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Post Conference Report

22nd International Conference on the Boundary Element Method

4 - 6 September 2000

New Hall College, Cambridge University, Cambridge, UK

Organised by **Wessex Institute of Technology, Southampton, UK**
Sponsored by **International Society of Boundary Elements (ISBE)**

The **22nd International Conference on Boundary Element Methods** has been recently held at New Hall, Cambridge University. The meeting was attended by participants from more than 20 countries, and attended by well known boundary element researchers from around the world as well as statistical members of significant number of younger scientists and engineers.

The meeting was under the Co-Chairmanship of **Professors Carlos Brebbia** and **Henry Power**, both of Wessex Institute of Technology and with the collaboration of **Dr C Y Cha** from Philips Display Components, USA as Industrial Chairman.



View of one of the sessions



Group photograph

Carlos opened the meeting with the following remarks:

"Welcome to you all to this the **22nd International Conference on Boundary Elements**, held this time in this beautiful and historical university town of Cambridge. Nearly a quarter of a century has passed since the first paper with the name of Boundary Elements appeared in 1977. I was privileged to be co-author of that seminal paper with my friend and colleague Jose Pepón, then working with me at Southampton University.

Since 1977 extraordinary advances have been made in the fundamental principles and the computational mathematics required to solve boundary integral equations using computers. Most of those papers were published in the International Journal of Engineering Analysis with Boundary elements (EABE), which continues to be the most respected and well-established vehicle for the publication of new papers on the

BEM formulations applied to gradient plasticity

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Abstract

In this paper BEM formulations applied to localization problems are discussed. The formulation is particularly dedicated to analysis of elastoplastic two-dimensional bodies for which the formulation has successfully implemented. First, we discuss the capability of BEM to deal with the localization phenomena. The proposed algorithms are formulated as usual by assuming an initial stress or strain field applied over the domain area where plastic strains take place. The initial stress integrals are approximated over triangular and quadrangular cells. The regularization of the plastic zone is achieved by following the gradient plasticity approach. The plastic multiplier integral representation is derived and several alternatives to obtain the corresponding algebraic equations are discussed. A simple numerical example is shown to illustrate the capability of the BEM technique to obtain regularized solutions in the context of gradient plasticity.

1 Introduction

Along the last three decades, the Boundary Element Method (BEM) has proved to be an appropriate numerical model to deal with an enormous number of engineering problems. The technique is nowadays a well-established procedure for the analysis of many practical engineering applications. In particular, the use of BEM to analyse non-linear problems has deserved special attention of the BEM community. Non-linear phenomena, such as plasticity, visco-plasticity and no-tension, for instance, were treated by BEM in the earlier eighties [1,2,3]. As the BEM formulations work on the stress space, it is expected that numerical solutions for non-linear analysis are better than other techniques that require differentiation of shape functions to compute the stress field.

Although proving to give good results, the BEM non-linear approaches, appearing before this decade, were all based on the very simple explicit scheme

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accomplished by constant matrix procedures. Implicit approaches have been proposed more recently. Jim et al. [4] have used implicit integration for BEM finite deformation plasticity. Telles & Carrer [5] have also proposed an implicit model to solve elastoplastic problems in the context of dynamic analysis for which they followed mass matrix approach. Bonnet & Mukherjee [6] have introduced the Consistent Tangent Operator - CTO in the boundary element technique by following a scheme similar to the one used in finite elements [7].

Localisation phenomena is certainly appropriate to be analysed by BEM; it exhibits small areas of interest inside the body, where the dissipation of energy occurs, as well as rather large displacement gradients. Though localization phenomena were and are still extensively analysed in the context of the Finite Element Method (FEM) [8,9], in an attempt to improve the numerical simulation of structural failures, there has been only limited interest in the context of the BEM [10].

In this paper, the Boundary Element Method (BEM) is applied to model non-linear structural problems where localization phenomena take place. A CTO scheme is proposed to solve J_2 elastoplastic problems. Examples are solved to illustrate that the BEM formulations based on local plasticity theories exhibit pathological mesh dependence, while the formulation based on gradient plasticity guarantees the solution regularization.

