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Identification of geodesic chirping Alfvén modes and q-factor estimation in hot core tokamak plasmas in ASDEX Upgrade

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Abstract
Alfvén eigenmodes (AE) driven by ion cyclotron resonance heating are usually registered by different diagnostic channels in the hot core plasmas of large tokamaks like JET and ASDEX Upgrade. These AE appear very near to the extremum points of Alfvén wave continuum, which is modified by the geodesic effect due to poloidal mode coupling. It is shown that the AE spectrum may be explored as the magnetic spectroscopy (like Alfvén cascades by Sharapov *et al* 2001 *Phys. Lett.* A **289** 127) to determine the q-factor minimum and geodesic frequency at the magnetic axis in standard sawtoothed discharges without reversed shear.

(Some figures in this article are in colour only in the electronic version)

A series of low-frequency Alfvén eigenmodes (AE) [1, 2] driven by ICRH are regularly observed in ASDEX Upgrade discharges with a long sawtooth period. One of them (dubbed as *sierpes* [1]) with poloidal/toroidal mode numbers $M/N = 1$ is excited at the $q = 1$ surface, with a frequency of ≈80 kHz, defined by a geodesic continuum in the hot plasma core [2, 3]. The *sierpes* frequency increases slightly during a sawtooth period. Other modes have a chirping frequency spectrum, which varies strongly in the band 50–170 kHz during one sawtooth. This behavior is very surprising and differs strongly from Alfvén cascades [4], which are also observed during the initial inversed shear stage of the ASDEX Upgrade discharges [2]. These chirping Alfvén modes are detected by core soft x-ray (SXR) diagnostics but magnetic probes do not detect them. The chirping spectrum is anticipated by decreasing frequency modes and both types of behavior may be presented by the maximum frequency of the geodesic Alfvén continuum (GAC) [3, 5], whose field is chosen in the form $\sim \exp i(M\theta + N\zeta - \omega t)$, and the spectrum is

$$\omega_{\mathrm{GAC}} = \frac{\omega_{\mathrm{i}}^{*}}{2} + \frac{NV}{R_0} + \sqrt{(\omega_{\mathrm{i}}^{*})^2 + (c_{\mathrm{A}} k_{\parallel})^2 + \omega_{\mathrm{geo}}^2}, \tag{1}$$

where

$$\omega_{\mathrm{geo}}^2 \approx \left[\frac{7}{2} + 2\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}} + \left(\frac{23}{8} + 2\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}} + \frac{T_{\mathrm{e}}^2}{2T_{\mathrm{i}}^2}\right)\frac{v_{T_{\mathrm{i}}}^2}{q^2 R_0^2 \omega_{\mathrm{geo}}^2}\right]\frac{v_{T_{\mathrm{i}}}^2}{R_0^2}$$

and

$$\omega_{\mathrm{i}}^* = \frac{k_{\mathrm{b}}}{\omega_{\mathrm{ci}} n_{\mathrm{i}}}\frac{\partial}{\partial r}(v_{T_{\mathrm{i}}}^2 n_{\mathrm{i}})$$

is the ion drift frequency, $k_{\mathrm{b}} \approx MB_{\zeta}/rB$, $k_{\parallel} = (B_{\zeta}/BR_0)(N - (M/q))$, $c_{\mathrm{A}} = B/\sqrt{4\pi n_{\mathrm{i}} m_{\mathrm{i}}}$ is the Alfvén speed, $v_{T_{\mathrm{e,i}}} = \sqrt{T_{\mathrm{e,i}}/m_{\mathrm{e,i}}}$ is the electron or ion thermal speed, V is the toroidal rotation velocity, M and N are the poloidal and toroidal mode numbers, respectively, $\boldsymbol{B}$ is the magnetic field, R_0 is the major radius and q is the safety factor. This equation is asymptotically valid in the limit $(R_0 q\omega_{\mathrm{geo}})^2 \gg (v_{T_{\mathrm{i}}})^2$ when kinetic effects on ion motion may be ignored [3, 6], so that we may use it in tokamak plasmas if $T_{\mathrm{e}} > T_{\mathrm{i}}$. Instead of GAC, there is a second geodesic continuum branch [3]. This low-frequency branch is related to the ion sound continuum, whose frequency is much below ω_{GAC}, which is the reason for not taking it into account in our analysis. We note that the eigenmodes may be excited at the upper branch of GAC and one of them at rational surfaces was named the geodesic ion induced Alfvén mode (GIAM) due to a geodesic ion motion effect on this mode [3]. In [2, 7], similar modes were named beta-induced AE without any specification to the form of the GAC.

