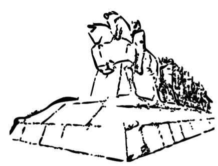
BOOK OF ABSTRACTS
LIVRO DE RESUMOS



XXI EBL

2025

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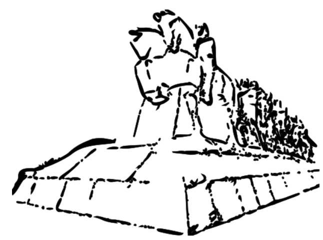

XXI BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XXI ENCONTRO BRASILEIRO DE LÓGICA

LIVRO DE RESUMOS

XXI Brazilian Logic Conference
XXI EBL

May 12-16, 2025 – Serra Negra, Brazil
May 9-11 (**Logic School** – São Paulo, Brazil)
https://ebl2025.ime.usp.br

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XXI BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XXI ENCONTRO BRASILEIRO DE LÓGICA

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Contents

Preface

The Brazilian Logic Conference (EBL) is the main event organized by the Brazilian Logic Society (SBL) and has been occurring since 1979. The EBL congregates logicians from different fields and the meeting is an important moment for the Brazilian and South-American community to come together and engage in a discussion about the state of the art of their subject. The areas of Logic covered span Foundations and Philosophy of Science, Mathematics, Computer Science, Informatics, Linguistics, and Artificial Intelligence.

The goal of the EBL meeting is to encourage the dissemination and discussion of research papers in Logic in a broad sense. It is expected to have among the participants several invited speakers from different continents.

In 2025, the 21st edition of EBL will be held from May 12 to May 16 at the city of Serra Negra, São Paulo State, preceded by the Logic School from May 9 to May 11 at São Paulo City.

The call for papers of this 21th EBL expected submissions on general topics of logic, including philosophical and mathematical logic and applications, history and philosophy of logic, non-classical logic and applications, philosophy of formal sciences, foundations of computer science, physics and mathematics, and logic teaching. The abstracts of the talks presented in this volume reflect the plurality of interests in Logic that comes across in the EBL. This volume brings abstracts of 101 session talks, 23 workshops, 12 invited speakers conferences, 2 round tables, 29 posters and 3 minicourses of the Logic School.

The 21th EBL has been sponsored by SBL, INCTMat, CAPES, CNPq, FAPERJ, FAPESP, USP, IME–USP, FFLCH-USP and City Hall of Serra Negra. It also had support from Unicamp.

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Minicourses (Logic School)
Minicursos (Escola de Lógica)

Neuro Probabilistic Logic Programming

Denis Deratani Mauá
IME–USP

Logic Programming is a successful declarative approach to symbolic knowledge representation and reasoning, with roots in scalable logic inference, database theory and constraint programming. More recently, the framework has been extended to cope with uncertainty and non-symbolic data, bridging the gap between symbolic and sub-symbolic schools. In this short course, we will go through the basics of logic programming formalisms including the most popular semantics, reasoning modes and modeling strategies. We will then look into probabilistic extensions of logic programming such as ProbLog and NeurASP and PASP, considering different semantics, challenges and applications. Finally, we will see how probabilities enables us to connect symbolic and sub-symbolic reasoning, by performing end-to-end gradient-based learning of neuro-logic programs. The course should be accessible to anyone with a basic knowledge of propositional logic and calculus.

Some Applications in Infinite Combinatorics

LEANDRO AURICHI
ICMC–USP

In this course we will present some techniques of logic and set-theory in graphs, mainly infinite graphs. We will show some examples with compactness arguments, elementary submodels, transfinite induction and extra ZFC axioms. The course will focus on select graph problems and show how these techniques can be used.

An Introduction to the Ecumenical Perspective in Logic

Luiz Carlos Pereira
UERJ

Ecumenism can be understood as a pursuit of unity, where diverse thoughts, ideas, or points of view coexist harmoniously. In logic, ecumenical systems refer, in a broad sense, to proof systems for combining logics. We will start our minicourse (Lecture 1) with a general introduction about the emergence of non-classical logics in the 20th century and about results on translations between logics and theories. Then, we will present some central ideas of the ecumenical perspective and different motivations for the development of ecumenical codifications. In Lecture 2, we will present different ecumenical systems for classical and intuitionistic logics (Natural Deduction, Sequent Calculi, Tableaux) and extensions to modal logics. We will also discuss some results about these systems (Normalization, Cut-Elimination). In Lecture 3, we will present two types of semantics for these systems: a semantics based on Kripke frames and an Ecumenical Proof-theoretical Semantics. In the fourth and last Lecture, we will discuss some additional topics (translations, Glivenko, pure systems) and give an overview of ongoing and future work.

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Invited Speakers
Palestrantes Convidados

Semiring Provenance for Knowledge Bases

Ana Ozaki*
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Knowledge bases (KBs) are widely used to represent the relevant knowledge of a domain of interest in a structured, logic-based format. They are often formed of two parts: one that contains facts about specific individuals (e.g., Renata is a professor), called ABox, and one that contains conceptual knowledge about the domain (e.g., professors are academic researchers), called TBox. One of the main benefits of working with KBs is that each piece of information has a well defined semantics and query answering combines the information present in both the ABox and the TBox. However, KBs can be very large, reaching hundreds of millions of data instances. So, when answering queries, one is interested in not only obtaining the answers but also knowing why/how these answers were obtained. Provenance here refers to the origin of a query result, in particular, which parts of a KB were relevant for answering a query. This corresponds to a notion of explanation explored in the context of databases using commutative semirings. In this talk, we present work on semiring provenance for explaining queries posed to KBs. We focus on KBs expressed in lightweight description logics and discuss algorithmic solutions to determine the provenance of a query w.r.t. a KB.

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On Valuation Semantics for Modal Logics

Cezar A. Mortari*
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In this expository talk we present the application, to some systems of modal logic, of the valuation semantics technique proposed by A. M. Loparić for the basic normal modal logic **K** ("The method of valuations in modal logic", 1977).

A *valuation* for a logic L is a function from the set $\mathcal{F}_L$ of all formulas of L to the set $\{1, 0\}$ of truth-values satisfying certain conditions (which vary depending on L). For classical propositional logic **PL**, for example, a valuation v is a function from $\mathcal{F}_{\mathbf{PL}}$ to the set $\{1, 0\}$ such that $v(\neg A) = 1$ iff $v(A) = 0$; $v(A \wedge B) = 1$ iff $v(A) = 1$ and $v(B) = 1$; and so on.

Valuation semantics were presented for many logics; among them we have, for instance, da Costa's paraconsistent logics $\mathbf{C}_n$ and $\mathbf{C}_\omega$, modal logics (normal, classical, nonnormal), temporal logics, Johansson's minimal logic and intuitionistic logic.

Valuations for **PL** can be easily defined because classical operators are truth-functional. For modal logics, however, we need to add conditions specifying how to deal with modal operators; which conditions precisely will depend on the logic in question. In general, for normal modal logics we would need something like the following:

- for any valuation v, if $v(\Box A) = 0$ then there is a valuation v' such that $v'(A) = 0$ and, for every formula $\Box B$ such that $v(\Box B) = 1$, $v'(B) = 1$;

and then we would add a clause dealing with the case in which $v(\Box A) = 1$. But evidently we cannot use this, on pain of circularity, to define a valuation. So we will need first to define certain functions and then define valuations in terms of them. This can be done in different ways: (i) directly in terms of valuations for **PL** (for instance, for some logics in which modal inference rules are restricted to tautologies), or (ii) we can use the modal degree of formulas, or (iii) we can use finite sequences of formulas closed under subformulas, as Loparić originally did for **K**. We will illustrate this for some logics, and point out how to modify the definitions to handle other systems. We finish by mentioning some open problems regarding valuation semantics.

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Combinatorial Properties of Ramsey Ideals

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An ideal on a set A is a colection $\mathcal{I}$ of subsets of A closed under subsets and finite unions.

Given an ideal $\mathcal{I}$ over A, a set $B \subseteq A$ is said to be $\mathcal{I}$-positive if it does not belong to the ideal. The coideal associated to $\mathcal{I}$ is the collection $\mathcal{I}^+ = \{B \subseteq A : B \notin \mathcal{I}\}$ of $\mathcal{I}$-positive sets.

We will examine several combinatorial properties of ideals on the set of natural numbers $\mathbb{N}$ and explore how they relate to each other.

Ramsey's theorem shows that for every partition of $[\mathbb{N}]^2$, the set of two-element subsets of $\mathbb{N}$, in finitely many parts, there is an infinite set $H \subseteq \mathbb{N}$ with all its two-element subsets in one of the parts. In some occasions we want to find such a set H in a coideal, and this motivates the main definition of this talk: An ideal on $\mathbb{N}$ is Ramsey if for every $A \in \mathcal{I}^+$ and every partition of $[A]^2$ in two parts, there is an $\mathcal{I}$-positive $B \subseteq A$ such that $[B]^2$ is contained in one of the parts. Ramsey ideals and ideals with other combinatorial properties provide a rich and active field of research.

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A Note on Lattices with a Symmetric Difference Like Operation

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The purpose of this note is to discuss the existence of a binary operation in lattices with properties analogous to that of symmetric difference in a Boolean algebra (BA).

The original intent of the authors was to

(1) Investigate the existence of reduced special groups (cf. [3]) inside distributive lattices.

(2) That this question might also be relevant regarding the coding of logical validity via polynomials (cf. [1], [2]) and the references therein).

We show that that under very mild hypothesis, the existence of such an operation entails the original lattice to be a BA and the proposed operation to be classical symmetric difference.

This result constituted motivation to approach the question from a different perspective: define a binary operation, $*$, which would be simulate the properties of symmetric difference in a Boolean algebra in a general distributive lattice, via an adjunction, leading us to examine the structure and basic properties of **$*$-lattices**.

After developing some of the basic properties of $*$-lattices, we give a number of equivalence conditions for a $*$-lattice to be a Boolean algebra.

We then prove that the class of $*$-lattices coincides with Brouwer (or Brouwerian lattices) as defined in [4]. We also give an explicit equational characterization of $*$-lattices (or Brouwer lattices) and show that the join operation can be obtained from the operations $*$ and meet, generalizing a well-known result in Boolean algebras.

It is explicitly stated in [4] that the notion of ideal in a Brouwer algebra will not be discussed therein. We go on to describe the basic properties of ideals and the congruence they generate, the $*$-algebra nature of quotients by ideals and then present a $*$-lattice version of Glivenko's Theorem.

It is then proven that there is an anti-equivalence (Duality) between the categories of $*$-Lattices and their morphisms with a certain category of Spectral Spaces and their morphisms (dubbed $*$-Spectral Spaces).

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Andrea Loparić and the Universes of Discourse

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Andrea Loparić (1941-2021), philosopher by training and a logician by vocation, educated generations of Brazilian philosophers, contributed to the consolidation of the Brazilian logic community, and played a significant role in the early developments of non-classical logic in South America. Among her last ongoing projects was a book celebrating her 80th birthday, an ensemble of her logical papers, original contributions about her work, plus an interview with Ítala D'Ottaviano and Evandro Gomes [1]. In my talk, I will examine a rather extraordinary text in her corpus – Loparić's contribution to the volume Lacan avec les philosophes [2]. My aim is to explore the peculiar status of this text by situating it as both, an example of the interdisciplinary powers of non-classical semantics and an instance of recent efforts to revise canonical narratives by including logical investigations made by women (e.g., [3]). Regarding the first, I will show how Loparić's frames the semantics with which she attests the logical readability of Lacan's controversial "formulae of sexuation". With respect to the historical dimension, I argue that attention to "minor works" like [2] reveal historiographical challenges and pedagogical possibilities common to retrieval programs in the histories of philosophy [4].

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An Algebraic Approach to Possibilistic Substructural Logics

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Substructural modal logics (extensions of substructural logics with various modal operators) have recently been studied in a wide range of contexts. These systems are the appropriate formalisms for dealing with reasoning that is both fuzzy and uncertain. Usually, the corresponding logics of these kind of systems are introduced either semantically via a relational semantics, or syntactically, via an axiomatization or class of algebraic structures, but general systematic accounts relating these two perspectives are mostly lacking in the literature.

In this talk we present some ideas to work with an algebraic semantics for a system that is substructural and possibilistc ('$KD45$-like' modal substructural logics) which is semantically determined by relational structures. These algebraic structures generalize Bezhanishivili's pseudomonadic algebras for the modal logic $KD45$.

We will discuss the relationship between the algebraic and the relational semantics, the advantages of the algebraic approach for the axiomatization of the systems, and the limitations of our approach in certain specific cases of substructural systems.

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On the Plurality of Logics, Inferential Affordances and Normative Abilities

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Keywords: Metaphysics of affordances, Ecological Psychology, Epistemology of Logic, Inferentialism, Expressivism, Neo pragmatism

In this contribution, I develop the metaphysical counterpart of a neo-pragmatist proposal to give a philosophical account of the plurality of logics, especially the emergence of non-classical logics. The goal is to connect two debates, one in the philosophy of logic and another in the metaphysics of affordances, that stand to gain much from each other. I will introduce and develop the notions of inferential affordances and normative abilities, inspired by Vetter (2018, 2023), to give a metaphysical counterpart for an inferentialist and expressivist view of logic (Brandom 1994, 2001, 2008). According to Gibson (1986), affordances are objective and real environmental opportunities to skillful agency. In this view, cognition is enacted by embodied and situated agents exploring affordances in their environment. For the epistemology of logic, we may apply this ecological insight as follows: we use non-classical reasoning to cope with inferential affordances objectively presented in our environment, by exercising our normative abilities and acting in the world. Inferential affordances are opportunities for us to apply our normative skills to restrict or allow logical rules. In this view, non-classical reasoning displays how our inferential practices have to cope with affordances that may aptly be taken as normatively restricting some classical rules of inferences and allowing for alternative inferential practices. Thus, logicians can study and systematize these norms, implicit in our daily inferential practices, expressing them in different formal and abstract systems. In this project, I will defend that logicians, on the inferentialist and expressivist side, build up formal systems to make explicit and systematize rules that express norms implicit in our skillful inferential practices, norms designed to manage, on the metaphysical side, different and objective normative affordances.

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How to Use Logic to Make Reinforcement Learning Safe

Natasha Alechina
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Reinforcement learning is famously about learning by trial and error. It is often quite difficult to specify a reward function in each state to encourage the agent to achieve its goal and not engage in “reward gaming”. It is also difficult to ensure that the agent’s behaviour is always safe, especially when training is on- rather than offline. I will talk about using logic to provide declarative specification of rewards and to ensure that unsafe actions are always blocked (both during training and during deployment of the agent).

Relational Semantics and Ordered Algebras for Monotone Propositional Logics

Ramon Jansana*
University of Barcelona, Barcelona, Spain

I will present part of the general theory of relational semantics for propositional logics developed together with Tommaso Moraschini in the recent past. More specifically, I will expound a relational semantics for monotone logics; a logic (as a consequence relation) is monotone if every connective is in each coordinate either increasing or decreasing with respect to the pre-order induced in the algebra of formulas by the consequence relation (i.e., the pre-order that declares a formula below another if the second follows from the first.) These logics include many of the well-known logics, in particular logics whose algebraic semantics is based on lattices not necessarily distributive and also their fragments whose algebras have no lattice reduct.

The semantics we develop is based on a duality between a class of ordered algebras associated with a monotone logic and a class of general frames for it. I will motivate the semantics by a brief historical introduction to dualities between algebras and relational semantics for logics, starting from Stone's dualities for Boolean algebras and distributive lattices.

The idea we use to turn an ordered algebra into a frame and conversely is inspired by the notion of relational dual of a function, coming from B. Jónsson and A. Tarski work on Boolean algebras with operators and M. Dunn's gaggle theory; it also is inspired by M. Gehrke's work on RS-frames.

The frames we use consist of a polarity – a set of positive states (worlds), a set of negative states (co-worlds), and a relation between them – and for each connective of the language of the logic a suitable relation between worlds and co-worlds in accordance to the logical behavior of the connective.

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Dividing Lines among Fields

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One of the objects of study of model theory are the complete first order theories and their classification. Shelah classified complete first order theories by their ability to encode certain combinatorial configurations. The importance of Shelah's classification theory lies in its ability to provide tools and conceptual frameworks to understand the structure of mathematical theories and to address a wide range of problems in model theory and related areas. His work has influenced numerous subsequent developments in the field and has significantly contributed to advancing knowledge in model theory and related disciplines.

In the first part of the talk, we will define some of the principal classes of theories (stable, simple, and NIP) and explore the current conjectures on classifying fields within these categories and the advancements achieved to date. In the second part, we will define the class of *theories without the tree property of the second kind* (NTP_2 theories), this class was defined also by Shelah and contains strictly the class of simple and NIP theories. Among the main known examples of NTP_2 fields are PRC and PpC bounded fields. We will then present joint work with Silvain Rideau-Kikuchi, in which we propose a unified framework for studying such fields: the class of *pseudo-I-closed fields*, where I is a set of enriched theories of fields. These fields satisfy a *local-global principle* for the existence of rational points on varieties, relative to models of the theories in I. Under certain hypotheses, we show that pseudo-I-closed fields yield new examples of NTP_2 fields.

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On Formulations for Group Theory*

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We examine two equational formulations for groups: the familiar formulation (with three symbols) and an alternative formulation (with a single symbol); we show that they are equivalent. This alternative formulation leads to an economy in notation and to a simpler statement of some concepts. One often regards a group as a non-empty set with a binary operation that is associative, has a neutral element and every element has an inverse. We also have an equational formulation for groups, by considering three symbols: $\cdot$ (for a binary operation), $\smile$ (for a unary inverse operation) and e (a constant for neutral element). It is interesting to ask whether one may decrease the number of symbols. Here, we investigate such a formulation: with a single binary operation symbol $\oslash$ (for division). Division can be defined in group terms by: $\mathrm{x} \oslash \mathrm{y} \coloneqq \mathrm{x} \cdot (\mathrm{y}^{\smile})$.

This economy leads to simpler formulations of some concepts, such as subgroup, homomorphism and congruence There is another motivation for this move. In a group, an equation $\mathrm{x} \cdot \mathrm{a} = \mathrm{b}$ has a (unique) solution. This solution is expressed by division: $\mathrm{x} \coloneqq \mathrm{b} \oslash \mathrm{a}$. We are thus using a basic intuition about groups as our starting point.

We will show that these two equational formulations are equivalent.

*This paper is dedicated to the memory of the late Paulo A. S. Veloso.

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§*in memoriam*

Normal Numbers and the Borel Hierarchy

Verónica Becher
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More than one hundred years ago Émile Borel defined the notion of normality for real numbers: A real number is normal to an integer base b greater than or equal to 2 if each block of digits 0, 1, $\ldots$, $b-1$ occurs in the expansion the number with the same asymptotic frequency. Many questions on normal number are still open, such as whether any of pi, e or $\sqrt{2}$ is normal in some base, as well as Borel's conjecture that the irrational algebraic numbers are normal to every base. In this talk I will highlight some theorems on the descriptive complexity of the set of normal numbers in the Borel hierarchy.

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Round Tables
Mesas Redondas

Tornando-se Professor de Lógica

Dra. Evelyn Erickson[*]
UFSC, Florianópolis, Brasil
Mahan Vaz Silva[†]
UNICAMP/RUB, Bochum, Alemanha
João Mendes[‡]
UFRN, Natal, Brasil
Prof. Bruno Lopes[§]
UFF, Niterói, Brasil

Palavras-chave: Ensino de Lógica, Lógica, Projeto de Extensão

Palestrantes convidados: Elaine Pimentel (UCL), Sheila Veloso (UERJ), Cassiano Terra Rodrigues (ITA)

Mediadores: Evelyn Erickson (UFSC), João Mendes (UFRN)

Resumo da mesa: Esta mesa é uma iniciativa do projeto de extensão Seminários de Orientação extra-Lógica (SOL). Ela tem como seu principal objetivo discutir de que modo uma pessoa que passou por uma pós-graduação em Lógica torna-se professora de lógica. A formação em Lógica é diversa, e as oportunidades de ensino também: há vários cursos em que pode-se ensinar lógica, em várias disciplinas e conteúdos muito diferentes que podem ser abordados, dependendo do perfil dos estudantes. Considerando esse cenário, esta mesa redonda busca explorar os diversos caminhos que nossos convidados percorreram e saber quais desafios enfrentam em suas práticas docentes. Abordaremos assuntos como o espaço da Lógica nos cursos de graduação, as ementas, planos de curso, escolhas pedagógicas, e sobre a diversidade de alunos a quem precisamos ensinar: estudantes em tipos de instituição diversos, de cursos diversos, com interesses diversos, assim como diferentes recortes sociais e geográficos. Também trataremos do desafio que o tempo traz no ensino de lógica, como a evolução do ensino acompanha a vida do professor em seus diferentes estágios e a evolução tecnológica da sociedade. Iremos conversar sobre o vir a ser professor de lógica, para ajudar a nova geração a saber que tarefa é essa de ensinar.

Resumo do projeto: A mesa é proposta como atividade do projeto de extensão SOL: Seminários de Orientação Extra-Lógica, da Universidade Federal Fluminense, com o apoio da Sociedade Brasileira de Lógica e do Grupo de Interesse em Lógica da Sociedade Brasileira de Computação. O projeto tem como objetivo cultivar a diversidade da lógica no Brasil apresentando a pesquisadores em formação ou início de carreira importantes aspectos da vida acadêmica que não se relacionam diretamente ao conteúdo da disciplina de lógica. O projeto promove atividades regulares, ouvindo participantes em diversas posições na comunidade de lógica brasileira, entre professores, pesquisadores de carreira e pós-doutorandos, de modo a abrir e instigar o debate de como melhor se formar fora das aulas e seminários.

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Redes Dialéticas: Teoria e Prática

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Palavras-chave: discussão crítica, lógica dialógica, representação diagramática

"Redes Dialéticas" é uma ferramenta para a anotação diagramática (o diagrama dialético) e não-diagramática (o grafo dialético) da argumentação resultante de discussões críticas. Ela foi desenvolvida por Sautter [4] e posteriormente utilizada para anotar a compilação de Hans Kelsen dos principais argumentos favoráveis e contrários à Tese da Logicalidade do Direito, também conhecida como "Tese da Aplicabilidade da Lógica ao Direito" [5].

Em sua versão original, ela contém representações primitivas para as asserções dos participantes de uma discussão crítica, denominados "Proponente" e "Oponente". Também contém representações primitivas para organizar os argumentos em ações de ataque e ações de defesa (suporte). Por fim, contém representações primitivas para distinguir os modos acoplado e múltiplo de uma ação.

Nessa versão original não havia diferença representacional entre uma ação dirigida a uma asserção – a defesa (suporte) ou o ataque a uma asserção – da ação dirigida a um argumento – a defesa (suporte) ou o ataque a um argumento. O bom senso do usuário poderia distinguir um caso do outro, mas a solução não era totalmente satisfatória. Um objetivo desta mesa redonda é propor e dicutir uma nova versão das Redes Dialéticas, que contém as mesmas representações primitivas da versão original, mas na qual uma ação dirigida a um argumento é claramente distinguida de uma ação dirigida a uma asserção.

A semântica da Lógica Dialógica [1] utiliza as noções de proponente e oponente, e de ataque e defesa. Outro objetivo desta mesa redonda é explorar essas semelhanças entre a Lógica Dialógica e as Redes Dialéticas para determinar se e em que medida podemos utilizar as Redes Dialéticas para representar operações de distintas lógicas.

Para exemplificar as qualidades representacionais das Redes Dialéticas será apresentada a imbricada argumentação resultante do famoso debate entre Herbert L. A. Hart e Patrick Devlin em torno da Tese da Separação entre o Direito e a Moral (TSDM), ocorrido na década de 1960. O debate foi suscitado pelo Relatório Wolfenden, um documento produzido por ocasião da reforma da legislação inglesa sobre crimes sexuais. A proposta do referido relatório – um relaxamento da legislação – estava apoiado na tese de que o Estado não deve interferir em questões de moralidade privada, e isso implica a TSDM. Devlin [2] escreveu ao menos sete ensaios posicionando-se contra a TSDM, enquanto que Hart [3] produz uma extensa defesa da TSDM.

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Seminars
Seminários

Seminários de História da Lógica
Apresentação

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Na sua primeira edição, os Seminários de História da Lógica visam contemplar o estudo da história da lógica realizado no Brasil. A justificativa se dá pela simples razão de que o conhecimento histórico é imprescindível para compreender por que chegamos a pensar, hoje, tal como o fazemos. pergunta decisiva: por que o passado nos conduz a pensar desta forma? Nesse sentido, levanta-se uma pergunta decisiva: por que o passado nos conduz a pensar desta forma? Para responder a essa pergunta, o conhecimento da história da lógica é essencial. Com ela, damos forma mais definida à nossa maneira de pensar no presente e, por conseguinte, conseguimos moldar mais conscientemente pesquisas futuras. Nesse sentido, o percurso elegido ressalta momentos representativos da história da lógica cuja importância se mostra pela produção brasileira na área.

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Seminário I: Antiguidade

A Lógica Estoica sob o Prisma da Linguagem Natural, da Ambiguidade e da Sabedoria

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Apresentarei a lógica estoica sob uma nova perspectiva. Em vez de usar os operadores vero-funcionais modernos, como o fizemos em nosso estudo prévio, apresentarei a dialética do Pórtico ao modo estoico propriamente dito, ou seja, por meio da linguagem natural. Farei isso porque os estoicos não conceberam uma linguagem formal para a sua lógica e visavam aplicar seu cálculo proposicional tanto para as questões do dia a dia quanto para analisar argumentos filosóficos, o que faz com que a questão da ambiguidade em particular e dos sofismas da lógica informal em geral tenham uma importância muito maior do que para a lógica moderna, o que decorre do fato de que a dialética era vista pelos estoicos como fundamental para o humano em sua busca pela sabedoria, havendo virtudes lógicas que decorrem do domínio e da prática da dialética estoica que são próprias do sábio e que devem ser buscadas pelos que progridem moralmente.

Stoic Logic through the Prism of Natural Language, Ambiguity, and Wisdom

I will present Stoic logic from a new perspective. Instead of using modern truth-functional operators, as we did in our previous study, I will present Porticus´ dialectic in the Stoic way, that is, through natural language. I will do this because the Stoics did not conceive of a formal language for their logic and aimed to apply their propositional calculus both to everyday issues and to analyze philosophical arguments, which makes the problem of ambiguity in particular and the sophisms of informal logic in general more important than for modern logic. The Stoics consider dialectic as fundamental for humans in their search for wisdom, with logical virtues that arise from the mastery and practice of Stoic dialectic that are proper to the wise and that should be sought by those who want to progress morally.

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Seminário II: Idade Média

Tópicos de Lógica Medieval

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A Idade Média é frequentemente caracterizada como um período de trevas e de pensamento estéril. Essa visão ignora, contudo, os inúmeros desenvolvimentos em lógica feitos pelos medievais. Este seminário pretende apresentar diversos aspectos de avanços teóricos desenvolvidos pelos medievais em lógica, que possibilitam lidar com problemas semânticos, modais e paradoxos. Por fim, pretende-se também apontar uma reavaliação da lógica no contexto institucional de sua produção, buscando uma melhor compreensão do papel e das funções da lógica para os medievais.

Topics in Medieval Logic

The Middle Ages are often characterized as a period of darkness and sterile thought. However, this view ignores the numerous developments in logic made by the medievals. This seminar aims to present various aspects of theoretical advances developed in medieval logic, which make it possible to deal with semantic, modal problems, and paradoxes. Additionally, the seminar will explore a reevaluation of logic within its institutional framework of production, aiming to clarify the role and purpose of logic for medieval thinkers.

