Technical Note

Entropic Friction Factor Modeling for Mineral Slurry Flow in Pressurized Pipes

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Abstract: The principle of maximum entropy (PME) relies on information theory, Shannon entropy, and four constraints, namely the total probability and conservation laws of mass, momentum, and energy. This paper applies PME theory to an exactly accurate data set available in the literature to develop a model that relates the friction factor (f) to the entropy parameter (M). The proposed model exhibits fair adherence to the experimental data, and it was validated by multiphase flow pumping tests with concentrated iron ore slurry. Extending the use of the proposed model for multiphase flow, particularly mineral slurries, allows for the determination of apparent viscosity and Reynolds number without resorting to rheological measurement. The deviations between the M parameter obtained from the proposed model and that reported in the literature were smaller than 5% for the iron ore slurry. It has been demonstrated that the PME is a particularly important tool for hydraulic systems, especially in multiphase flow such as that of mineral slurries containing a high content of solids. **DOI: 10.1061/(ASCE)HY.1943-7900.0001934.** © *2021 American Society of Civil Engineers.*

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Introduction

The design of hydraulic transport systems for mineral slurries in pressurized pipes is based on the rheological behavior of fluid in pipelines (Wilson et al. 2006; Peker and Helvaci 2008). The head loss relies on the well-known Darcy-Weisbach equation (Bombardelli and García 2003; Zeghadnia et al. 2019), whereas the velocity profile is usually represented by the Prandtl-von Kárman universal law, expressed in Eq. (1), applied to turbulent flow through pipes with axial symmetry (Schlichting 1979; Chiu and Hsu 2006). However, determination of the apparent viscosity is a challenging task because not only is it a molecular property of the fluid but it also depends on the flow regime and turbulence effects (Souza and Moraes 2017)

$$\frac{(u_{\max} - u)}{u^*} = -\frac{1}{k}\ln\left(1 - \frac{r}{R}\right) \quad (1)$$

As for the velocity distribution proposed by Prandtl-von Kárman, physical inconsistencies lead to inaccuracies in both the regions near the wall and at the pipe center (Chiu et al. 1993; Souza and Moraes 2017).

The probabilistic approach based on information theory and Shannon entropy has recently received increased attention because it combines both the deterministic and probabilistic domains, allowing the construction of analytical models for the velocity distribution, shear rate, and friction factor without any physical inconsistencies (Singh 2013, 2014).

Entropy can be interpreted as a measure of the uncertainty, density of information, or degree of uniformity of a given probability distribution (Singh 1997, 2011, 2014). The principle of maximum entropy (PME) is based on information theory, Shannon entropy, and four constraints, namely, the total probability and the laws of conservation of mass, momentum, and energy (Singh 1997; Lofrano et al. 2019). The entropy maximization method, based on Lagrange multipliers, allows a given probability distribution to become as uniform as possible (Chiu 1987). The PME was disseminated in the realm of hydraulic engineering in connection with the work of Chao-Lin Chiu and coworkers (Chiu and Chiou 1986; Chiu 1987, 1988, 1989), whose proposed conceptual models were based on the theory of probability. The variational principle is based on Shannon entropy (H), corresponding to information on a random variable or to the measure of uncertainty represented by a probability distribution. In hydraulic engineering, the random variable to be considered is the fluid flow velocity whose uncertainty should be the maximum, thus requiring a method to maximize the Shannon entropy of the velocity (Singh 2014). The success of applying the PME to either open channels or pressurized pipes was consolidated with the proposal of conceptual mathematical models for velocity distribution, shear stress, and particle concentration (Chiu 1987; Chiu et al. 1993, 2000; Chiu and Said 1995). In their investigation into flow through pipes, Chiu et al. (1993) developed a theoretical model for the distribution of velocity,