Here, we analyze the experimental conditions for chirping AE excitation in the 50–180 kHz frequency band that appears during a monster sawtooth in H-mode discharges #22325 and #23828 in ASDEX Upgrade. The characteristics of the latter discharge are similar to those of #23824, discussed in [2]. A theoretical interpretation of the chirping AE involves the theory of modes [5] excited at the maximum of the GAC in equation (1), which is modified by the geodesic effect [2, 3, 6, 7] where the magnetic surface elongation [8] is ignored due to the mode position at the plasma core. The AE propagation at the GAC extremum depends strongly on the relation between logarithmic gradients of plasma pressure profile and the magnetic shear [5]. When the plasma pressure gradients are strong, the AE may propagate at the maximum of the GAC; however, if the pressure gradients are weak, especially after a sawtooth, the modes may appear at the continuum minimum [2, 3]. Finally, we are going to use the observation of these modes to estimate the central q-value and the geodesic frequency.

We start by analyzing the discharge #22325, whose parameters are $R_0 = 1.7$ m, minor radius $a = 0.5$ m, magnetic field 2 T, plasma current 0.8 MA, ICRH power 4.5 MW, density 4×10^{19} m^3 of rather flat profile, and electron temperature profile well represented by the expression $1.91(1 - r^2/a^2)^{2.5}$ keV for 2/3 of the radius after a sawtooth crash at $t = 1.815$ s. This profile is modified to sharp $3\exp(-r^2/(0.18^2 + 0.18r))$ keV before a sawtooth crash at $t = 1.85$ s. The ion temperature profile measured at $t = 1.85$ s by the neutral beam diagnostic is reproduced by the expression $2.5\ \exp(-r^2/(0.05 + 0.12r))$ keV that gives the fundamental ion drift frequency $f_1^* \approx 2.5$ kHz in the plasma core. At $t = 1.848$ s, the frequencies $f_{\mathrm{GA}} = 103$, 112.1, 122, 134.5 and 151.3 kHz are found for the respective lowest toroidal mode numbers $N = 3$, 4, 5, 6 and 7 in the spectrogram of the magnetic probes that are similar to the SXR spectrogram presented in figure 1. The $q = 1$ position is defined by the inversion radius $r_{\mathrm{in}} = R_{\mathrm{in}} - R_0 \approx 0.135$ m from the J_049 and J_057 channels of the SXR system. These important details suggest finding the q_0-value at the GAC maximum, $r = 0$.

Employing the idea of AC excitation at the maximum of GAC where $k_N = N\delta q/R_0 q_0$ for $M = N$, we exclude the combined drift and rotation frequency $f_{\mathrm{rot}} = (f_i^*/2M) + (V/2\pi R_0)$ from the GAC equation (1) to simplify solutions, and calculate the q-factor deviation

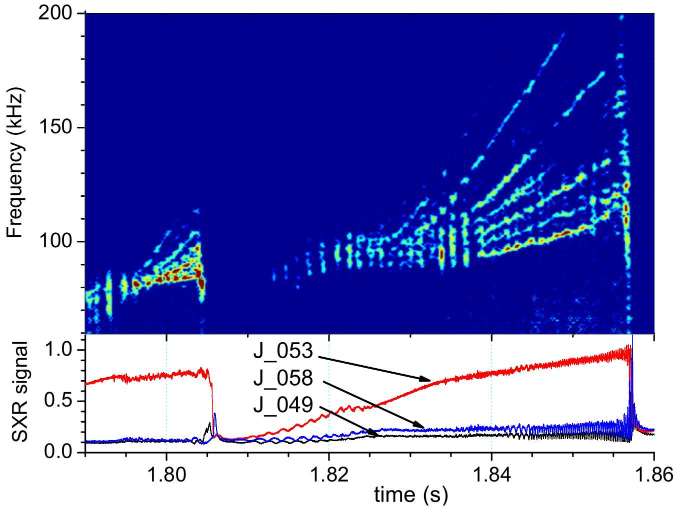

Figure 1. Spectrogram of J_053, and amplitudes of J_049, J_053 and J_058 signals in #22325.

