RTMAT 91-01

Derivations of Upper Triangular Matrix Rings

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junho 1991

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1 Introduction.

Let R be a ring with unity and denote by $T_n(R)$ the ring of upper triangular $n \times n$ matrices over R. The group of automorphisms of this ring, given certain restrictions on R, has been the object of several recent papers (see [1], [2], [5]). In this note we shall give a description of the derivations in $T_n(R)$ assuming no restriction on R other than the existence of an identity element. We shall show that they are obtained in a very natural way.

First, note that if $\delta : R \to R$ is a derivation of R then the map $\overline{\delta} : T_n(R) \to T_n(R)$ defined by $\overline{\delta}(x_{ij}) = ((\delta(x_{ij})), \forall (x_{ij}) \in T_n(R)$ is a derivation. Also, if $A \in T_n(R)$, we shall denote by d_A the inner derivation induced by A. In what follows, we shall show that all derivations of $T_n(R)$ arise from these; namely, we prove the following

Theorem. *Let R be a ring with unity and let $d : T_n(R) \to T_n(R)$ be a derivation. Then, there exists a derivation $\delta : R \to R$ and a matrix $A \in T_n(R)$ such that $d = \bar{\delta} + d_A$.*

A similar result for full matrix rings appears in [4] and the special case where R is an algebra over a field, with $char(R) \neq 2,3$ and $n > 2$ is given in [6].

The proof we give below is very simple and constructive in nature: we show how the matrix A is determined by the values of d on the usual matrix units $e_{ij} \in T_n(R)$. Actually,this technique also applies to the ring $M_n(R)$ so, it gives an elementary proof for the description of the derivations of this ring.

2 Proof of the Theorem.

Notice that a matrix unit e_{ij} belongs to $T_n(R)$ provided that $i \leq j$. Hence, in particular, $e_{kk} \in T_n(R), 1 \leq k \leq n$.

Since e_{kk} is an idempotent, we have that $d(e_{kk}) = e_{kk}d(e_{kk}) + d(e_{kk})e_{kk}$. Thus, if we write $d(e_{kk}) = \sum_{i \leq j} x_{ij}(k)e_{ij}$, we have that:

$$d(e_{kk}) = \sum_{i \leq j} x_{ij}(k)e_{ij} = \sum_{j \geq k} x_{kj}(k)e_{kj} + \sum_{i \leq k} x_{ik}(k)e_{ik}$$

which shows readily that $x_{kk}(k) = 0$ and that $x_{ij}(k) = 0$ if $i \leq j$ are both different from k.

Given a matrix $X \in T_n(R)$, we shall denote by $[X]_{ij}$ the i j entry of this matrix. If $i < j$ we define $a_{ij} = -[e_{ii}d(e_{ii})e_{jj}]$ i.e. $a_{ij} = -x_{ij}(i)$. Also we set $a_{ij} = 0$ if $i > j$.

Lema 1. *Let A be a matrix with entries chosen as above and arbitrary diagonal entries. Set $h = d - d_A$; then:*
(i) $h(e_{kk}) = 0, 1 \le k \le n$.
(ii)Furthermore, if we set $a_{11} = 0$ and $a_{jj} = -[d(e_{1j})]_{1j}$, $1 < j \le n$, then $h(e_{ij}) = 0$, for $i < j$.

Proof. (i) We compute

$$d_A(e_{kk}) = Ae_{kk} - e_{kk}A = \sum_{i \le k} a_{ik}e_{ik} - \sum_{j \ge k} a_{kj}e_{kj}$$

By our definition $-a_{kj} = x_{kj}(k)$. Also, notice that if $e, f \in T_n(R)$ are orthogonal idempotents, we have that $ed(f) + d(e)f = 0$ which implies $ed(f)f = -ed(e)f$. Hence, in particular, $e_{ii}d(e_{jj})e_{jj} = -e_{ii}d(e_{ii})e_{kk}$ so, if $i \le k$ then $a_{ik} = e_{ii}d(e_{jj})e_{jj} = -x_{ik}(i)$. Hence $d_A(e_{kk}) = d(e_{kk}), 1 \le k \le n$ and (i) follows.

(ii) We shall show first that $h(e_{1j}) = 0, 1 < j \le n$.

Since $e_{1j} = e_{11}e_{1j}e_{jj}$, using (i) we obtain:

$$\begin{aligned} h(e_{1j}) &= e_{11}h(e_{1j})e_{jj} \\ &= e_{11}(d(e_{1j}) - Ae_{1j} + e_{1j}A)e_{jj} \\ &= -a_{jj}e_{1j} - a_{11}e_{1j} + a_{jj}e_{1j} = 0 \end{aligned}$$

Now we prove that $h(e_{ij}) = 0$, $1 < i < j$. In fact, since $e_{1j} = e_{1i}e_{ij}$ we see that

$$0 = h(e_{1j}) = e_{1i}h(e_{ij}) = e_{1i}d(e_{ij}) - e_{1i}Ae_{ij} + e_{1i}e_{ij}A.$$

Hence:

$$[d(e_{ij})]_{jj} = a_{ii} - a_{jj}.$$

Finally, we have that $e_{ij} = e_{ii}e_{ij}e_{jj}$ so we can write:

$$\begin{aligned} h(e_{ij}) &= e_{ii}h(e_{ij})e_{jj} \\ &= e_{ii}d(e_{ij})e_{jj} - e_{ii}Ae_{ij} + e_{ij}Ae_{jj} \\ &= [d(e_{ii})]_{ij}e_{ij} - a_{ii}e_{ij} + a_{jj}e_{ij} = 0 \end{aligned}$$

□

From now on, we shall assume that the elements in the diagonal of A are chosen as in (ii) above and hence, that $h(e_{ij}) = 0$, $i \leq j$.