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Seminário III: Modernidade

Recepção da Lógica na América Ibérica

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Que forma de lógica foi cultivada na América Latina desde o século XVI? Ela foi desenvolvida formalmente? Quais as relações entre lógica, filosofia e ciências? Considerando o contexto geral da lógica ibero-europeia do século XVI ao final do século XVIII, que questões dessa disciplina específica podem ser identificadas no processo de ensino da lógica na América Latina? Qual era o papel da lógica nesse contexto? De fato, os estudiosos latino-americanos, assim como seus colegas europeus, esperam da lógica um suporte para as ciências e a racionalidade, dando-lhes uma teoria geral da argumentação e da verdade, do método e das ciências. Aqui como lá, a compreensão da lógica subjugou a abordagem formal.

Reception of Logic in Ibero-America

What kind of logic has been cultivated in Latin America since the 16th century? Was it developed formally? What were the relationships between logic, philosophy, and the sciences? Considering the general context of Ibero-European logic from the 16th century to the end of the 18th century, what issues of this specific discipline can be identified in the process of teaching logic in Latin America? What was the role of logic in this context? In fact, Latin American scholars, like their European colleagues, expected logic to provide support for the sciences and rationality, giving them a general theory of argumentation and truth, method, and the sciences. Here, as there, the understanding of logic has subjugated the formal approach.

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Seminário IV: Contemporaneidade I

O Lugar de Peirce na Gênese da Lógica Matemática

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É famosa, a ponto de tornar-se praticamente um lugar comum na historiografia da área, a tese de que a lógica matemática – também chamada de lógica simbólica ou formal – nasceu do gênio de Gottlob Frege (1848-1925), tal qual Atenas teria brotado da cabeça de Zeus. Segundo tal concepção, as inovações e características que distinguem a lógica matemática da aristotélica aparecem na *Begriffsschrift* em 1879, para posteriormente ganharem melhor expressão por Bertrand Russell (1872-1970) e Ludwig Wittgenstein (1889-1951).

Essa narrativa inclui os sistemas de lógica algébrica de Augustus De Morgan (1806-1871), George Boole (1815-1864), Charles Sanders Peirce (1838-1914) e Ernst Schröder (1841-1902) como pertencentes à tradição aristotélica. Dessa perspectiva, a silogística aristotélica teria sido apenas reescrita com símbolos algébricos pelos "booleanos", que nada de original teriam produzido em termos lógicos.

O objetivo deste módulo é mostrar que essa narrativa, apesar de muito difundida, é falsificadora, de tão simplista.

Para tanto, num primeiro momento, serão consideradas as inovações de Frege e as características que a distinguem da silogística aristotélica; em seguida, será mostrado como Peirce também desenvolveu independentemente cada um desses aspectos da lógica. Ao fim, serão consideradas algumas concepções filosóficas que se tornaram centrais para a elaboração dessa narrativa, como cálculo, linguagem, universo, modelo etc.

Peirce's Place in the Genesis of Mathematical Logic

A widely repeated thesis in the historiography of logic, almost to the point of becoming commonplace, is that mathematical logic (also termed symbolic or formal logic) originated solely from the genius of Gottlob Frege (1848-1925), as if it had sprung fully formed, like Athena from the head of Zeus. According to this narrative, the defining innovations that distinguish mathematical logic from Aristotelian logic first emerged in Frege's *Begriffsschrift* (1879), later refined by Bertrand Russell (1872-1970) and Ludwig Wittgenstein (1889-1951).

This account relegates the algebraic logic systems of George Boole (1815-1864), Charles Sanders Peirce (1838-1914), and Ernst Schröder (1841-1902) to the Aristotelian tradition, dismissing their contributions as mere symbolic rewritings of syllogistic logic devoid of originality.

My talk challenges this oversimplified and misleading narrative. First, it examines Frege's innovations and their contrast with the Aristotelian syllogistic. Next, it demonstrates how Peirce independently developed parallel logical advances. Finally, it critiques the philosophical underpinnings of the dominant narrative's concepts like *calculus*, *language*, *universe*, and *model*¿that have shaped its persistence.

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Seminário V: Contemporaneidade II

Horizontes da Lógica para o Século 20
As Contribuições Russas à Lógica Não Clássica – Andrei Kolmogorov

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Nesta palestra, relembraremos os importantes, originais e inovadores trabalhos de alguns pensadores russos: Vasiliev, Florensky, Kolmogorov, Glivenko, Orlov e Bochvar.

Discutiremos o artigo publicado por Kolmogorov em 1925, sobre o princípio do terceiro excluído, uma contribuição fundamental para o debate entre Brouwer e Hilbert sobre os fundamentos da matemática.

Kolmogorov propôs a primeira formalização para a lógica intuicionista e a primeira tradução da lógica clássica para a intuicionista. Seu sistema formal também é em sentido amplo paraconsistente.

Horizons of Logic for the 20th Century:
Russian Contributions to Non-Classical Logic – Andrei Kolmogorov

In this talk we will recall the important, original and innovative works of some Russian thinkers, Vasiliev, Florensky, Kolmogorov, Glivenko, Orlov and Bochvar. We will discuss the paper published by Kolmogorov in 1925, On the principle of the excluded middle, a fundamental contribution to the debate between Brouwer and Hilbert on the foundations of mathematics. Kolmogorov proposed the first formalization for intuitionistic logic and the first translation from classical to intuitionistic logic. His formal system is also paraconsistent in a broad sense.

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Workshops
Gaps and Gluts

Workshop: Gaps and Gluts 4

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Keywords: Non-classical negations, paraconsistency, paracompleteness, logical pluralism, belief revision, proof-theory.

Logics that allow for gaps and gluts, i.e. paracomplete and paraconsistent logics, now occupy a central place in logic and the philosophy of logic. Research into their philosophical and conceptual aspects, as well as their potential applications, extends to other fields, particularly computer science and mathematics.

This workshop, which is a smaller-scale version of the workshops related to the research project *Negations, Gaps, and Gluts* (CNPq 408040/2021, `https://gapsandgluts.wordpress.com/`), aims to bring together researchers from different fields working on topics related to paracomplete and paraconsistent formal systems. The goal is to create an environment where technical and conceptual discussions can interact, enabling the refinement of ideas that strengthen the connection between technical results and the intuitive interpretations of proposed formal systems. Additionally, the workshop seeks to foster philosophical discussions on key issues in the philosophy of logic.

Topics of interest include (but are not limited to): logical pluralism; the interpretation of paraconsistency and paracompleteness; gaps and gluts in modal logics; proof theory of non-classical logics; paraconsistent belief revision; paraconsistent logics and truth theories; automated reasoning in paraconsistent logics. Nine 30-minute talks are planned, as listed below, initially scheduled to take up a morning and an afternoon.

1. Marcelo Coniglio, *An AGM-like paraconsistent belief revision system defined from epistemic entrenchment.*
2. Cezar Mortari, *Valuation semantics for failed axiomatizations of K and E.*
3. Walter Carnielli, *Constructing justifications: evidence-based reasoning via lambda terms* (joint work with J.C. Agudelo).
4. Abilio Rodrigues, *Many-logic modal structures and information based logics.*
5. Evelyn Erickson, *Gaps, gluts, and theory choice in logic.*
6. Guilherme Cardoso, *Gluts without dialetheias.*
7. Marcos Silva, *How to revise logic: on reflective equilibrium, upward normative pressures and a posteriori revisions.*
8. André Porto, *Intuitionism and the problem of the continuum.*
9. Luiz Carlos Pereira, TBA.

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On Many-Logic Modal Structures Based on the Lattice $L6$

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Keywords: logics of evidence and truth, many-logic modal structures, non-classical negations, Kripke models.

This talk proposes an approach to information-based logics using the many-logic modal structures introduced by Freire and Martins in [3]. Such structures can express accessibility relations between worlds with different underlying logics, anchoring them in a base lattice that contains the semantics of each logic as a sublattice. The base lattice allows us to transfer semantic information between different logics in a natural way. Many-logic modal structures are suitable for representing connections between information states interpreted as configurations of databases and the evolution of such information states over time. We will illustrate the application of many-logic structures through the six-valued logic of evidence and truth LET_K^+ [1], related to the lattice $L6$, which extends the lattice $L4$ defined by Belnap-Dunn four-valued logic [2] with a new top and a new bottom, that are intuitively interpreted respectively as reliable positive and reliable negative information. From LET_K^+ and $L6$, we obtain a family of matrix logics capable of representing paracomplete, paraconsistent, and classical contexts with six, four, three, and two scenarios. In this setting, modalities are redefined in a non-standard manner (joint work with Manuel Martins, Marcelo Coniglio, and Alfredo Freire).

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Intuitionism and the Problem of the Continuum

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Keywords: Intuitionism, The Continuum, Identity

Our presentation's first objective will be to argue against the idea that intuitionism should be viewed as basically a rejection of the principle of the excluded middle. There are many aspects involving that approach to mathematics, some quite traditional, stemming all the way back to Aristotle's view that the continuous should never be construed as being composed out or "points". And some more recent, such as the constructivist's idea that "existence" should be construed as "obtainability", i.e., the possession of an algorithm which allowed one to generate a witness to some required property (we are thinking of Kronecker, of instance). But there is also one crucial very recent aspect of contemporary intuitionism which involves a thorough reconstrual of the very notion of identity, rejecting the typically classical idea of one single universal notion in favor of a plurality of specialized identities, one for each single semantical domain.

This discussion of the notion of "identity" brings us to our second goal within our presentation. We will also try to argue that one should never attempt to access the merits (or demerits) of intuitionistic mathematics in abstraction from the crucial arena with respect to which those merits can be properly evaluated. We are referring here to the old Problem of the Continuum, the challenge to offer an adequate logico-semantical treatment of "continuous magnitudes". If we are correct, the full force of intuitionistic proposals can only become evident when regarded with respect to this crucial undertaking.

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Valuation Semantics for Failed Axiomatizations of K and E

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Keywords: modal logics, valuation semantics, paraconsistency

In [4], H. Omori and D. Skurt, among other things, presented a sixteen-valued non-deterministic semantics for a system they called $\mathsf{K_f}$, proving correction and completeness. This system was first presented by L. Humberstone in [2], and consists in taking $\Diamond$ as the only primitive modal operator and replacing, in a standard axiomatization for K, all ocurrences of $\Box$ by $\neg\Diamond\neg$. Humberstone showed that $\mathsf{K_f}$ is not after all a complete axiomatization of K, since $\Diamond p \to \Diamond\neg\neg p$ is not derivable. A semantics for $\mathsf{K_f}$, however, was first provided by Omori and Skurt. In this work, we provide a two-valued valuation semantics for $\mathsf{K_f}$. We discuss some extensions of $\mathsf{K_f}$ and also present another system, which we call $\mathsf{E_f}$, obtained from an axiomatization for the minimal classical modal logic E by the same procedure of Humberstone's. For similar reasons, $\mathsf{E_f}$ fails to axiomatize E. Finally, we show that, in both systems, neither $\Diamond\neg p \to (p \to q)$ nor $p \vee \Diamond\neg p$ are theses, and discuss whether systems like $\mathsf{K_f}$ and $\mathsf{E_f}$ can be seen as paraconsistent and/or paracomplete logics, should we understand $\Diamond\neg$ as a kind of "weak negation" (cf. [1], [3]).

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Gaps, Gluts and Theory Choice in Logic

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Palavras-chave: epistemologia da lógica, paraconsistência, paracompletude, pluralismo lógico

While two recent discussions in the epistemology of logic, pluralism [2] and anti-exceptionalism, have centered around the philosophical significance of non-classical logics (in particular a reactions to gappy and glutty logics), they have not yet been put on a par. The present contributing aims to characterize pluralism and anti-exceptionailism with the same discussion, one about logical disagreement and theory choice.

Logical abductivist [?, 1, 4] are motivated by logical disagreements into using Inference to the Best Explanation to select the theory which can best provide an account of deductive validity. Pluralists present both an account of validity and of logical disagreement, but instead of putting forth an argument why one logical theory is better than another, they take it that more than one logic may be correct at once.

The present contribution aims to discussing both pluralism and anti-exceptioanlism (in particular, logical abductivism [?, 1, 4]) in terms of their reactions to non-classicality, and argue that these two "isms" are different reactions to the same phenomenon, and are incompatible. Counter Hjortland [1], it will be argued that there cannot be an anti-exceptionalist pluralist, because each doctrine suggest a different response in dealing with gaps and gluts. While logical pluralism puts forth the idea that more than one logic can, in some sense, be correct, discussions around anti-exceptionalism point to one theory being revised in favor of another one. While a pluralist might say that classical logic and a glutty/gappy logic can both be equally legitimate (what this legitimacy amounts to is also up for debate), an anti-exceptionalist might say that classical logic should be replaced by a glutty/gappy one (in the one legitimate context).

By exploring the assumptions of logical pluralism and logical anti-excptionalism, the presentation aims at showing that both offer only a limited framework for discussing the epistemology of logic, as well as proposing a better way to value the conceptual developments that gappy and glutty logics have to offer.

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Gluts without Dialetheias

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Keywords: Dialetheism, Gaps, Gluts, The Paradox of the Liar

Many-valued logics have frequently been invoked to address philosophical puzzles surrounding the concept of truth. Consider, for example, Łukasiewicz's [3] Ł$_3$ on future contingent truths, Kripke's [2] interpretation of Strong Kleene K_3 for ungrounded sentences, and Priest's [4] notion of a dialetheia, based on his Logic of Paradox (LP). The first two approaches involve what has been called truth-value gaps (a statement that is neither true nor false), while the last involves truth-value gluts (statements that are both true and false). A promising candidate for both gaps and gluts is the Liar sentence. Some might argue that it is neither true nor false; otherwise, it would be both true and false. Dialetheism offers an approach to the Liar, according to which it is a true contradiction (i.e., a dialetheia). In this paper, I shall argue for an alternative approach, one that incorporates both gaps and gluts to avoid dialetheias. My approach is based on Austin?s notion of truth [1] and an extension of First-Degree Entailment (FDE). According to this view, the Liar (and other potential cases of gluts) is a true contradiction, but not a dialetheia. The contradiction is true in the sense that the sentence accurately classifies a situation, though it is a situation that is not real. Regarding real situations, the Liar sentence is neither true nor false; thus, it represents a gap. Therefore, in a sense, the Liar sentence is both a real gap and an unreal glut.

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Revisiting Full Intuitionistic Linear Logic and Other (Strange) Exponentials and Quantifiers

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Keywords: Linear Logic, Intuitionism, Quantifiers

In 1991, during the LMPS conference organized in Uppsala, Sweden, Valeria de Paiva gave a presentation in which there was a proposal for a categorical semantics for what was then called "Complete Intuitionist Linear Logic". This logic was named FILL. As far as I know, this was the first public mention to such a complete intuitionistic linear logic. The main idea was the following: to propose a categorical semantics for an intuitionistic linear logic which also contained a multiplicative disjunction $\parr$ and the exponential "?" (Why not?). According to the de Paiva, this Logic could be easily described by a Gentzen-like sequent calculus where the usual restriction to mono-succedent is imposed only on the rule for $\multimap$-right. Using this idea, there would be a natural analogy between [1] Gentzen's LK and Classical Linear Logic (CLL), [2] Gentzen's LJ and Intuitionistic Linear Logic (ILL), and [3] Maehara's system LJ' and $FILL$. In fact, the real analogy would not be between LJ' and $FILL$, but between a sequent calculus based on LJ' for the logic Constant Domains (CD) and $FILL$ and, given this analogy, simple counter-examples for cut-elimination for $FILL$ were found. A solution to the cut-elimination problem for Full Intuitionistic Linear Logic was based on a technique explored by Kashima and Shimura with respect to the logic CD of *Constant Domains*: to replace a cardinality restriction by a control of dependency relations. In this way, a new version of the $FILL$ system for complete intuitionistic linear logic was proposed.
My modest goals will be: [1] to show that this new version of $FILL$ is not really intuitionist, but rather an intermediate system between a complete intuitionistic linear logic and classical Linear Logic (CLL), in the same sense in which the logic CD is intermediate betwen Intuitionistic Logic (IL) and classical logic (CL); and [2] to investigate some relations between FILL and other heterodoxical exponentials (as in Dissymetrical Linear Logic - DLL) and quantifiers (like CD, and Dissymetrical First Order Logic - DFOL).

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An AGM-like Paraconsistent Belief Revision System Defined from Epistemic Entrenchment

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Keywords: belief revision, paraconsistent logic, Logics of Formal Inconsistency, AGM, epistemic entrenchment

Belief Revision is the study the dynamics of agents's epistemic states. The most influential paradigm in this area is the AGM model, introduced by C. Alchourrón, P. Gärdenfors and D. Makinson in 1985, in which *epistemic states* are represented as theories (sets of sentences closed under logical consequence in an underlying logic which expands classical logic) called *belief sets*. One of the main operations is *contraction*, which consist of the retraction of a sentence from a given belief set K, while maintaining as much as possible of K. A very natural way to induce a contraction is by means of *epistemic entrenchment*, based on the idea that contractions in a belief set K should be guided by an ordering of sentences according to their epistemic importance. In rough terms, if γ is less entrenched than β in K then it is easier to discard γ than β in a given contraction of K.

In this talk we address the integration of epistemic entrenchment into the AGM-like paraconsistent belief revision system AGM$\circ$, introduced in [3], which is based on Logics of Formal Inconsistency (**LFI**s, see for instance [1]). AGM$\circ$ explicitly incorporates the 'consistency' (or 'robustness') operator "$\circ$" as a way to introduce new epistemic attitudes – namely, *strong acceptance* (both $\circ\alpha$ and α belong to the belief set K) and *strong rejection* (both $\circ\alpha$ and $\neg\alpha$ belong to the belief set K) of a belief-representing sentence α. In AGM$\circ$, the epistemic robustness of a sentence α, expressed by $\circ\alpha$, ensures that α, when accepted, cannot be retracted from the belief set through contraction unless $\circ\alpha$ itself is first removed. This mechanism guarantees the protection of strongly accepted sentences, thereby structuring the belief revision process around the preservation of core beliefs.

A symmetry exists between the underlying concept of non-retractability of strongly accepted sentences in AGM$\circ$ and the notion of *entrenchment*, where strongly accepted sentences (as well as the valid ones) could naturally correspond to the most entrenched beliefs. However, this symmetry has remained unexplored due to the inability to define *epistemic entrenchment* in paraconsistent belief revision systems based on **LFI**s. This limitation stems from the absence of the *replacement property* – crucial to define epistemic entrenchment – in most **LFI**s. This property guarantees that if two formulas are logically equivalent, one can replace the other in any context without affecting derivability or inferential behavior. From this, the lack of another expected properties – such as the preservation of equivalences involving the consistency operator – hinders the formulation of a coherent framework for *epistemic entrenchment* with this kind of **LFI**s as underlying logic.

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To address these challenges, two paraconsistent logics will be introduced in this talk: **Cbr** and **RCBr**. The logic **Cbr** (already defined in [3], although only a few technical results about it have been proven there) is specifically designed to support belief revision in AGM$\circ$ by incorporating properties that are essential for the formal characterization of epistemic attitudes such as strong acceptance and strong rejection. In particular, the consistency operator $\circ$ satisfies $\circ\alpha \equiv \circ\neg\alpha$, reflecting a symmetry in the evaluation of consistency for a formula and its negation. Since $\alpha \equiv \neg\neg\alpha$ also holds in **Cbr**, it follows from both properties that, for every sentence α and belief set K: α is strongly accepted in K if and only if $\neg\alpha$ is strongly rejected in K; and α is strongly rejected in K if and only if $\neg\alpha$ is strongly accepted in K. Both properties concerning epistemic attitudes are very natural and expected. Another interesting feature of **Cbr** is that $\circ$ preserves logical equivalence under certain reasonable assumptions, namely: if $\alpha \equiv \beta$ and $\neg\alpha \equiv \neg\beta$, then $\circ\alpha \equiv \circ\beta$. These properties ensure that $\circ$ behaves coherently in relation to logical equivalences and provides a suitable foundation for modeling strong epistemic attitudes in belief revision systems. We prove that **Cbr** is characterized by a 3-valued non-deterministic matrix.

We further introduce **RCbr**, the extension of **Cbr** that incorporates the *replacement property*, by using the technique introduced in [2]. The (algebraic) semantics corresponding to **RCbr** is, therefore, by means of a class (a variety) of expansions of Boolean algebras with **LFI** operators (BALFIs, see [2]). The first challenge now is to prove that **RCbr** is still paraconsistent. To this end, we construct a BALFI over the Boolean algebra $\wp(\mathbb{Z})$ which models **RCie**, the axiomatic extension of **RCbr** by adding axiom (ci) $\neg\circ\alpha \to (\alpha \wedge \neg\alpha)$, and invalidates the inference $\{p, \neg p\} \vdash q$ for two different propositional variables p, q (showing that **RCie**, and so **RCbr**, are indeed paraconsistent). The second challenge is to prove that (cp) $\circ\alpha \to \circ\circ\alpha$ is not valid in **RCbr**. This is a fundamental question: in AGM$\circ$, if $\circ\alpha \in K$ and either $\alpha \in K$ or $\neg\alpha \in K$, it cannot be eliminated by contraction (as occurs with the tautologies of the logic). Hence, if (cp) holds in **RCbr** and α is boldly accepted (or boldly rejected) in K then this situation will last forever. Indeed, α (or $\neg\alpha$) cannot be removed from K, by definition of AGM$\circ$. A solution would be remove first $\circ\alpha$ from K and then remove α (or $\neg\alpha$) from the resulting set. But this is impossible, since $\circ\circ\alpha \in K$ by (cp). This is why this second challenge is so important. By constructing a somewhat sophisticated BALFI structure over $\wp(\mathbb{Z})$ using modular arithmetic, we prove that (cp) does not hold in **RCbr**.

By resolving the limitations of previous **LFI**s, **RCbr** allows for the substitution of logically equivalent formulas, supporting the construction of a robust and natural notion of *epistemic entrenchment*, extending the classical setting to AGM$\circ$. This, in turn, enables the definition of a principled and coherent operation of contraction based on *epistemic entrenchment*, which respects consistency, prioritizes entrenched beliefs, and accommodates the dynamic nature of paraconsistent reasoning.

Through the integration of **LFI**s and belief revision principles, this work aims to provide a flexible and coherent framework that captures the reasoning processes of agents operating in dynamic and contradictory environments. This contribution advance the theory of paraconsistent belief revision and pave the way for applications in domains such as multi-agent systems and inconsistent knowledge bases.

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How to Revise Logic: On Reflective Equilibrium, Upward Normative Pressures and a posteriori Revisions

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How could we justify our logical principles if the very possibility of rational justification presupposes them? To what extent is it possible to revise something as fundamental as logical principles? Otherwise stated, how can a set of basic principles of logic be shown to be correct without lapsing into vicious circularity or resulting in an infinite regress? We argue that logic is a science analogous to normative disciplines, a view also promoted by Prawitz (2007). In order to do so, we will use lessons from semantic inferentialism and logical expressivism, as proposed by Brandom (1994, 2000). In light of this context, we outline a method that bases the revision of logic on a neopragmatist version of reflective equilibrium between general theoretical considerations and particular inferences. Then, we develop the notion of normative bidirectionality and argue that what we call upward normative pressure adequately expresses the dynamical and a posteriori aspect of the revision of some logical principles. Upward normative pressures show how to see the rise of non-classical logic in analogy of how political and ethical systems might be revised as well. My aim here is not to defend Reflective Equilibrium against detractors. This contribution will defend that Reflective Equilibrium can be used as a method to revise logic in a horizon where the normativity of logic is due to the normativity of natural language, as in a usual inferentialist point of view.

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Constructing Justifications: Evidence-Based Reasoning via Lambda Terms

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Keywords: Logic of Evidence and Truth, lambda terms, constructive evidence

Paraconsistent and paracomplete logics provide connectives for consistency and determinacy or determinedness that enable the independent recovery of explosiveness and the law of excluded middle for specific propositions [2]). We discuss the logic LET_C, a kind of Logic of Evidence and Truth (LET, cf. [3]), which preserves full symmetry between truth and falsity, allowing direct proofs of both the truth and falsity of a proposition. Furthermore, LET_C introduces a distinction between truth and the impossibility of falsity, as well as between falsity and the impossibility of truth-concepts that are indistinguishible in classical logic and whose distinction appears blurred in Intuitionistic Logic. In this framework, dual connectives for inconsistency and undeterminedness are defined within LET_C. Evidence is explicitly formalized by integrating lambda calculus terms into LET_C, resulting in the type system LET_C^λ. In this system, lambda calculus terms represent procedures for constructing evidence for compound formulas based on the evidence of their constituent parts. Several properties of type systems are proven for LET_C^λ lemmas on generation, free variables, uniqueness of types (under a strong equivalence defined on formulas), substitution, subject reduction, and a normalization theorem. A realizability interpretation is provided for LET_C, establishing a strong connection between deductions in this system and recursive functions.

Our approach contrasts with others in the literature, such as those by M. Fitting and S. Artemov, in the sense that in LET_C^λ, lambda-calculus terms not only represent evidence but also describe algorithmic procedures for deriving evidence for compound formulas based on evidence for their constituent parts. Extending LET_C to first-order logic is not particularly difficult. This can be achieved by incorporating the rules for the quantifiers along with their negations, as done in quantified Nelson's logic.

It is compelling to suggest that the concept of constructive justification may be particularly well-suited for exploring algorithmic explainability in the context of Artificial Intelligence, a field of development yet to be fully realized. This research ([1]) is conducted in collaboration with J.C. Agudelo-Agudelo..

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Workshops
Logic, Conditionals and Probability

Workshop: Logic, Conditionals and Probabilities

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Keywords: probability , conditionals, logic,induction

This workshop aims to bring together researchers and practitioners to explore the intersections of logical systems, probabilistic reasoning, and conditionals in philosophy and language. Our intention is to provide an exciting platform to discuss foundational, theoretical, and applied advancements in these fields, which play a crucial role in understanding decision-making, artificial intelligence, and formal reasoning. We invite contributions within the following topics (but not limited to):

- Logical Foundations of Conditionals: Analyzing the nature and structure of conditionals in various logical systems, including classical, non-classical, and paraconsistent logics, including their relation to conditional probability
- Probabilistic Reasoning and Logic: Integrating probability theory with logical frameworks, including Bayesian inference, probabilistic logics, and decision theory.
- Conditionals and Causality: Exploring the relationship between conditionals and causal reasoning in both philosophical and computational contexts, including counterfactuals.
- Non-Classical Logics and Probability: Investigating non-classical logical systems (e.g., many-valued logic, intuitionistic logic, paraconsistent logics) and their interactions with probabilistic models.
- Induction versus Probability: Philosophical implications on the relationship between inductive reasoning and probability, especially in the context of scientific discovery and reasoning under uncertainty.
- Extended Bayesian network models: conditional dependency and independency in non-classical logics
- Probabilistic Satisfiability over diverse logical systems
- Applications in AI and Machine Learning: how logic, conditionals, and probabilities contribute to reasoning under uncertainty, decision-making processes, and probabilistic programming.

We encourage submissions that advance both the theoretical understanding and practical applications of logic in conjunction with probability theory and conditionals. Contributions from a broad spectrum of discipline - including philosophy, computer science, mathematics, and artificial intelligence - are highly welcomed.

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On a Paraconsistent and Paracomplete Non-Deterministic Fuzzy Logic

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Keywords: paraconsistency, paracompleteness, fuzzy logics, evidence.

In this talk, I present the guiding ideas of a paraconsistent and paracomplete fuzzy logic designed to represent the degrees of evidence, positive and negative, available for a sentence A. The proposal aims to capture paraconsistent, paracomplete, and classical scenarios within a fuzzy semantics using a recovery operator $\circ$ as is done by logics of evidence and truth (see e.g. [1]). A preliminary investigation of the semantic clauses for the binary connectives of Gödel's fuzzy logic and product logic (cf. [2]) suggests that they align well with the interpretation in terms of evidence.