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based on probability and entropy concepts, applicable to any flow condition, in either a laminar or turbulent flow regime, within smooth or rough pipes. That model was obtained by maximizing the Shannon entropy of the velocity $H(u)$ subject to only two constraints (total probability and mass conservation) by means of the Lagrange multiplier method, as depicted by Eq. (2) (Chiu and Hsu 2006; Singh 2014)

$$u = \left(\frac{u_{\max}}{M}\right) \ln\left\{1 + [e^M - 1]\left[1 - \left(\frac{r}{R}\right)^2\right]\right\} \tag{2}$$

According to Chiu et al. (1993), the velocity distribution represented by Eq. (2) is consistent, satisfying all the premises of flow through pipes, such as maximum velocity at the center of the pipe and null velocity at the wall in addition to null velocity gradient at the center and nonnull velocity gradient at the wall. The entropy parameter (M) is mathematically defined as the product of maximum flow velocity ($u_{\max}$) and the second Lagrange multiplier (λ_2), indicating the uniformity of the velocity distribution (Chiu 1988).

Three models are found in the literature to determine the entropy parameter (M). The model represented by Eq. (3) relates M to fluid velocities, according to Chiu et al. (1993). The approach pursued by Souza and Moraes (2017) allows for the determination of M based on the friction factor (f) and Reynolds number, according to Eq. (4). Finally, Chiu et al. (1993) propose a model based solely on the friction factor, expressed as follows by Eq. (5):

$$\frac{\bar{u}}{u_{\max}} = \frac{e^M}{(e^M - 1)} - \frac{1}{M} \tag{3}$$

$$f = \left(\frac{32}{\mathrm{Re}_a}\right)\left[\frac{(e^M - 1)^2}{(Me^M - e^M + 1)}\right] \tag{4}$$

$$f = 0.0983\left(\frac{0.17Me^M + e^M - 1.17M - 1}{Me^M - e^M + 1}\right)^2 \tag{5}$$

The three models available in the literature to determine the entropy parameter (M) are unsuitable when applied to concentrated mineral slurries. The application of Eq. (3) requires prior knowledge of the value of $u_{\max}$, whose determination is not trivial because not only do solid particles at high concentrations in water block probes but they also interfere with indirect techniques. According to Whiten et al. (1993) and Senapati and Mishra (2014), the measurement of slurry viscosity in rotational rheometers is jeopardized by particle settling, wall slip, and turbulence, thereby hindering reliable determination of the Reynolds number with Eq. (4). Although Eq. (5) overcomes the need for previous knowledge of slurry viscosity and maximum fluid velocity, it exhibits physical inconsistencies at the pipe center and close to the wall because it is based on the Nikuradse empirical velocity distribution (Bogue and Metzner 1963). In view of the aforementioned reasons, mineral slurries call for a model to determine the entropy parameter (M), and this paper represents an attempt to fill for this gap in the literature.

Research groups from the University of Oregon (Swanson et al. 2002) and Princeton University (Zagarola and Smits 1998; McKeon et al. 2004a, 2005) yielded highly accurate results for the friction factor and Reynolds number, for different fluids and pumping facilities. Swanson et al. (2002) conducted experiments in a tubular device with a 4,672-mm nominal diameter using different fluids (helium, oxygen, nitrogen, carbon dioxide, and sulfur hexafluoride) in a wide range of Reynolds numbers ($10 \leq \mathrm{Re} \leq 10^6$). Experiments from the Princeton University resorted to an experimental apparatus with a 129-mm-nominal-diameter pipe, thereby adopting compressed air as the work fluid. Using the results maintained by both research groups, Mckeon et al. (2004a) plotted the friction factor (f) versus the Reynolds number, verifying coincident curves under a turbulent flow regime, although both sets of data were obtained for a wide variety of Newtonian fluids and different pumping systems. The quality of the data maintained by the research groups coupled with the PME approached by Chiu et al. (1993) support the model proposed in this paper.