$\delta q = 1 - q_0$ through the frequency difference between the N and N_1 AC modes as follows:

$$\frac{\omega_{N1}}{N_1} - \frac{\omega_N}{N} \approx \sqrt{\left(\frac{\omega_{\rm geo}}{N_1}\right)^2 + \left(\frac{c_{\rm A}\delta q}{R_0 q_0}\right)^2} - \sqrt{\left(\frac{\omega_{\rm geo}}{N}\right)^2 + \left(\frac{c_{\rm A}\delta q}{R_0 q_0}\right)^2}, \tag{2}$$

where drift term $(\omega_{\rm I}^*)^2$ is disregarded under the square root as small in equation (1). Initially, we have five equations for three variables ($f_{\rm fl}$, $f_{\rm geo}$ and δq) that gives ten independent solutions, which may be statistically treated. Here, using ten independent pairs of the equations in (2) for different $N = 3$–7 modes, we get ten solutions and find the geodesic frequency mean value with the respective deviation $f_{\rm geo} = 88 \pm 1$ kHz, which yields $q_0 \approx 0.962 \pm 0.002$ at $t = 1.848$ s. The same procedure gives $f_{\rm geo} = 95.6 \pm 1$ kHz and $q_0 \approx 0.955 \pm 0.002$ before the sawtooth crash at $t = 1.856$ s. Tracking the three lowest modes in figure 1 in the time interval $t = 1.84$–1.856 s, we find the equation for q-variation as $\delta q = 0.028 + 1.413 \times (t - 1.84) - 19.7 \times (t - 1.84)^2$. We note that the general accuracy of the δq estimation appears as 5%, which is defined by the central density measured by Thomson scattering and the combined drift, and the rotation corrections $f_{\rm rot}$ appear to be smaller than 1 kHz and they are ignored in our analysis. Using the electron and ion temperatures, the geodesic frequency of this discharge $f_{\rm geo} \approx 86.5 \pm 1.5$ kHz is independently estimated from equation (1) at $t = 1.848$ s whose satisfactory coincidence gives support to our approach.

We note that the spectra represent the eigenmode frequencies in figure 1 but we calculate frequency maxima of GAC, which are plotted for modes $N = 0, 3$–6 in figure 2 at $t = 1.848$ s where the $N = 0$ curve represents the classical geodesic mode frequency. Usually, the frequencies of the continuum maxima should stay very close to the observed AE frequencies. To confirm the existence of AE at the continuum maxima, we use the quasi-cylindrical tokamak model [3, 5] with geodesic effect taken into account for plasmas with hot electrons and cold ions, $v_{T_{\rm e}} \gg \omega R_0 \gg v_{T_{\rm i}}$. The relevant equation for AE is represented in the Hain–Lust form [3, 5] as follows:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(rD\frac{\mathrm{d}F}{\mathrm{d}r}\right) + \left[Q - \frac{M^2 D}{r^2}\right]F = 0 \tag{3}$$

