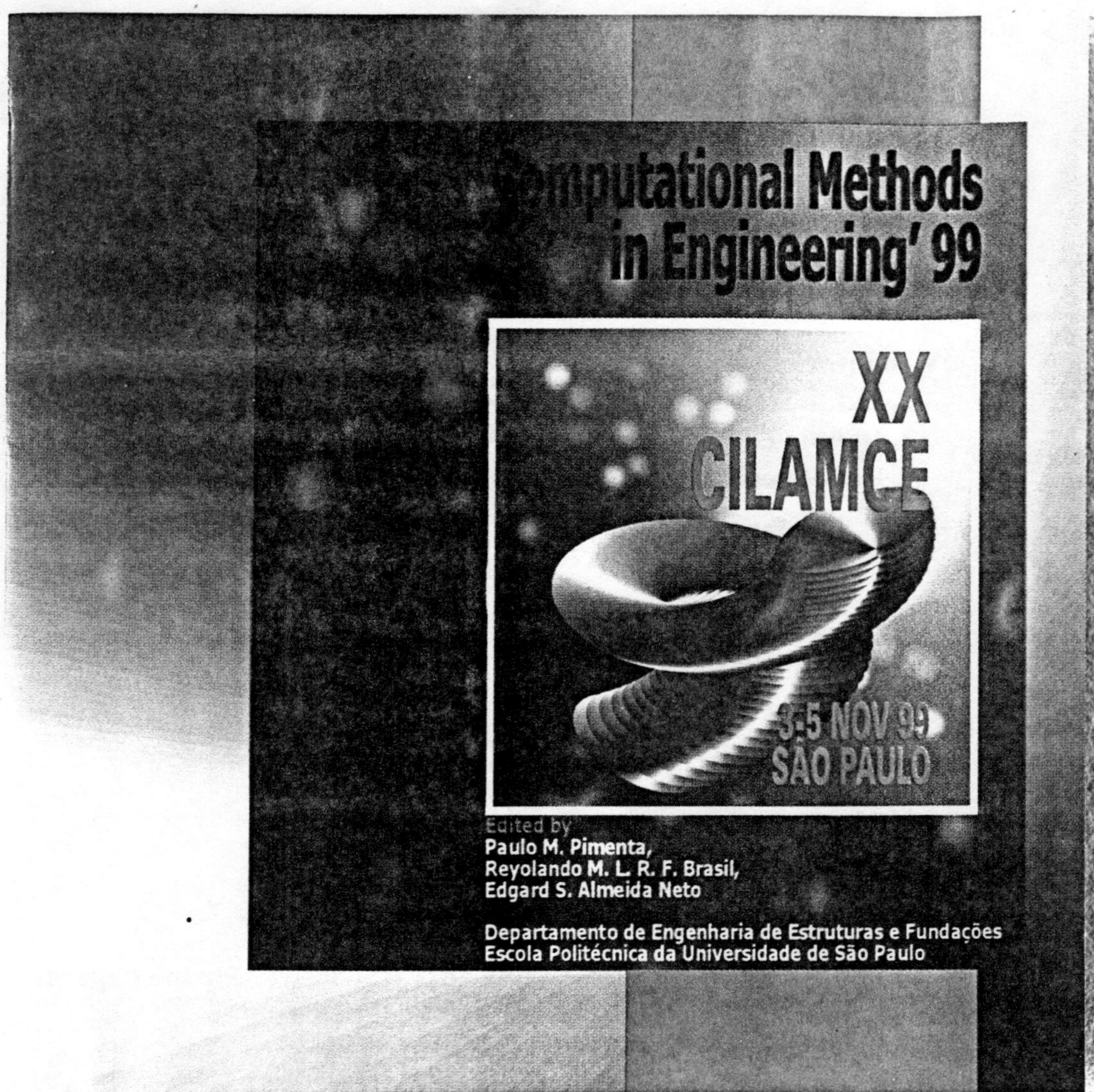
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SOIL-STRUCTURE INTERACTION WITH THE CONSIDERATION OF A RIGID LAYER IN THE SOIL

