

# Preservation of Topological Properties by Forcing

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**Keywords:** Forcing, Cohen Reals, Preservation of Topological Properties

Forcing is a technique (or series of techniques) in set theory that gives us an extension of a model of set theory. By adding new functions to the universe, it can break topologies, as they can stop being closed by unions. However, we can use the initial topology as a base and obtain a new topological space. A natural question arises: when does this new space retain the properties of the original. In [1], Watson was one of the first to ask such questions about Cohen forcing, the original form of forcing.

The Lindelof property, a generalization of compactness, is a topological property that is not necessarily preserved by forcing in general. However, it is preserved by Cohen forcing. This was shown in [2], using the technique known as endowments.

Our objective is to provide an overview of recent results, such as attempts to generalize the spaces [3], or the forcing [4], and the techniques used. We aim to offer insight into the current state of research in this area.

## References

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