Proposed Model

To build the model proposed in this paper, the entropy parameter (M) was determined by Eq. (4) using Reynolds number values

Table 1. Entropy parameter calculated from results (Reynolds number and friction factor) reported by McKeon et al. (2004b)

Reynolds	Friction factor	M[a]	(e^M-1)[a]
4,835	0.03797	2.291	8.89
5,959	0.03610	2.559	11.92
8,162	0.03364	2.952	18.15
10,900	0.03088	3.265	25.19
13,650	0.02903	3.509	32.41
18,990	0.02670	3.865	46.71
29,430	0.02386	4.319	74.11
31,310	0.02364	4.391	79.70
40,850	0.02086	4.581	96.62
41,440	0.02216	4.681	106.90
56,360	0.02061	4.991	146.12
59,220	0.02000	5.017	149.90
73,970	0.01929	5.258	191.09
84,760	0.01805	5.347	209.07
98,460	0.01815	5.545	255.05
120,000	0.01686	5.702	298.50
145,600	0.01666	5.929	374.85
176,000	0.01594	6.110	449.15
184,800	0.01594	6.170	477.09
229,600	0.01529	6.385	591.96
237,700	0.01511	6.413	608.73
298,200	0.01462	6.649	770.75
308,500	0.01461	6.689	802.46
408,100	0.01384	6.961	1,053.22
467,800	0.01365	7.107	1,219.85
537,800	0.01324	7.237	1,389.00
587,500	0.01313	7.332	1,527.56
750,700	0.01249	7.563	1,924.08
824,200	0.01244	7.668	2,137.60
1,024,000	0.01183	7.863	2,599.36
1,050,000	0.01198	7.907	2,716.41
1,342,000	0.01131	8.126	3,381.23
1,791,000	0.01079	8.406	4,473.77
2,352,000	0.01028	8.664	5,792.23
3,109,000	0.00989	8.940	7630.52
4,438,000	0.00941	9.289	10,818.86
6,103,000	0.00897	9.596	14,706.94
7,757,000	0.00862	9.822	18,435.20
10,310,000	0.00825	10.093	24,170.55
13,680,000	0.00798	10.373	31,975.83
18,300,000	0.00767	10.654	42,344.60
24,130,000	0.00740	10.922	55,364.92
30,150,000	0.00720	11.139	68,779.31
35,540,000	0.00708	11.302	81,010.80

Source: Data from Swanson et al. (2002), Zagarola and Smits (1998), and McKeon et al. (2004a, b, 2005).
[a]Results yielded in present work.

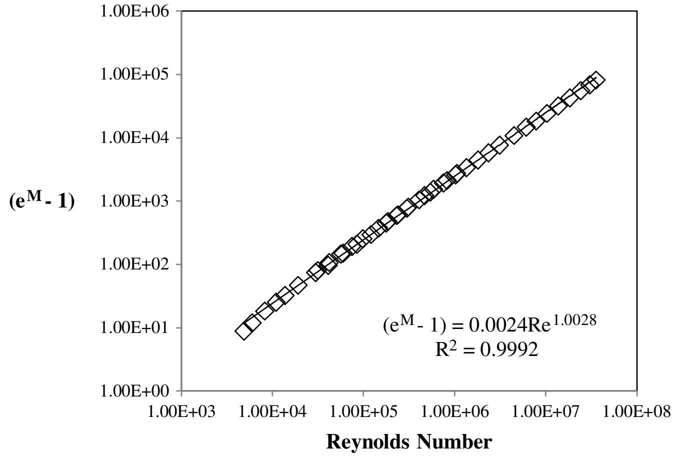

Fig. 1. Entropy parameter versus Reynolds number calculated from data published by McKeon et al. (2004b).

