ be a generator-cogenerator of mod$A$ satisfying the r-resolution property, $\bar{X} = (D^m, X, f)$ with X in mod$B$ and $f : M \otimes D^n \longrightarrow X$ a morphism of B-modules.

Let $\bar{Y} = \bigoplus_{i=1}^{t} Z_i \oplus X$. If follows from the description of the projective and the injective modules in mod$A$, ([4], III, Prop. 2.5), that $B \oplus D(B)$ is in add$\bar{Y}$ because $A \oplus D(A)$ is in add$\bar{X}$. So $\bar{Y}$ is generator cogenerator

of mod$B$, and we will prove that it satisfies the r-resolution property, and thus rep.dim$B \leq r+1 =$ rep.dim$A$. This amounts to prove that $\mathrm{pd}_B(-,N) \leq r-1$ for each N in ind$B$, where ${}_B(-,N)$ is considered as an element of $\mathcal{F}_Y$, as observed in (1.6).
We will also consider ${}_A(-,N) \in \mathcal{F}_{\bar{X}}$, and show that $\mathrm{pd}_B(-,N) \leq \mathrm{pd}_A(-,N) \leq r-1$. This will end the proof of (a). We will prove the inequality by induction on $k = \mathrm{pd}_A(-,N)$. We may assume that $N \notin$ add $\bar{Y}$.
If $k=0$, then the result clearly holds. So let $k>0$ and consider a right $\bar{X}$-approximation $X_1 \longrightarrow N$ of N. Let $0 \longrightarrow K \longrightarrow X_1 \longrightarrow N \longrightarrow 0$ be exact. Then $0 \longrightarrow {}_A(-,K) \longrightarrow {}_A(-,X_1) \longrightarrow {}_A(-,N) \longrightarrow 0$ is exact in add$\bar{X}$, and $\mathrm{pd}_A(-,K) < k = \mathrm{pd}_A(-,N)$.
We write $K = (D^n, K_1, f)$ and $X_1 = (D^d, Y, g)$ with $Y \in \mathrm{add}X \subseteq \mathrm{add}\bar{Y}$. We have an exact sequence $0 \longrightarrow K_1 \longrightarrow Y \xrightarrow{\alpha} N \longrightarrow 0$. Moreover, as we observed above, $K = (D^n, Z, h) \oplus (0, N_1, 0)$ with $Z \in \mathrm{add}(Z_1 \oplus \cdots \oplus Z_t) \subseteq$ add $\bar{Y}$. Since pd ${}_A(-,N_1) \leq \mathrm{pd}_A(-,K) < k$ we can apply the induction hypothesis to conclude that $\mathrm{pd}_B(-,N_1) < k$. On the other hand, $K_1 = Z \oplus N_1$, and $Z \in add\bar{Y}$. So pd ${}_B(-,K_1) = \mathrm{pd}_B(-,N_1) < k$. Thus, to prove that $\mathrm{pd}_B(-,N) \leq k$, we only need to show that the sequence

$$(*) \qquad 0 \longrightarrow {}_B(-,K_1) \longrightarrow {}_B(-,Y) \xrightarrow{(-,\alpha)} {}_B(-,N) \longrightarrow 0$$

is exact in add$\bar{Y}$. Since $N \notin$ add$\bar{Y}$ we have that N is not a successor of M and therefore ${}_B(Z_i,N) = 0$ for each $i = 1, \cdots, t$. So we only need to prove that $(*)$ is exact in add$X$.
Let $\theta : X \longrightarrow N$ be a map in mod$A$. Then the composition $M^m \simeq M \otimes D^m \longrightarrow X \longrightarrow N$ is zero because ${}_B(M,N) = 0$. So $(0,\theta) : \bar{X} = (D^m, X, f) \longrightarrow N$ is a morphism in mod$A$, and thus it can be lifted through $X_1 \longrightarrow N$, because ${}_A(-,X_1) \longrightarrow {}_A(-,N)$ is surjective in add$\bar{X}$. Since $X_1 = (D^d, Y, g)$, then θ can be lifted through $Y \xrightarrow{\alpha} N$. This proves that $(-,\alpha)$ is surjective, as desired.