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Abstract. *The consideration of soil-structure interaction in the structural analysis of buildings can be very important in some situations, as the case of buildings on shallow foundations. Therefore, a building can be better represented as a superstructure/substructure/soil massif system. The superstructure can be modelled with bar elements. The substructure should be modelled in order to guarantee the interaction among the parts of the system. For the soil, a practical mathematical model for daily application should be used. This model should also represent the soil satisfactorily, since this medium is quite heterogeneous. A. special element of rigid footing based on the Boundary Element Method has been adopted to represent the substructure together with the soil massif. Its formulation has been improved for a better representation of the soil, including the existence of a rigid layer in the massif. Therefore, the real importance of the rigid layer consideration can be verified by means of a numerical example.*

1 INTRODUCTION

The consideration of the soil-structure interaction in the structural analysis of buildings can be very important in some situations, as the case of buildings on shallow foundations. In order to verify the real influence of that consideration on the effort and displacement results, it is necessary that the construction modelling represents the reality well.

Therefore, a building can be better represented as a superstructure–substructure–soil massif system, as shown in the scheme of Figure 1. The parts of the system should be well modelled so that the results of the analysis are close to reality.

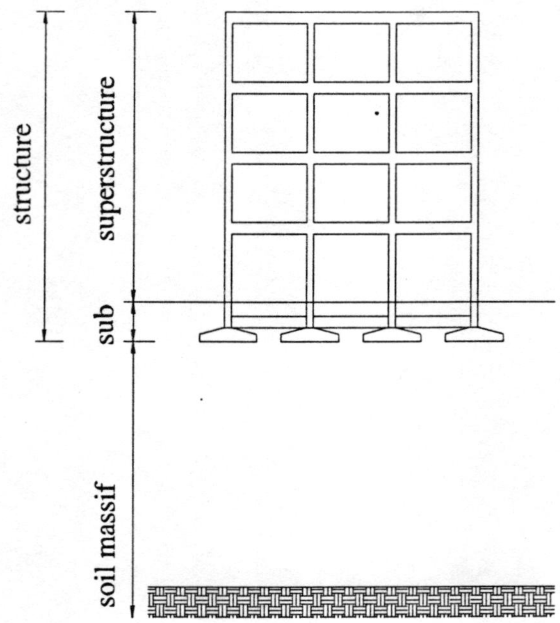

Figure 1 Superstructure–substructure–soil massif system.

Using the Finite Element Method, the superstructure can be modelled with bar elements, with six degrees of freedom at each end. The infrastructure should be modelled in order to guarantee the interaction among the parts of the system. For the soil, a practical mathematical model for daily application should be used. This model should also represent the soil satisfactorily, since this medium is quite heterogeneous.

Therefore, a special element of rigid footing has been adopted to represent the infrastructure together with the foundation soil. That element is based on the Boundary Element Method and has been developed by Ramalho[1]. Its formulation has been improved for a better representation of the soil, including the existence of a rigid layer in the massif. Not rarely, a layer with such characteristic might be found in soil, at a certain depth, for the applied load level. For example, it can be rock or a much-compacted soil.

The aim of this paper is to present the modification implemented in the "rigid footing" element formulation and to verify the real importance of the consideration of a rigid layer in soil by means of a numerical example. Effort has been made in order to produce satisfactory results avoiding harming the practicality of the element.

2 IMPLEMENTATION

In the element formulation, the soil was originally considered a continuum semi-infinite, linear elastic, isotropic and homogeneous medium. A domain such as the soil taken as a semi-infinite space leads to greater settlements, since there would not be resistance to vertical displacements at any point in the medium. The modification consists of defining the depth of a rigid surface, which is the upper surface of the rigid layer. Since it is an adaptation of an existing program, that surface is supposed to be horizontal.

With such purpose, an artifice proposed by Steinbrenner[2] has been applied, which is described below:

When a vertical load is applied to the flat boundary of a continuum semi-infinite, isotropic, homogeneous and elastic domain, the variation of the induced displacements of the points in the line of action of the resultant with the depth can be represented in the diagram of Figure 2. The curve tends to zero at an infinite distance of the point of application of the force. This displacement configuration corresponds to Boussinesq's solution[3], which can be considered as a simplification of Mindlin's solution[4], when the load is applied to the boundary. In this study, Mindlin's solution, whose formulation was in the program, has been adopted.