Regarding negation, the starting point is to consider it as a primitive symbol, that is, $\neg A$ is not defined as $A \to \mathbf{0}$, where $\mathbf{0}$ is a formula that is always assigned the value 0. A fuzzy valuation is a function v from the set of *literals* (formulas p and $\neg p$) to $[0, 1]$, thus allowing for a free assignment of values to p and $\neg p$. The semantics of negation, therefore, is non-deterministic (i.e. it is not truth-functional). Double negation is valid because it is assumed that A and $\neg\neg A$ have, so to speak, the same meaning – that is, positive evidence for $\neg A$ amounts to negative evidence for A, and positive evidence for A amounts to negative evidence for $\neg A$. In addition, all directions of De Morgan's laws are valid, and it is desirable that the equivalence between $\neg(A \to B)$ and $A \wedge \neg B$ holds, as it does in Nelson's *N4*.

With respect to the connective $\circ$, the idea is to replace it with the connectives $\olessthan$ and $\ogreaterthan$, representing gaps and gluts, so that $\olessthan A$ and $\ogreaterthan A$ express, respectively, the degree of incompleteness and inconsistency of A. Thus:

$$v(\olessthan A) = \max\{1 - (v(A) + v(\neg A)), 0\} \quad \text{and} \quad v(\ogreaterthan A) = \max\{v(A) + v(\neg A) - 1, 0\}.$$

My idea is to combine the original clauses of the binary connectives with negative clauses similar to those of *N4*, in order to obtain a fuzzy treatment of scenarios with gaps and gluts suitable for the interpretation in terms of degrees of evidence. It is likely that the success of this proposal will depend mainly (although not exclusively) on the behavior of negation, which must be consistent with the usual properties of fuzzy operations, defined by *t-norms* and *t-conorms* (joint work with Manuel Martins).

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Actuality Modal Operators without Kripke Semantics: A Non-Deterministic Approach

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Keywords: non-deterministic matrices, modal logic, Yuriy Ivlev.

In 1988, J. Ivlev develops a four-valued semantics with multifunctions to characterize non-normal modal logics [5] preceding the works of non-deterministic semantics made by Lev and Avron in 2005 [1]. In 2015, Coniglio, del Cerro, and Perón extend a hierarchy of Ivlev-like systems, also indicating the interpretation of modal, epistemological, and deontic concepts in these systems [2].

In this presentation, we start by considering a non-normal (Ivlev-like) version of $S5$ called $T45m$, characterized by a 4-valued non-deterministic matrix (Nmatrix, for short), and we extend it with an actuality operator "A". The actuality operator follows the commonground rules established in the literature relative to actuality operators: thus, we define a Hilbert calculus for a (non-normal) modal system for actuality in terms of the axioms introduced in Crossley & Humberstone's $S5A$ [3], with exception of the first one, (Ax1): $\mathsf{A}(\mathsf{A}\alpha \to \alpha)$. The resulting logic $T45Am^-$ is characterized by a 6-valued Nmatrix. In order to satisfy axiom (Ax1) it is required to restrict the valuations of the Nmatrix for $T45Am^-$ to the ones that validate (Ax1). This produces a system called $T45Am$ characterized by a 6-valued restricted Nmatrix (RNmatrix, for short), which constitutes a non-normal version of $S5A$.

Given the obtained results, we argue against some of the properties attributed to actuality operators. In particular, the redundancy of the operator such as presented in [4] is addressed and we contend its hyperintensional nature. We argue that the results presented here support a formal semantic approach to actuality modal operators as an alternative to Kripke's possible worlds semantics.

Finally, we aim to outline future research directions for characterizing these operators with swap structures semantics, as described in [6], and to explore Ivlev-like deontic systems incorporating actuality operators. This is a joint work with Marcelo E. Coniglio.

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Multialgebras' Evolution: From Mathematics to Logic and Artificial Intelligence

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Keywords: multioperation, non-deterministic matrices, artificial intelligence

The idea of constructing algebraic hyperstructures, based on a broader definition of *multioperations*, which differ from the usual operations by returning a set of elements instead of a single element, was first proposed by the French mathematician Frédéric Marty in 1934 [1]. However, previous works may have already explored concepts surrounding these structures, since they are known by different names in different parts of the world. The class of structures, composed by at least one multioperation, is what we call of *algebraic hyperstructure*. The elements in the class of hyperstructures are *multialgebras*, that is algebras equipped with at least one multioperation. There are at least three areas of knowledge that use such algebraic hyperstructures: mathematics, logic and computer science. In mathematics, they are studied from the point of view of universal algebra, and more recently in quadratic forms; in logic, they are used to define non-deterministic matrices; and in computer science they are used in formal languages and, more recently, to model systems that deal with incomplete information, as is the case of some artificial intelligence systems. In this work we propose to establish a link between the various forms of application of multialgebras in the various areas of knowledge addressed, with an emphasis on the field of artificial intelligence, where the use of such structures is still little explored. This research was supported by the São Paulo Research Foundation (FAPESP, Brazil) trough the Thematic Project "Rationality, logic and probability" - RatioLog (grant #2020/16353-3).

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A Non-Deterministic Approach to Fictional Logic

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Keywords: logic of fiction, non-deterministic semantics, swap structures

According to Frege's *Principle of Compositionality*, for a sentence to have a truth value, the singular terms that occur in it must refer to objects. This principle, however, seems too strong when we deal with sentences in which fictional terms occur. For example, the sentence "Sherlock Holmes is a man" seems to be true when contrasted with "Sherlock Holmes is a lawyer", though we not seeM to be dealing with the same notion of truth at stake when one says that "Socrates is a man" is true.

There are several alternative ways of modelling the logical behavior of sentence in which fictional terms occur, e. g., using classical logic, free logic or modal logic. In [2] an alternative approach was proposed using a family of four-valued logics. The key idea of the proposal in [2] is to assign the values T and F to sentences in which singular terms occur that refer to flesh and blood individuals, while reserving the values t and f to fictional sentences (the former pair of values corresponding to the notions of a factually true/false sentence, and the latter pair to the notion of a fictionally true/false sentence).

Obviously, the greatest difficulty is how to deal with *mixed sentences*, i.e., sentences in which both fictional and non-fictional terms occur. For example, on the one hand, it seems reasonable to consider the sentence "Sherlock Holmes admires Isaac Newton" to be either fictionally true or fictionally false (and so to assign it one of the values in $\{\mathsf{t}, \mathsf{f}\}$); on the other hand, the sentence "Albert Einstein admires Sherlock Holmes" appears to be either factually true or factually false (and so should be assigned one of the values in $\{\mathsf{T}, \mathsf{F}\}$). Since there is no formal criterion to distinguish these two cases, in [1] two types of logic were proposed. In one of them, mixed sentences receive the value T or F; while in the other they receive t or f.

In this talk, we will present a third possibility. Our proposal is that mixed sentences should receive an indeterminate set of values $\{\mathsf{T}, \mathsf{t}\}$ or $\{\mathsf{F}, \mathsf{f}\}$. This indeterminacy, occurring at the atomic level, propagates across complex sentences, involving propositional operators and quantifiers.

From a technical point of view, correctness and completeness can be easily achieved by means of swap structures, inspired in [1], in which these four truth values can be understood as ordered pairs: the first coordinate informs whether the truth value would be designated or not, while the second coordinate informs whether it is a factual or fictional truth value. Propositional operators and quantifiers are defined as multi-algebraic functions.

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The Problem of Conditionals in the Philosophy of Paraconsistency

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Paraconsistent logics, in their LFI tradition, generally consist of systems that take into account the role of negation to show how contradiction can be supported within rationality. Technically, these systems are constructed by extending positive propositional calculus, as is the case with systems such as mbC, bC, Ci, Cie, LFI1, among others (cf. [1]). More recently, this way of conceiving systems has been altered, and attention has shifted to understanding how the consequence relation can be comprehended within the context of rationality when dealing with inferences involving contradiction. To this end, the concept of truth preservation has been replaced by that of evidence preservation. This concept has become the theoretical foundation for the development of LETs (cf. [2] and [3]). However, issues inherent to the conditional have not been thoroughly explored in the context of paraconsistency. Why should a conditional with a false antecedent be considered true? To say that a conditional holds means that, in the actual world, whenever the antecedent occurs, the consequent also occurs. It is invalid when the antecedent occurs but the consequent does not (cf. [6]). If the antecedent does not occur, however, what is the reason for considering the conditional true in this case? This debate leads us to adopt a three-valued logic, or more generally a many-valued logic, in which conditionals with a false antecedent are seen as indeterminate rather than true, as in the case of material implication. Is it possible to handle contradictions in contexts involving this type of conditional? Could the language of LFIs serve as a formal apparatus to express these situations? This kind of problem is becoming more and more relevant in the philosophy of human rationality (cf. [4] and [5]). In this talk, we aim to discuss the feasibility of introducing paraconsistent systems in which the conditional better aligns with the facts of the world, investigating how the concept of evidence would be affected by this type of analysis.

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The e-value and the Full Bayesian Significance Test: Logical Properties and Philosophical Consequences

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Keywords: Bayesian inference 1, Logical coherence 2, Test of scientific hypotheses 3.

The e-value, $\text{ev}(H|X)$ – also named the *epistemic-value* of hypothesis H given observations X, or the evidence-value of observations X in favor (or in support) of hypothesis H – is a Bayesian statistical significance measure introduced in 1999 by Carlos Alberto de Bragança Pereira and Julio Michael Stern, together with the FBST – the Full Bayesian Significance Test, see [5]. The definitions of e-value and the FBST were further refined and generalized by subsequent works of several researchers at USP – the University of São Paulo, and UFSCar – the Federal University of São Carlos, in Brazil, including Wagner Borges, Luís Gustavo Esteves, Rafael Izbicki, Regina Madruga, Rafael Bassi Stern, and Sergio Wechsler, see [1–4, 6].

The e-value was specially designed to assess the *statistical significance* or the *logical truth value* of *sharp* or *precise* hypotheses in the context of Bayesian statistics. The e-value has desirable asymptotic, geometrical (invariance), and logical (compositional) properties that allow consistent and coherent evaluation and testing of sharp statistical hypotheses. Furthermore, in applied modeling, the FBST offers an easy to implement and powerful statistical test that is fully compliant with Bayesian principles of good inference, like the likelihood principle.

In the context of statistical test of hypotheses, a *compositional logic* is conveyed by an algebraic formalism that allows the evaluation of truth-functions of composite models and truth-values of composite hypotheses by algebraic operations on the corresponding truth functions of elementary models and truth values of elementary hypotheses. The e-value and the FBST have a rich, expressive and intuitive compositional logic, while traditional truth-values and accompanying tests offered by either frequentist (classical) statistics, like the p-value, or by Bayesian statistics, like Bayes factors, have important and well-known deficiencies in this regard, specially in cases involving sharp statistical hypotheses. Furthermore, *logically coherent* evaluations and testing of sets and subsets of statistical hypotheses should render sequences of inferential reasoning that do not generate internal contradictions or anti-intuitive results. As expected, the e-value and the FBST comply with well-established rules of logical coherence, even in the case of sharp hypotheses, while traditional

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alternatives often fail to do so. Asymptotically, $\text{sev}(H|X)$ – the standarized e-value – shares several properties of the p-value, the well-known significance measure of frequentist statistics; Nevertheless, $\text{sev}(H|X)$ retains many theoretical characteristics of the Bayesian framework. For several theoretical developments and practical applications of the e-value and the FBST, see [7], [12].

Section 2 reviews the Bayesian statistical framework; Section 3 defines the e-value; Sections 4, 5 and 6 explain the invariance, asymptotic and compositional properties of the e-value; Section 7 defines the GFBST - the Generalized Full Bayesian Significance Test and its logical properties; Sections 8 and 9 comment on computational implementation and give a detailed numerical example in model selection; Section 10 lists a representative assortment of articles from many practical applications of the e-value and the FBST already published in the scientific literature. Section 11 considers the philosophical consequences of the aforementioned developments by briefly commenting the *Objective cognitive constructivism* epistemological framework, that was specifically developed to accomodate the formal properties of the e-value and the FBST, and renders a naturalized approach to ontology and metaphysics. Section 12 presents some topics for further research at the interface between Logic and Statistics.

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On Morgado and Sette's Implicative Hyperlattices as Models of da Costa logic C_ω

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Keywords: hyper lattice, reticuloid, swap structure, da Costa $C\omega$ algebra, LFI

Paraconsistent logics, particularly the logics of formal inconsistency (LFIs), were developed to address scenarios where contradictory but non-trivial information must be handled, avoiding the explosion principle of classical logic ([2], [4]). Unlike many classical logics, LFIs are generally not algebraizable in the sense of Blok and Pigozzi, which poses challenges for their semantic characterization. However, alternative frameworks, such as non-deterministic matrices (Nmatrices [1]) and hyperalgebraic structures (e.g., swap structures [3]), have proven effective in providing semantics for LFIs. These approaches offer powerful tools for reasoning in inconsistent and non-deterministic contexts, bypassing the limitations of algebraizability.

In 1962, José Morgado introduced the concept of hyperlattices (which he called *reticuloides*), extending classical lattice structures ([6]). Building on this, Antonio M. Sette proposed in his 1971 Master's thesis, supervised by Newton da Costa, the notion of implicative hyperlattices (here called SIHLs) ([8]). Sette further extended SIHLs by defining a unary hyperoperator, leading to the formulation of a class of hyperalgebras (here called SHCωs) that correspond to da Costa algebras for $C\omega$, providing an algebraic characterization of the paraconsistent logic C_ω.

Building on recent advances in the theory of hyperstructures ([7], [5]), and inspired by the ideas of Sette and Morgado, this paper revisits Sette's implicative hyperlattices, providing a lattice-theoretic characterization and establishing foundational results on SIHLs. we introduce a class of swap structures, an special class of hyperalgebras over the signature of C_ω naturally induced by implicative lattices. It is proven that these swap structures are SHC$_\omega$s, being so a very intuitive class of models for these hyperalgebras. Finally, it is proven that the class of SHC$_\omega$s, as well as the above mentioned class of swap structures, characterize the logic C_ω.

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On Probability as a Linguistic Model

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Keywords: Probability, theoretical-linguistic model, probability theories.

This talk explores the thesis that probability is best understood as a theoretical-linguistic model (TLM), rather than as a subjective, ontological, or frequentist theory. Gillies [**?**] discusses five main interpretations of probability and defends three: subjective theory (ST), where probability is an individual degree of belief (DB); propensity theory (PT), where probability is a tendency in repeatable conditions; and inter subjective theory (IST), where probability reflects social consensus. Hacking [2] distinguishes ”frequency-type” and ”belief-type” probabilities. However, these perspectives fail to explain probability from a historical and philosophical standpoint.

The proposal of probability as a TLM is supported by the interpretation from Schurz' [3] work, which bridges ”statistical” and ST perspectives using principles like the ”Nearest Reference Class Principle”. Following Schurz, I prefer 'statistical' over 'frequentist' because the model concerns limits of frequencies, not actual frequencies. When analyzing sample frequency, we focus on the distribution of characteristics (e.g., mean or proportion) within a sample, while limiting distribution describes statistical behavior as the sample size approaches infinity, aligning with the Central Limit Theorem.

A theoretical model based on limiting distributions serves as a tool for statistical decision-making, rather than relying on sample frequencies. This distinction makes the model ”statistical” rather than ”frequentist.” Schurz unifies probability theory for both objective statistics and degrees of belief, which can be interpreted as a TLM. While Schurz sees a dualistic theory, we propose a linguistic model with two applications: beliefs and decisions.

We argue that probability is a TLM because it relies on idealized abstractions, such as perfect dice and symmetric distributions. Schurz's bridging principles enable practical applications in DB reasoning and statistical inference, but these applications do not define probability itself. What defines probability is how it is historically constructed.

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The Philosophy of Markov Chain Monte Carlo

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Keywords: Philosophy of Bayesian statistics; Markov chain Monte Carlo (MCMC); inductive reasoning.

Due to the fallacy of affirming the consequent, we know that we cannot deductively infer that a hypothesis is true based on the favorable outcome of hypothesis testing, no matter how many tests we run. Despite this, some philosophers of science argue that favorable outcomes can provide partial support, corroboration, or confirmation for a hypothesis [1, 7].

In the latter half of the 20th century, statistical methods like Markov chain Monte Carlo (MCMC) [3], along with variations such as the Hastings-Metropolis algorithm and Gibbs sampling, have expanded the scope and complexity of hypothesis testing. As McGrayne notes, "Today, computers routinely use the Hastings-Metropolis algorithm to work on problems involving more than 500,000 hypotheses and thousands of parallel inference problems" [5].

This talk raises two main questions. First, can these new statistical methods transform or expand traditional notions in the philosophy of science regarding hypothesis testing and corroboration? Second, how might these methods contribute to inductive logic — or, as Skyrms [6] puts it, might "Bayesian statistics [have] useful things to contribute to inductive logic"? We will approach these questions by reflecting on the development of MCMC, considering both its historical trajectory [4] and recent applications [2], while drawing connections to longstanding philosophical debates surrounding inference, corroboration, and the logic of scientific discovery.

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PolCa II: Polynomial Ring Calculus

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Keywords: Algebraic Proof, Software, nonclassical logics.

The method for automatic theorem proving proposed by Walter Carnielli in [1] - called Polynomial Ring Calculus (PRC) - is an algebraic proof mechanism based on handling polynomials over finite fields. During the last few years, a series of papers and notes concerning the PRC classical and nonclassical logics, in particular for propositional (deterministic and non-deterministic) many-valued logics, paraconsistent logics, and modal logics have been published ([5], [4], [2], [3]).

In 2014, in [3] the PoLCa software was developed with the aim of translating sentences of several logics (such as many-valued, paraconsistent, etc.), whose semantics are deterministic (truth-functional) or non-deterministic (controlled non-truth-functional) into simple polynomials, whose registration was granted by the National Institute of Industrial Property (INPI) in 2023. In this work, I present the preliminary ideas for the development of PolCa II, which, in addition to translating formulas into polynomial rings, performs operations on them, in a more intuitive programming language. This research was also supported by the São Paulo Research Foundation (FAPESP, Brazil) trough the Thematic Project Rationality, logic and probability - RatioLog(grant #2020/16353-3).

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On n-Valued Post Algebras and n-Valued Post Logics: Twist-style Representation and Proof Theory

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Keywords: Post algebras. Post logics, twist-style representation. proof theory

Mathematical Fuzzy Logic studies fuzzyness from a foundational perspective based on many-valued logics. In the early twenties of the past century, almost simultaneously, systems of many-valued logic were introduced in the respective articles of Jan Łukasiewicz and Emil L. Post, making many-valued logics a respectable field of study. Emil L. Post gave a definition of many-valued logics that was a generalization of two-valued classical logic. He defined the most important operations and discussed some of their properties by means of truth-tables. Two decades later Paul C. Rosenbloom gave a definition of an algebraic structure that served as an interpretation of Post's system; these structures are called Post algebras. Post algebras were meant to capture the algebraic properties of Post's systems. On the other hand, in his 1969 PhD dissertation, R. Cignoli showed that n-valued ($n \geq 2$) Post algebras are Łukasiewicz-Moisil algebras containing $n-2$ additional constants $\mathbf{e}_i$, $1 \leq i \leq n-2$ ($\mathbf{e}_0 = 0$ and $\mathbf{e}_{n-1} = 1$).

In this talk, in first place, we give a representation of n-valued Post algebras by means of *generalized twist structures*. More precisely, we define n-valued twist structures as certain subsets of the product of $2n$ lattices satisfying a number of conditions; and present a suitable notion of morphism between these structures. Then, we prove that the category of n-valued Post algebras with its morphisms (homomorphisms in the sense of Universal Algebra) and the category of generalized twist structures with its morphisms are equivalent. On the other hand, we study some logics that arise from n-valued Post algebras. More precisely, we present different n-valued logics that naturally can be associated to n-valued Post algebras, namely, the non–falsity preserving logic, the truth–preserving logic, and the logic that preserves degrees of truth associated to n-valued Post algebras. We provide cut-free sequent calculus for the first two logics and a sequent calculus for the last one that presumably does not enjoy the cut-elimination property. Finally, it is shown that the logic that preserves degrees of truth w.r.t n-valued Post algebras is a logic determined precisely by logical n-matrices.

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Controlling Consistency: A Hierarchy of LFIs

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Keywords: paraconsistent logic, Logics of Formal Inconsistency, swap structures

Logics of Formal Inconsistency (LFIs) are a family of paraconsistent logics that include a unary *consistency* or *recovery* operator $\circ$ that locally recovers the explosion law w.r.t. its paraconsistent negation $\neg$ ([1]). In formal terms: every formula follows from $\{\alpha, \neg\alpha, \circ\alpha\}$, for any α, although $\{\alpha, \neg\alpha\}$ may be non-trivial, for some α. By introducing such primitive consistency operator, LFIs are able to distinguish many scenarios in which consistency (in the sense above) plays a role. We can consider stronger contexts in which non-contradiction implies consistency and consistency implies the consistency of consistency, such as the case of the robust three-valued logic **LFI1** (which is equivalent to D'Ottaviano-da Costa's $J3$). However, we can also weaken the notion of consistency in such a way that consistency is sometimes contradictory: in mbC, the minimal LFI based on classical logic considered in [1], consistency implies non-contradiction, which is guaranteed by the basic axiom (bc1) $\circ\alpha \to (\alpha \to (\neg\alpha \to \beta))$. However, neither non-contradiction implies consistency nor consistency implies consistency of consistency in mbC [1, Prop. 2.3.3. and Th. 3.6.9].

In intermediate LFIs, more axioms are introduced to restrict the behavior of $\circ$: mbCciw is the logic resulting from mbC with the addition of the axiom (ciw) $\circ\alpha \vee (\alpha \wedge \neg\alpha)$. Adding (ciw) makes consistency determined in terms of non-contradiction. However, to determine inconsistency in terms of contradiction, we need to add to mbC the axiom (ci) $\neg\circ\alpha \to (\alpha \wedge \neg\alpha)$ instead of (ciw), obtaining the (strictly) stronger system mbCci [1, Def. 3.1.7, Prop. 3.1.10].

The logics mbCciw and mbCci share a close relationship: mbcCi is equivalent to mbCciw plus the axiom (cc) $\circ\circ\alpha$ [1, Prop. 3.1.10]. Thus, they are an important step in the standard construction of LFIs, because the jump from mbCciw to mbCci is an important change in the behavior of consistency: in mbCci, any judgement of consistency is required to be itself consistent, which is exactly the meaning of axiom (cc).

The jump from mbCciw to mbCci regarding consistency is not small: any formula of the form $\circ\circ\alpha$ is valid in mbCci. In particular, take a formula $\alpha = \circ^n\beta$, for $n \geq 0$ (where $\circ^0\beta = \beta$). By (cc), $\vdash_{mbCci} \circ^{n+2}\beta$. By a straightforward procedure, we can show that, for any formula α, and for any $k \geq 2$, $\vdash_{mbCci} \circ^k\alpha$.

This means that consistency in mbCci is only allowed to be inconsistent regarding judgements that are not about the consistency of some other proposition. Any level of meta-consistency, that is, consistency about the consistency of a statement, is not allowed to include divergencies about meta-consistency. The situation in mbCciw is the opposite: consistency is possibly contradictory at any level. There is no iteration of $k \geq 2$ instances of $\circ$ such that $\vdash_{mbCciw} \circ^k\alpha$, for all formula α. Therefore, at this point either you can choose consistency to behave inconsistently all the way (in mbCciw), or you can choose consistency to be consistent in all meta-consistent situations (in mbCci).

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The above comparison already suggests that the construction of LFIs in this point should be supplemented. In many situations, meta-consistent statements are required to be not always consistent. For example, when a lawyer talks about the testimony of someone who he regards unreliable, the prosecutor may talk about his/her statements as being unreliable themselves. That is, the prosecutor considers the lawyer's statements about the consistency of a testimony as being inconsistent themselves. In general, the context also dictates to which extent we can question the consistency of judgements. For example, when playing a game of detective in which the players can lie only about their current location, the players can question the consistency of each other's statement about their location. However, it makes no sense within this game to question the consistency of the statements concerning the consistency of their location, because they can only lie about their location (not about the consistency of this kind of statements). In this situation, consistency would be controlled at two iterations of the consistency operator. There are other situations in which we want to control consistency at four iterations, for example. In general, we want to control consistency statements to the point that we can control consistency at any level n, for $n \in \mathbb{N}$. In addition to the formal advantage, an area of research that can benefit from this development is the area of paraconsistent belief revision. For each level of control n, we present an LFI to characterize such context. We will see that mbCciw and mbCci stand as extreme points of the hierarchy of logics to be proposed here.

In this presentation, we will present a hierarchy of LFIs that capture the idea of the control of consistency. Starting from mbCciw, for each logic L^0_n, we add the axiom (cc^n) $\circ^{n+2}\alpha$, for $n \geq 1$. Thus, L^0_1 is mbCciw plus $\circ^3\alpha$, L^0_2 is mbCciw plus $\circ^4\alpha$, and so on. We recover mbCci as the limit L^0_ω = mbCciw plus (cc) (i.e., L^0_0). In that sense, the construction of the L^0_k hierachy of logics is a supplement between mbCciw and mbCci. Concerning their expresive power, let us consider 0-1 valuations (a.k.a. bivaluations) v characterizing these systems (then, $v(\circ\alpha) = 1$ iff $v(\alpha) \neq v(\neg\alpha)$). In L^0_1 it can happen that $v(\circ\alpha) = v(\neg\circ\alpha)$ but $v(\circ^n\alpha) \neq v(\neg\circ^n\alpha)$ for $n \geq 2$. In L^0_2 it can happen that $v(\circ\alpha) = v(\neg\circ\alpha)$ and $v(\circ^2\alpha) = v(\neg\circ^2\alpha)$, but $v(\circ^n\alpha) \neq v(\neg\circ^n\alpha)$ for $n \geq 3$; and so on.

But we can also go beyond this hierarchy. We can also strenghten mbCciw in other directions, while preserving the (cc^n) axiom. Thus, the systems L^k_n will be defined as an extension of L^0_n, to deal with the equivalence between $\circ^m\alpha$ and $\circ^m\neg\alpha$, for $1 \leq m \leq k+1$. Let L^1_n be the system L^0_n plus axiom (dn) $\neg\neg\alpha \leftrightarrow \alpha$. It is easy to see that $\circ\alpha \equiv \circ\neg\alpha$, but $\neg\circ\alpha \not\equiv \neg\circ\neg\alpha$ in L^1_n. Let L^2_n be L^1_n plus $\neg\circ\alpha \leftrightarrow \neg\circ\neg\alpha$; then $\circ^2\alpha \equiv \circ^2\neg\alpha$ in L^2_n. In general, let L^{k+1}_n be L^k_n plus $\neg\circ^k\alpha \leftrightarrow \neg\circ^k\neg\alpha$, for $1 \leq k \leq n$. Then $\circ^m\alpha \equiv \circ^m\neg\alpha$ in L^{k+1}_n, for $1 \leq m \leq k+1$. Observe that, in L^k_n, $\circ^k\alpha \equiv \circ^k\neg\alpha$ if $k \geq n+2$, hence $\mathrm{L}^k_n = \mathrm{L}^{n+1}_n$ if $k \geq n+2$.