2 Constitutive equations for gradient plasticity

The small strain gradient plasticity model taken for this work is a simple modification of the flow theory of plasticity [7]. We will consider only the J_2 flow theory. The following relations describe the adopted elastoplastic model:

$$\sigma = C : (\epsilon - \epsilon^p), \quad f(\sigma, R(p)) \leq 0 \quad (1,2)$$

$$\dot{p} = \sqrt{\frac{2}{3} \dot{\epsilon}^p : \dot{\epsilon}^p}, \quad \dot{\epsilon}^p = \dot{\lambda} \partial F / \partial \sigma, \quad \dot{p} = \dot{\lambda} \partial F / \partial R \quad (3,4,5)$$

where $\dot{\sigma}$, $\dot{\epsilon}$ and $\dot{\epsilon}^p$ are the stress rate, total strain rate and the plastic strain rate, respectively; C is the matrix of elastic moduli, F gives the plastic potential, while R and p represent the size of the yield surface and the cumulated plastic strain, respectively,

The plastic multiplier $\dot{\lambda}$, defined in equations (4) and (5), satisfies the complementary (Kuhn-Tucker) and the consistency conditions, i.e.

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} f = 0, \quad \dot{f} = 0 \quad (6a,b,c,7)$$

Assuming that R depends just on p and its Laplacian $\nabla^2 p$, the yield criterion, equation (3), becomes:

$$f(\sigma, p, \nabla^2 p) = 0. \quad (8)$$

The consistency condition gives now,

$$\frac{\partial f}{\partial \sigma} : C : \dot{\epsilon} - H \dot{\lambda} + \omega \Delta \dot{\lambda} = 0 \quad (9)$$

where $(\partial F / \partial R)$ is assumed constant and

$$H = h + \frac{\partial f}{\partial \sigma} : C : \frac{\partial F}{\partial \sigma}, \quad h = -\frac{\partial f}{\partial R} \frac{\partial R}{\partial p} \frac{\partial F}{\partial R}, \quad \omega = \frac{\partial f}{\partial R} \frac{\partial R}{\partial (\Delta p)} \frac{\partial F}{\partial R} \quad (10a,b,c)$$

where, h is the plastic modulus.

The dimension of ω is H times squared length, which gives: $\omega = \alpha \ell^2$, being ℓ a characteristic length and α a material parameter.

3 Integral representation of displacements and stresses

The integral representations required for the analysis of non-linear problems, defined in a domain Ω with boundary Γ , can be easily obtained by applying Betti's principle. Splitting the total strain into its elastic and plastic components, i.e. $\epsilon_{kt} = \epsilon_{kt}^e + \epsilon_{kt}^p$, being ϵ_{kt}^e and ϵ_{kt}^p the elastic and plastic parts, respectively, the Betti's principle leads to [3]:

$$c_{ik} \dot{u}_k = \int_{\Gamma} u_{ik}^* \dot{p}_k d\Gamma - \int_{\Gamma} p_{ik}^* \dot{u}_k d\Gamma + \int_{\Omega} u_{ik}^* \dot{b}_k d\Omega + \int_{\Omega} \sigma_{ijk}^* \dot{\epsilon}_{jk}^p d\Omega \quad (11)$$

$$\dot{\epsilon}_{ij} = \int_{\Gamma} D_{ijk} \dot{p}_k d\Gamma - \int_{\Gamma} S_{ijk} \dot{u}_k d\Gamma + \int_{\Omega} D_{ijk} \dot{b}_k d\Omega + \int_{\Omega} F_{ijmk} \dot{\epsilon}_{mk}^p d\Omega + \dot{f}_{ij} \quad (12)$$

where p_k and u_k are traction and displacement components respectively; the symbol "*" is related to the fundamental solution corresponding to a Dirac delta load applied in the collocation point; the free terms c_{ik} is dependent upon the boundary geometry; D_{ijk} , S_{ijk} and F_{ijmk} are well-known kernels and f_{ij} is a free-term that appears due to the differentiation of a strong singular kernel given by:

$$f_{ij} = \frac{-1}{16(1-\nu)G} [(3-4\nu)(\delta_{ik}\delta_{jt} + \delta_{it}\delta_{jk}) - \delta_{ij}\delta_{kt}] E_{ktrs} \epsilon_{rs}^p \quad (13)$$

being E_{ktrs} the elastic tensor.