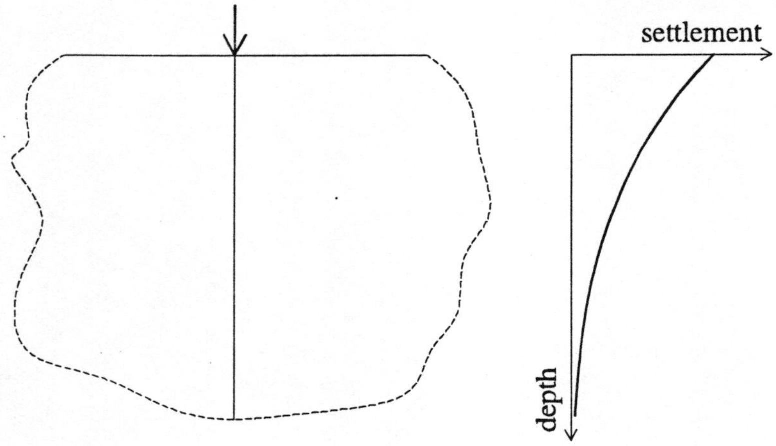

Figure 2 Settlement–depth curve for a continuum, semi-infinite, isotropic, homogeneous, elastic medium.

If there is a rigid layer at a certain depth in the massif, the displacement of the point in its surface should be zero. Thus, Steinbrenner proposed to impose the null value to the settlement of that point, and to recalculate the displacements at the other points of the medium. For each point, the previously calculated displacement ought to be reduced by the prior determined settlement of the point in the rigid surface right above it. A scheme is shown in Figure 3.

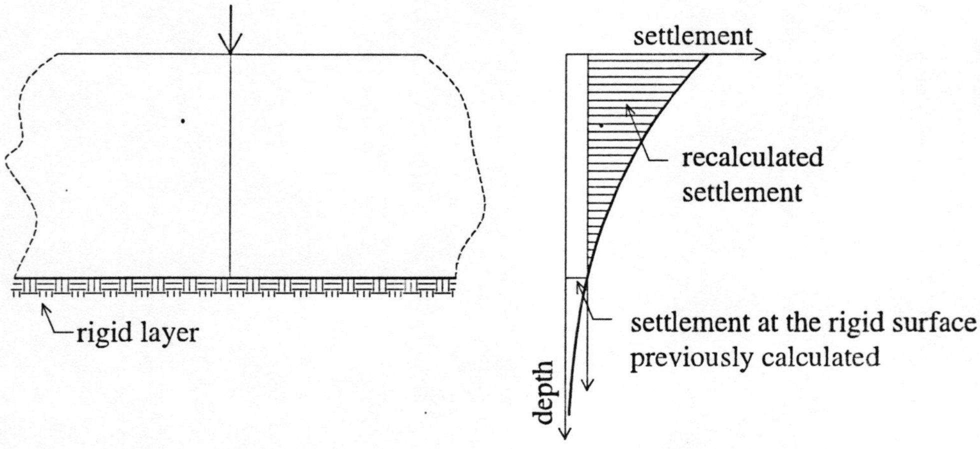

Figure 3 Settlement recalculation according to Steinbrenner's artifice.

For a better understanding of the accomplished modification, a brief introduction to the fundamental solutions of the three-dimensional problem is presented next.

2.1 Three-dimensional elastic problem

Defining a body $\Omega + \Gamma$, in which Ω is a three-dimensional region and Γ is its bounding contour, consider that it is filled with an elastic, linear and isotropic material, characterised by its Young's modulus (E) and its Poisson's ratio (ν). In this case, the equations of equilibrium according to the theory of elasticity, in index notation, can be written as follows:

$$\sigma_{ij,j} + b_i = 0 \tag{1}$$

where σ_{ij} is stress components and b_i is body forces.

The stress-strain relationship according to Hooke's law is given by:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G\varepsilon_{ij} \tag{2}$$

where: ε_{ij} is strains;

δ_{ij} is the Kronecker delta, which is equal to zero if $i \neq j$ and to 1 if $i = j$;

λ and G are, respectively, Lamé's constant and the modulus of elasticity in shear, defined by:

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$G = \frac{E}{2(1+\nu)}$$

The relations among strains and displacements (u_i) are expressed by:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

Finally, for the entire definition of the elastic problem, the boundary conditions at a point $Q \in \Gamma$ must be determined:

$$u_i(Q) = \bar{u}_i(Q) \tag{4}$$

$$p_i(Q) = \bar{p}_i(Q) \tag{5}$$

where u_i and p_i are displacements and surface forces defined at the boundary.