Lema 2. *There exists a derivation $\delta : R \to R$ such that $h(rI) = \delta(r)I$, $\forall r \in R$.*

Proof. Set $r \in R$. Notice first that

$$h(re_{11}) = h(rIe_{11}) = h(rI)e_{11} = r'e_{11}, \text{ for some } r' \in R$$

Also, we have that:

$$h(re_{12}) = h(re_{11}e_{12}) = h(re_{11})e_{12} = r'e_{12}.$$

Now, we wish to compute $h(re_{22})$. To do so, we write:

$r'e_{12} = h(re_{12}) = h(re_{12}e_{22}) = e_{12}h(re_{22})$;

$h(re_{22}) = h(rI)e_{22} = r_1e_{12} + r_2e_{22}$.

Hence,

$r'e_{12} = e_{12}(r_1e_{12} + r_2e_{22}) = r_2e_{12}$, thus $r' = r_2$.

Since $e_{11}re_{22} = 0$ we have that $e_{11}h(re_{22}) = 0$ i.e. $r_1e_{12} = 0$.

Consequently $r_1 = 0$ and $h(re_{22}) = r'e_{22}$.

Repeating the argument above we see, inductively, that $h(re_{ii}) = r'e_{ii}, 1 \leq i \leq n$. Hence, $h(rI) = r'I$. It is now easy to see that the map $\delta : R \to R$ given by $r \mapsto r'$ is a derivation of R. $\square$

Proof of the Theorem:

Set $\alpha = (r_{ij})_{i\leq j} \in T_n(R)$. We have that:

$$\begin{aligned}(d - d_A)(\alpha) &= h(r_{ij}) = h(\sum_{i\leq j} r_{ij}Ie_{ij}) \\ &= \sum_{i\leq j}\delta(r_{ij})Ie_{ij} = \sum_{i\leq j}\delta(r_{ij})e_{ij} = \overline{\delta}(r_{ij}) = \overline{\delta}(\alpha).\end{aligned}$$

Hence, $d(\alpha) = d_A(\alpha) + \overline{\delta}(\alpha)$, $\forall \alpha \in T_n(R)$. $\square$

It may be interesting to note that the derivation δ and the matrix A are unique up to inner derivations.

Proposition. *Let $d : T_n(R) \to T_n(R)$ be a derivation and let A, δ be as in the theorem above. If $A' \in T_n(R)$ and $\delta' : R \to R$ are such that $d = d_{A'} + \overline{\delta'}$ then, there exists an element $a \in R$ such that $\delta' = \delta - \delta_a$ and $A' = A + aI$ (so $d_{A'} = d_A + d_{aI}$).*

Proof.-We have that $d_{A'} - d_A = \overline{\delta} - \overline{\delta'}$ so, computing in an arbitrary elementary matrix e_{ij} we obtain:

$$(A' - A)e_{ij} - e_{ij}(A' - A) = 0$$

i.e.

$$\sum_h (a'_{hi} - a_{hi})e_{hi} = \sum_h (a'_{jh} - a_{jh})e_{jh}.$$

Hence, $a'_{hk} = a_{hk}$ if $h \neq k$ and $a'_{ii} - a_{ii} = a'_{jj} - a_{jj}$.

If we set $j = 1$ we obtain $a'_{ii} = a'_{11} + a_{ii}$ so, if we denote $a = a'_{11}$ we have that $A' = A + aI$.

If we choose $r \in R$ we can compute:

$$(d_{A'} - d_A)(rI) = (\overline{\delta} - \overline{\delta}')(rI)$$

which gives:

$$(ar - ra)I = (\delta(r) - \delta'(r))I$$

so $\delta' = \delta - \delta_a$.

We note that the derivation $d : T_n(R) \to T_n(R)$ is inner if and only if the corresponding derivation $\delta : R \to R$ is inner. This fact, together with the well-known theorem of Skolem-Noether, readily gives:

Theorem. *Let R be a finite dimensional central simple algebra. Then, all linear derivations of $T_n(R)$ are inner.*

As we mentioned before, the same techniques also prove the following.

Theorem ([4], [6]) Let R be a ring with unity and let $d : M_n(R) \rightarrow M_n(R)$ be a derivation. Then, there exists a derivation $\delta : R \rightarrow R$ and a matrix $A \in M_n(R)$ such that $d = \overline{\delta} + d_A$.

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