Although equation (12) is valid only for internal points, it can be properly used to describe the strain field over the whole domain. In this work however, we decided to work with boundary strain values, but avoiding the usual and imprecise procedure based on computing the normal strain parallel to the boundary by finite differences. Equation (12) can be extended to the boundary carrying out the proper limits, as shown by Guiggiani [11] for the linear case. Then, one finds that a constant equal to 0.5 multiplies the left-hand side and the independent term f_{ij} .

In order to solve a gradient plasticity problem, equation (9), which governs the plastic multiplier, has also to be taken into consideration. Following the collocation scheme, one may adopt the simplest fundamental solution to derive an integral representation; i.e. one can adopt the fundamental solution of the Laplacian equation ($\Delta \lambda^*(q, p) + \delta(q, p) = 0$), in which $\delta(q, p)$ is the Dirac's delta.

Assuming that the plastic multiplier problem, equation (9), is valid only on the region Ω_p with boundary Γ_p , the plastic region, the integral representation of the plastic multiplier λ is obtained:

$$c\dot{\lambda} = -\int_{\Gamma_p} \frac{\partial \lambda^*}{\partial n} \dot{\lambda} d\Gamma + \int_{\Gamma_p} \lambda^* \frac{\partial \dot{\lambda}}{\partial n} d\Gamma - \frac{H}{\omega_{\Omega_p}} \int_{\Omega_p} \lambda^* \dot{\lambda} d\Omega + \frac{1}{\omega_{\Omega_p}} \int_{\Omega_p} \lambda^* \frac{\partial f}{\partial \sigma} : E : \dot{\epsilon} d\Omega \quad (14)$$

where c is a free term similar the one used in equation to (11).

Note that the third integral is eliminated if one uses a proper fundamental solution of equation (9) that can be obtained from:

$$\Delta \lambda^*(q, p) - \frac{H}{\omega} \lambda^*(q, p) + \delta(q, p) = 0 \quad (15)$$

Although the fundamental solution given by equation (15) eliminates one domain integral in equation (14), it would be applied only to the particular case of constant H/ω . Thus, we continue the formulation using the simplest fundamental solution. Moreover, as the kernel of both domain integrals in equation (14) is the same, therefore only one integral has to be performed and conveniently used.

In order to complete the problem boundary conditions along the actual boundary of the body and along the interface between the plastic and elastic regions have to be assigned. Along the actual boundary we assume $\partial \lambda / \partial n = 0$, while $\lambda = 0$ is assumed for points defined over the interface.

4 Time discretization. Finite step problem

The relations presented in the previous sections were written in rates. Thus, transforming them into their incremental forms is the starting point of the numerical formulation of the boundary value problem in gradient plasticity.

Let $\Delta t = t_{n+1} - t_n$ be a typical time-step in the time discretization. The finite step boundary value problem consists of searching the solution at the end t_{n+1} of the time step when it is known at the beginning t_n of the time step. Thus, developing the procedure only for the total implicit case the given relations become:

- The plastic strain increment:

$$\Delta \epsilon^p = \Delta \lambda \left. \frac{\partial F}{\partial \sigma} \right|_{n+1} = \Delta \lambda_n \mathbf{m}_{n+1} \quad (16)$$

- The integral representations (11), (12) and (14):

$$c_{ik}(\Delta u_k)_n = \int_{\Gamma} u_{ik}^*(p_k)_n d\Gamma - \int_{\Gamma} p_{ik}^*(\Delta u_k)_n d\Gamma + \int_{\Omega} u_{ik}^*(\Delta b_k)_n d\Omega + \int_{\Omega} \epsilon_{ijk}^*(\Delta \sigma_{jk}^p)_n d\Omega \quad (17)$$

$$(\Delta \epsilon_{ij})_n = \int_{\Gamma} D_{ijk}(\Delta p_k)_n d\Gamma - \int_{\Gamma} S_{ijk}(\Delta u_k)_n d\Gamma + \int_{\Omega} D_{ijk}(\Delta b_k)_n d\Omega + \int_{\Omega} E_{ijmk}(\Delta \sigma_{mk}^p)_n d\Omega + (\Delta f_{ij})_n \quad (18)$$

$$c(\Delta \lambda)_n = \int_{\Gamma} \lambda^* \left(\Delta \frac{\partial \lambda}{\partial n} \right)_n d\Gamma - \int_{\Gamma} (\Delta \lambda)_n \frac{\partial \lambda^*}{\partial n} d\Gamma - \frac{h}{\omega} \int_{\Omega} \lambda^* (\Delta \lambda)_n d\Omega + \frac{1}{\omega} \int_{\Omega} \lambda^* \left(\frac{\partial f}{\partial \sigma} \mathbf{E} \Delta \epsilon \right)_n d\Omega \quad (19)$$