Besides bivaluations, each system L^k_n can be semantically characterized by a non-deterministic matrix (Nmatrix) defined by means of swap structures [1, Chap. 6], which produces a decision procedure for each system. However, the size of the Nmatrix for L^k_n increases with n. A better semantic account (producing a more efficient decision procedure) can be given by means of restricted Nmatrices (RNmatrices [3]). Thus, let $\mathcal{M}_{mbCciw}$ [1, Sec. 6.5] and $\mathcal{M}_{Cbr}$ [2, Def. 3.2] be the 3-valued characteristic Nmatrices for mbCciw and Cbr = mbCciw plus (dn). Then, each system L^0_n can be characterized by taking suitable restrictions on the Nmatrix $\mathcal{M}_{mbCciw}$ (i.e., an RNmatrix based on $\mathcal{M}_{mbCciw}$). In turn, L^k_n $(1 \leq k \leq n+1)$ can be characterized by an RNmatrix based on $\mathcal{M}_{Cbr}$. Hence, while Nmatrices present these logics in terms of many valuedness, providing an intuitive semantics, RNmatrices improves their decidability problem, maintaining the intuitive appealing.

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Abductive Reasoning as a Paraconsistent AGM Belief Change Operation

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Keywords: Abduction, AGM, Belief Revision, Paraconsistency, LFIs

Abduction is the reasoning responsible for formulating and selecting hypotheses capable of explaining, even in a rudimentary and conjectural way, intriguing and surprising facts. The theory of abduction is the result of more than 50 years of thought by the philosopher and logician Charles Sanders Peirce. The belief revision system **AGM** - initially introduced by Carlos Alchourrón, Peter Gärdenfors and David Makinson [1], hence the acronym **AGM** -, in turn, concerns a formal modeling of epistemic states and the belief change operations related to them, namely, expansion, contraction and revision. Despite the differences between Peirce's and the AGM trio's thinking, and the profound philosophical distinctions between coherentism/foundationalism and pragmatism principles, the choice to adopt the **AGM** system to model abductive reasoning is based on two initial justifications: (i) Peirce's understanding of the nature of inference is closer, I believe, to the **AGM** belief change operations than to the classical logical consequence notions; (ii) both abduction and the revision operation **AGM** are non-monotonic reasoning [9]. I will briefly dedicate the first part of the exposition to these philosophical issues.

The first abduction modeling as an **AGM** operation was introduced by Pagnucco [10]. The author establishes, among other developments, the **AGM** *abductive expansion operation*, with its postulates and constructions based on *partial meet* selection functions and on abductive epistemic entrenchment - similar to the analogous constructions for the classical **AGM** contraction operation [8] [7]. On the other hand, Rafael Testa [11] - and, later improved by Testa, Marcelo Coniglio and Márcio Ribeiro [12] - formally introduces the **AGM**$\circ$ system: a *paraconsistent belief revision system* that extends the expressive power of the classical **AGM** language with the **LFI**s (*Logic of Formal Inconsistency* [2], [3], paraconsistent [4]) negation $\neg$ and consistency $\circ$ operators, and that enables the epistemic agent to (i) prevent strongly accepted beliefs derogation and (ii) coherently tolerate contradictory beliefs. The AGM$\circ$ system deals predominantly with the **AGM** contraction and revisions (internal, external and semi-revision) operations, establishing the appropriate postulates and the construction based exclusively on the *partial meet* selection function.

Recently, though, Coniglio, Figallo and Testa [6] introduced a paraconsistent epistemic entrenchment construction for the **AGM**$\circ$ system. In order to obtain the appropriate belief ordering in the epistemic state, the authors established two new **LFI**s, **RCie** and **RCbr**, that satisfies the *replacement* property - generally not satisfied by **LFI**s - without losing paraconsistency. To satisfy the *replacement* property, the new **RCie** and **RCBbr** logics introduce a new inference rule in their axiomatic system and use a *BALFI* (*Boolean Algebra with LFI operators*) based semantics, accordingly to the previous development of Carnielli, Coniglio and Fuemayor [5].

I believe that from the abductive expansion operation established by Pagnucco, on the one hand, and from the extension of the expressive power of the underlying language with the paraconsistent **LFI**s $\neg$ and $\circ$ operators, on the other hand, *Paraconsistent **AGM** abductive expansion and revision operations* - in the sense of coherently supporting contradictory hypotheses for the same surprising

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phenomenon - may be obtained. Analogously to Testa, Coniglio and Ribeiro [11] [12] work and Coniglio, Figallo and Testa [6] article, postulates and constructions such based on *partial meet* selection functions as another based on paraconsistent abductive epistemic entrenchment may be, I think, respectively adapted. I will dedicate a second moment of the exposition to elucidate such discussions and present recent results in this direction.

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On Universal Functionally Complete Many-Valued Logics

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Keywords: functional completeness, Polynomial Ring Calculi, many-valued logics

This proposal aims to present a simple, intuitive, and human-centered algorithm that enables the derivation of any connective from a functionally complete many-valued logic. Since the famous C. Peirce's one-page manuscript of 1909, passing through J. Łukasiewicz with contributions from K. Gödel, D. A. Bochvar, S. Kleene, E. Post, N.C. A da Costa, M. Sette, I. D'Ottaviano, S. Hallden, and B. de Finetti, there are some obvious connections between trivalent logics and philosophy, conditionals, probability and computation. D. Knuth, for instance, advocates his 'trits' (ternary digits) instead of bits 0 and 1 for data storage and processing, and the three-valued computer Setun, a computer developed in 1958 at Moscow State University is an interesting case.

This justifies the focus on the special case of three-valued logics, making it clear that the concept can be extended to all finite many-valued logics with a prime or prime power number of truth values, since the underlying idea depends on finite fields.

The problem of functional completeness in many-valued logics has been much investigated, though not always with good success. In [4], two axiom systems proposed by Jerzy Słupecki from 1936 and from 1946 (cf. [5], [6], and [7]) for functionally complete three-valued propositional logic are critically examined and refuted. It is shown that both systems are inadequate: the first is semantically incomplete, while the second is functionally incomplete. This is quite surprising, as several papers and classical books in many-valued logics cite J. Słupecki's results as a definitive solution, now seen to be flawed.

The Polynomial Ring Calculus (PRC), introduced by [3], is a formal framework that intersects mathematical logic and algebra, particularly using coefficients from finite (Galois) fields in proofs. This technique is highly versatile and was initially applied to the modal logic **S5** in [2], with subsequent extensions in [1] to other modal logics, such as **K**, **KD**, **T**, **S4**, and intuitionistic logic. The semantics derived using this method represent the truth conditions of modal formulas via polynomials, allowing logical deductions to be performed through polynomial manipulations. Additionally, the study explores the connection to algebraic semantics in modal logic, equational logics, and rewriting systems,including Gröbner bases used in algebraic geometry computational mathematics.

It is a folklore result that the finite fields $\mathbf{F}_{p^n}$ form a functionally closed algebraic structure under the operations and constants $\{+, \cdot, 1\}$. The interest here is to reorient this structure in terms of logic.

The Webb functions $W_m(x, y) = max\{x, y\} + 1(mod\ m)$ (cf. [8]) describe one of the simplest and most useful functionally complete connectives in many-valued logics. For $m = 3$, this reduces to the following function $W_3(x, y)$:

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W_3	2	1	0
2	0	0	0
1	0	2	2
0	0	2	1

In terms of polynomials over $\mathbf{Z}_3$, the ternary Webb function $W_3(x, y)$ turns out to be:

$$W_3(x, y) = x^2y^2 + x^2y + xy^2 + 2xy + x + y + 1$$

whose table is depicted above. From $W_3(x, y)$ it is possible to define: $\Pi(x, y) = x \cdot y$ and $\Sigma(x, y)x+y$. Obviously, $\Pi(x, y)$, $\Sigma(x, y)$, and 0, 1, and 2 also define $W_3(x, y)$. By introducing appropriate logical ingredients, we can define a universal trivalent logic SP such that:

Theorem 1. *The trivalent logic SP, whose connectives are* $\Pi(x, y)$, $\Sigma(x, y)$, 0, 1, *and* 2 *(where* 0, 1, *and* 2 *are understood as 0-ary (constant) operators), is functionally complete.*

Starting from the logic SP one can easily obtain any other three-valued connective. The same approach can be used to define analogous versions of SP for several other many-valued logics with $n > 3$, thereby providing universal many-valued logics capable of defining any other logics. This is a joint work with J.C. Agudelo-Agudelo.

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Irreducible Features of Set and Class Theories

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Keywords: Multiverso, Foundations of Mathematics, Set Theory

The universalist position maintains that there is just one maximal universe of sets or classes. It maintains that all sentences about these subjects are either true or false. In practice, however, set and class theorists investigate a myriad of alternative structures, hence a universalist who does not wish to simply dismiss the diversity of mathematical practice must somehow account for those alternative structures. It is unclear how the universalist would do so without relying upon *interpretations* of those alternative universes in a base universe. Indeed, this seems a core feature of *universalism* itself. Nonetheless, recent results on set-theoretic interpretations in set theories by Enayat, by Freire and Hamkins, and by Freire and Williams put serious limitations on the universalist's ability to produce such interpretations. The universalist, we argue, thus cannot account for the real diversity of mathematical practice. We finish this presentation with remarks on what can be preserved from the universalist position, namely a justification for the mathematicians' ability to intensionaly discuss non-algebraic structures.

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A Lógica dos Problemas Computacionais

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Palavras-chave: Lógica, problemas computacionais, NP-completude

Como todos sabem, a gente prova a NP-completude de um problema computacional fazendo a redução (polinomial) de um problema reconhecidamente NP-completo para o nosso problema [1]. Mas, uma vez que isso tenha sido feito, nós passamos a ter dois problemas NP-completos. E agora, nós podemos fazer reduções para os dois lados. É claro que o propósito dessas reduções já não é mais provar a NP-completude dos problemas. O seu propósito é investigar as reduções elas mesmas: *o que está envolvido em uma redução?*, *porque a redução em uma direção é mais fácil do que na outra?*, *o que falta a um problema polinomial para que se possa reduzir um problema NP-completo para ele?*, *existe alguma maneira sistemática de produzir essas reduções?*, etc. O nosso entendimento é que a literatura não joga muita luz sobre essas questões. E que a tarefa de provar a NP-completude de um problema ainda é uma arte. Uma arte relativamente bem difundida, mas ainda assim uma arte.

A metáfora que nós encontramos para começar a pensar sobre reduções foi a ideia de que um problema computacional é uma linguagem. Essa ideia faz sentido quando nós pensamos que em uma redução nós utilizamos os elementos básicos de um problema para descrever um outro problema. Não apenas isso mas, uma vez que nós tenhamos sido bem sucedidos e o problema foi provado NP-completo, em princípio nós podemos descrever qualquer outro problema da classe NP na sua linguagem. Nesse sentido, nós podemos pensar que um problema NP-completo corresponde a uma linguagem completa: nós podemos dizer qualquer coisa com ela. Essa observação despertou o nosso interesse em investigar a lógica associada às linguagens dos problemas computacionais. E aqui nós temos a motivação para o título do trabalho.

Quando nós falamos em lógica nesse contexto, o que nós temos em mente é um conjunto de elementos, propriedades e relações básicas, cuja articulação permite descrever as coisas. Para o problema SAT, por exemplo, nós temos as variáveis booleanas, os valores verdade, a negação e as cláusulas disjuntivas. Daí, como toda redução para SAT envolve articulação desses elementos básicos, com o tempo a gente acaba descobrindo alguns poucos truques úteis para fazer descrições com esses elementos. E acaba observando que frequentemente as reduções para SAT são muito semelhantes entre si.

Em princípio, a mesma coisa poderia acontecer com qualquer outro problema NP-completo. Quer dizer, em um primeiro momento a gente identifica os elementos, propriedades e relações básicas, cuja articulação nos permite fazer descrições. Daí, por meio do exercício de produzir uma série de reduções, a gente acaba descobrindo alguns poucos truques úteis de descrição com esses elementos. E daí, é plausível que a gente também encontre uma grande semelhança entre as reduções para esse

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problema. Em outras palavras, nós vamos ter um método sistemático para produzir reduções, o que corresponderia a aprender a falar a linguagem do problema.

Mas, aqui há uma pequena sutiliza. Como o problema com que nós estamos trabalhando é NP-completo, nós podemos reduzir SAT para ele. Fazendo isso, nós vamos ter os componentes básicos de SAT à nossa disposição — na forma de *gadgets* lógicos. E a partir de agora, nós podemos produzir as reduções articulando os *gadgets* lógicos. Só que isso não tem muita graça: a semelhança que nós vamos ver em nossas reduções é apenas um reflexo da lógica. E nós não vamos poder dizer que nós aprendemos a falar a linguagem do nosso problema.

Em vista disso, nós adotamos uma estratégia diferente. Quer dizer, ao invés de escolher um problema e fazer muitas reduções para ele, nós escolhemos uma coleção de problemas e fizemos a redução de 3SAT para todos eles — em geral, adaptando reduções conhecidas. Em todos os casos, as reduções consistiram na construção de *gadgets* lógicos para as variáveis booleanas, a negação, o OU, e o TROU (i.e., a disjunção tripla). E o fato surpreendente é que mais uma vez as reduções apresentaram muitas semelhanças entre si.

É certo que nós deveríamos esperar ver alguma semelhança, porque o problema de origem é sempre o mesmo. Mas, as linguagens destino são todas diferentes. E chama atenção a semelhança que nós encontramos na estrutura e modo de articulação interno dos *gadgets* em todas as reduções. A contribuição desse trabalho consiste na apresentação desse resultado, e em uma breve discussão sobre ele.

Quando nós fazemos muitas reduções para SAT e encontramos semelhanças entre elas, nós podemos dizer que estamos vendo ali alguma coisa da estrutura e modo de funcionamento da lógica. Da mesma maneira, quando nós fazemos muitas reduções a partir de SAT para outros problemas NP-completos e encontramos semelhanças entre elas, nós podemos dizer que estamos vendo ali o reflexo de alguma coisa da estrutura e modo de funcionamento da lógica. A nossa intuição é que examinando esses reflexos, nós podemos aprender alguma coisa sobre a natureza da lógica — porque os reflexos apresentam a lógica sob um ângulo diferente daquele que nos é usual. Nós pretendemos fazer essa investigação em um trabalho futuro.

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Sheaves on Quantales and their Truth Values

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Keywords: sheaves, quantales, subobject classifier

Sheaf Theory is a well-established area of research with applications that goes from Algebraic Geometry [9] and Logic [10] to recent developments in Machine Learning [1]. A presheaf on a locale (or, equivalently, a complete Heyting algebra) L is a functor $F : L^{op} \to Set$. Given $u, v \in L$ such that $v \leq u$, we consider restriction maps $\rho^u_v : F(u) \to F(v)$ and denote $\rho^u_v(t) = t_{\restriction_v}$, for any $t \in F(u)$. Quantales are a non-idempotent and non-commutative generalization of locales, introduced by C.J. Mulvey [2]. Explicitly, a quantale Q is a complete lattice endowed with a binary associate operation $\odot$ that distributes over arbitrary suprema. A quantale Q is *semicartesian*, whenever $u \odot v \leq u, v$. Examples of semicartesian quantales include locales, the poset of ideals of a commutative ring, and the interval $[0, \infty]^{op}$ (where $\odot = +$). There are distinct definitions of sheaves on quantales [3–7], generalizing different ways that one can approach sheaf theory.

In [8,11], we proposed the following novel definition of a sheaf on a semicartesian (commutative) quantale Q:

A presheaf $F : Q^{op} \to Set$ is a **sheaf** if for any cover $u = \bigvee_{i \in I} u_i$ of any element $u \in Q$, the following diagram is an equalizer

$$F(u) \xrightarrow{e} \prod_{i \in I} F(u_i) \overset{p}{\underset{q}{\rightrightarrows}} \prod_{(i,j) \in I \times I} F(u_i \odot u_j)$$

where

$$e(t) = \{t_{\restriction_{u_i}} \ : \ i \in I\}, \quad p((t_k)_{k \in I}) = (t_{i_{\restriction_{u_i \odot u_j}}})_{(i,j) \in I \times I}$$
$$q((t_k)_{k \in I}) = (t_{j_{\restriction_{u_i \odot u_j}}})_{(i,j) \in I \times I}$$

This definition recovers cohomological [13] and logical aspects [8,12] of sheaf theory. In this talk, we present the category $Sh(Q)$ whose objects are sheaves on Q and whose morphisms are natural transformations. The main results concerning the logical properties of this category are:

- $Sh(Q)$ is not a topos in general, although it behaves like one in a certain sense.
- The lattice of external truth values of $Sh(Q)$, that is, the lattice of subobjcts of the terminal sheaf, is canonically isomorphic to the quantale Q.
- Assuming certain extra conditions on Q, there is a sheaf that essentially classifies a class of subobjects in the category $Sh(Q)$. In other words, there is a candidate for an "internal truth value object" in $Sh(Q)$.

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The above places this work as part of a greater project towards a monoidal closed but non-cartesian closed version of elementary toposes. We hope to obtain a category more general and strongly related to a topos (whose internal logic is intuitionistc) but with an internal logic that has a linear flavor.

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Synalgebraic Computability for Machine Reasoning and Learning

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Keywords: Algebra, Computability, Reasoning, Learning

Leibniz principle of identity states that $x = y$ if, and only if, for every property α, it is the case that $\alpha(x)$ happens exactly in the situations in which $\alpha(y)$ does. This principle has two implications. The implication $x = y \to (\alpha(x) \leftrightarrow \alpha(y))$ is known as the *indiscernability of identicals*, whereas $(\alpha(x) \leftrightarrow \alpha(y)) \to x = y$ is named *identity of indiscernables*. The indiscernalibility of identicals is considered a logical principle and appears in any logical system with equality, while the identity of indiscernables only holds in specific contexts. Indeed, the vast majority of algebraic theories works upon the indiscernability of identicals. We assume that some identities define an algebraic structure, specify some proprierties over the terms occuring in these identities and, then, apply the indiscernability of identicals to show that if some properties hold about some terms, then another properties must hold too. In this sense, algebras are *analytic theories* because they analyse properties using the indiscernalibity of identicals. Can we develop *synthetic theories* of algebras which synthesize properties using the identity of indiscernables? That is what *synalgebra* is about.

Synalgebra is a synthetic theory of equations. It develops an idea implicit in Peirce's works on algebra, namely, equations are just deducible equivalences. To create synthetic theories of equations, we adapt Gentzen's sequent calculus rules to distinguish between axiomatic and postulative deductions [1]. In the context of deduction of equations this distinction establishes the notions of identities and similarities, respectively. In particular, synalgebra enables the logical representation of calculations involving switching and quantum circuits. In this work, we demonstrate how synalgebra can integrate machine reasoning and learning. The combination of symbolic methods with connectionist approaches remains an open problem in AI. While there are various proposals – mainly aimed at integrating logical systems with neural networks – none, so far, provides a foundational approach grounded in first principles. That is precisely our objective.

First, we introduce an extension of the standard notion of computation with switching circuits, referred to as *synalgebraic computability*. Switching circuits are an extension of Boolean circuits, formulated by Shannon to represent the functioning of digital electronic computers, in which loops are allowed - something lacking in Boolean algebra. Up to now there is no Turing complete model based on switching circuits because they are non-uniform devices, that is to say, they only compute inputs of fixed sizes. However, we will present a synalgebraic version of switching circuits that is Turing complete. The synagebraic computability model also allows to define rules for calculating digital differentiation and integration. Using these rules, in the present work we derive a synalgebraic version of the backpropagation algorithm for binary neural networks. We present preliminary computational results based on a Rust implementation to solve a classification problem. Additionally, we analyze the relationship between the interpretability enabled by synalgebraic computability and the characteristics of learnability.

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Towards a Syntactical Proof of the Undecidability of FOL

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Keywords: Entscheidungsproblem, First-order logic, connection tableau calculus

Turing proved the unsolvability of the decision problem (*Entscheidungsproblem*) for first-order logic (FOL) in his famous paper *On Computable Numbers, with an Application to the Entscheidungsproblem*. In general, undecidability proofs of FOL based on Turing machines involve encoding a Turing machine as an FOL formula and demonstrating that the decidability of FOL would imply the decidability of some problem known to be unsolvable for Turing machines. Like Turing's proof, these proofs are semantic insofar as they rely on the soundness of first-order logic. Here, we present a step toward a purely syntactic proof of the undecidability of FOL. More specifically, we show how to mimic the behavior of a certain kind of universal Turing machine – namely, splitting Turing machines (STMs) – through sequences of inference steps in an exhaustive proof search within a complete calculus, the tight connection tableau calculus, initialized by clauses containing only negative literals. We restrict our focus to a simple undecidable class of FOL formulas, namely those expressible as a set of Krom-Horn clauses. We demonstrate that a given STM halts if, and only if, the corresponding formula is refutable. Since the halting problem for STMs is undecidable, no algorithm can decide the refutability of Krom-Horn clauses within the aforementioned calculus. Although the deterministic proof search for Krom-Horn clauses mirrors the behavior of STMs, our approach does not involve encoding STMs in FOL, as is typically done in undecidability proofs. Instead, it remains purely syntactic. We conjecture that the core ideas of our proof can be applied to any complete calculus for FOL, thereby establishing the undecidability of FOL by purely syntactic means.

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What is Logic Anyway?

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Keywords: deduction, necessity, logical pluralism

We propose four interrelated questions regarding the nature of logic: (i) Why are logical relations held as "necessary", or what exactly is the necessity involved in logical analysis? (ii) What is the relation between logic and 'metaphysics' or 'ontology', these terms taken as denoting the study of the most fundamental aspects of an independently existing reality? (iii) Is logic intrinsically related to truth? And finally, (iv) How to properly interpret the plurality of 'logics'? Our answers to these questions proceed in the following way: for (i), we rehabilitate an argument by Ernest Nagel (cf. [5]) in order to conclude that logical principles and rules are actually implicit definitions for specifying the structure of some type of consistent and meaningful discourse, otherwise they would have to be construed as general hypotheses in need of empirical validation, which would preclude any characterization of logic as necessary and a priori. We complement this argument with considerations on the thesis of the underdetermination of theory by evidence (the Duhem/Quine thesis) in order to answer (ii), criticizing any conception of '*the* objective structure of *the* system of fact' capable of being known independently of any particular theoretical (and thus human) activity. That is, we criticize the conceivability of an unconditional intrinsic and pervasive structure of things in general, to which logic as a theory would then fit as an appropriate description. These answers, in turn, lead us to a negative answer to question (iii), to which we notice that despite the more contemporary abstract mathematical views of logical consequence, there still pervades in the literature a general conception of consequence, or of the 'validity' of arguments, invariably as 'preservation of truth', which we take as a source of many misconceptions. For we intend to show, in answering (iv), that to each particular logical system, defined as a structural consequence relation, there corresponds a particular class of sentential (or propositional) properties which its rules preserve and implicitly define. As examples, we show that classical logic can be interpreted as defining truth and falsity as commonly understood, while intuitionistic logic would define notions of 'necessity' or 'provability', and dual-intuitionistic logic (an example of paraconsistent logic), in its turn, notions of 'possibility' or 'non-refutability'. We finish with a discussion of the intrinsically circular character of any proposal for the (epistemological) justification of deduction, following an argument by Susan Haack [2], complemented with a comparison between Tarski's denotational and Popper's (cf. [1]) 'proof-theoretical' semantical approaches to logic.

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Logical Translations and Categorical Extensions

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Keywords: Logical Translations, Category Theory, Universal Algebra.

Logics may be understood as systems, algebras, spaces, or categories, depending on a chosen perspective (logical, algebraic, topological, categorical). We can also profit from many combinations of the various approaches, using correlative tools from category and topology, in particular, sheaves, topos, allegories, and intermediate categories. In these combinations, a functorial paradigm emerges, where adjoints, equivalences, isomorphisms, monads, allow to elucidate the structure of the logics involved. On another hand, translations between logics emphasize deductions and closure operators, capturing the logical-algebraic-topological characteristics of the logics involved. Orienting ourselves towards categorical contexts, translations become functors between categories, and one can try to use many constructions around functors and categorical hierarchization, in order to situate translations in a wider panorama. Our project circulates around a thorough and unifying study of the various mathematical layers present in logical translations. *Categorizing and functorializing logical translations* offers a good possibility to attain such an unification and deepening of perspectives. As a consequence of a Many-One, local-global, particular-universal, mathematical understanding of logical translations, we will also explore the philosophical significance of our layered categorical procedure.

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Critical Analysis of AGM Postulates for Ontology Repair

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Keywords: Belief Revision, AGM model, Ontology Repair

Belief revision [4, 6] is an area of knowledge representation research that aims to understand the rationality of an agent's belief changes in the face of new information and how to reflect these changes in the agent's belief set. Several models have been proposed to formalize and characterize the dynamics of belief changes, but the most prominent model was proposed by Carlos Alchourrón, Peter Gärdenfors and David Makinson in [1], giving rise to the AGM model. In this model, an individual's belief state is represented by a belief set closed under logical consequence [1].

The AGM model is a simple and elegant theory in which all changes result exclusively from inputs, which take the form of a sentence accompanied by a specific instruction on what to do with it [10]. This instruction determines whether a specific sentence should be included or excluded from the resulting belief set, as explained in [4]. One of the pillars of AGM theory is that changes in beliefs should occur with minimal loss with respect to the previous belief state [4].

In the AGM paradigm, there are three fundamental forms of belief revision: expansions, revisions, and contractions. Specifically, expansions consist of adding a sentence to a belief set, which may result in an inconsistency. In contrast, revisions consist of adding a sentence to a belief set but preserving consistency. Some previous beliefs may have to be removed. Finally, contractions consist of the removal of a sentence from a belief set. These are the main forms of belief change in the AGM model, and the symbols $+$, $*$ and $-$, respectively, are used to denote them [4]. The contraction and revision operations are characterized in the AGM model by sets of rationality postulates.

Although considered the standard model of belief revision, the AGM model has become the target of criticism [4]. Firstly, because it is a simplified and idealized theory of rational belief change, there are characteristics of real belief systems that are not captured by the model. The logical closure of belief sets is also seen as problematic, given that the agent must accept all consequences of the beliefs they hold and ensure that their beliefs are consistent [10].The representation of the agent's beliefs as belief sets is also cause for criticism, not only because they are too large, but also because they do not contain information that adds informational value for the agent [4, 10].

According to Fermé and Hansson [4], the main criticisms are associated with the postulates of revision and contraction operations. The recovery postulate has been the subject of intense questioning, as its acceptability may be doubtful under its original interpretation, given that there are counter-examples in which recovery seems to contradict the intuition of preserving the belief set [5, 8, 10].

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Fermé and Hansson [4] point out that although the expansion postulate of revision is less debated than recovery, it is just as problematic as revision. In this way, both postulates have motivated the investigation and development of alternative structures for contraction and revision that do not satisfy these postulates [4, 5].

In this paper, we will discuss the suitability of the AGM postulates for ontology repair and revision applications. Recent work in the literature has explored the connection between belief change and ontology repair (see [2, 3, 7, 11]), especially contraction operations in belief change and ontology repair, since both areas explore how to modify a knowledge base so that an unintended consequence no longer occurs [2, 3].

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A Indefinibilidade da Noção de Informação Algorítmica na Aritmética

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Palavras-chave: Indefinibilidade, Informação Algorítmica, Lógica Filosófica, Sentenças tipo-Berry.

Nesta apresentação, iremos expor como interpretamos e formalizamos por meio de um sistema formal contendo aritmética elementar (PA) a definição de informação algorítmica de Gregory Chaitin em Chaitin (1976) – culminando em sua versão da incompletude em Chaitin (1974) – aplicando-a ao comprimento mínimo de fórmulas que definem números em PA. Utilizamos como base os artigos Boolos (1989), Kikuchi (1994), Caicedo (1993) e Roy (2003), voltados a sentenças tipo-Berry. Com isso, realizamos uma análise de certa arbitrariedade valorativa nas formalizações de Boolos/ Kikuchi e Caicedo, acrescentando outras formas de expressividade numérica e determinando definições com base nesta interpretação, transformando assim funções recursivas que representam numerais em fórmulas que definem este numeral, substituindo-o em uma outra fórmula que o contém.