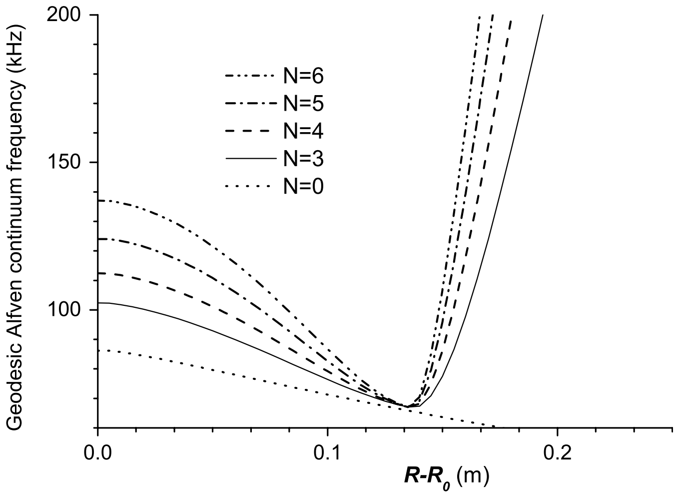

Figure 2. Plot of the geodesic Alfvén continuum for $N = 3, 4, 5, 6$ at $t = 1.848$ s (solid, dashed–dotted, dashed and chain lines, respectively), and the dotted line ($N = 0$) marks the kinetic geodesic continuum ω_{geo}.

where

$$D = \left(\frac{\omega}{c_{\text{A}}} - \frac{NV}{c_{\text{A}}R}\right)^2 - \frac{\omega_{\text{i}}^*\omega + \omega_{\text{geo}}^2}{c_{\text{A}}^2} - k_N^2,$$

$$Q = \frac{M}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{2k_N}{R_0 q} + (\omega_{\text{i}}^* - \omega_{\text{e}}^*)\frac{\omega_{\text{ci}}}{c_{\text{A}}^2}\right],$$

and $E_{\text{b}} = F/r$ is the binormal component of the perturbed electric field. The equation $D = 0$ describes the geodesic Alfvén wave continuum shown in equation (1). Generally, solutions of equation (3) may be obtained numerically but the second-order Taylor series of the plasma parameters at the magnetic axis exactly reduces equation (3) to the equation for AC discussed in [9]. It should be noted in our case that the plasma pressure [5] defines the AE dispersion, whose effect is most important when the reversed shear is absent or small.

To begin the AE study, we use the formal eikonal solution of equation (3) in the Bessel form [5] $F = F_0 J_M\left(\int_0^r \kappa\,\mathrm{d}r\right)$ for large $\kappa^2 = Q/D \gg 1/r_f^2$ valid in the plasma core. We assume the q-profile variation in the form $q(r,t) = 1 - \delta q(t)(1 - (r/r_{\text{in}})^2)$ in the interval from the magnetic axis to the inversion radius r_{in} and the approximation

$$q(r) = 1 - (q_a - 1)\frac{r^2 - r_{\text{in}}^2}{a^2 - r_{\text{in}}^2}$$

is used in the outer region of the $q = 1$ surface. Due to the sharp T_{e} profile typical for ICRH, the Q-function depends mainly on the electron temperature in the plasma core. In spite of that, due to the rapid T_{e} decrease with radius and the large q-derivative at $q \approx 1$ surface shown in figure 2, the Q-function may change sign giving a reflection point $r = r_f$ under the condition $Q = 0$. Finally, we obtain the equation for the AE frequency, $\int_0^{r_f} \kappa\,\mathrm{d}r = ((|M|/2)+s)\pi$. Generally, to have an eigenmode structure of the electric field, the D-function has to be very small at $r \approx 0$ to give enough contribution to the AE integral due to the rapid decrease in Q-function with radius. This means that the mode frequency should be very close to the respective GAC maximum. Now, we begin to discuss AE variation during a sawtooth period. After the sawtooth crash at $t = 1.805$ s in figure 1, the plasma pressure profile becomes flat, and the q-profile that