- The stress increment:

$$\Delta \sigma_n = \mathbf{E} : (\Delta \epsilon_n - \Delta \epsilon_n^p) \quad (20)$$

- The Kuhn-Tucker conditions:

$$(\Delta \lambda)_n \geq 0, \quad f(\sigma_{n+1}, \lambda_{n+1}) \leq 0, \quad (\Delta \lambda)_n f_{n+1} = 0 \quad (21)$$

With the adopted time marching scheme the initial boundary value problem for gradient plasticity is reduced to the three global and coupled integral equations, (17), (18) and (19), complemented by the local expression (20) for the stresses and the constraints (21). With the relevant boundary conditions this constitutes the finite step boundary value problem.

5 Space discretization. Boundary element formulation

The continuous finite step problem described above should be discretized now in space. This is carried out here by using the Boundary Element Method. The boundary Γ of the solid is discretized into a series of elements Γ_s , over which displacement and traction increments, Δu_k and Δp_k , are interpolated in terms of their values at a series of nodal points and approximation functions. Due to the presence of domain integrals in (17), (18) and (19), the domain Ω is also divided into cells Ω_s over which body forces and the plastic strains are approximated and then the corresponding domain integrals computed numerically.

Writing the discretized form of equations (17) and (18) for the selected collocation points, solving the system of algebraic equations, one can express boundary value increments and strain increments in terms of the plastic strain increments (see Brebbia et al.). Thus, equations (17) and (18) become:

$$\{\Delta X_n\} = \{\Delta M_n\} + [R^n] \{E : \Delta \epsilon_n^p\} \quad (22)$$

$$\{\Delta \epsilon_n\} = \{\Delta N_n\} + [S^n] \{E : \Delta \epsilon_n^p\} \quad (23)$$

where $\{\Delta M_n\}$ and $\{\Delta N_n\}$ represent the elastic solution (boundary value and strain increments); the two remaining products are contributions of the plastic strain field.

Similarly, after discretizing the boundary (moving) and the relevant domain, the gradient plasticity integral relation (19) is also transformed into its algebraic form. One may write two system of equation one related with boundary nodes and other to express the plastic multiplier at internal points. After convenient matrix operations they can be expressed by the following algebraic equation:

$$\{\Delta \lambda_n\} = -[H^\lambda] \{\Delta \lambda_n^r\} + [G^\lambda] \left\{ \frac{\partial \Delta \lambda}{\partial n} \right\}_n^\Gamma = [Q^\lambda] \left\{ \frac{\partial f}{\partial \sigma} \right\}_{n+1} : E : \Delta \epsilon_n \quad (24)$$

Particular attention must be paid when choosing the boundary element and cell approximations. Straight linear elements with linear approximations have been adopted to transform the boundary terms. The domain integrals have been performed using linear approximations over triangular cells. Moreover, strains have been computed at boundary nodes defined inside the elements (given by the dimensionless co-ordinates equal to $\xi = \pm 0.5$).

6 Moving boundary problem

Equations (22), (23) and (24) define the incremental problem under consideration. They have to be solved taking into account that the plastic region varies through the iterative process. The domain denote by Ω_n^p with boundary Γ_n^p at iteration n will be Ω_{n+1}^p and Γ_{n+1}^p at iteration $n+1$. As a consequence, all matrices the matrices computed at n must be modified to include (or eliminate) elements and cells. Figure 1 illustrates the domain definition scheme to be followed; active plastic points must be inside the plastic region, which is defined by internal boundary constituted only by elastic nodes.

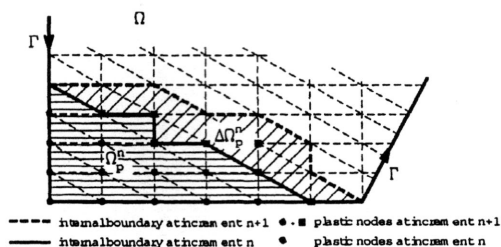


Figure 1. Internal boundary definition.