(b) Let $Z_1, \cdots, Z_t$ be the successors of M in ind$B$, let ind$A \backslash$ind$B = \{D_1, \cdots, D_s\}$, and assume that rep.dim $B = r+1$.
Let $\bar{Y}$ be a generator cogenerator of mod$B$ with the r-resolution property, and let $\bar{X} = \bar{Y} \oplus \bigoplus_{i=1}^{s} D_i \oplus \bigoplus_{i=1}^{t} Z_i$.
Let N in ind$A$ but not in add$\bar{X}$. Then N is in ind$B$. Considering again ${}_A(-,N) \in \mathcal{F}_{\bar{X}}$ and ${}_B(-,N) \in \mathcal{F}_{\bar{Y}}$, we will prove that $\mathrm{pd}_A(-,N) \leq pd_B(-,N) \leq r-1$, by induction on $k = \mathrm{pd}_B(-,N)$. The result holds for $k=0$, so we assume $k>0$.
Let $0 \longrightarrow K \longrightarrow Y_1 \longrightarrow N \longrightarrow 0$ be an exact sequence in mod$B$ such

that $Y_1 \in \text{add}\bar{Y}$ and

$$0 \longrightarrow {}_B(-,K) \longrightarrow {}_B(-,Y_1) \longrightarrow {}_B(-,N) \longrightarrow 0$$

is exact in $\text{add}\bar{Y}$. Since ${}_B(-,N)$ is not projective we have that $\text{pd}_B(-,K) < k = \text{pd}_B(-,N)$ and then by the induction hypothesis we conclude that $\text{pd}_A(-,K) < k$.
Let $D_i = (D^{n_i}, U_i, f_i)$. Then $(M,U_i) \neq 0$ for all i, and $U_i \in \text{add}(Z_1 \oplus \cdots \oplus Z_t)$. Then ${}_B(U_i,N) = 0$ and consequently ${}_B(D_i,N) = 0$. Since ${}_A(-,Y_1) \longrightarrow {}_A(-,N) \longrightarrow 0$ is exact in $\text{add}\bar{Y}$, it follows that it is also exact in $\text{add}\bar{X}$. Since $\text{pd}_A(-,K) < k$ we get that $\text{pd}_A(-,N) \leq k$, as desired, ending the proof of the theorem. □

4.2. The next result extends [11](6.1).

COROLLARY. *Let B be an artin algebra, D a division ring, M a $B-D$ bimodule and $A = B[M]$ the one point extension of B by M. If M is a simple injective module then rep.dim$A$ = rep.dim B.*

REFERENCES

1. I. Assem, F. U. Coelho, *Glueings of tilted algebras*, J. Pure Appl. Algebra **96**(3) (1994) 225-243.
2. I. Assem, F. U. Coelho, *Two-sided glueings of tilted algebras*, preprint 2002.
3. M. Auslander, *Representation dimension of artin algebras*, Queen Mary College Mathematical Notes, London, 1971.
4. M. Auslander, I. Reiten, S. Smalø, *Representation theory of artin algebras*, Cambridge Studies in Advanced Mathematics **36**, Cambridge Univ. Press, 1995.
5. R. Fossum, P. Griffith, I. Reiten, *Trivial extensions of abelian categories*, Lecture Notes in Math. **456**, Springer-Verlag, Berlin- Heidelberg-New York (1975).
6. K. Igusa, G. Todorov, *On the finitistic global dimension conjecture*, preprint.
7. D. Happel, *Triangulated categories in the representation theory of finite dimensional algebras*, London Math. Soc. Lecture Note Series **119** (1988).
8. D. Happel, *Selforthogonal modules*, in: Abelian groups and modules, Proc. of Padova Conf. (ed. Facchini et all) Math. Appl. **343** (1995), 257-276.
9. O. Iyama, *Finiteness of representation dimension*, preprint 2002.
10. H. Tachikawa, *Representations of trivial extensions of hereditary algebras*, Lecture Notes in Math. 832, Springer-Verlag, Berlin- Heidelberg-New York (1980), 579-599.
11. C. Xi, *On the representation dimension of finite dimensional algebras*, J. Algebra **226**, (2000), 332-346.
12. C. Xi, *Representation dimension and quasi-hereditary algebras*, Advances in Math. **168**, (2002), 193-212.

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