Por meio destes recursos, se obtivermos no sistema formal a demonstração de uma fórmula que, substituindo a sua variável livre por um numeral – e apenas por este numeral – que expressa um número p em PA, tal fórmula define aritmeticamente p. Quando o comprimento desta fórmula é mínimo, temos no valor deste comprimento o que denominamos de *informação numérico-algorítmica de* p. Porém, mostramos que quanto maior for o comprimento do numeral do número expresso, a fórmula que o define é, em proporção, cada vez menor (algo já apontado em Caicedo (1993)). Assim, chegamos a uma família de sentenças tipo-Berry onde, inevitavelmente, temos uma contradição. Com isso, observamos por conclusão que a noção de informação numérico-algorítmica é indefinível, e sendo esta elaboração formal um recorte da informação algorítmica de Chaitin em PA delimitada a definibilidade numérica, tal noção mais ampla torna-se, também indefinível.

Na última parte da apresentação, realizamos críticas sobre definições formais similares a nossa com base nos artigos Raatikainen (1998) e Salehi and Seraji (2018). Por fim, destacamos as críticas à incompletude de Chaitin e sua própria interpretação em Chaitin (1982), feitas propriamente em Raatikainen (1998) e nos artigos de Van Lambalgen (1989) e Franzén (2005), fornecendo assim uma diferente interpretação de sua concepção algorítmica de informação, defendendo que o mesmo é um conceito essencialmente semântico – tal como a noção de verdade.

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Formalization of the XMPP Protocol Using UPPAAL*

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Keywords: XMPP UPPAAL Timed Automata Formal Verification

We present a detailed analysis and formal modeling of the XMPP [3] protocol using UPPAAL [2], which operates based on the concept of Timed Automata [1]. The paper delineates the message flow between interacting entities and sheds light on the associated states and transitions during the process of message exchange through the XMPP protocol. UPPAAL was employed to validate specific properties of the protocol, ensuring scenarios where both clients would reach the stream's end, verifying the return of the Closing Stream Header to the sender, and checking continuous message transmission. We also provide a tool for model parameter variation in order to simulate multiple scenarios of protocol usage. This work aims to serve as a valuable resource for researchers and professionals seeking a deeper understanding of formal modeling of client-server architecture protocols, exemplified by XMPP, and provides a foundation and framework for the formal verification of their requirements.

The UPPAAL model represents communication between a sender and multiple receivers using the XMPP protocol, mediated by a central server. The model depicts stream establishment, message exchange with varying types and priorities, and error handling.

The sender model manages a `sendQueue` of messages with priorities, sequentially sending them, retrying on errors, and closing the stream with each receiver after sending all messages.

The receiver model maintains a `receiveQueue`, processes incoming messages based on type (e.g., `IQ_SET_MESSAGE`, `NORMAL_MESSAGE`), simulates processing errors with a configurable probability, and responds with result or error messages.

The server model uses a `buffer` to intermediate communication between sender and receivers, managing message flow. Separate models, `SenderStreamSetup` and `ReceiverStreamSetup`, handle the handshake process for stream establishment using `ISH_MESSAGE` and `RSH_MESSAGE`.

Communication flow involves stream setup, message sending from the sender to the server, routing by the server to the receiver, processing and response by the receiver, delivery of the response to the sender, retransmission upon error, and stream closure initiated by the sender. While simplifying aspects like error management and resource negotiation, the model captures core XMPP communication flows. A Python code snippet is included, designed to generate the UPPAAL configuration for the XMPP system, including component specification and random message generation for simulation. This automates the creation of the system configuration, facilitating the analysis and simulation of different XMPP communication scenarios.

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A Gênese da Teoria de Modelos de Peirce a Löwenheim

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Palavras-chave: Peirce, Schroeder, Teorema de Löwenheim-Skolem, teoria de modelos, álgebra da lógica.

Este trabalho busca apresentar os elementos da filosofia da lógica e da matemática de C. S. Peirce pertinentes à teoria dos modelos na lógica formal. Assim, buscarei refazer um percurso conhecido que filia Löwenheim à tradição da álgebra da lógica. A narrativa mais conhecida liga o assim denominado teorema de Löwenheim-Skolem aos trabalhos seminais de Peirce no século XIX, principalmente sua lógica dos relativos e a invenção dos quantificadores. No entanto, um exame mais cuidadoso revela que essa continuidade não é direta nem linear. É necessário considerar, nessa narrativa, não apenas as contribuições de Schroeder, mas sobretudo o pouco conhecido diálogo epistolar de Peirce e Cantor. Com isso, ganhamos não apenas uma melhor contextualização dos resultados de Peirce, fundamentais para o entendimento de como se desenvolveu a tradição da álgebra da lógica, como uma mais bem informada discussão sobre a natureza do contínuo.

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Fraïssé Limit and Ramsey Theorem for MV-Algebras

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Keywords: Ramsey Theorem, Fraïssé Theory, MV-algebra

1 Introduction

Structural Ramsey theory was introduced by Nešetřil and Rödl [5] as a more general approach to Ramsey theory [6] by means of some category-theoretical language.

Since then, the Ramsey and the Dual Ramsey properties have been proved to hold or not for many classes of mathematical structures, including a number of relevant algebraic ones. Structural Ramsey Theory is, on its turn, connected with Fraïssé Theory [2], and is receiving quite a lot of interest since the discovery of its connection with topological dynamics [3].

MV-algebras are the equivalent algebraic semantics of Łukasiewicz propositional logic and are intimately connected to Boolean algebras. As a matter of fact, they can be seen as a sort of non-idempotent generalization of Boolean algebras, and many properties of Boolean algebras either hold for MV-algebras or have a suitable MV-algebraic version. Such a connection has also a topological counterpart: Stone duality extends to a fuzzy topological duality between a class of MV-algebras and a suitable fuzzy generalization of Stone spaces [7].

In this talk, which is based on [8], we shall describe the Fraïssé limit of the class of finite MV-algebras as a basis of rational-valued fuzzy clopens of the Cantor set equipped with the fuzzy topology induced by the usual one of $\mathbb{R}$, then we will show that the finite Ramsey property holds for the class of MV-algebras. Finally, we will observe that the way Ramsey property for Boolean algebras is inherited by MV-algebras is actually an instance of a more general fact.

We recall that the real unit interval $[0,1]$ is an MV-algebra with the sum $x \oplus y := \min\{x+y, 1\}$ and the involution $\neg x := 1 - x$. For any positive natural number n the MV-algebra $Ł_n = \{k/n \mid k = 0, \ldots, n\}$ is the $n+1$-element subalgebra of $[0,1]$. Obviously, $Ł_1$ is the two-element Boolean algebra $\mathbf{2}$.

We refer the reader to [1] for the main definitions and properties of MV-algebras.

2 Main results

Fraïssé theory was introduced in 1954 [2] and boasts applications in various areas of mathematics, including functional analysis, topological dynamics, and Ramsey theory.

In [8], we proved that the class of finite MV-algebras enjoys the joint embedding property and the amalgamation property. Such results, along with other ones already known, imply that the class $\mathcal{MV}_f$ of finite MV-algebras is indeed a Fraïssé class which obviously contains the one of finite Boolean algebras. On the other hand, it is well-known that the Fraïssé limit of finite Boolean algebras is the countable atomless Boolean algebra B_∞.

We will now describe the Fraïssé limit of finite MV-algebras and show that its automorphism group is isomorphic to the one of B_∞, which happens to be its Boolean center.

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Let us consider the Cantor set (C, τ) as a subspace of $\mathbb{R}$ with the standard topology, and let τ' be the MV-topology on C induced by the standard fuzzy topology on $\mathbb{R}$.

Lemma 2.1. *Let, for all $q \in [0,1] \cap \mathbb{Q}$,*

$$\overline{q}_U : x \in C \mapsto \begin{cases} q & \text{if } x \in U \\ 0 & \text{if } x \notin U \end{cases} .$$

Then the set

$$A = \left\{ \bigvee_{i=1}^{n} \overline{q_i}_{U_i} \mid n < \omega \& \forall i \leq n (q_i \in [0,1] \cap \mathbb{Q} \ \& \ U_i \in \operatorname{Clop} \tau) \right\} \tag{1}$$

is an MV-subalgebra of $[0,1]^C$ and a clopen basis for τ'.

Lemma 2.2. *A map $f : C \to C$ is a self-homeomorphism w.r.t. τ' if and only if it is a self-homeomorphism w.r.t. τ.*

Theorem 2.3. *The following hold.*

(i) $B(\operatorname{Flim}(\mathcal{MV}_f)) \cong B_\infty$;

(ii) $\operatorname{Flim}(\mathcal{MV}_f) \cong A$;

(iii) $\operatorname{Aut}(\operatorname{Flim}(\mathcal{MV}_f)) \cong \operatorname{Aut}(B_\infty)$.

We will now move to (structural) Ramsey Theory. For the pertinent notions, the reader may refer to [4]. The next three results essentially state that the Dual Ramsey property for finite sets implies the Ramsey property for finite MV-algebras.

For any set X of cardinality $m < \omega$ and for any $n < \omega$, we shall denote by $\tau_{X,n}$ the set $Ł_n^X$. Note that $\tau_{X,n}$ is the finest n-valued MV-topology on X.

Lemma 2.4. *Let $n < \omega$, $\langle X, \tau_X \rangle$ and $\langle Y, \tau_Y \rangle$ n-valued MV-topological spaces, and let us denote by $j(X)$ and $j(Y)$ the skeleton topological spaces $\langle X, j(\tau_X) \rangle$ and $\langle Y, j(\tau_Y) \rangle$ respectively.*

Then $\hom_{{}^{\mathrm{MV}}\mathcal{T}\mathrm{op}}(X,Y) \subseteq \hom_{\mathcal{T}\mathrm{op}}(j(X), j(Y))$. Moreover, the equality holds if $\tau_X = \tau_{X,n}$.

We recall that epimorphisms in the categories ${}^{\mathrm{MV}}\mathcal{T}\mathrm{op}$, $\mathcal{T}\mathrm{op}$, and $\mathcal{S}\mathrm{et}$ are exactly the morphisms whose underlying map is surjective.

Theorem 2.5. *Dual Ramsey Theorem holds for finite-valued finite Stone MV-topological spaces.*

Corollary 2.6. *Ramsey Theorem holds for finite MV-algebras.*

The validity of Ramsey property for finite MV-algebras strongly relies on the one for finite Boolean algebras, as we saw in the previous results. It turns out that this is just a special case of a definitely more general fact, namely, that Ramsey property is preserved under categorical completions, given obvious additional hypotheses meant to ensure that the given categories are Fraïssé classes, as we show in Theorem 2.7. Our result strongly relies on the so-called KPT (Kechris-Pestov-Todorčević) correspondence [3].

Theorem 2.7. *Let $\mathcal{D}$ be a category and $\mathcal{C}$ be a completion for $\mathcal{D}$, i.e., there exist functors $F : \mathcal{C} \to \mathcal{D}$ and $G : \mathcal{D} \to \mathcal{C}$ such that:*

(i) F is a full embedding,

(ii) G is left inverse and left adjoint to F,

(iii) G is faithful.

Moreover, assume that $\mathcal{K}$ and $\mathcal{K}'$ are Fraïssé classes with a distinguished total order, made of objects of $\mathcal{C}$ and $\mathcal{D}$ respectively, such that $G[\mathcal{K}'] = \mathcal{K}$.

Then $\mathcal{K}$ satisfies the Ramsey property if and only if so does $\mathcal{K}'$.

As a consequence of Theorem 2.7, we can immediately obtain once again Corollary 2.6 by simply observing that the category of Boolean algebras with Boolean embeddings as morphisms is a completion of the one of MV-algebras with MV-algebra embeddings, where the completion functor is the one that associates to each MV-algebra its own Boolean center and acts as the restriction on morphisms, and its right adjoint is simply the inclusion functor.

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An Approach to Explainability in AI with Abduction

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Keywords: lstm, artificial intelligence, solomonoff, abduction

Large language models (LLMs) and generative artificial intelligence are at the forefront of current AI research. However, the opaque *black box* nature of these models has raised concerns among users, especially in contexts where generated texts, programs, or other outputs may not meet expectations or even produce untruthful results, a phenomenon commonly known as hallucination. Research in explainable AI (XAI) has thus explored a variety of approaches, such as feature-oriented methods, global techniques, concept models, surrogate models, local interpretations, and human-centric strategies, all aimed at shedding light on these complex systems [1]. Despite these advances, there is still the issue of narrowing down, from potential explanations, the most meaningful insights about the model's behavior. In other words, we look for the "best explanations" that would allow us to better understand the outputs of these models. This concept aligns with Harman's [4] approach to Peirce's later account of the *best explanation* in the epistemology of science. recent work suggests that transformers, the architecture underpinning LLMs, approximate Solomonoff Induction [3], which could help in explainability of the models [3, 5], due to its nature of prioritizing hypotheses by algorithmic simplicity. This idea could be interesting for operationalize Harman's ideas on XAI. In this talk we want to discuss the theoretical foundations on how to improve explainability by formalizing the abductive reasoning in XAI. We want to discuss how Solomonoff Induction could help us in this context. By following the Curry-Howard isomorphism, we are able to define these models (or programs) as proofs we argue that by treating a set of programs (or proofs) as candidate explanations, Solomonoff induction and abductive reasoning could help eliminate less plausible options for models' explanation thus allowing both users and the training process in improving LLMs' results.

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Philosophical Commitments Implicit in the Translation-Calculation Analysis of Informal Arguments

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Keywords: informal Logic, translation-calculation, theories of vagueness.

Informal Logic is the study of informal arguments, those formulated in natural language. If we assume that formal Logic can contribute to the analysis procedures of informal Logic, one of its contributions is the translation-calculation strategy. The first step in the translation-calculation strategy is to convert an informal argument into an argument of a certain formal logic, in a way that does not misrepresent the original argument. The second step is to evaluate the validity of the formal argument with the chosen formal logic, and then extend these findings back to the original, informal argument. For example, a certain informal argument could be considered fallacious if its formal counterpart is found invalid.

A possible caveat in the translation step is that natural language is inherently vague, whereas formal propositions are precise objects. The sorites paradox is arguably the best example of how vagueness may challenge these translations. Theories of vagueness are philosophical accounts of what vagueness is and what the semantics of vague statements is.

I argue that the translation-calculation strategy presupposes the adoption of a theory of vagueness, even if tacitly. The teaching of formal Logic often uses translation-calculation as a pedagogical tool. In doing so, I argue, this teaching inadvertently endorses a particular theory of vagueness: the semantic nihilism of Braun & Sider [1]. As Logic textbooks are not generally expected to commit us to philosophical stances on vagueness, these arguments show that formal Logic is more philosophically laden than usually assumed. The discussion does not intend to address the philosophical problem of vagueness in depth. The goal is to contribute to the epistemology of Logic and to illustrate what sort of repercussions vagueness, and its theories, have on the relationship between formal and informal Logic.

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Lógica Suspensiva
Uma Semântica para Asserção de Dúvida

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Palavras-chave: Lógica, Fundamentação da Matemática, Dúvida, Polivalência, Semimonotonicidade.

Embora usualmente seja tomada como padrão de certeza, a matemática é repleta de dúvidas. Antes de serem demonstradas ou refutadas, as conjecturas carregam a marca da incerteza e costumam estar entre os temas nucleares pelos quais gravita a atividade matemática. A Lógica Suspensiva pretende fazer juz à importância desse fenômeno ao possibilitar a asserção explícita da dúvida. Para tanto, acrescenta aos símbolos lógicos clássicos um operador unário para exprimir dúvida e outro para negar fracamente, rejeitando enunciados apenas provisoriamente (asserções de dúvida são tidas como provisoriamente rejeitadas porque, surgindo uma demonstração ou uma refutação, passam a ser afirmadas ou negadas). Tem-se, assim, a linguagem para uma lógica semimonotônica - uma vez que é monotônica em relação a afirmações e negações, mas não-monotônica em relação a aserções de dúvida. Esta apresentação cuida de uma semântica trivalorada para interpretar essa linguagem.

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O Que a Justificação de Inferências Lógicas Tem a Ver com Teorias do Significado

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Palavras-chave: justificação, dedução, significado

Quando fazemos lógica, assumimos que as regras de inferência utilizadas são corretas uma vez feita a prova de correção para o sistema. Essa prova arregimenta internamente a noção de consequência lógica, no entanto, ela não confere justificação num sentido forte àquelas regras, apenas uma relação adequada entre sintaxe e semântica daquela linguagem.

Há uma boa razão para pensarmos que é possível apresentar uma justificação mais forte do que uma prova de correção para algumas regras de inferência. Quando provamos a correção de algum sistema de lógica clássica que contém, por exemplo, o *Modus Ponens*, é comum que raciocinemos de acordo com esta regra na metalinguagem (Isto é, no interior da prova de correção). Neste caso, estamos assumindo o que queremos provar, com isso incorrendo em uma falácia de circularidade e não apresentando justificação adequada. Mas este resultado é inaceitável, porque se nós não temos uma justificação para as regras de inferência, não há justificação a ser transmitida para aquilo que temos a partir da aplicação dessas regras (fórmulas, sentenças, etc). Assim, algum tipo de consideração mais substantiva precisa ser feita a respeito de regras de inferência.

Em *Justification of Deduction*, Michael Dummett dá a seguinte dica: "a significância de uma prova de correção ou de completude, em termos de uma semântica bi-valorada para uma certa sistematização da lógica depende, portanto, de uma tese que não pertence à lógica e que não pode ser testada por ela, mas que pertence, pelo contrário, à teoria do significado" [1]. Nesse espírito, Paul Boghossian escreve *Knowledge of Logic* [2], em que propõe que a justificação decorre, em parte, do significado das constantes lógicas: qualquer pessoa que entenda o significado da implicação deve inferir de acordo com o *Modus Ponens*. A resposta de Boghossian, apesar de intuitiva e plausível à primeira vista, envolve comprometimentos que trazem sérios problemas, alguns apontados por Williamson, que conclui que "questões epistemológicas não podem ser reduzidas a questões sobre teoria do significado" [3]. Na presente comunicação vou examinar tanto os compromissos envolvidos na proposta de Boghossian quanto a crítica de Williamson. Por fim, proponho o abandono da noção da justificação neste contexto, em troca da de *entitlement* [4] (Usando a terminologia de Crispin Wright), e também que o significado das constantes lógicas desempenha um papel muito diferente daquele que Boghossian pensava na estrutura "justificatória"das regras de inferência.

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Conectando Revisão de Crenças e Reparo de Ontologias

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Palavras-chave: Representação de Conhecimento, Revisão de Crenças, Reparo de Ontologias

Em Representação de Conhecimento, podemos representar conhecimentos sobre o mundo por meio de um conjunto de crenças. Como o conhecimento é dinâmico [3], mesmo pequenas mudanças podem ter um impacto significativo em toda a base de crenças. A Revisão de Crenças estuda formas de lidar com essas mudanças.

O marco inicial dessa área foi o modelo AGM [1]. Esse modelo propôs a representação das crenças por meio de um conjunto de fórmulas da lógica proposicional, além de princípios e operações criados para tratar potenciais mudanças no conjunto de crenças. A partir do modelo AGM, diversas generalizações foram criadas, incluindo adaptações para outras lógicas, como as lógicas de descrição.

A Engenharia de Ontologias, por sua vez, estuda como modelar uma ontologia para que ela represente adequadamente um domínio específico. Durante a construção de uma ontologia, reparos se mostraram de grande importância para a manutenção da consistência. A área de Reparos de Ontologias explora diferentes formas para reparar uma ontologia [2, 5]. Abordagens clássicas de reparo resultavam em uma grande perda de informações durante o processo. Para mitigar o impacto dos reparos, algumas estratégias foram criadas [4], bem como abordagens com o objetivo de se encontrar reparos ótimos [8].

Diversos estudos buscaram mapear as semelhanças entre as áreas Revisão de Crenças e Reparos de Ontologias [6,7]. Baader e Wassermann iniciaram a conexão entre contrações e reparos ótimos de ontologias [9]. Semelhantemente, Souza explorou as diferenças entre reparos ótimos de ontologias e os resultados das operações de pseudo-contração em Revisão de Crenças, contribuindo para a integração entre as áreas, já iniciada em trabalhos anteriores [7, 9]. Souza ainda propôs um método de saturação da base de conhecimento, que permite que operações de pseudo-contração gerem resultados mais próximos aos reparos ótimos. Nesta apresentação, queremos discorrer sobre os resultados obtidos em [10], na tentativa de se aproximar essas duas áreas.

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On Forcing Axioms and Weakenings of the Axiom of Choice

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Keywords: forcing axioms, choice principles, principle of dependent choices.

We prove forcing axiom equivalents of two families of *weakenings of the axiom of choice* (like the hundreds listed in the standard reference [2]): a trichotomy principle for cardinals isolated by Lévy (in [3]), H_κ, and DC_κ, the principle of dependent choices generalized to cardinals κ, for regular cardinals κ. Using these equivalents we obtain new forcing axiom formulations of the Axiom of Choice, AC (in a similar fashion of [4]).

A point of interest is that we use a new template for forcing axioms. For the class of forcings to which we asks that the axioms apply, we do not ask that they apply to all collections of dense sets of a certain cardinality (as in Viale, op. cit, or in [1]), but rather only for each particular forcing to a *specific* family of dense sets of the cardinality in question.

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Adapting Lean Tutorials to the Brazilian Case

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Keywords: Proof Assistants, Lean

Lean4 ([1]) is a proof assistant that is also a general-purpose language, and that has "metaprogramming facilities" ([2]) that let people extend its syntax in many ways. It is becoming very popular, and the page [3] has links to more than 40 courses on Lean, or using Lean.

I don't think that these courses are adequate for Brazilians. They all start from some point like "everybody knows VSCode" and they move to non-trivial ideas very quickly. But "everybody knows the programs such and such" is not true in Brazil at all; half of the Brazilian logicians that I know only use computers in a very basic way, and here "knowledge about programs does not propagate" (see [4]).

In this presentation I will show how I adapted the approach in [4] – that I use to teach Maxima to undergraduate students with *very* little experience with computers – to create a short workshop on Lean ([5]) that teaches people how to install Lean4, how navigate its manuals, how to understand its interface, and how to run and modify simplified versions of some examples from the main books on Lean.

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Quantum Polynomial Certification of Non-Hamiltonian Graphs

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Keywords: Implicational Logic, Jozsa-Deutsch, Proof Theory

This article presents a novel approach to verifying the non-Hamiltonicity of graphs by combining standard proof theory and quantum computing. Starting from a non-Hamiltonian graph G, we construct a formula α_G provable in minimal implicational logic. From this, a natural deduction proof Π is generated, which is then transformed into a directed acyclic graph (DAG) proof ∇. This DAG proof is converted into a logical circuit C_∇, which outputs 1 (true) for every input boolean path vector, verifying the non-Hamiltonian property. By applying the Jozsa-Deutsch quantum algorithm to C_∇, we efficiently obtain a quantum polynomial certificate that verifies with 100% probability that G is non-Hamiltonian. This method bridges proof systems and quantum algorithms, offering a quantum advantage in graph non-Hamiltonicity verification.

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Quantum Agency: A Sheaf-Theoretical Approach

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Keywords: Quantum Turing Machines(QTM), Quantum Agency, Sheaf Theory

This paper explores the integration of Quantum Turing Machines (QTMs) into the concept of computational agency through a sheaf-theoretical approach, extending the classical Solomonoff semi-distribution into the quantum domain. By combining principles of quantum mechanics with algorithmic probability, we propose a novel framework that redefines QTMs as autonomous agents capable of leveraging quantum phenomena such as superposition, entanglement, and interference to enhance decision-making and computational efficiency.

The study begins by formalizing QTMs as agents, emphasizing their unique ability to explore multiple computational paths simultaneously, a feature that distinguishes them from classical Turing machines. This parallelism enables QTMs to address problems involving uncertainty and to evaluate multiple possibilities concurrently, making them particularly suited for applications in quantum machine learning and cryptography [1, 2] .

A key contribution of this work is the adaptation of the Solomonoff semi-distribution to quantum systems. Unlike classical systems, where probabilities are assigned to deterministic programs, QTMs operate on quantum states that encode multiple computations simultaneously. This requires a refined probabilistic framework that accounts for the complexity of all quantum paths and their associated amplitudes [3].

To illustrate the practical implications of this framework, we provide a detailed example of a QTM designed to determine whether a quantum superposition of states is balanced. The machine's operation is depicted through a state transition diagram, highlighting the initialization of quantum states, the application of quantum gates (e.g., the Hadamard gate), and the measurement process that concludes the computation. This example underscores the role of interference in amplifying or suppressing computational paths, optimizing decision-making in quantum systems.

Furthermore, the paper introduces geometrical and probabilistic interpretations of QTMs, leveraging concepts from Hilbert spaces and sheaf theory. These tools provide a deeper understanding of the local consistency and global structure of quantum computational processes, offering insights into the behavior of QTMs as agents [4].

The implications of this framework are far-reaching. In quantum machine learning, QTMs can adapt to data in ways that mimic classical agents but with enhanced computational power. In cryptography, the ability to predict quantum system behavior and explore simpler quantum processes opens new avenues for secure communication and data protection. Additionally, when multiple QTMs are entangled, their agency becomes intertwined, necessitating probabilistic models that reflect joint computational complexity [2].

In conclusion, this work establishes a foundational model for quantum agency, highlighting the non-deterministic and probabilistic nature of QTMs while emphasizing the importance of simpler quantum processes. Future research could explore deeper connections between quantum agency

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and practical quantum algorithms, as well as the application of sheaf theory to other quantum computational frameworks.

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Abstract Logics as Classifications of Abstract Structures

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Keywords: abstract logic, classification, abstract structure

It is standard to consider a logic as a structure of type $\mathcal{L} = (\mathbb{F}; \vdash)$ where $\mathbb{F}$ is a set of formulas and $\vdash$ is a binary consequence relation between theories (sets of formulas) and formulas, i.e. $\models \subseteq \mathcal{P}(\mathbb{F}) \times \mathbb{F}$ [4, 21]. From a more general perspective, "logics are characterized as pairs constituted by an arbitrary set (without the usual requirement of dealing with formulas of a formal language) and a consequence operator" [5, 4]. In abstract (generalized) model theory [2], an abstract (model-theoretic) logic is defined differently: "*An abstract logic* is a triple $L = (S, F, \models)$ where $\models \subseteq S \times F$. Elements of the class S are called the structures of L, elements of F are called the sentences of L, and the relation $\models$ is called the satisfaction relation of L" [8, 21]. Abstract logics satisfy the *Isomorphism Property*: If $U \models_L \phi$ and $B \cong U$, then $B \models_L \phi$. Since isomorphism types are extensions of generalized quantifiers, these logics are called logics with generalized quantifiers.

In this talk, I suggest considering abstract (model-theoretic) logics as classifications (sf. [3, 28]) of abstract structures (isomorphism types): A = $\langle$*tok* (A), *typ* (A), $\models_A\rangle$ where *tok* (A) is a set of tokens, i.e., isomorphism types, *typ* (A) is a set of types, i.e., sentences of the language, and $\models_A$ is a binary relation between them. We may read a $\models_A$ $\mathfrak{a}$ as "a is of type $\mathfrak{a}$ in A". Thus, an abstract (model-theoretic) logic can be seen as a classification where the *token set* is an isomorphism type and the *type set* is the set of sentences which are true in the structures of this isomorphism type (i.e., the theory of the corresponding abstract structure).