(Re > 4,000) and their corresponding friction factor (f) maintained by Swanson et al. (2002), Zagarola and Smits (1998), and McKeon et al. (2004a, b, 2005). Because Eq. (4) is of nonlinear form, M was numerically calculated using the Microsoft Excel Office version 2019. Subsequently, the exponential term (e^M–1) was calculated and correlated to the Reynolds number, as depicted by Eq. (6). The results are summarized in Table 1 and Fig. 1, where satisfactory fit was found ($R^2 = 0.99$). Fig. S1 shows a block diagram to illustrate the methodology/rationale adopted to yield the model proposed in this paper.

An algebraic manipulation of Eq. (6) generates an expression that relates Re to M, as depicted by Eq. (7). The substitution of Eq. (7) into Eq. (4) yields Eq. (8), which represents the model proposed in this paper

$$(e^M - 1) = 0.0024(\mathrm{Re})^{1.0028} \quad (R^2 = 0.99) \tag{6}$$

$$\mathrm{Re} = [416.667(e^M - 1)]^{\frac{1}{1.0028}} \tag{7}$$

$$f = \frac{32}{[416.667(e^M - 1)]^{\frac{1}{1.0028}}}\left[\frac{(e^M - 1)^2}{(Me^M - e^M + 1)}\right] \tag{8}$$

The model expressed by Eq. (8) offers the following advantages:

1. It is applicable to any flow regime (laminar or turbulent) within either smooth or rough pipes.
2. Although the model is based on the flow of different gases in pipes, it is applicable to Newtonian and non-Newtonian fluids (as concentrated mineral slurries) since it makes use of the PME.
3. In situations in which it is possible to measure $\bar{u}$ and $u_{\max}$, the entropy parameter (M) can be calculated by Eq. (3), according to the rationale depicted in Fig. S2. Thus, the friction factor (f) can be determined using Eq. (8) without previous knowledge of pipe roughness.
4. According to the rationale depicted in Fig. S3, results from pumping experiments ($\bar{u}$, ΔP) may feed the Darcy-Weisbach equation to calculate f and, subsequently, M, by applying Eq. (8). Then one can determine the wall shear rate ($\dot{\gamma}_w$) using Eq. (9) or Re and μ_a by means of Eq. (7) for any sort of fluid.
5. The proposed model allows the determination of head loss (ΔH) when the values of the flow rate (Q), pipe diameter (D), length (L), and fluid rheological behavior are known. This rationale is illustrated by Fig. S4. For instance, if a fluid obeys the power law, the rheological model is expressed by Eq (10). If one relates Eq. (10) to the generalized Newton's law, a relation between apparent viscosity (μ_a) and velocity gradient (du/dr) at the wall is obtained, according to Eq. (11). Since apparent viscosity is the ratio $(u\bar{D}\rho)/\mathrm{Re}$ and the entropy parameter (M) is related to the Reynolds number by Eq. (7), it is possible to write Eq. (12) by substituting Eqs. (7) and (9) into Eq. (11)

$$\dot{\gamma}_w = \left(-\frac{du}{dr}\right)\bigg|_{r=R} = \left(\frac{8\bar{u}}{D}\right)\left[\frac{(e^M - 1)^2}{2(Me^M - e^M + 1)}\right] \tag{9}$$

$$\tau_w = K\left(\frac{du}{dr}\right)_R^n = \left[K\left(\frac{du}{dr}\right)_R^{n-1}\right]\left(\frac{du}{dr}\right)_R \tag{10}$$

$$\mu_a = K\left(\frac{du}{dr}\right)_R^{n-1} \tag{11}$$

$$\frac{\bar{u}D\rho}{[416.667(e^M - 1)]^{\frac{1}{1.0028}}} = K\left\{\left(\frac{8\bar{u}}{D}\right)\left[\frac{(e^M - 1)^2}{2(Me^M - e^M + 1)}\right]\right\}^{n-1} \tag{12}$$

Table 2 provides a summary of the expressions for the friction factor (f) that allow the determination of the entropy parameter (M), as well as their respective advantages and limitations.