The boundary conditions, to be imposed at iteration level, have also to be considered to define the final set of algebraic equilibrium equations. The usual scheme to define the boundary conditions is schematized in Figure 2a. In this article, we have adopted a simplified scheme that reduces significantly the computations. No internal boundary is defined; instead the plastic zone is governed by enforcing the plastic multiplier to be zero over the elastic region. Using this simplification, computing new matrices is avoided; they are only rearranged through the iterative process.

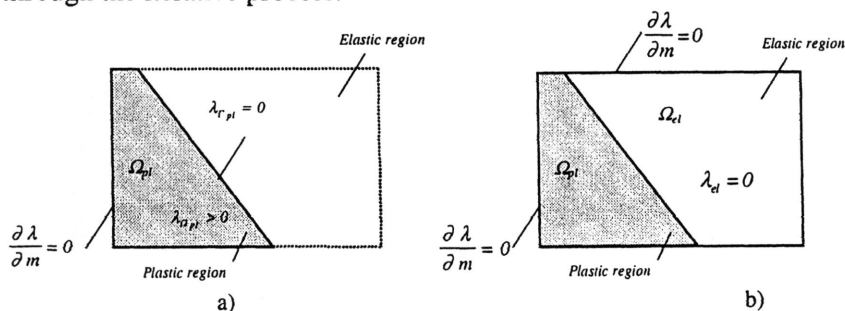


Figure 2. Boundary and internal conditions. a) with definition of internal boundary; b) without defining the internal boundary.

7 BEM Implicit formulations

7.1 Local plasticity

To obtain the implicit algorithm with the consistent tangent operator - CTO, we can start by rearranging equation (23) in its incremental form, as follows:

$$\{Y(\Delta \epsilon_n)\} = [C]\{\Delta \epsilon_n\} - \{\Delta N_n\} - [\bar{S}][C][\Delta \epsilon_n^p] = 0 \quad (25)$$

where $[C]$ is the usual Hookean elastic tensor, $[\bar{S}] = [S] + [I]$ with $[I]$ being the identity matrix

Equation (25) is solved by a Newton-Raphson scheme. Therefore, the additive corrections $\delta \epsilon_n^i = \Delta \epsilon_n^{i+1} - \Delta \epsilon_n^i$ to $\Delta \epsilon_n^i$ must be considered leading to the consistent tangent operator, which is given by the derivative of equation (25) with respect to $\Delta \epsilon$. Thus, for the correction i , the CTO matrix is given by the approximation:

$$\{Y(\Delta \epsilon_n^i)\} + \frac{\partial \{Y(\Delta \epsilon_n^i)\}}{\partial \Delta \epsilon_n^i} \delta \Delta \epsilon_n^i = 0 \quad (26)$$

with

$$C^{ep} = \partial Y(\Delta \epsilon_n^i) / \partial \Delta \epsilon_n \quad (27)$$

which, in closed form, can be written as:

$$C^{ep} = \mathbf{I} - \mathbf{S}^u \Delta \lambda_n \mathbf{E} : \partial \mathbf{m}_{n+1} \partial \Delta \epsilon_n \quad (28)$$

In the sequel, the derivative of the yield surface normal $\mathbf{m}_{n+1}$ is given in its closed form for the particular case of J_2 criterion, which is the same one presented in Simo & Taylor [7].

$$\frac{\partial \mathbf{m}_{n+1}}{\partial \Delta \epsilon_n} = \frac{2G}{\|\mathbf{s}_{n+1}\| + 2G\Delta \lambda_n} \left[\mathbf{I} - \frac{1}{3} \mathbf{1} \otimes \mathbf{1} - \mathbf{m}_{n+1} \otimes \mathbf{m}_{n+1} \right] \quad (29)$$

The yield criterion must be checked at the correction level, i.e.

$$f_{n+1}^t = J_2(\sigma_{n+1}^t) - H\lambda_n \leq 0 \quad (30)$$

where the superscript t means trial.