Adequate manipulation of equations (1) to (3) leads to the differential equations of the elastic problem in terms of displacements or Navier's equations, whose solution allows the calculation of all important parameters for the case of a three-dimensional domain:

$$u_{j,ii} + \frac{1}{1-2\nu} u_{i,ij} + \frac{1}{G} b_j = 0 \tag{6}$$

2.2 Mindlin's fundamental solution

To formulate the integral equations of the elastic problem the use of a fundamental solution of the differential equation (6) becomes necessary. Consider points s and q in a domain Ω^*, which contains Ω. This solution may be physically understood as responses in q to the application of concentrated forces F_i^* at s.

Different solutions are given for each domain with different boundary conditions. Therefore, there is Kelvin's solution for an infinite elastic solid, Mindlin's solution for a semi-infinite domain and Boussinesq's solution, which can be considered as a singular case of Mindlin's, when loads are applied to the bounding surface.

In this study, Mindlin's fundamental solution has been used. It includes all situations covered by Boussinesq. This solution is obtained from Navier's equation:

$$u^*_{j,ii} + \frac{1}{1-2\nu} u^*_{i,ij} + \frac{1}{G} \Delta(s,q) F^*_j = 0 \tag{7}$$

In the mathematical formulation of the prior equation, the body forces have been substituted by the expression:

$$b^*_j(q) = \Delta(s,q) F^*_j(s) \tag{8}$$

where $\Delta(s,q)$, Dirac's delta, is so defined:

$\Delta(s,q) = 0 \quad \text{if} \quad s \neq q$

$\Delta(s,q) = \infty \quad \text{if} \quad s = q$

$\int_V \Delta(s,q) dV = 1$

Representing the displacements u_i^* of point q due to the applied forces at s gives:

$$u^*_i(q) = U^*_{ji}(s,q) F^*_j(s) \tag{9}$$

where in array U_{ji}^*, the first index corresponds to the unit load direction, and the second index corresponds to the displacement component, as shown in Figure 4. The surface forces are not mentioned here, as they are not used.

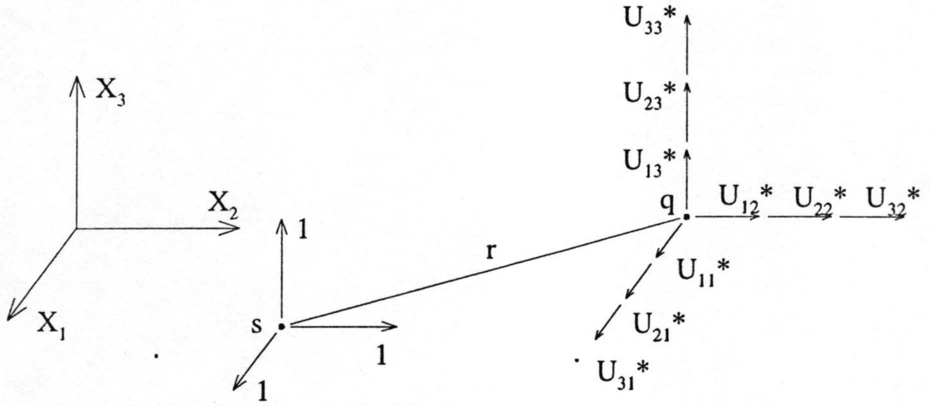

Figure 4 Fundamental displacement matrix components.

Arranged in an array, the fundamental displacement matrix of the three-dimensional problem becomes:

$$U_{ji}^{*}(s,q) = \begin{bmatrix} U_{11}^{*} & U_{12}^{*} & U_{13}^{*} \\ U_{21}^{*} & U_{22}^{*} & U_{23}^{*} \\ U_{31}^{*} & U_{32}^{*} & U_{33}^{*} \end{bmatrix} \tag{10}$$

Next, the expressions of each component above are presented, as they had been determined by Mindlin for a semi-infinite, elastic, linear, homogeneous and isotropic domain. Point *s*, where loads are applied at, is an internal point of the domain located at a depth *C* from the bounding surface, according to the outline of Figure 5.