Determination of Entropy Parameter for Iron Ore Slurry

A sample of iron ore from Serra da Serpentina in Brazil was supplied by the mining company Vale S.A. The solids were dried and submitted for characterization (Table 2). The dried sample is composed of 44.5% Fe and 31.9% SiO_2 and shows a specific gravity (ρ) of $3,850\ \mathrm{kg/m^3}$. Particle size distribution is represented by an average size of 64.3 μm. Iron ore was mixed with tap water to produce a slurry containing 67% solids (w/w) and a specific gravity of $1,984\ \mathrm{kg/m^3}$.

Pumping experiments were conducted in a test-loop experimental facility containing rough pipes (carbon steel) with Perspex sections to visualize the flow and with pipe diameters of 0.0762 and 0.1016 m. Volumetric flow rates were measured with Krohne Conaut electromagnetic flowmeters. The pressure gradient was measured along a test section, where Siemens differential pressure transducers were installed 2 m apart. All the instruments were connected to a Presys data acquisition system.

Table 2. Particle properties and chemical composition

Properties	Techniques	Value
Specific gravity	Pycnometry—Ultrapyc 1200e—Quantachrome Instruments	3,850 $\mathrm{kg/m^3}$
Average particle size ($D50$)	Laser diffraction—particle size analyzer Mastersizer 2000—Malvern Instruments	64.3 μm
Chemical composition	X-ray fluorescence—Zetium Panalytical	44.5% of Fe, 31.9% of SiO_2

Table 3. Entropy parameters determined for iron ore slurry in tubular device ($D = 0.0762$ m)

$\bar{u}$ (m/s)	$(\Delta P/L)$ (kPa/m)	f	M_1	M_2
2.23	1.88	0.0294	3.73	3.58
2.05	1.58	0.0293	3.74	3.59
1.94	1.46	0.0303	3.64	3.49
1.80	1.28	0.0307	3.61	3.46
1.71	1.18	0.0315	3.53	3.38
1.58	1.02	0.0319	3.50	3.35
1.40	0.86	0.0342	3.31	3.16
1.21	0.65	0.0346	3.28	3.13
0.96	0.46	0.0389	2.99	2.85
0.74	0.35	0.0498	2.41	2.33

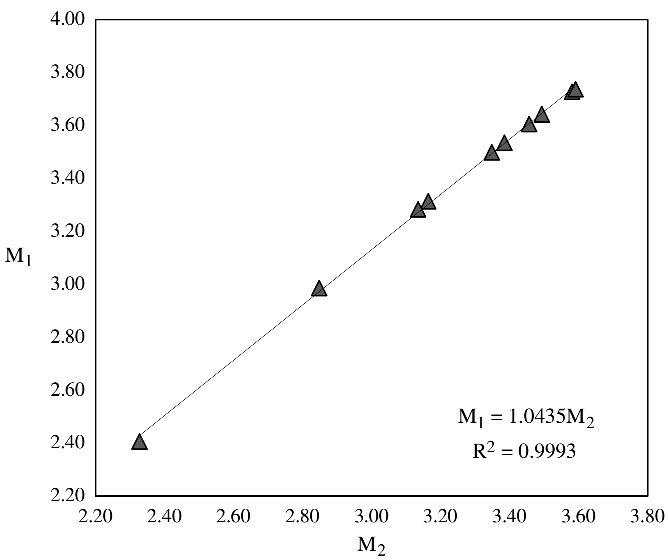

Fig. 2. Entropy parameters obtained for hematite slurry flow in pipe with 0.0762 m inner diameter.

Iron ore was mechanically mixed with tap water in a stirring 1.2–m^3 tank to feed the pumping system. The slurry circulated through the pipeline test loop by means of a Warman 4/3C – AH centrifugal pump (Weir, Todmorden, UK). The pressure gradient $(\Delta P/L)$ and mean velocity $(\bar{u})$ data were obtained by varying the pump speed with a Weg CFW 700 variable-frequency driver (Weg, State of Santa Catarina, Brazil). Once the fully developed slurry flow was visually observed through the Perspex section and set velocity achieved, the pressure gradient measurement started to be recorded in the data acquisition system using VR-2000 version 3.3 software.