One should notice that the plastic correction of the adopted algorithm corresponds to a radial return to the yield surface from the elastic trial. Indeed, this elastic trial is:

$$\Delta \sigma^t = \sigma_{n+1}^t - \sigma_n = \mathbf{E} : \Delta \epsilon_n \quad (31)$$

from which we can also find the deviatoric stress trial $\mathbf{S}_{n+1}^t$ as

$$\mathbf{S}_{n+1}^t - \mathbf{S}_n = \mathbf{E} : \Delta \mathbf{e}_n = 2G\Delta \mathbf{e}_n \quad (32)$$

where $\Delta \mathbf{e}_n$ is the deviatoric part of the strain increment.

Taking into account the constitutive equation (20), one can compute the deviatoric stress at the end of the increment as

$$\mathbf{S}_{n+1} = \mathbf{S}_{n+1}^t - 2G\Delta \lambda_n \mathbf{m}_{n+1} \quad (33)$$

7.2 Gradient plasticity

For the gradient plasticity case little modification is required in the above formulation. Now, one has to solve the two coupled non-linear algebraic equations, (23) and (24), for the strains and the plastic multiplier, conveniently rewritten as follows,

$$\{Y^u(\Delta \epsilon_n, \Delta \lambda_n)\} = [\mathbf{C}]\{\Delta \epsilon_n\} - \{\mathbf{N}_n\} - [\mathbf{S}^u]\{[\mathbf{C}][\Delta \epsilon_n^p]\} = 0 \quad (34a)$$

$$\{Y^\lambda(\Delta \epsilon_n, \Delta \lambda_n)\} = \{\Delta \lambda_n\} - \{\Delta \mathbf{Z}_n\} - [\mathbf{S}^\lambda]\{\mathbf{m}_{n+1} : \mathbf{E} : \Delta \epsilon_n^p\} = 0 \quad (34b)$$

The consistent tangent operator is now given by differentiating equations (34) with respect to $\Delta \epsilon$ and $\Delta \lambda$. For the particular case of J_2 yield surface one can find that

$$\partial \mathbf{m}_{n+1} / \partial \Delta \lambda_n = 0 \quad (35)$$

therefore the obtained tangent matrix is the same one already shown for the local plasticity case.

The complementary relations also are valid replacing equation (29), which now reads:

$$f_{n+1}^i = J_2(\sigma_{n+1}^i) - H\lambda_n + \omega \nabla^2 \lambda_n \leq 0 \quad (36)$$

8 Numerical example

The formulation described here is validated by analysing a rectangular block subjected to compression along its upper face. The analysis is carried out under plane strain conditions, assuming the J_2 yield surfaces and using both the local and gradient models described in the previous sections. The data used for this analysis are: Young modulus $E = 2000$ Mpa; Poisson's ratio $\nu = 0.00$; softening modulus $h = -0.0125E$; yield stress $\sigma_y = 2$ Mpa.

Figure 3 shows the dimensions of the block, the loading and the boundary conditions and also the coarse mesh (containing 128 cells) used in the analyses. Two other finer discretizations, containing 512 and 2048 cells respectively were also considered to illustrate the mesh dependency.

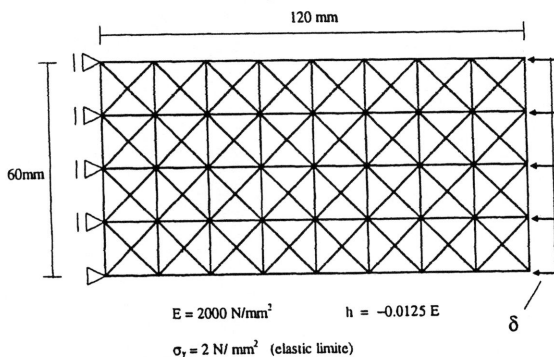


Figure 3. Rectangular block: dimensions, boundary conditions and loading

To trigger the localisation phenomenon, a weaker zone near the center of the block was considered. The yield stress of 2.0Mpa is assumed everywhere, except over the weaker internal square near the center, where the yielding stress is reduced by 0.2Mpa. The plate is loaded by applying a uniform displacement along the block vertical side as indicated in Figure 3.

Figure 4a shows the lack of objectivity of the solution obtained by using the classical local plasticity theory. The dissipation zone is defined over a narrow region one-cell length wide. Figure 5 illustrates the plastic strain developed over the domain with very large values along the localized narrow region. As the plastic strains are concentrated over the localized zone defined by the mesh length, they are it very high for the finer meshes.