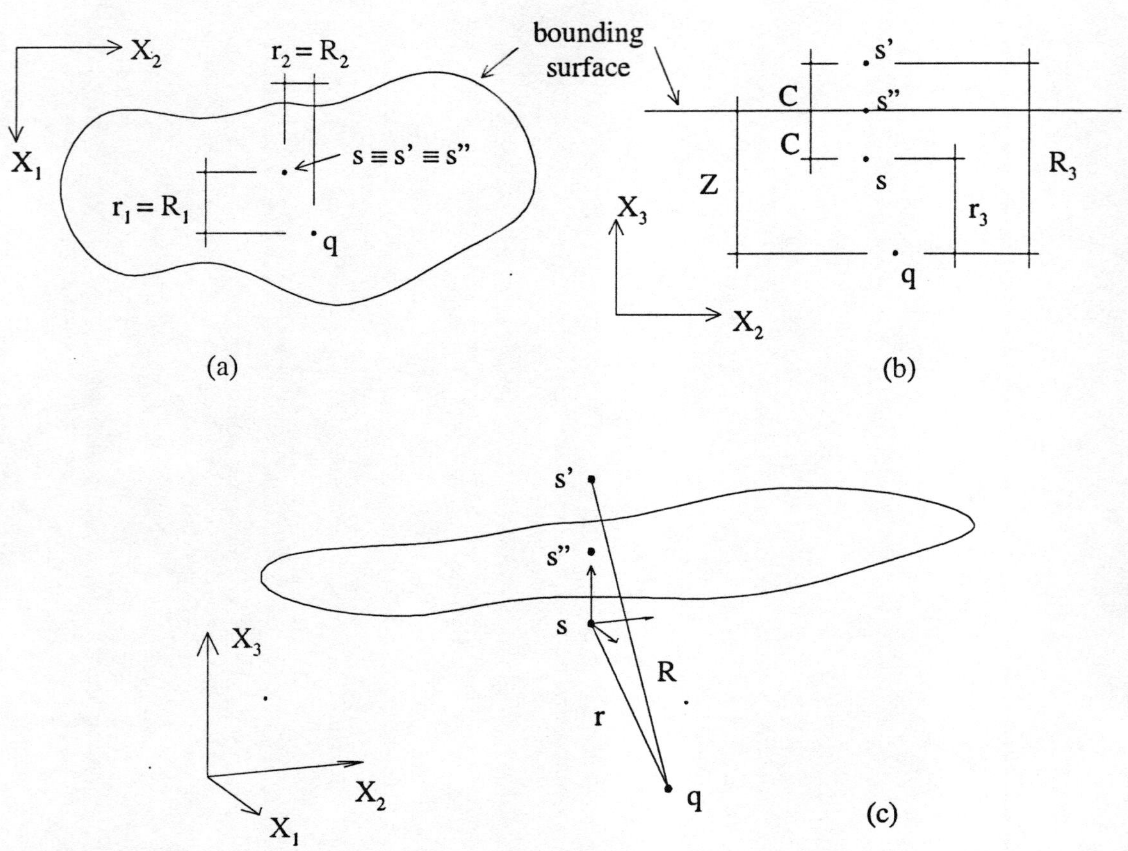

Figure 5 Mindlin's semi-infinite space. (a) Plane X_1X_2 view. (b) Plane X_2X_3 view. (c) Perspective.

$$U_{11}^{*} = K_d\left\{\frac{3-4\nu}{r}+\frac{1}{R}+\frac{r_1^2}{r^3}+\frac{(3-4\nu)r_1^2}{R^3}+\frac{2CZ}{R^3}\left(1-\frac{3r_1^2}{R^2}\right)+\frac{4(1-\nu)(1-2\nu)}{R+R_3}\left[1-\frac{r_1^2}{R(R+r_3)}\right]\right\} \tag{11}$$

$$U_{12}^{*} = K_d r_1 r_2\left[\frac{1}{r^3}+\frac{3-4\nu}{R^3}-\frac{6CZ}{R^5}-\frac{4(1-\nu)(1-2\nu)}{R(R+R_3)^2}\right] \tag{12}$$

$$U_{13}^{*} = -K_d r_1\left[\frac{r_3}{r^3}+\frac{(3-4\nu)r_3}{R^3}-\frac{6CZR_3}{R^5}+\frac{4(1-\nu)(1-2\nu)}{R(R+R_3)}\right] \tag{13}$$

$$U_{21}^{*} = U_{12}^{*} \tag{14}$$

$$U_{22}^{*} = K_d\left\{\frac{3-4\nu}{r}+\frac{1}{R}+\frac{r_2^2}{r^3}+\frac{(3-4\nu)r_2^2}{R^3}+\frac{2CZ}{R^3}\left(1-\frac{3r_2^2}{R^2}\right)+\frac{4(1-\nu)(1-2\nu)}{R+R_3}\left[1-\frac{r_2^2}{R(R+r_3)}\right]\right\} \tag{15}$$