Pumping experiments yielded values of pressure gradient $(\Delta P/L)$ plus mean fluid velocity $(\bar{u})$, which allowed for the calculation of the friction factor (f) via the Darcy-Weisbach equation. The rationale depicted in Fig. S5 was adopted to determine the entropy parameter by applying our model, Eq. (8), and the model maintained by Chiu et al. (1993), Eq. (5). Table 3 and Fig. 2 (for a pipe of $D = 0.0762$ m) as well as Table 4 and Fig. 3 (for a pipe of $D = 0.1016$ m) exhibit and compare values of the entropy parameter yielded by our model (M_2) against those (M_1) obtained by the model proposed by Chiu et al. (1993).

According to the results presented in both Tables 3 and 4, the values of the entropy parameter (M_1) calculated using the model proposed by Chiu et al. (1993) are slightly and systematically

Table 4. Entropy parameters determined for haematite slurry in tubular device ($D = 0.1016$ m)

$\bar{u}$ (m/s)	$(\Delta P/L)$ (kPa/m)	f	M_1	M_2
1.92	0.77	0.0217	4.69	4.60
1.75	0.64	0.0216	4.70	4.62
1.63	0.56	0.0217	4.69	4.60
1.55	0.53	0.0229	4.50	4.40
1.43	0.50	0.0251	4.20	4.08
1.33	0.42	0.0248	4.24	4.12
1.24	0.40	0.0268	4.00	3.86
1.12	0.35	0.0286	3.81	3.66
0.97	0.29	0.0317	3.52	3.37
0.86	0.31	0.0434	2.72	2.60

Note: M_1 = entropy parameter determined with Eq. (11); and M_2 = entropy parameter determined with model proposed herein, expressed by Eq. (15).

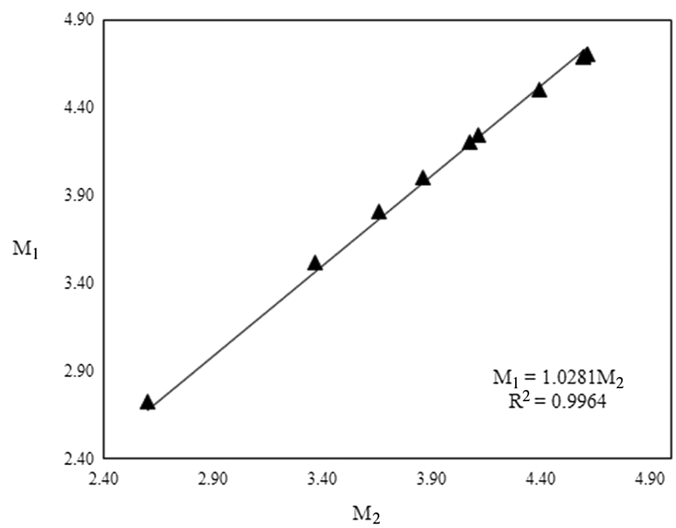

Fig. 3. Entropy parameters obtained for hematite slurry flow in pipe with 0.1016 m inner diameter.

higher than those (M_2) calculated by our model. However, deviations are smaller than 5%. The slope (α) of the straight lines shown in Fig. 2 $(\alpha = 1.0435)$ and Fig. 3 $(\alpha = 1.0281)$ illustrates the same trend. Because M_1 was obtained from a model based on the approach pursued by Nikuradse (1933), where the pipe roughness was artificially controlled with sand grains (average diameter of 0.8 mm), turbulence was promoted by both velocity profile instability and pipe roughness. Conversely, the model proposed in this paper is based on accurate data obtained in smooth pipes (Zagarola and Smits 1998; McKeon et al. 2004a, b, 2005), where only the velocity profile instability contributes to the turbulence.