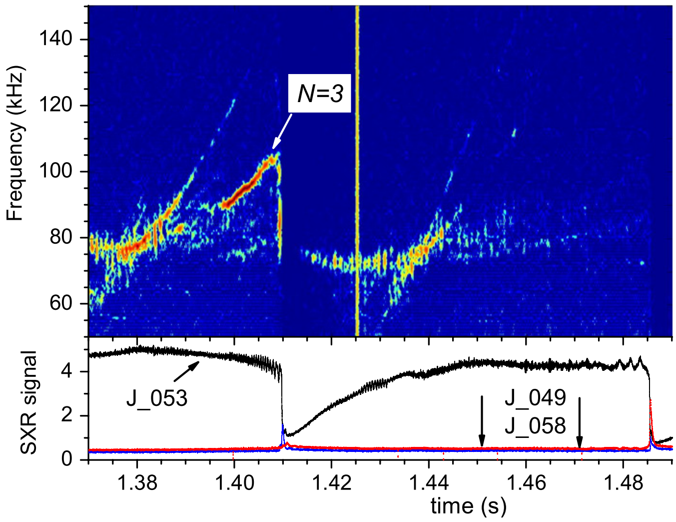

Figure 3. Spectrogram of J_053, and amplitudes of J_049, J_053 and J_058 signals in #23828 showing that the inversion radius $R_{in} - R_0 \approx 0.14$ m is defined by the J_049 and J_058 SXR channels.

produces $Q \approx 0$, $\kappa \approx 0$ and equation (3) has no eigenmode solutions. Further, the electron temperature increases and it begins to be peaked at $t = 1.815$ s. In this case, the Q-function, which is proportional to the pressure due to very small q-corrections, begins to be positive and peaked, $\kappa^2 \approx 4M^2((T_i + T_e)/m_i c_A^2 D)$. In this case, the D-function should be positive to form the eigenmode above the GAC. To confirm the existence of the eigenmodes, a numerical solution of equation (3) is also employed to find the respective eigenvalues at the maxima of GAC in figure 2. First, we defined the position of the AE reflection point $r_{in} = 0.131$ m at the $q = 1$ rational surface due to strong q-profile variation at this surface. Then, using the plasma profiles at $t = 1.848$ s, we numerically solve equation (3) for $N = 3$, and find that the AE frequency has $\omega = \omega_{GAC} + \delta\omega$ with the deviation frequency $\delta\omega/\omega \approx 0.01$.The eigenfunction found numerically is very similar to the $M = 3$ Bessel eikonal solution which has a similar deviation frequency $\delta\omega/\omega \approx 0.012$. The field has a maximum at the position $r_m = 0.75 r_{in}\sqrt{\delta\omega/\omega}$, and then, the amplitude begins to drop exponentially for $r > 3r_m$.

In addition to the analysis of the discharge #22325, we apply the same procedure to the H-mode discharge #23828, whose spectrogram of the central SXR channel (J_053) for the first sawtooth period is shown in figure 3. The parameters of this discharge are similar to those given above with a similar electron temperature profile $2.9\exp(-r^2/(0.05^2 + 0.16r))$ keV before a sawtooth crash; however, it has a higher central density 5.2×10^{19} m^3. Unfortunately, the toroidal mode numbers are not directly identified for this discharge due to weak magnetic probe signals. Nevertheless, we may calculate δq through the difference between three neighbor modes at $t = 1.398$ s using equation (2). From the calculations we found $\delta q = 0.042$ and $f_{geo} \approx 74.5$ kHz, and $N = 3$ for the lower branch that gives a small value of the rotation frequency ($\ll$1 kHz). Then, assuming that the ion temperature is not changed and the geodesic frequency is correctly defined by equation (1), and using the frequency $\approx$103.6 kHz of the $N = 3$ Alfvén mode in figure 3 before the sawtooth crash, we get $\delta q \approx 0.06$, $q_0 \approx 0.94$.We also find $N = 5$ for the highest visible AE mode using equation (1).

Finally, we may conclude that the low-frequency modes, which appear in the sawtoothed discharges heated by ICRH in ASDEX Upgrade, are identified as Alfvén eigenmodes excited at the geodesic Alfvén continuum maximum. Based on the simplified equations for this continuum, analyses of spectrograms of SXR and magnetic probe channels for these modes, we calculate the deviation δq of the q-factor from one and geodesic frequency during a sawtooth period that can serve as indirect experimental confirmation of the geodesic effect in the hot plasma core. We note that some applications of the method for q_0 calculations may clarify the physics of incomplete sawtooth crash [10] in discharges heated by ICRH.

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