An abstract structure is the result of an abstraction operation on isomorphism types. An abstract structure can be viewed as a form which is shared by all structures in an isomorphism type or as an isomorphism type itself which is represented by any of its token. I'll try to show that this dichotomy is rooted in the prehistory of modern model theory. On the one hand, Edmund Husserl postulates a special region of abstract forms in his transcendental justification of logic. For him, the theory of definite manifolds is the highest task of formal logic. In contemporary debates, there is no consensus on the exact meaning of Husserl's "definite manifold". Categoricity, syntactic completeness, and semantic completeness are considered as terms that best capture Husserl's "definiteness" ([1], [6], [7], [9]). Although the term "isomorphism" does not appear in his early works on definite manifolds (e.g., in *Doppelvortrag*), he used it later to describe (unfortunately, imprecisely) "formally equivalent" abstract structures as pure forms of possible theories. On the other hand, Rudolf Carnap explicitly defined structures shared by isomorphic models (i.e., model structures) as isomorphism types that can be specified by means of "definitions by abstraction" [10, 384]. The differences between Husserl's "definiteness" and Carnap's "monomorphicity" will be discussed.

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On Semantics of First-Order Justification Logic with Binding Modalities

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Keywords: justification logic, logic of proofs, first-order logic

The talk will focus on the semantics of first-order justification logic. Propositional justification logics were introduced by S. N. Artemov (in [1], [2]). They are formulated in an extension of the propositional language with formulas of the form $t:\Phi$, where t is a justification term and Φ is a formula. The interpretation of such formulas is the following:

"t is a justification for Φ".

For these logics, both Kripke-style semantics and arithmetical semantics have been studied, along with their connection to modal logic [3].

First-order justification logic was introduced in [4]. In this logic, justification formulas $t:\Phi$ are redefined to enable a distinction between local and global parameters. Specifically, they consider formulas of the form $t{:}_X\, \Phi$, where X represents a set of global parameters (i.e., free variables) that are open for substitution.

In [4], S. Artemov and T. Yavorskaya described arithmetical semantics for first-order justification logic, also known as first-order logic of proofs (FOLP). Subsequently, M. Fitting introduced possible world semantics for FOLP and proved a completeness theorem in [5].

In the current work, we present first-order logic of proofs $FOLP^{\Box}$ within a language extended by the modality $\Box_X$, which also distinguishes between global and local parameters. We describe Fitting models for $FOLP^{\Box}$ and establish its strong completeness and soundness with respect to these models. Unlike the approach taken by M. Fitting, we provide models with a valuation of individual variables without extending the language with additional constants. Our approach enables us to assign semantic meaning to formulas containing free variables. The main results are the soundness and completeness of $FOLP^{\Box}$ with respect to the described semantics.

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On Generalized Functors in the Category of Consequence Operations

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Keywords: Consequence structures, Category theory, Paraconsistency, Universal logic

In his 1930 article [1], Tarski started the investigation of the notion of logical consequence from an abstract point of view. He thought consequence as an operation that acts on sets of formulas subjected to certain axioms. From this, a study of multiple logical systems has been conducted by the community of logicians, including paraconsistent, paracomplete, classic, modal logics. De Souza, Edelcio G., in [4] observed that Tarski's initial condition on finiteness could be generalized, as the finiteness hypothesis was not necessary to derive results such as monotonicity — an axiom ussed for modern account of the notion of logical consequence as in [2]. Instead, a more general axiom was investigated.

Also, De Souza, Edelcio G.; Costa-Leite, A.; and Dias, Diogo H. B., in [3] also investigated the notion of consequence operation from an categorical perspective. That is, they considered a category **CON** with objects being consequence operations on a set $\wp(X)$ and arrows being consequence *homomorphisms* between these consequence operations. From this, they formulated a paraconsistentization endofunctor that acts on this category and allows one to pass from a consistent set of formulas to a paraconsistent one.

We propose, in this talk, to continue this investigation by proposing that the paraconsistentization functor can be derived as an instance of an generalized functor conceptualized on the more general axiom proposed in [4]. This can be of interest because it can generalize investigations of transformations of logical systems beyond the case of paraconsistentization. For instance, it allows to transform a consequence operations that acts classically to a consequence operations that blocks some rule or other; or it allows to transform into operators with special attributes such as finiteness, structurality[1], analyticity, etc...

To be more specific. Given an consequence operation $Cn : \wp(X) \to \wp(X)$, we define a new consequence operation:

$$Cn_R(A) = \bigcup\{Cn(A') : A' \subseteq A \text{ and } R(A')\},$$

where $R \subseteq \wp(X)$ is considered a property of the set of formulas X. We argue that the paraconsistentization functor can be derived considering R a specific property[2]. Also other transformations of Cn-operators can be achieved as we change R.

In this talk, we will analyze this functor, describing it through the framework of category theory, following [3], and exploring its connections with other logical investigations.

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Florensky's Insights on Truth, Antinomy, and Paraconsistency

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Pavel Aleksandrovich Florensky (1882–1937) was an extremely original thinker who dedicated himself to theology, philosophy, logic, mathematics, physics, and electrical engineering. He was an inventor, polymath, Orthodox priest, and martyr. His fruitful intellectual career, the largeness of his personality, and the multiplicity of his interests, earned him the nickname of the "Russian Da Vinci". *The Pillar and Ground of the Truth* [*Stolp i utverzhdenie istiny*, published in 1914] is Florensky's main work and was not designed as a traditional philosophical treatise. Florensky's style exalts emotion and ranges from logic to lyricism. One of Florensky's main resources is argumentation from language and its etymological roots. We will analyze some key concepts of Florensky's epistemology, in particular the notions of truth and antinomy. Indeed, it is from the tension of the contrast between human truths and the total and eternal Truth that Florensky develops an epistemology whose consequences lead him, in a *sui generis* manner, to a broad sense paraconsistent approach. Rationality is unreasonable; for Florensky, reasonableness is antinomic: reason speaks of life, of flow, and of the non-self-identical. Therefore, reason and rationality do not coincide. Reason is opposed to rationality and rationality is opposed to reason, because reason and rationality have distinct and opposing requirements. Rationality and reason operate at different levels, with different logicalities. But what then is truth? *Truth is an antinomy.* Florensky recognizes that "what is needed is a formal logical theory of antinomy", which he will try to outline. He recognizes the need to develop logical systems capable of handling antinomies, and also recognizes that they must be non-trivial so that they do not lose their reasonableness. Reason absorbs the canons of rationality by expanding them. In Florensky's formulation of the notion of antinomy there is an epistemological component of Orthodoxy which is based on apophatic theology. There is also a link with tradition that brings together Heraclitus of Ephesus, Plato, Nicholas of Cusa, Hegel, Fichte, Shelling, Nietzsche, and Florensky: the Truth is discontinuous; the Truth is supralogical; the true nature of Truth can only be antinomic. Christian doctrine itself is seen by Florensky as a network of antinomic statements about this Truth. We argue that Pavel Florensky can not only be considered as a paraconsistent thinker in the broad sense, but as a precursor who anticipated the need for a paraconsistent formal logic that could deal with contradiction without trivialization.

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Ensino de Lógica na Filosofia: O Quê, Como e Por Quê

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Palavras-chave: ensino de lógica, ensino de filosofia

Propõe-se apresentar alguns dos resultados da "Consulta: Ensino de Lógica em cursos de graduação em Filosofia", realizada com apoio da SBL entre os dias 9 de outubro e 8 de novembro de 2024.

Professores de lógica encontram-se espalhados por departamentos de filosofia, matemática e computação, e há amplo escopo de assuntos e técnicas a serem ensinadas sob o escopo da Lógica. Relatos anedóticos têm sugerido que a área tem perdido espaço nos cursos de Exatas, enquanto que em Filosofia, aponta-se sempre a dificuldade tanto no ensino quanto no aprendizado. A consulta foi elaborada como forma de substanciar mais discussões a respeito de ensino de lógica a nível superior, em particular na Filosofia, e fomentar uma discussão mais ampla com a comunidade.

A consulta buscou investigar qual é a percebida função do ensino de lógica em cursos de graduação de Filosofia no Brasil e estabeleber quais são os esforços atuais no ensino de lógica, para assim refletir em novas e necessárias possibilidades. Em particular, exploramos quais são as estratégias de ensino, materiais, conteúdos e objetivos almejados pelos cursos obrigatórios em lógica em graduações de Filosofia no Brasil.

A consulta recebeu 61 respostas de docentes de 49 instituições de ensino superior localizadas em 22 unidades da federação. Foram 21 respondentes da região sudeste, 20 do nordeste, 11 do sul, 6 do centro-oeste e 3 do norte. Embora os dados não possam ser generalizados, eles fornecem uma imagem ilustrativa do ensino de Lógica nos cursos de Filosofia brasileiros que confirma muitas das impressões que a comunidade tem de si mesma.

Visamos apresentar alguns dos achados da consulta, e refletir criticamente sobre eles, constrastando práticas docentes, e explorando diferentes perfis de ensino contidos na consulta. Há diferenças em docentes com mais especalização na área *versus* docentes menos especializados? Há contraste em relação a práticas por tempo de ensino? Os conteúdos das disciplinas condizem com os resultados esperados? As competências (gerais) e habilidades (específicas) que os professores de Lógica pretendem desenvolver em seus alunos são condizentes com os conteúdos por eles abordados?

Com este trabalho, esperamos contribuir para reflexões sobre práticas pedagógicas tanto de professores de nível superior quanto estudantes em processo de formação.

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Conceitos Deônticos, Axiológicos e Valores em Ética Formal

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Este trabalho apresenta uma proposta de Ética Formal inspirada nos estudos de Franz von Kutschera, com ênfase em conceitos deônticos e axiológicos, organizados em categorias classificatórias e comparativas. O objetivo central é demonstrar que todos esses conceitos podem ser reduzidos à noção fundamental de valor. A abordagem adota uma estrutura lógica que conecta dimensões normativas e axiológicas, permitindo compreender de forma sistemática as relações entre obrigações, permissões, proibições e valores. Dessa forma, busca-se construir uma base formal consistente para a análise e aplicação de princípios éticos.

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A Tableaux for the iALC Legal Reasoning Logic

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Keywords: Tableaux, Legal Reasoning, Proofs of Compliance

Ensuring that a knowledge base with public administration acts contains only facts in accordance with its legislation becomes a challenge for any public manager. To achieve this, given the large volume of data generated by public companies, it is necessary to apply technological resources that assist in the process of analyzing the compliance of these acts. In [1], the author presents a computational architecture capable of extracting information published in official gazettes and then serializing it into two knowledge bases, RDF/XML triples of facts and RDF/XML triples of rules formalized in *iALC* logic [2,3], an intuitionistic description logic aimed to perform Legal Reasoning, according Kelsen's Jurisprudence [4]. To ensure the consistency of this knowledge base, a SAT Solver for *iALC* was developed in the form of an intuitionistic semantic tableau. It is extension of the first-order intuitionist tableau presented by Fitting (1960). This SAT Solver is part of a module that generates models and counter-examples for rules formalized in *iALC* and generates a preliminary query code in SPARQL [5]. This approach allows the architecture to infer and certify the quality of the data available in the RDF/XML knowledge base of facts. To guarantee the quality of our SAT Solver, we carry out the soundness proof of its rules. The completeness, on the other hand, supports the scope of our approach facing all possible Compliance Proofs. The purpose of this talk is to show both, the proofs of soundness and completeness and how the implementation of the Tableaux was used to prove the compliance of some concrete KB's built from official gazettes.

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A Paracoherent Interpretation of Tarski-Grothendieck Set Theory

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Keywords: Cantor, category theory, coherent, contrasistent, Gödel, Grothendieck, heritor, incompleteness, liar paradox, librationism, manifestation set, paracoherent, paraconsistent, paradox, Ramsey, Russell, set theory

Let X be an enlargement of system Y just if X is an extension of Y whilst Y is not an extension of X; it is assumed that needed clauses on language are satisfied. Let X be a *coherent* enlargement of classical logic just if X enlarges classical logic and for no q does X prove $\neg q$ if classical logic proves q; notice that in [1] the word "sedate" was used instead of "coherent", as here. X is *consistent* just if it for no q proves $q \wedge \neg q$. Notice that coherent theories are consistent. X is *contrasistent* just if there is some q such that X proves q and X as well proves $\neg q$. "paracoherency" is defined more precisely than in [1] by stipulating that theory X is *paracoherent* just if X is a coherent and contrasistent enlargement of classical logic. Librationist set theory £ is paracoherent, and therefore a coherent and consistent enlargement of classical logic. Paraconsistent systems proposed by others are not coherent and consistent enlargements of classical logic, so £ is either not a paraconsistent system or it is a quite distinguished paraconsistent system which coherently and consistently enlarges classical logic.

A *numerary* policy is adopted so that formulas and terms of £ *are* von Neumann ordinals of the meta language, while formal expressions are identified with bijective base-8 numerals corresponding with the primitive Polish formal symbols presupposed. Given the *numerary* policy, metasets of formulas are metasets of natural numbers, and the latter are ontologically identified with real numbers. So the semi inductive semantic process is a function from ordinal numbers to real numbers interpreted according to the numerary policy presupposed. A real number is *stroked* just if it contains $\downarrow$AB precisely if it neither contains A nor B. That real number x is *exhaustive* means $\forall v$A is in x just if $A^{b}_{v} \varepsilon x$ for all b substitutable for v in A. Real number x is *full* just if x is stroked and exhaustive. £ is fully impredicative, and the constructible hierarchy at level L_{β_0+2} for ordinal β_0 of *ramified analysis* is needed to pass the closure ordinal and include all full L_{β_0+2}-constructible reals as possible initial values for the semantic process at the first ordinal. The latter possibility is needed to delimit the presupposed satisfaction notions.

A sentence is an anti-thesis just if its negation is a thesis, and a sentence is a *maxim* just if it is a thesis and it is not also an anti-thesis. A sentence is *minor* just if it is a thesis and it is also an anti-thesis.

A coding $\ulcorner\ \urcorner$ of formal expressions, into von Neumann ordinals which denote object language sets, is defined, and truth set $\mathcal{T}$ is introduced so that $\mathcal{T}\ulcorner A \urcorner \leftrightarrow \exists x(x \in \{y|A\})$ is a maxim for any sentence A. The predicative form $\mathcal{T}\ulcorner A \urcorner$ is preferred to $\ulcorner A \urcorner \in \mathcal{T}$ here, to conform with the literature on theories of truth.

£ is incomplete e.g. in that neither $s \in s$ nor $s \notin s$ for $s = \{x|x \in x\}$. Although neither the Diagonal Lemma nor the related Löb-formula holds in £, Theorem 16.01 of [3] establishes that $\mathcal{T}$ always fulfils the Hilbert-Bernays-Löb derivability conditions, so that one may marshall philosophical

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arguments for the view that $\vdash \mathcal{T}\ulcorner \mathrm{A}\urcorner$ signifies that sentence A has been proven to be true. Still and still, £ treats of the paradoxes, and it has the sentence L for which it is a maxim that $\mathrm{L} \leftrightarrow \neg\mathcal{T}\ulcorner \mathrm{L}\urcorner$. As a consequence of the logical apparatus of £, L is a minor, so notice that £ is not incomplete for Gödelian reasons, as £ is contrasistent. Interestingly, the definition which introduces the semantic Liar sentence is, *pace* Ramsey, $\mathrm{L} \overset{\mathrm{def}}{=} \boldsymbol{r} \notin \boldsymbol{r}$, where $\boldsymbol{r}$ is Russell's paradoxical set $\{x|x \notin x\}$.

Define *manifestation set* $\llparenthesis^{v}_{\substack{w\\x\\y}} A(\vec{p}, w, \{y|(y,x) \in x\})\rrparenthesis$, with parameters in $\vec{p}$, as

$\{v|(v, \{(w,x)|A(\vec{p}, w, \{y|(y,x) \in x\})\}) \in \{(w,x)|A(\vec{p}, w, \{y|(y,x) \in x\})\}\}$.

If it is a maxim that $\forall w(\acute{z} \in \{z|\mathrm{A}(\vec{p}, w, z)\} \leftrightarrow \mathrm{A}(\vec{p}, w, \acute{z}))$ according to £, then it is also a maxim according to £ that

$\forall\vec{p}\forall u(u \in \llparenthesis^{v}_{\substack{w\\x\\y}} \mathrm{A}(\vec{p}, w, \{y|(y,x) \in x\})\rrparenthesis \leftrightarrow \mathrm{A}(\vec{p}, u, \llparenthesis^{v}_{\substack{w\\x\\y}} \mathrm{A}(\vec{p}, w, \{y|(y,x) \in x\})\rrparenthesis))$.

The construction which builds an interpretation of Tarski-Grothendieck set theory depends heavily upon the manifestation sets, and it postulates extensively on heritors, which are the members of $\{x|x = \{y|y \in x\}\}$. The postulates on heritors amount to conditions upon which sets are allowed as initial in the semantic process, and the formidable strength £ gains to interpret Tarski-Grothendieck set theory, and as a consequence category theory in a natural manner, stems from the mentioned conditions and inducing postulates. The author will relate the construction used to interpret Tarski-Grothendieck set theory. All topics covered here, and related ones, may be discussed. The author will be especially interested in communication that may help research to isolate manifestation sets for larger *relatively* inaccessible cardinals. Notice that the adverbial modifier "relatively" was used, as according to the author's librationist philosophy, defended in many publications, and most notably [2], all sets are countable. For Cantorian arguments for uncountability are blocked on account of £'s contrasistency.

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Universal Families of Rayless Graphs

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Keywords: Infinite graph, Universal, Rayless

Although present in a more general setting on model theory, the study of universal elements for classes of graphs has a very particular tradition. One that started in 1964 by Rado, when he constructed an universal element for the class of countable graphs. Meanwhile he also investigated for classes of graphs with larger cardinalities. Ever since, various different classes were studied for the existence of universal elements, and in case there was no such graph, what was the least universal family, whose cardinality is called the complexity of that class.

A common class in this type of research is when there are fixed forbidden subgraphs. It is known that there can be no universal element for the countable rayless graphs. Not only that, via Schmidt's hierarchy as presented in his PhD thesis [6], if we consider the class of rayless graphs of cardinality up to κ, its complexity is somewhere between κ^+ and 2^κ, for κ any infinite cardinal number.

We have proven that the such complexity is always small, that is, there is a strong universal family of rayless graph of size up to κ, whose cardinality is κ^+. We have also verified that when we forbid finitely many finite subgraphs, the complexity remains small. On the other hand, we present an infinite graph of order 1 such that the class of countable rayless graphs with it forbidden has complexity $2^{\aleph_0}$. Other cases of rayless were seen to also have small complexity, such as when it is forbidden every cycle, only even cycles or only odd cycles.

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Demonstração Métrica da Compacidade da Lógica S4

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Palavras-chave: lógica modal **S4**, compacidade, métrica

Em um artigo de 1944, Tarski e Mckinsey [5] demonstraram, utilizando álgebras de interior, que a lógica **S4** é correta e adequada (completude) em relação a semânticas construídas sobre espaços topológicos. Com este resultado podemos afirmar que toda lógica que contém **S4** é completa em relação a alguma estrutura topológica. Posteriormente, Kremer [3] demonstrou que **S4** é fortemente completa sob qualquer espaço métrico denso-em-si-mesmo.

Sabe-se que existe uma forte conexão entre compacidade, como propriedade topológica, e compacidade como propriedade lógica. Em [2] os autores construíram um espaço métrico sobre a coleção de todos os os modelos de teorias consistentes e maximais para a lógica proposicional, utilizando a compacidade topológica desse espação para realizar uma demonstração métrica da compacidade da lógica proposicional.

Neste trabalho a ser comunicado, temos dois objetivos principais:

i. exibirmos a métrica que gera o modelo topo-$S4$-canônico da lógica modal **S4**, adaptando a prova em [2] para o caso modal;

ii. utilizarmos as ferramentas disponíveis em espaços métricos para demonstrar a compacidade desta lógica, apresentando uma demonstração alternativa para tal resultado.

Denomina-se aqui como modelo Topo-$S4$-Canônico para a lógica modal proposicional **S4** (linguagem enumerável e finitária) a estrutura construída em [1]. Sabemos que tal modelo é um espaço topológico, com topologia induzida pela interpretação do operador $\Box$ por meio da operação de interior. Por ser um espaço denso-em-si-mesmo e possuir outras propriedades topológicas, podemos demonstrar que tal espaço é metrizável.

A partir da adaptação das ferramentas apresentadas em [2] para a coleção de modelos consistentes e maximais da lógica proposicional, encontramos uma métrica sobre o cojunto de todas as teorias $S4$-consitentes e maximais, gerando assim um espaço métrico. Demonstramos que esse espaço métrico é isomórfico ao modelo Topo-$S4$-Canônico, i.é. a métrica que exibimos de fato induz a topologia do modelo canônico para a lógica modal **S4**.

Com a definição explícita da métrica deste espaço, apresentamos uma demonstração alternativa para a compacidade da lógica modal proposicional **S4** (enumerável e finitária), utilizando para isso a compacidade (topológica) do espaço métrico construído. De fato, pode-se demonstrar que tal espaço métrico é compacto se, e somente se, a lógica **S4** for compacta.

Em resumo:

Seja $\mathcal{L}$ linguagem modal proposicional, enumerável e finitária, estendida de uma linguagem proposicional, com $\neg$, $\wedge$ e $\Box$ operadores primitivos e P uma enumeração fixada das variáveis proposicionais da linguagem.

Define-se recursivamente sobre o comprimento de uma fórmula $\varphi \in \mathcal{L}$ e das variáveis em φ, a partir da enumeração P, a **altura** $h(\varphi)$ da fórmula φ.

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Considerando o conjunto $For_n(\Box\mathcal{L}) = \{\Box\varphi(p_1, ..., p_k) \mid (1 \leq k \leq n) \text{ e } h(\varphi) \leq n\}$, define-se uma relação de equivalência $\equiv_n$ em W, coleção de conjuntos $S4$-consistentes e maximais, para todo natural n, como $w_1 \equiv_n w_2$ se e somente se $\forall\psi \in For_n(\Box\mathcal{L})$: $\psi \in w_1 \Longleftrightarrow \psi \in w_2$.

Definição: Define-se $d : W \times W \longrightarrow \mathbb{R}$ função, que é métrica, como:

$$d(w_1, w_2) = inf\{\frac{1}{n+1} \mid w_1 \equiv_n w_2\}$$

Definição: Sejam (W, τ_C) frame do modelo Topo-$S4$-Canônico, que é espaço topológico, e (W, τ_d) o espaço métrico gerado pela métrica d.

Proposição: Os espaços (W, τ_C) e (W, τ_d) são isomórficos.

Proposição: O espaço (W, τ_d) é cauchy-completo.

Proposição: O espaço (W, τ_d) é totalmente limitado.

Proposição: O espaço (W, τ_C) é compacto.

Teorema: A lógica **S4** é compacta se, e somente se, o espaço (W, τ_C) for compacto.

Corolário: A lógica **S4** é compacta.

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Number Theory Combination: Natural Density and SMT

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Keywords: theory combination, finite models, natural density

Satisfiability modulo theories (SMT) is an area in computer science that studies algorithms to determine whether a formula is satisfied in a given first-order theory [2]. Theory combination is a subarea of SMT where we combine these algorithms in order to obtain one that works for the combined theory: that is, for example, the combination of the theories of lists and the theory of numbers is a theory of lists of numbers. Methods for theory combination include Nelson-Oppen [9], strong politeness [8], shininess [10], and gentleness [6], and depend on the theories to be combined having certain model-theoretical properties.

On the other hand, probabilities in finite models is an area in model theory initiated by Carnap [3], Glebskii [7], and Fagin [5]: given a class of models (often a theory) and a formula, one can consider the set of all models that satisfy the formula, and their respective cardinalities; this then becomes a set of natural numbers, and we can calculate (in several ways) the probability that a random number is in this set. The area started by calculating which probabilities could be found as a result of these calculations, but evolved to include rates of convergence, non-classical logics, and the properties of the probabilities in relation to those of the class of models considered [1, 4]: and it is in this last line of inquiry that one can find the present work.

We consider model-theoretic properties of a theory that allow it to be combined according to any of the aforementioned theory combination methods, and its interaction with the probabilities calculated by use of the natural density of a set: some sufficient results are found (showing that if the probability equals some specific values, then the theory has some specific properties), many necessary ones, and we construct a plethora of examples to show the sharpness of these results. This analysis is at first restricted to theories over signatures without functions and predicates, as the results become more relevant in this context, but we also extend some of them to all signatures, and prove that the others cannot be generalized in this sense. In addition, we consider probabilities calculated by use of Schnirelmann's [11] and Dirichlet's [12] densities.

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Modal Languages as First-order Languages

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Keywords: modal logics, first-order logics, translations

In this talk I will compare modal logics, counterpart theory and hybrid logics. In a standard way, modal logics are expansions of classical logics. However, according to Van Benthem, modal logics can be translated into a decidable fragment of first-order logic (FOL), with a binary predicate for accessibility relations. So, in this way, modal logics are fragments of FOL. Counterpart theory was firstly proposed by David Lewis as an explicit attempt to translate quantified modal logic sentences into a new language, which eliminates alethic modal operators $\Box$ and $\Diamond$ by adding four primitive predicates – Ww (w is a possible world), Ixw (x is in the possible world w), Cxy (x is a counterpart of y), and Ax (x is actual) – and eight initial postulates to the theory – which will be introduced in the presentation. Among the things that this theory proposed to do is to try to translate modal sentences and to show that we can not translate some sentences from the counterpart-theoretic language into a properly modal language, with the operators $\Box$ and $\Diamond$. In this sense, counterpart theory can be seen as an attempt to expand standard quantified modal logic. In the presentation, I will give the full language of counterpart theory, presenting slightly different from what was originally proposed by Lewis, and providing a more cohesive formal language with a many-sorted approach for variables. For other reasons, hybrid logics are also expansions of modal logic. This type of logic adds a satisfaction operator $@_a\varphi$ – read as φ is true *at* a –, binders $\forall$ and $\downarrow$ and nominals a, b, c to the language of modal logic, allowing us to formalize true sentences about specific points; sentences that would be false at any other point, i.e. we can name the worlds using nominals and every nominal names an specific world. Besides, like the case of modal logics, both counterpart theory and hybrid logics can be described using a fragment of first-order logic. In this sense, the focus of my presentation will be talk about how some modal languages can be characterized and translated into a first-order language. The presentation is based on my PhD research proposal, showing the state of the art on this topic, being original in raising some criticism – especially against what Lewis proposed for counterpart theory and his translation schemes – and pointing out gaps in the literature that can be pursued in the course of my research.

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Is Mereology Logic?

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Keywords: Mereology, Logicality, Ontological Innocence

Mereology (*classical extensional* mereology, in particular its plurals-based incarnation – a framework that raises a range of related problems of its own) is sometimes claimed to be more than just a theory of part-whole relations, but a kind of *logic*. Is this idea plausible? In fact, what does it even come down to?

One might begin to shed some light on the issue by disentangling the question whether mereological *notions* are logical from the question whether mereology qua *theory* is logic. Clearly, very distinct considerations must be employed in order to assess each of them. A positive answer to the former is by no means sufficient for a positive answer to the latter, given certain natural counterexamples from first-order logic with identity. Nor is it strictly speaking necessary: one might countenance logics for such clearly non-logical subject-matter as propositional attitudes, duties etc.

Nevertheless, we may be fairly confident that the claim that mereology is logic is usually meant as a positive answer to both questions. That is, it is meant as the claim mereology is *pure* rather than applied logic. Hence the chracteristically bold theses put forward in its favor: that mereological notions are "topic-neutral"; or that they are, in some elusive sense, analogous to paradigmatically logical notions like identity; and that mereology is therefore ontologically innocent despite appearances; that it applies to absolutely everything, is "utterly clear and well-understood", formal (in some sense), etc.

The purpose of this talk will be to briefly survey and examine some of these suggestions and how well they fare in justifying the claim mereology is, after all, logic. We shall be particularly concerned with the thesis, or family of theses, known as "composition as identity"; and what it, or they, imply with respect to the existential commitments of mereology.