Conclusions

The application of the PME to accurate sets of data available in the literature allowed for the development of a model represented by Eq. (8), which relates the friction factor (f) to the entropy parameter (M). The proposed model was validated by multiphase flow pumping tests with concentrated iron ore slurry (67% of solids), yielding values of pressure gradient plus mean fluid velocity, which were used to calculate the friction factor (f) and, subsequently, entropy parameter (M). The values of the entropy parameter determined by our model (M_2) were compared to those (M_1) obtained by another model maintained by Chiu et al. (1993) and based on an approach pursued by Nikuradse (1933). Although deviations were smaller than 5%, the values of M_1 were systematically higher than those of M_2. Such a difference likely stems from the inaccuracy of the Nikuradse velocity profile in regions close to the wall and at the center of the pipe. It has also been demonstrated that the PME is an important tool for hydraulic systems, especially in multiphase flow, such as that of mineral slurries containing a high content of solids. Extending the use of the proposed model for multiphase flow, particularly mineral slurries, allows for the determination of apparent viscosity and Reynolds number without resorting to rheological measurement.

Appendix. Advantages and Limitations of Models for Entropy Parameter Determination

Entropic friction factor	Authors	Advantages	Limitations
$f=\left(\frac{32}{\mathrm{Re}}\right)\left[\frac{(e^M-1)^2}{(Me^M-e^M+1)}\right]\left(\frac{\varepsilon_w}{v}\right)$	Chiu et al. (1993)	Analytical model without inconsistencies	Requires prior knowledge of both apparent viscosity and Reynolds number
$f=\left(\frac{32}{\mathrm{Re}_a}\right)\left[\frac{(e^M-1)^2}{(Me^M-e^M+1)}\right]$	Chiu et al. (1993)	Analytical model without inconsistencies	Requires prior knowledge of the apparent Reynolds number or apparent viscosity
$f=0.0983\left(\frac{0.17Me^M+e^M-1.17M-1}{Me^M-e^M+1}\right)^{\mathbf{2}}$	Chiu et al. (1993)	Does not require prior knowledge of apparent fluid viscosity. Darcy-Weisbach equation is used to determine f	Derived from Nikuradse velocity distribution, which is inaccurate in pipe center and near-wall regions
$f=\frac{32}{[416.667(e^M-1)]^{\frac{1}{1.0028}}}\left[\frac{(e^M-1)^2}{(Me^M-e^M+1)}\right]$	Present work	Derived from accurate experimental results without inconsistencies and does not require prior knowledge of the apparent viscosity. Darcy-Weisbach equation is employed to determine f	—

Data Availability Statement

All data, models, and codes generated by or used in the study appear in the published article.

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Notation

The following symbols are used in this paper:

D = pipe internal diameter;
f = friction factor;
g = gravitational acceleration;
K = fluid consistency coefficient;
k = von Kárman constant;
L = pipe length;
M = entropy parameter;
n = flow behavior index;
Q = volumetric flow rate;
R = pipe radius;
Re = Reynolds number;
$\mathrm{Re_a}$ = apparent Reynolds number;
r = radial distance;
u = velocity distribution;
$u_{\max}$ = maximum velocity;
$\bar{u}$ = mean velocity;
u^* = shear velocity;
$\dot{\gamma}_w$ = wall shear rate;
ε_w = momentum transfer coefficient at wall;
ν = kinematic viscosity;
μ_a = apparent viscosity;
$\bar{H}$ = head loss due to friction;
$\left(\frac{\Delta P}{L}\right)$ = pressure gradient;
τ_w = wall shear stress; and
ρ = fluid-specific mass.

Supplemental Materials

Figs. S1–S5 are available online in the ASCE Library (www.ascelibrary.org).

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