The capability of the proposed gradient plasticity BEM model to capture the stabilised zone is clearly demonstrated in Figure 6. One can see that the plastic strain zone is now a narrow zone that contains several row of cells, even if the

weaker region (the marked square) is greater than the localized band. The same regularized zone has been captured for all experimented discretizations.

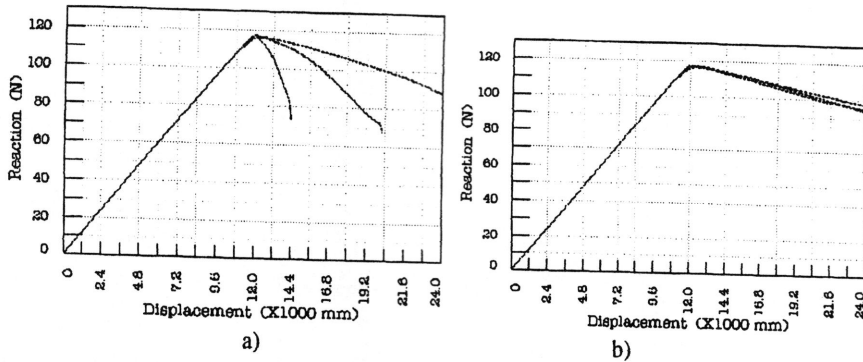


Figure 4. Load-displacement curves. a) local model; b) gradient plasticity model

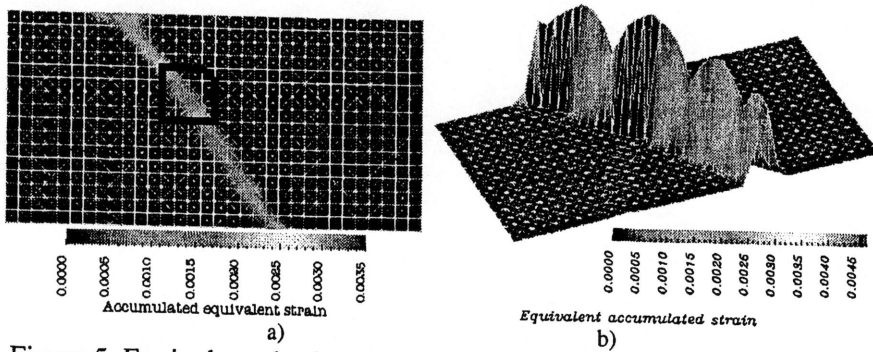


Figure 5. Equivalent plastic strain. a) Narrow region; b) 3D visualization.

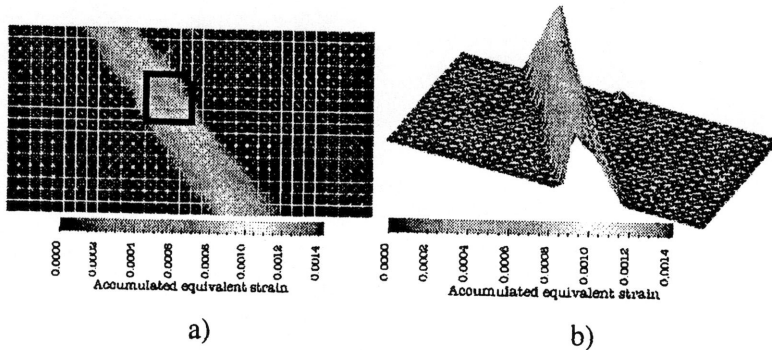


Figure 6. Stabilized solution. a) Plastic strain; b) 3D visualization.

Figure 6 is shown in order to better visualize the plastic strain concentration, now clearly defined over the narrow dissipation zone formed by several rows of cells. The same distribution of plastic strains is also computed when other discretizations were used, demonstrating the objectivity of the computations. Figure 4b shows the load-displacement responses for the three discretizations when using the gradient plasticity model. The three responses are almost the same, exhibiting therefore regularized effects and objectivity of the computations.

8 Conclusions

Implicit BEM formulations for local and gradient plasticity theories have been presented. The plastic multiplier algebraic equation for gradient plasticity has been coupled with the usual non-linear BEM equations. The global consistent tangent operator has been derived. The selected example has demonstrated that the local plasticity scheme gives solutions with clear lack of objectivity, while the gradient plasticity model regularizes the numerical responses.

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