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BL Logic with an Interior Operator

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Keywords: BL logic, BL algebra, Interior operator, Modal logic.

The BL logic, that is denoted by **BL**, was introduced by P. Hájek [4] in 1998, like a basic logic or basic fuzzy logic, which presents a common fragment of important many-valued logics, as Lukasiewicz Logic, Gödel Logic, and Product Logic.

It is built over a propositional language determined by the set of symbols $L = \{\bot, \odot, \to\}$, in which the propositional operators $\odot$ and $\to$ denote, respectively, the notions of conjunction and conditional; and the constant $\bot$ denote the *falsum*.

In the original paper it was used the symbol & in the place of $\odot$.

The BL algebras are algebraic models of the basic fuzzy logic **BL**, that can be specified for some particular cases of many-valued logics.

In this paper, we extend the BL algebras with an unary operator for the notion of interior, as in the following notions.

Definition 1. *Let $\mathcal{A} = (A, 0, 1, \odot, \wedge, \vee, \to)$ be a BL algebra. An interior operator $\circ$ over $\mathcal{A}$ is a unary function $\circ : A \to A$, such that:*
(i) $\circ a \leq a$
(ii) $\circ a \leq \circ(a \vee b)$
(iii) $\circ \circ a \leq \circ a$.

Proposition 2. *For an interior operator $\circ$ on $\mathcal{A}$, the following conditions hold:*
(i) $\circ 0 = 0$
(ii) $\circ \circ a = \circ a$
(iii) $a \leq b \Rightarrow \circ a \leq \circ b$
(iv) $\circ(a \odot b) \leq \circ(a \wedge b) \leq \circ a \wedge \circ b$
(v) $\circ a \vee \circ b \leq \circ(a \vee b)$.

Definition 3. *If $\mathcal{A} = (A, 0, 1, \odot, \wedge, \vee, \to)$ is a BL algebra and $\circ$ is an interior operator over $\mathcal{A}$, then $\mathcal{A}_\circ = (A, 0, 1, \circ, \odot, \wedge, \vee, \to)$ is a BL algebra with an interior operator.*

Proposition 4. *In any algebra $\mathcal{A}_\circ$, the following conditions hold:*
(i) $\circ(a^-) \leq a^- \leq (\circ a)^-$.

Definition 5. *An element a of $\mathcal{A}_\circ$ is open if $\circ a = a$.*

Proposition 6. *In $\mathcal{A}_\circ$, for each $a \in A$, $\circ a$ is the biggest open element that proceeds a.*

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In this way, we obtain a new algebraic structure composed by a BL algebra with an interior operator that must be the algebraic model for a new modal logic, which extends the **BL** logic with an interior operator.

Finally, in the context of algebraic logic, we show the adequacy of this new logical system generated by the composition of two algebraic structures a BL algebra and an interior operator.

This is a new modal logic that is not a Kripke logic. In a first observation, a BL algebra is not a Boolean algebra, for what needs more laws, but even the operator $\circ$ does not satisfies the usual Kripke axiom K.

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Gelfand-Kirillov Conjecture as a First-Order Formula

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Keywords: first-order axiomatization, algebraic geometry, non-commutative algebra.

Connections between Algebra and Logic are well known. In the specific topic of Algebraic Geometry, this line of inquiry began with the work of Alfred Tarski on the decidability via quantifier elimination in the theory of algebraically closed fields, and have achieved a remarkable development through the years using more sophisticated methods of Model Theory in Algebraic Geometry, such as the work of Ax, Kochen and Ershov on Artin's Conjecture, or the celebrated proof of Mordell-Lang Conjecture by Hrushovski.

Let's fix some conventions. All our rings and fields will be algebras over a base field k.

One of the main problems of algebraic geometry is the birational classification of varieties ([4]). In case k is algebraically closed and we work in the category of affine irreducible varieties the situation is rather simple: given two finitely generated domains A and B and corresponding varieties $X = \text{Spec } A$, $Y = \text{Spec } B$, they are birationally equivalent if and only if $\text{Frac } A = \text{Frac } B$. In general, two varieties are birationally equivalent if their function fields are isomorphic fields [4]. For a state-of-the-art introduction to the subject, see [6].

In the 1966 the study of birational geometry of noncommutative objects began. In his adress at the 1966 ICM in Moscow, A. A. Kirillov proposed to classify, up to birational equivalence, the enveloping algebras $U(\mathfrak{g})$ of finite dimensional algebraic Lie algebras $\mathfrak{g}$ when k is algebraically closed of zero characteristic. This means to find canonical division rings such that every skew field $\text{Frac } U(\mathfrak{g})$ of the enveloping algebras, which are an Ore domain, is isomorphic to one of them.

The idea became mature in the groundbreaking paper [2], where A. A. Kirillov and I. M. Gelfand formulated the celebrated Gelfand-Kirillov Conjecture. Before we formulate it, lets recall some definitions:

Definition. *The rank n Weyl algebra $A_n(\mathsf{k})$ is the algebra given by generators $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ and relations $[x_i, x_j] = [y_i, y_j] = 0; [y_i, x_j] = \delta_{ij},\ i, j = 1, \ldots, n$. We denote by $A_{n,s}(\mathsf{k})$ the algebra $A_n(\mathsf{k}(t_1, \ldots, t_s))$, for $n \geq 1, s \geq 0$. For the sake of notational simplicity, call $A_{0,s}(\mathsf{k}) = \mathsf{k}(t_1, \ldots, t_s)$. In characteristic 0, as is well known, the Weyl algebras are finitely generated simple Noetherian domains [7]. We denote by $\mathbb{D}_{n,s}(\mathsf{k}), \mathbb{D}_n(\mathsf{k})$ the skew field of fractions of $A_{n,s}(\mathsf{k}), A_n(\mathsf{k})$, respectively. These skew fields are called the Weyl fields.*

Conjecture. (Gelfand-Kirillov Conjecture)*: Consider the enveloping algebra $U(\mathfrak{g})$, $\mathfrak{g}$ a finite dimensional algebraic Lie algebra over k algebraically closed of zero characteristic. Its skew field of fractions,* $\text{Frac } U(\mathfrak{g})$*, is of the the form $\mathbb{D}_{n,s}(\mathsf{k})$, for some $n, s \geq 0$.*

The purpose of this work is to show that, surprisingly, given a (reduced) root system Σ (cf. [1, 11.1]) and *any* algebraically closed field k with zero characteristic, the validity of the Gelfand-Kirillov Conjecture for the finite dimensional Lie algebra $\mathfrak{g}_{\mathsf{k},\Sigma}$ — that is, the only semisimple Lie algebra

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over k whose associated root system is Σ — is equivalent to the provability of a certain first-order sentence in the language of rings $\mathcal{L}(0, 1, +, *, -)$ in the theory of algebraically closed fields of zero characteristic — ACF_0 (cf. [5]).

Being more precise, we have:

Theorem. *Given a root system Σ, there is a first-order sentence ϕ_Σ in the language of rings $\mathcal{L}(1, 0, +, *, -)$ such that the below are equivalent:*

1. *For some algebraically closed field k of zero characteristic, the Gelfand-Kirillov Conjecture holds for $\mathfrak{g}_{\mathsf{k},\Sigma}$.*
2. *For all algebraically closed of zero characteristic k , the Gelfand-Kirillov Conjecture holds for $\mathfrak{g}_{\mathsf{k},\Sigma}$.*
3. *$ACF_0 \vdash \phi_\Sigma$.*

Moreover, ϕ_Σ is **naturally** *constructed as an existential closure of boolean combinations of atomic formulas in the language.*

This theorem is surprising because there is no a priori reason for the Gelfand-Kirillov Conjecture to be expressed in a first-order formula. In what follows, $\mathbb{A}$ will denote the field of algebraic numbers.

Let Σ be a root system and k an algebraically closed field of zero characteristic. We want to define the predicate $\mathcal{GK}(\mathsf{k}, \Sigma)$, that means that the Gelfand-Kirillov Conjecture is true for $\mathfrak{g}_{\mathsf{k},\Sigma}$. The initial definition is in ZFC.

If, for each k, we had a formula $\theta_{\mathsf{k},\Sigma}$ in the language $\mathcal{R}$ such that $\mathcal{GK}(\mathsf{k}, \Sigma)$ if and only if $\mathsf{k} \models \theta_{\mathsf{k},\Sigma}$, we would already have a remarkable fact.

However, there is a first-order formula θ_Σ in the language $\mathcal{R}$ such that $\mathcal{GK}(\mathsf{k}, \Sigma)$, for arbitrary k if and only if $\mathbb{A} \models \theta_\Sigma$. As AFC_0 is a complete theory, this holds if and only if $AFC_0 \vdash \theta_\Sigma$.

We remark also that the expression of a statement as a first-order sentence in ACF_0 is a very important question. One of the main applications of this idea is Lefchetz's Principle from algebraic geometry, since ACF_0 is a complete theory ([5]), in order to prove a statement for a variety over an algebraically closed field of zero characteristic, it suffices to show it for $\mathsf{k} = \mathbb{C}$, where transcendental methods are applicable. The Gelfand-Kirillov Conjecture is obviously in the realm of noncommutative algebraic geometry.

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Continuous Hyperfiniteness

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Keywords: descriptive set theory, coutable borel equivalence relation, continuous embedding into E_0, continuous asymptotic dimension, continuous toast structure.

It has been a long-lasting problem, posed by Weiss (1984) [1], whether any Borel action of a countable amenable group on a standard Borel space gives rise to a hyperfinite Borel equivalence relation. Since then, the groundbreaking work by Gao and Jackson (2015) [2] has shown that any Borel action of a countable abelian group gives rise to a hyperfinite Borel equivalence relation. More recently, Conley, Jackson, Marks, Seward and Tucker-Drob (2021) [3] advanced this line of research by introducing the notion of Borel asymptotic dimension to establish the various hyperfiniteness results for Borel group actions.

It is possible to consider these questions in a continuous setting. Weiss's problem can be reformulated as the continuous embedding into E_0 problem, whether any continuous action of a countable amenable group on a zero dimension second countable Hausdorff space can be continuously embedded into E_0. Using the notion of G-hyperfinite equivalence relations—where the induced orbit equivalence relation is the union of an increasing G-clopen finite equivalence relation (an equivalence class being G-clopen if it is "relatively clopen" in a G-action sense)—Kang and Jackson have recently made the progress that 1. for any continuous action of a countable group on a zero dimensional second countable Hausdorff space, if its induced orbit equivalence relation is the lim inf of a sequence of G-clopen finite equivalence relations (i.e. two elements are related by an element in G if and only if they are eventually related in a sequence of equivalence relations), then it can be continuously embedded into E_0, and 2. if G is a finitely generated countably infinite group and the compact zero dimensional second countable Hausdorff space admits a hyperaperiodic element, then the induced orbit equivalence relation cannot be G-hyperfinite.

Kang and Jackson also introduced the continuous analogue of the Borel asymptotic dimension and studied its various consequences. This work continues the study on the continuous embedding of the shift action of $\mathbb{Z}^n$ on $2^{\mathbb{Z}^n}$ into E_0 in [2], and the standard asymptotic dimension and the continuous asymptotic dimension coincide for a polycyclic continuous action on a zero dimensional second countable Hausdorff space [3]. By modifying the proofs in [3], Kang and Jackson have shown that if the action is free and the continuous asymptotic dimension is finite (or even when it is the increasing union of the subgroups with finite continuous asymptotic dimension), then it can be continuously embedded into E_0. Kang and Jackson have also shown that for the shift action of $\mathbb{Z}^n$ on $2^{\mathbb{Z}^n}$, extending continuous asymptotic dimension to the non-free part gives infinity, which justifies the assumption of free action of the continuous asymptotic dimension. We introduce further on the challenge of embedding with the non-free part.

Finally, we introduce the notion of continuous toast structure and show that for a free action with finite continuous asymptotic dimension, there exists a piecewise continuous toast structure, leading to the piecewise continuous 3-coloring, at least in the $G = \mathbb{Z}^n$ case.

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Ivan Orlov: Relevance and Paraconsistency

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Ivan Efimovich Orlov (1886-1936), a Soviet philosopher and industrial chemist, familiar with the philosophical and scientific scholarship of his time, wrote about the foundations and philosophy of mathematics and logic, and specifically on dialectical logic and the logic of natural sciences, the theory of probability, psychology, the theory of music, and chemical engineering. His ideas, developed in the quest for a special substantive logic of natural science that would correspond to the spirit of the dialectic, led him to the formulation of the logic of propositional consistency, which was an important milestone on the path of development of modern relevant logic. In this presentation we will discuss Orlov's paper "Calculus of the compatibility of propositions (Ischislenic sovmestnosti prediozheni)", published in 1928 in Mathematicheskii Sbornik (Récueil Mathématique), a leading Russian mathematical journal. Orlov not only introduced what is now considered the first formal system of relevance logic in the literature, but also presented, two years before Becker, the modal axioms for the axiomatization of the system S 4 and, five years before Godel, an interpretation of intuitionistic logic into the modal system S4, basing intuitionistic logic not on classical logic but on his relevance logic. We will analyze the role of Orlov in the history of paraconsistency.

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A First-Order Characterization and Topological Properties of Gel'fand $\mathcal{C}^\infty-$Rings

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Keywords: $\mathcal{C}^\infty$-ring, Gel'fand rings, Smooth Spectra

Definition 1. *A $\mathcal{C}^\infty-$**structure** on a set A is a pair $\mathfrak{A} = (A, \Phi)$, where:*

$$\begin{array}{rccc} \Phi: & \bigcup_{n\in\mathbb{N}} \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R}) & \to & \bigcup_{n\in\mathbb{N}} \mathrm{Func}\,(A^n; A) \\ & (f: \mathbb{R}^n \stackrel{\mathcal{C}^\infty}{\to} \mathbb{R}) & \mapsto & \Phi(f) := (f^A : A^n \to A) \end{array},$$

*that is, Φ interprets the **symbols** –here considered simply as syntactic symbols rather than functions– of all smooth real functions of n variables as $n-$ary function symbols on A.*

A $\mathcal{C}^\infty-$structure $\mathfrak{A} = (A, \Phi)$ is a $\underline{\mathcal{C}^\infty-\textbf{ring}}$ whenever it preserves projections and all equations between smooth functions.

In this work we introduce the concept of a "Gel'fand $\mathcal{C}^\infty-$ring" in this first-order language as follows:

Definition 2. *A $\mathcal{C}^\infty$-ring A is a **Gelfand $\mathcal{C}^\infty$-ring** whenever the following formula is true in it:*

$$(\forall x \in A)(\exists y \in A)(\exists y' \in A)((1 - x \cdot y) \cdot (1 - (1 - x) \cdot y') = 0) \tag{1}$$

Gel'fand $\mathcal{C}^\infty-$rings compose a full subcategory of $\mathcal{C}^\infty\mathbf{Ring}$, which we denote by $\mathcal{C}^\infty\mathbf{GfRing}$, and have the following remarkable properties:

- A $\mathcal{C}^\infty-$ring, A, is a Gel'fand $\mathcal{C}^\infty$ if, and only if, every prime and $\mathcal{C}^\infty-$radical prime ideal (see [2]) is contained in a unique maximal ideal;
- $\mathcal{C}^\infty\mathbf{GfRing}$ is closed under products, quotients and directed colimits (for their definition, see [3]);
- Every $\mathcal{C}^\infty-$domain (for the definition, see [3]) is a Gel'fand $\mathcal{C}^\infty$ ring (the converse is not true);
- Every von Neumann regular $\mathcal{C}^\infty$-ring (see [1]) is a Gel'fand $\mathcal{C}^\infty$-Ring;
- A $\mathcal{C}^\infty$-domain is a Gel'fand $\mathcal{C}^\infty$-ring if, and only if, it is a local $\mathcal{C}^\infty$-ring;

Moreover, we show the following topological result concerning the topology of the smooth versions of the prime and maximal *spectra* (for the definitions, see [1]) of Gel'fand $\mathcal{C}^\infty$-rings:

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- Whenever A is a Gel'fand $\mathcal{C}^\infty$-ring, the map:

$$\begin{array}{rccc} \mu: & \mathrm{Spec}^\infty(A) & \to & \mathrm{Specm}^\infty(A) \\ & \mathfrak{m} & \mapsto & \mathfrak{m}_\mathfrak{p} \end{array}$$

which maps every prime ideal to the unique maximal ideal in which it is contained, is a continuous retraction; Conversely, if there is a continous retraction from $\mathrm{Spec}^\infty(A)$ to $\mathrm{Specm}^\infty(A)$, then A is necessarily a Gel'fand $\mathcal{C}^\infty$-ring;

- A $\mathcal{C}^\infty$-ring is Gel'fand if, and only if, $\mathrm{Spec}^\infty(A)$ is a normal topological space.

The main contribution of this work is, thus, to show that the first order notion of a Gel'fand $\mathcal{C}^\infty$-ring proposed in **Definition 2** is a "fair" one, showing some of its interesting unfoldings.

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Considerações Acerca do Debate Excepcionalismo vs. Anti-Excepcionalismo Lógico

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Palavras-chave: Epistemologia da Lógica, Excepcionalismo Lógico, Anti-excepcionalismo Lógico.

A lógica tem sido frequentemente considerada a base sólida sobre a qual toda ciência deve ser construída. Por isso, consolidou-se na tradição filosófica como uma disciplina com um status especial, devido ao seu modo distinto de justificação epistemológica. Nesse contexto, o debate entre excepcionalistas e anti-excepcionalistas na filosofia da lógica gira em torno de algumas questões como: A justificação da lógica é *a priori* ou *a posteriori*? A lógica é passível de revisão? Existe uma continuidade metodológica entre a lógica e as ciências empíricas? Os excepcionalistas sustentam que a justificação das teorias lógicas ocorre por meio da intuição racional ou da análise conceitual, pois nenhum dado observável pode demonstrar diretamente a validade de uma regra de inferência [1] [2] [3]. Assim, defendem que a lógica é uma disciplina *a priori*, analítica e necessária, diferenciando-se das ciências empíricas. Por outro lado, os anti-excepcionalistas defendem que a lógica seria justificada a partir de mecanismos de escolha de teorias similares ao de teorias científicas, e as decisões dessa escolha são orientadas por fontes de evidências que não são totalmente *a priori.* As teorias lógicas, assim como as teorias científicas, poderiam estar sujeitas a uma revisão desbancando seu caráter *a priori*, desafiando a ideia de que a lógica possui um estatuto epistemológico especial. Essa abordagem visa desmistificar a epistemologia da lógica, assim, afastando-se de problemas relacionados a analiticidade epistêmica e os apelos à intuição [4] [6] [8] [7].

Alguns filósofos, como Hjortland [4], Priest [9] e Williamson [10], defendem que a seleção entre teorias lógicas deve ocorrer por meio da inferência à melhor explicação, empregando a metodologia abdutiva. Segundo essa posição, as teorias lógicas são justificadas com base em sua capacidade de se ajustar melhor aos dados e dispor de determinadas virtudes epistêmicas em comparação com suas concorrentes. Dessa forma, ao adotar a abdução como método para a escolha de teorias lógicas, implica que a lógica é fundamentada *a posteriori.* Contudo, Priest [9] argumenta que as intuições ainda desempenham um papel na justificação dos dados utilizados na escolha teórica. Através delas, teríamos acesso a certas validades lógicas, o que sugere que os dados relevantes para a escolha de teorias poderiam ser, em parte, *a priori.* Isso levanta a possibilidade de que o anti-excepcionalismo possa ser compatível com a ideia de que a lógica possui uma fundamentação *a priori.*

Diante desse cenário, a presente pesquisa busca examinar algumas questões: Se o anti excepcionalismo aceitar a possibilidade de justificação *a priori*, em quais aspectos ele diverge do excepcionalismo lógico? É possível sustentar um anti-excepcionalismo parcial? Seria coerente defender diferentes versões do anti-excepcionalismo de forma isolada? Tentando esclarecer se há espaço para uma posicão intermediária ou se o anti-excepcionalismo deve rejeitar qualquer fundamentação excepcionalista. Afinal, defender uma forma parcial de anti-excepcionalismo pareceria contraditório, pois isso implicaria considerar a lógica excepcional em alguns casos, o que entraria em conflito com a própria definição de anti-excepcionalismo.

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Simulating a Smile

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Keywords: Paraconsistent logic, bisimulations, van Benthem characterization theorem

According to the classical consistency assumption, non-trivial logical theories cannot contain both a formula and its negation. Such a strong assumption is averted by paraconsistent logics, which for this very reason also end up failing some classically-valid inferences.

A paraconsistent behaviour can be achieved by changing the classical negation to an intensional one, which we denote here by $\smile$ ('smile'). More precisely, we consider $\mathcal{L}_\smile$ as the language generated by the grammar:

$$\varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \smile\varphi \mid \bot \mid \top,$$

where p ranges over some set of proposition letters. This language can be interpreted in a Kripke model $\mathfrak{M} := (W, R, V)$ by setting $\mathfrak{M}, w \Vdash \smile\varphi$ iff $\mathfrak{M}, v \nVdash \varphi$ for some $v \in W$ such that wRv. Note that the interpretation of $\smile$ is dual to that of intuitionistic negation in Kripke models.

One can ask how expressive this logic is, and how it relates to other known logics. An informative answer to this is obtained via a van Benthem-style characterization theorem, which describes a modal logic as the (bi)simulation-invariant fragment of some suitable first-order logic [6]. In order to achieve this in our context, we modify the notion of a bisimulation to that of a *simulation pair*. This is reminiscent of the characterisation result for intuitionistic logic from [3, Section 5], but differs from the latter because we work with a different language and we do not restrict our class of models.

Definition. A *simulation pair* between Kripke models $\mathfrak{M} := (W, R, V)$ and $\mathfrak{M}' := (W', R', V')$ is a pair (F, B) of relations $F \subseteq W \times W'$ and $B \subseteq W' \times W$ such that for all $w, v \in W$ and $w', u' \in W'$:

(i) if wFw' and $\mathfrak{M}, w \Vdash p$, then $\mathfrak{M}', w' \Vdash p$, for all $p \in \mathrm{Prop}$;

(ii) if $w'Bw$ and $\mathfrak{M}', w' \Vdash p$, then $\mathfrak{M}, w \Vdash p$, for all $p \in \mathrm{Prop}$;

(iii) if wFw' and wRv then there is a $v' \in W'$ such that $w'R'v'$ and $v'Bv$;

(iv) if $w'Bw$ and $w'R'u'$ then there is a $u \in W$ such that wRu and uFu'.

We write $(F, B)\colon \mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w'$ (resp. $(F, B)\colon \mathfrak{M}, w \underline{\leftarrow} \mathfrak{M}', w'$) to indicate that (F, B) is a simulation pair between $\mathfrak{M}$ and $\mathfrak{M}'$ such that wFw' (resp. $w'Bw$). Furthermore, we write $\mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w'$ if there exists some simulation pair (F, B) such that wFw', and similarly for $\mathfrak{M}, w \underline{\leftarrow} \mathfrak{M}', w'$.

Observe that if (F, B) is a simulation pair between $\mathfrak{M}$ and $\mathfrak{M}'$, then (B, F) is a simulation pair between $\mathfrak{M}'$ and $\mathfrak{M}$, and we have $(F, B) : \mathfrak{M}, w \underline{\leftarrow} \mathfrak{M}', w'$ if and only if $(B, F) : \mathfrak{M}', w' \underline{\rightarrow} \mathfrak{M}, w$. We say that a world w is *subsumed* by w', and write $\mathfrak{M}, w \rightsquigarrow \mathfrak{M}', w'$, if $\mathfrak{M}, w \Vdash \varphi$ implies $\mathfrak{M}', w' \Vdash \varphi$ for every $\varphi \in \mathcal{L}_\smile$. Being connected by a simulation pair implies subsumption, that is:

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Adequacy Theorem. If $\mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w'$ then $\mathfrak{M}, w \rightsquigarrow \mathfrak{M}', w'$.

Given our observation above, we also get that $\mathfrak{M}, w \underline{\leftarrow} \mathfrak{M}', w'$ implies $\mathfrak{M}, w \leftsquigarrow \mathfrak{M}', w'$. (In fact, the adequacy theorem is obtained by proving both these implications simultaneously.) It is now natural to ask whether the converse of the adequacy theorem is true as well. That is, does $\mathfrak{M}, w \rightsquigarrow \mathfrak{M}', w'$ imply $\mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w'$? While this does not hold in general, we can find classes of Kripke models for which it does, called *Hennessy-Milner classes*. One such class is given by the finite models. Another consists of the so-called *negatively-saturated* models, a variation of the modally saturated models (see e.g. [1, Definition 2.53]) tailored to our logic. In fact, it can be proven that every countably saturated Kripke model is negatively-saturated, and this plays an important role in the proof of the characterisation theorem.

Hennessy-Milner Theorem. If $\mathfrak{M}$ and $\mathfrak{M}'$ are negatively saturated models, then

$$\mathfrak{M}, w \rightsquigarrow \mathfrak{M}', w' \quad \text{implies} \quad \mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w'.$$

Let $\mathcal{FOL}$ be the language of first-order logic with a binary predicate symbol, and with a unary predicate symbol for each proposition letter of $\mathcal{L}_{\smile}$. In this case, Kripke models correspond precisely to the first-order structures used to interpret $\mathcal{FOL}$. We say that a formula $\varphi(x) \in \mathcal{FOL}$ with at most one free variable, x, is *preserved by simulation pairs* if for every simulation pair (F, B) between Kripke models $\mathfrak{M}$ and $\mathfrak{M}'$ we have:

$$\text{if} \quad \mathfrak{M}, w \underline{\rightarrow} \mathfrak{M}', w' \quad \text{and} \quad \mathfrak{M} \models \varphi(x)[w] \quad \text{then} \quad \mathfrak{M}' \models \varphi(x)[w'].$$

(The notation $\varphi(z)[a]$ means that the free variable z is interpreted as the individual a.)

Defining the standard translation of formulas of the language $\mathcal{L}_{\smile}$ to formulas of the language $\mathcal{FOL}$ as expected, we obtain the following characterisation theorem:

van Benthem Theorem. A first-order formula $\varphi(x)$ with one free variable x is preserved by simulation pairs if and only if $\varphi(x)$ is equivalent to the standard translation of a formula in $\mathcal{L}_{\smile}$.

Further work. As mentioned above, paraconsistent logics lack some classically-valid inferences. One way of recovering those inferences is by the addition of a (modally interpreted) *consistency connective* to our language [2,4,5]. A future direction of investigation for our present study consists in finding analogous results for the language $\mathcal{L}_{\smile}$ expanded with a *standard* consistency connective ☺, interpreted by setting $\mathfrak{M}, w \nVdash ☺\varphi$ iff $\mathfrak{M}, v \Vdash \varphi$ and $\mathfrak{M}, v \Vdash \smile\varphi$.

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Semantic Indeterminacy as an Objection to Mathematical Structuralism

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Keywords: Mathematical structuralism, Semantic indeterminacy, Newman objection

Since its conception, *mathematical structuralism* has relied on vagueness to substantiate its account of the nature of mathematical entities and our knowledge of them. From Benacerraf's view of numbers as 'positions in structures' to more contemporary versions motivated by category-theoretic considerations, indeterminacy of reference has been central to the claim that mathematics is the study of abstract structures rather than the science of numbers and quantities. This work challenges this claim by going in the opposite direction: I explore how semantic indeterminacy can support an *objection* to mathematical structuralism.