$$U^*_{23} = \frac{r_2}{r_1} U^*_{13} \tag{16}$$

$$U^*_{31} = -K_d r_1 \left[\frac{r_3}{r^3} + \frac{(3-4\nu) r_3}{R^3} + \frac{6CZR_3}{R^5} - \frac{4(1-\nu)(1-2\nu)}{R(R+R_3)} \right] \tag{17}$$

$$U^*_{32} = \frac{r_2}{r_1} U^*_{31} \tag{18}$$

$$U^*_{33} = K_d \left[\frac{3-4\nu}{r} + \frac{8(1-\nu)^2 - (3-4\nu)}{R} + \frac{r_3^2}{r^3} + \frac{(3-4\nu)R_3^2 - 2CZ}{R^3} + \frac{6CZR_3^2}{R^5} \right] \tag{19}$$

where: $K_d = \dfrac{1+\nu}{8\pi E(1-\nu)}$

$r_i = x_i(q) - x_i(s)$

$R_i = x_i(q) - x_i(s')$

$C = \dfrac{|X_3(s) - X_3(s')|}{2}$

$Z = |X_3(s'') - X_3(q)|$

2.3 Subroutine introduced

The necessary modification of the rigid footing element has been effectively made including a new subroutine. Its function is to calculate the settlement value at the rigid layer upper surface depth and to reduce it from the footing vertexes determined settlements. Applying equation (9) to vertical displacements, the recalculation may be mathematically explained.

$$u^*_3(q) = U^*_{j3}(s,q)F^*_j(s) \tag{20}$$

Let *r* be a point at the rigid surface located in the same vertical line as *q*:

$$u^*_3(r) = U^*_{j3}(s,r)F^*_j(s) \tag{21}$$

Simply reducing the settlement at *r* from the settlement at *q*:

$$u^*_3(q)_{\text{recalculated}} = u^*_3(q) - u^*_3(r)$$

$$u^*_3(q)_{\text{recalculated}} = U^*_{j3}(s,q)F^*_j(s) - U^*_{j3}(s,r)F^*_j(s)$$

$$u^*_3(q)_{\text{recalculated}} = [U^*_{j3}(s,q) - U^*_{j3}(s,r)]F^*_j(s)$$

Finally, it results in:

$$U^{*}_{j3}(s,q)_{recalculated} = U^{*}_{j3}(s,q) - U^{*}_{j3}(s,r) \tag{22}$$

It is worth noticing again that Steinbrenner's artifice is only suitable for settlements. That is the reason to use only displacements in direction 3, defined as vertical.

3 RIGID LAYER INFLUENCE

To verify the real importance of considering a rigid layer in soil, a 21-storey building has been taken as an example. It has been submitted to two different analyses considering soil-structure interaction: first with a semi-infinite soil massif and then with a rigid surface located at 15 m of depth.

By comparing the results, it has been possible to observe that all nodal vertical displacements have been reduced when considering the rigid layer existence. At the structure base that reduction ranged from -18,16% to -36,69% of the previously determined value for semi-infinite soil. At the building top, the vertical displacements have been reduced within a range from -10,68% to -28,38%.

Vertical reactions at column bases have also been modified characterising effort redistribution. As a general behaviour, loads have been transferred from columns tending to present greater settlements to the neighbouring ones. Although those changes have occurred, they were not significant.

Considerable changes occurred in bending moment values of almost all columns. Percentage differences ranged between -70,86% and 75,11%. Only columns P13, P14, P15 and P16 have not presented significant differences.

The same could be noticed in the beams efforts. Percentage differences ranged between -93,53% and 120,53% for bending moment and between -95,56% and 73,40% for shear force. Changes have not been important only in beams V11 and V12.

4 CONCLUSION

Considering a rigid layer existence at 15 m of depth in soil massif for the analysed structure, it was observed that all settlements were reduced, reaching percentage differences up to -36,67%. Bending moments in columns and beams and shear forces in beams have also presented very significant changes. Among all verified parameters, only vertical reactions at column bases did not changed considerably.

Therefore, the changes were, generally, very significant. This fact leads to the conclusion that the rigid layer location in the soil massif plays an important role in soil-structure interaction process, and should be included in such analyses.

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