The structuralist view centers on the assumption that we can characterize and distinguish entities only up to isomorphism. This aligns with mathematical practice, since its subject matter is often equated with the (informal) notion of structure, that is, a set with relations and functions defined on it; and insofar as these are preserved under bijection, we are dealing with the same structure—regardless of what the objects in the domain may look like. Yet, the more canonical motivation for structuralism is to be found in how we address and account for our knowledge of, say, *the* natural numbers, or some *specific* mathematical structure. The idea is that we are actually referring to their corresponding classes of isomorphic structures—which contain (possibly many) other structures. This vagueness extends to the level of the individuals as well: since a member of a structure can always be mapped to a different one from another structure of the same isomorphism class, there is no privileged interpretation. This referential elusiveness thus seems to support the structuralist view, which advocates not taking mathematical entities as 'fixed' things to which we can unambiguously refer and have knowledge of.

Contrary to this assessment, I argue that semantic indeterminacy can in fact substantiate the opposite claim, namely, that mathematics is about numbers and quantities, after all. To do so, I explore a rationale that has been almost exclusively employed to counter scientific structuralism, namely, the so-called Newman's objection. The basic idea is that isomorphic structures can always be induced on carriers of (at least) equal cardinality (in the case of a model-theoretic formulation, we must also add the constraint imposed by the relevant signature). As a result, it can be argue that knowledge of mathematical objects under structuralism is ultimately equivalent to knowledge of quantities.

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Approximations for Boolean Satisfiability

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Keywords: approximation algorithms, boolean satisfiability, linear programming

The Boolean satisfiability problem is the most notorious and first ever discovered NP-Complete problem. To solve it, state-of-the-art methods still rely on techniques that yield exponential running time in the worst case. The goal of this work, is to study methods used on correlated problems in order to approach solving a SAT instance from an optimization perspective. In particular, linear programming methods have obtained good results when dealing with a closely related problem, the satisfiability problem in many valued logics [1] and, more specifically, the satisfiability problem in Łukasiewicz Logic [2].

We want to develop a family of optimization settings that are simpler to solve than the original SAT, but as the complexity of these simpler problems grows, the family converges to the original problem. Therefore, positively solving one of this easier problems should imply solving the original harder problem.

Stating the problem formally: For a given boolean formula $\varphi(\mathbf{x})$, in clausal normal form, where $\mathbf{x} = (x_1, ..., x_n) \in \{0,1\}^n$. For each boolean variable x_i, we introduce a real variable $z_i \in [0,1]$, thus giving us $\mathbf{z} = (z_1, ..., z_n) \in [0,1]^n$. Let $\mathcal{C}$ be the set of clauses of φ. For every clause c of φ, we produce the following constraint

$$\sum_{x_j \in c} \nu(x_j) \geq 1$$

where

$$\nu(x_j) = \begin{cases} z_j, \text{ if } x_j \text{ appears in } c \\ 1 - z_j, \text{ if } \neg x_j \text{ appears in } c \end{cases} .$$

Lastly, let

$$g_c(\mathbf{x}) = \sum_{x_j \in c} \nu(x_j) - 1.$$

This transforms our constraint into $g_c(\mathbf{x}) \geq 0$.

We claim to want to study the satisfiability of φ by studying the following program

$$\begin{aligned} \text{maximize } & \sum_{i=0}^{n} |z_i - 0.5| \\ \text{subject to } & g_c(x) \geq 0, \forall c \in \mathcal{C} \\ & 0 \leq z_i \leq 1, i = 1, ..., n \end{aligned} \tag{1}$$

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It was previously thought that any absolute value programming problem could be easily solved by noticing that $|x| = |x_i^+ - x_i^-| = x_i^+ + x_i^-$, where $x^-, x^+ \geq 0$, if and only if not both x_i^+ and x_i^- are nonzero.

However, in [3] a discussion was given about optimizing absolute value functions, highlighting that, in fact, that technique is only applicable to a maximization problem with nonpositive coefficients (or a minimization problem with nonnegative coefficients); otherwise there are local maxima that the simplex method will converge to [4].

Even though we are assured to not converge directly to an optimal solution (which in this context may or may not be integral), we aim to tinker with the procedure in order to influence it towards an integral solution incrementally.

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Classical Recapture through Modal Logic

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Keywords: modal logic, subclassical connectives, classical recapture

There are numerous ways in which a logical system can recapture another logical system. This recapture effect may be achieved, for instance, by making use of different kinds of translations, simulations, semantical constraints, or derivability adjustments. In all cases, L1 is claimed to recapture L2 if L1 can somehow 'reproduce without loss' the inferential behavior that is characteristic of L2, or that L2 can somehow be seen as 'living inside' L1, meaning that there is a part of L1 that allows for the reasoning typical of L2 to be fully recovered.

A typical example of recapture through derivability adjustments lies at the very heart of a class of paraconsistent logics known as *Logics of Formal Inconsistency* (**LFI**s). Being paraconsistent, **LFI**s reject the classical consistency assumption, which disallows the simultaneous assertion of both a sentence and its negation; in addition, the language of an **LFI** allows for consistency to be internalized directly at the object-language level. The underlying intuition is that consistent reasoning should be recoverable by an **LFI** when explicit assertions about the consistency of the involved sentences are available: if a sentence is assumed to behave consistently, then that very sentence and its negation are not to be simultaneously asserted. Paraconsistent logics are bound to fail some classically-valid inferences, such as disjunctive syllogism: if asserting A provides grounds for asserting A-or-B, then asserting at once A and not-A, whenever this turn out to be justifiable, does not imply the assertion of B. However, once A is assumed to behave consistently, ruling out the previous negation-glutty scenario, then asserting both not-A and A-or-B implies, as in classical logic, that B is not to be denied.

In the above example, from the perspective of a classical reasoner there is a strong expectation concerning the behavior of negation: asserting A implies denying not-A. In order to minimally acknowledge the *legitimacy* of the classical logician's aspiration, the non-classical logician who differs on this point should at least allow for some situation in which not-A is denied while A is asserted. When a sentence A is marked by a *basic* consistency connective, though, the relevant expected classical behavior is restored: asserting the consistency of A is incompatible with an assertion-glut involving A and not-A. For a stronger version of this consistency connective, that we may call *standard*, denying either A or not-A also suffices to guarantee the consistency of A. Either way, a *coherent* restoration connective for recapturing consistent behavior should allow for the consistency of A to be asserted alongside A itself, as well as alongside not-A. In the dual scenario, involving paracomplete logics and negation gaps, while the classical logician insists that the denial of A implies the assertion of not-A, the rival non-classical logician acknowledges the legitimacy of this aspiration and introduces coherent restoration connectives, either basic or standard, to recapture the expected classical behavior. In both scenarios, the classically-valid inferences that are lost due to the weakening of the meaning of negation are recovered with the use of auxiliary connectives that allow for the lost classical assumptions to be explicitly added to those inferences, thus enabling appropriate derivability adjustments.

A simple way of arriving to a legitimate paraconsistent negation through semantical means consists in introducing some non-determinism: just allow for the negation of a true atomic sentence to be either true or false. A standard consistency connective is then obtained by stipulating that consistent-A is false under a certain valuation if, and only if, both A and not-A are true under this valuation. Alternatively, three-valued deterministic semantics are also often employed to achieve this effect: just consider two different degrees of truth, one that exhibits a classic-like behavior with respect to negation and another one that permits negation-gluts. Yet another semantical path towards the definition of legitimate paraconsistent negations, with the added advantage of being *congruential* (namely, by treating logically equivalent sentences as synonymous) —a property that rarely holds in three-valued scenarios— goes through modal semantics. By dualizing intuitionistic negation (a paracomplete negation according to which not-A means that 'A must fail to be true'), a legitimate paraconsistent negation is introduced in a modal environment by interpreting not-A as 'A might be false'). A standard consistency connective, then, is straightforwardly defined by setting consistent-A to be false at a world if, and only if, at that world A is true yet possibly false. In either of the above approaches, accompanying coherence conditions are easily imposed so as to give full meaning to the respective restoration connectives.

The above pretty much takes care of *subclassical* negations —connectives that do not disagree with the underlying assumptions that confer meaning to classical negation, but that may still fail to endorse the full package of classical assumptions— and the restoration connectives that accompany them, enabling the recovery of relevant lost classical inferences and the recapture of corresponding aspects of classical behavior. Generalizing this approach, in this talk I will focus on demonstrating that all kinds of 'classically-defective connectives', of any arity, may be obtained through modal means. To this end, I will explain how to associate legitimacy conditions with any specific class of subclassical connectives, and I will show how standard restoration companions can be linked to all such subclassical connectives, along with coherence conditions that ensure that the said restoration companions have the intended meaning. As a byproduct, general results concerning classical recapture will be available in all cases.

A Computational Procedure for the Generation of Proof Exercises with a Comparable Level of Complexity

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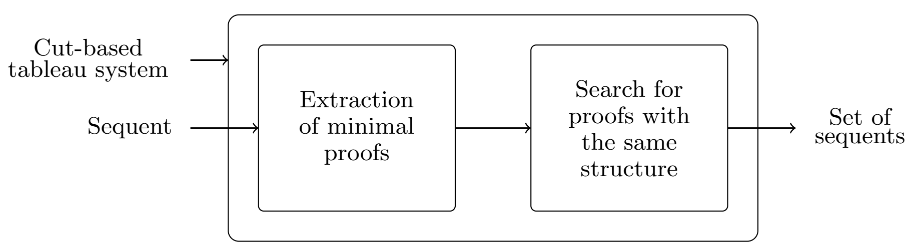
Keywords: Teaching Logic, Cut-based tableaux, Automatic Question Generation

Besides the time in a classroom, the schedule of an educator is filled with several other activities, such as preparing lessons, extraclass support to students and delivering exercises and assessments. In particular, the manual construction of exercises can be particularly time-consuming. This was what motivated us to present in [2] a method that allows for the automated generation of proof exercises with a comparable level of complexity. In this work, we present a computational procedure that implements this method to exercises described for first-order languages, specifically tailored for introductory higher Mathematics courses. Additionally, we provide a proof of the procedure's correctness and termination.

In our procedure, the inputs are a cut-based tableau [1] system R with which we can provide a proof for the other input, a sequent Seq_{inp}. The sequent Seq_{inp} acts as a logical description of a hand-curated proof exercise provided by the user. For example, the sequent $a \subseteq b, b \subseteq c \vdash a \subseteq c$ is a description for an exercise such as "For any sets a, b and c, prove that $a \subseteq c$ given that $a \subseteq b$ and $b \subseteq c$". As the output, we have a set of conjectures which have a comparable level of complexity to that of Seq_{inp}. The big picture of the method is described in the diagram below:



Two proof exercises are said to have a comparable level of proof complexity when the minimal effort to prove both exercises produce proofs with the same structure. That is why we divide our procedure into two parts. In the first step, we extract proofs that capture the minimal effort one puts into proving Seq_{inp}. In the second step, we search for proofs with the same structure to the previously extracted minimal proofs.

The strategy we employ in the extraction of minimal proofs is to check if there is a proof among all tableaux constructed with the application of one rule for the Seq_{inp}. If so, the set of minimal

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proofs has been found. Otherwise, we check among all tableaux constructed with the application of two rules and so on until the set of minimal proofs is extracted. The analytic character of R enables the extraction of the set of all possible tableaux with the application of n rules for any natural number n in a finite amount of time. Moreover, since the exercise given as input is provable, it is guaranteed that, by using this strategy, this extraction procedure terminates.

The search for proofs with the same structure is implemented via a combinatorial approach. For every minimal proof M extracted in the first step, we check, for every possible combination of predicate and function symbols of the formulas in M, if we have a minimal proof for some sequent Seq_{out}. If this is the case, Seq_{out} is added to the set of output sequents.

As a future work, we aim to adapt this method to also deal with refutable exercises. Then, we intend to integrate these implementations in a software that enables students to solve such exercises in a proof assistant, using a controlled natural language for proofs, such as Verbose Lean[1].

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Benchmark of State Tracking Reasoning

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Keywords: large language models, reasoning benchmark, state tracking, natural semantics

Large Language Models (LLMs)—like **ChatGPT** or **Gemini**—owe their popularity largely to their unexpected success in tasks for which they were not trained for, such as development and documentation of software [1, 5]. Soon it was conjectured that LLMs could deploy complex capabilities, or that these could emerge from their behaviour under training not focused on them. Reasoning is one of the most desired capabilities to emerge from their behaviour, given its relation with our understanding of what intelligence is, but also for *alignment* and *explainability* purposes.

Many philosophical frameworks can be then applied for the assessment of their capabilities, each of them with the corresponding assumptions, as is the case for those who think that LLMs can attribute intentional states to agents and model them, i.e. they have a *theory of mind*. Since LLMs are very complex systems, and some times they are better viewed as black boxes [3], the only evaluation we can perform is to interact with a LLM and assess the quality of their responses, as we would do with any human agent. Evaluations on theory of mind generally involve stories, and questions about these, where agents interact with other agents or perform actions on objects, and so on [2]. Results on theory of mind usually are far from human performance, and one possible explanation is that LLMs could be bad at tracking states [6].

Currently, many standardized tests, *benchmarks*, for reasoning as a capability face some problems and the assessment is still a challenge [3, § 8.4]. Even when a LLM performs well in one of these benchmarks, it still fails in some other evaluations [4]. Results indicate that LLMs do not generalize what they learned from their training—which often includes the reasoning benchmarks themselves—and they rely instead in statistical artifacts.

Benchmarks work with human generated data or synthetic tests. Working with the former usually involves scalability—too few data—and saturation—LLMs can overfit on them—issues. Artifacts emerge more easily when working with synthetic data, but they can be reported and then allow to control better the generation. In addition, an advantage of synthetic data is that it can be as much as wanted, it can be increased with new instances, and it can be generated on demand. Among these benchmarks, we can highlight **bAbI** [8] and **Dyna-bAbI** [7], which generate stories adequate to different capabilities to be tested. The first one, **bAbI**, tacitly tests reasoning about states, for which LLMs have a low performance, but its continuation, **Dyna-bAbI**, just explores more convoluted ways of state tracking, without controlling the complexity of these tasks.

We present here the design of a new benchmark for reasoning that includes tests on state tracking. Generation depends on specific vocabularies and can be controlled to produce tests of different complexities. It is also modular, in the sense that it tests capabilities of different types, which can be extended with both simpler and more complex tests. In particular, we take a subset

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of [8]'s tasks and give more precise definitions, so we can isolate groups of interdependent capabilities within a test, and lay a hierarchy of state tracking complexity.

Every test consists in taking terms which refer to entities from given collections, and describing their relations as a state of the world, so we can later define a sequence of these states and ask something about it. The simpler test we define is **Simple State Tracking**, for which a state is a total or partial description, so entities from a set are linked to entities from another, which we treat as their coordinates; e.g. we could treat a locations as the coordinates for people, and people as the coordinates for objects. **Complex State Tracking** works with two pairs of sets related as before, such they share a set that plays the role of coordinate in one of the pairs; e.g. people can be placed in locations as coordinates, at the same time they can be coordinates for objects. Other tracking tests can be defined; we provide as an example one of the tasks from [8], "Time Reasoning", which we redefine as **Time Tracking**, since it tests not only state tracking, but also temporal relations between entities of the same type (events).

Each test is fed with a vocabulary and some parameters, such as how many states, how many terms from each vocabulary to use, and what answer we want to warrant as a ground-truth. With that, we generate structures that describe a sequence of epistemic states, for which we can report adjacent differences. For these structures we define a natural semantics limited to a restricted use of the vocabulary in play, so we can warrant ground-truth. We then generate a list of sentences from these sequences using a first order syntax, which can be later be translated to natural languages.

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Categorical Constructions of Sets Valued on Semicartesian and Involutive Quantales

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Keywords: involutive quantales, quantale valued sets, monoidal categories

This work mainly concerns the—here introduced—category of $\mathcal{Q}$-sets and their functional morphisms, for $\mathcal{Q}$ an involutive and semicartesian quantale. In particular, we describe in detail the limits and colimits of this complete and cocomplete category, and establish that it is locally presentable.

In the 1970s, the topos of sheaves over a locale/complete Heyting algebra $\mathbb{H}$, denoted as $\mathrm{Sh}(\mathbb{H})$, was described, alternatively, as a category of $\mathbb{H}$-sets [4]. More precisely, in [3], there were three categories, whose objects were locale valued sets, that are equivalent to the category $\mathrm{Sh}(\mathbb{H})$.

Initially separate from the world of sheaves, there was a non-commutative and non-idempotent generalization of locales called "quantales", introduced in the mid 1980s by C.J. Mulvey [6]. In the early 1990s, quantales show up in logic, and in the study of C^*-algebras. Sheaves over certain quantales have been considered in [1] and, recently, by A. Tenório, C. Mendes and H. Mariano in [7]. Categories of sheaves on *involutive quantale*—identified with some enriched categories—hearkening to the work of R. Walters [8], were revisited by H. Heymans, I. Stubbe, and P. Resende. Examples of involutive quantales aren't few: binary relations over any set, maximal spectra of non-commutative C^* algebras, quantales of ideals of a ring endowed with an involution, etc.

Categories of sheaves over (certain subclasses of) quantales, in the sense of categories of quantale valued sets, have been proposed over the years in attempts to expand the celebrated notion of $\mathbb{H}$-sets (for complete Heyting algebras) to the broader category of parameter algebras consisting of certain quantales. For instance, categories of sets valued on right-sided idempotent quantales, were considered by M. Coniglio, F. Miraglia, and U. Solitro in the latte 1990s, [5].

In this work we deal with involutive and semicartesian quantales. Semicartesianess means that the quantale admits projections ($a \otimes b \leq a, b$). The logical meaning of an involution on a quantale can be extracted by inspecting the involution-free general case and then adding the involution: non-commutative substructural logic. Non-commutative logic is *temporal* since the relative order of premises of an entailment are logically relevant. As such, the involution reflects, or rather imposes, a way to temporally rearrange terms while preserving entailment.

Definition: Let $\mathcal{Q}$ be an involutive semicartesian quantale. A right $\mathcal{Q}$-set is a pair $X = (|X|, \delta)$ where $|X|$ is a usual set and $\delta : |X| \times |X| \to \mathcal{Q}$ is a function such that, for all x, y and $z \in |X|$: $\delta(x,y)^* = \delta(y,x)$ (symmetry), $\delta(x,y) \otimes \delta(y,z) \leq \delta(x,z)$ (transitivity), $\delta(x,y) \otimes \delta(y,y) = \delta(x,y)$

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(local right identity). A left $\mathcal{Q}$-set satisfies the first two axioms and a symmetrically analogous version of the third axiom. We also define $\mathrm{E}x := \delta(x, x)$, the extent of x.

Our theory of semicartesian involutive $\mathcal{Q}$-sets describes objects that capture a certain notion of "becoming", where the involution will play a role in "reversing a process", in that $\delta^*(x, y) = \delta(y, x)$, like how homotopies are reversible. This "becoming" expression is perhaps overly poetic, since the $\mathcal{Q}$-set itself is static and has no dynamics, rather, it describes an "equality with direction".

Definition: Let $\mathcal{Q}$ a quantale and $X = (|X|, \delta_X), Y = (|Y|, \delta_Y)$ $\mathcal{Q}$-sets. A funcional morphism $f : X \to Y$ is a function $|f| : |X| \to |Y|$ such that: $\delta(x, y) \leq \delta(f(x), f(y))$ (increasing), $\mathrm{E}x = \mathrm{E}f(x)$ (extent preservation). The identity morphism and the morphism composition are defined as the usual function identity and function composition respectively. The corresponding category will be denoted by $\mathcal{Q}$-Set.

Concerning the category of $\mathcal{Q}$-sets:

1. We describe, in detail, the limits and colimits of this complete and cocomplete category;
2. We describe generators;
3. We prove that it is a κ-locally presentable category (where $\kappa = \max\{|\mathcal{Q}|^+, \aleph_0\}$);
4. We investigate if the category has some form of subobjets classifier;
5. We investigate a certain family of monoidal products defined over this category;
6. We discuss the issue of "change of basis" induced by appropriate morphisms between the parametrizing quantales involved in the definition of $\mathcal{Q}$-sets.

We will also discuss the important notions of *relational morphisms*, *singletons*, and *Scott-Completion*, that play an important role in the connection between the categories $\mathrm{Sh}(\mathbb{H})$ and $\mathbb{H}-$*sets*, in the vein of [2].

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Rough Sets Semantics for a Three-Valued Logic

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Keywords: three-valued logics, rough sets, semantics

In [2] we studied a three-valued first-order logic which may be given in the signature $\{\wedge, \vee, \neg, D\}$. In this talk we present a rough sets semantics for the mentioned logic.

Rough sets were introduced by Pawlak and his co-workers in the early 1980s (for instance, see [3] and [4]).

Given a first-order language L (which, for simplicity, we assume only with a non-empty set of n-ary predicate letters), we define an *L-structure* as a non-empty set U (its *universe*) together with a partition of U. This partition of U, suitably defines one on U^n, for every $n \in \mathbb{N}$. We associate an n-ary relation $R^U \subseteq U^n$ to every n-ary predicate letter R in L. Note that each $R^U \subseteq U^n$ may be viewed as a rough set (on U^n). As usual, we indicate the upper approximation of R^U by $\overline{R^U}$, and its lower approximation by $\underline{R^U}$. An *interpretation* of a language L is a pair $\mathcal{I} = (\mathcal{A}, e)$, where $\mathcal{A}$ is an L-structure and $e : Var_L \to U$ a function assigning an element of U to each variable of L.

Definition 1. *Let $\mathfrak{F}_L$ be the set of formulas of the language L, let $\mathbf{3}$ be the three element algebra $(3; \wedge, \vee, \neg, D)$ as in [1] or [2], and let $\mathcal{I} = (\mathcal{A}, e)$ be an interpretation for L. We recursively define the function $v_{\mathcal{I}} : \mathfrak{F}_L \to \mathbf{3}$ which we will call the* $\mathbf{3}$-valuation associated to $\mathcal{I}$ *as follows:*

For every n-ary predicate letter R, we stipulate

$$v_{\mathcal{I}}(R(x_1, \ldots, x_n)) := \begin{cases} 1, & \text{if } (e(x_1), \cdots, e(x_n)) \in \underline{R^U}, \\ \frac{1}{2}, & \text{if } (e(x_1), \cdots, e(x_n)) \in \overline{R^U} - \underline{R^U}, \\ 0, & \text{if } (e(x_1), \cdots, e(x_n)) \notin \overline{R^U}. \end{cases}$$

Let now α, β be L formulas. We stipulate

$v_{\mathcal{I}}(\neg\alpha) := \neg(v_{\mathcal{I}}(\alpha))$,
$v_{\mathcal{I}}(D\alpha) := D(v_{\mathcal{I}}(\alpha))$,
$v_{\mathcal{I}}(\alpha \wedge \beta) := v_{\mathcal{I}}(\alpha) \wedge v_{\mathcal{I}}(\beta)$, *and*
$v_{\mathcal{I}}(\alpha \vee \beta) := v_{\mathcal{I}}(\alpha) \vee v_{\mathcal{I}}(\beta)$.

Finally, for any L formula α we define

$v_{\mathcal{I}}(\forall x\alpha) := min\{v_{\mathcal{I}^{x/a}}(\alpha) : a \in U\}$ *and*

$v_{\mathcal{I}}(\exists x\alpha) := max\{v_{\mathcal{I}^{x/a}}(\alpha) : a \in U\}$. *Here $\mathcal{I}^{x/a}$ indicates the interpretation with the same L-structure from $\mathcal{I}$ but with an assignment e_a such that $e_a(x) = a$ and $e_a(y) = e(y)$, for $y \neq x$.*

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We define the notion of semantic consequence as for classical logic.

Definition 2. *Let* $\Gamma \cup \{\alpha\} \subseteq L$. *We define* $\Gamma \Vdash \alpha$ *if for every interpretation* $\mathcal{I}$ *of* L, *it holds that* $min\{v_I(\gamma)\} \leq v_I(\alpha)$.

Now, to any interpretation $\mathcal{I} = (\mathcal{A}, e)$ we can associate a Kripke model $K_{\mathcal{I}}$. This model (as those studied in [2]) has as underlying frame one of the form $\{1 < \frac{1}{2}\}$. We call these two element Kripke models, *the Kripke model associated to the interpretation* $\mathcal{I}$. As we have already said, its frame is the two element poset. Furthermore, its universe $U_{\mathcal{I}}$ is the set of equivalence classes $\{[x] : x \in U\}$, and for every n-ary predicate letter we stipulate

$$1 \vDash_{\mathcal{I}} R(x_1, \ldots x_n) \text{ iff } (e(x_1), \ldots, e(x_n)) \in \underline{R},$$

$$\tfrac{1}{2} \vDash_{\mathcal{I}} R(x_1, \ldots x_n) \text{ iff } (e(x_1), \ldots, e(x_n)) \in \overline{R},$$

and extend in the usual way this forcing relation to any L-formula.

It is possible to prove the following fact.

Proposition 1. *For every formula* α *and* $k = \frac{1}{2}, 1$, *it holds that if* $k \leq v_{\mathcal{I}}(\alpha)$, *then* $k \Vdash \alpha$.

Reciprocally, given a Kripke model $\mathbf{K} = (K, \leq, U, \Vdash)$, with $K = \{1 < \frac{1}{2}\}$) and of the form of those studied in [2], we define an interpretation $\mathcal{I}_{\mathbf{K}}$ of L as follows.

The universe of $\mathcal{I}_{\mathbf{K}}$ is the set $U' = U \times \{0, 1\}$.

The partition of the set U' has as classes the subsets of the form $\{(u, 0), (u, 1)\}$, for each $u \in U$.

For any n-ary predicate letter R in L, a relation $R_{\mathbf{K}} \in (U')^n$ given by

$$\begin{aligned} R_{\mathbf{K}} := \; & \big\{((u_1, 0), \ldots, (u_n, 0)) \mid \tfrac{1}{2} \vDash R(u_1, \ldots, u_n)\big\} \cup \\ & \big\{((u_1, \varepsilon_1), \ldots, (u_n, \varepsilon_n)) \mid 1 \vDash R(u_1, \ldots, u_n) \text{ and } \varepsilon_i \in \{0, 1\} \text{ for } i \in \{1, \ldots, n\}\big\}. \end{aligned}$$

We can prove the reciprocal of Proposition 1, that is, the following fact.

Proposition 2. *For every formula* α *and* $k = \frac{1}{2}, 1$ *we have*
(i) if $1 \Vdash \alpha$, *then* $v_{\mathcal{I}_{\mathbf{K}}}(\alpha) = 1$,
(ii) if $\frac{1}{2} \Vdash \alpha$, *then* $\frac{1}{2} \leq v_{\mathcal{I}_{\mathbf{K}}}(\alpha)$.

We finally get the following result.

Theorem. $\Gamma \Vdash \alpha$ *if and only if* $\Gamma \vDash \alpha$.

As in [2] we proved that the first-order logic considered in this talk is sound and complete relative to the two-element Kripke models considered above, it follows that it is also sound and complete relative to the rough sets semantics considered in this talk.

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Quasi-Truth and Approximation to the Truth: A Bilattice-Based Framework for Integrating Semantics and Epistemology

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Keywords: Quasi-Truth, Approximation to Truth, Bilattices, Tarskian Semantics, Paraconsistent Logic.

Tarski's widely known semantic concept of truth [6] and its interpretation in structures and models offer an efficient method for defining truth in formalized languages and avoiding semantic paradoxes. However, this approach fails to capture crucial aspects of scientific activity, which is often subject to errors, failures, biases, and chance, resulting in only provisional success. As a result, the concept of truth, at least in the context of empirical sciences, should be understood as a regulative or approximate ideal, where some theories are only partially true throughout the course of investigation.

This motivation underpins the development of the theory of quasi-truth or pragmatic truth, which is a pragmatic notion of truth accommodating partiality in scientific theories, pioneered by Newton da Costa and collaborators. In the seminal paper by Mikenberg, Da Costa, and Chuaqui [5], the concept of partial structures was introduced to define quasi-truth. Later, Bueno and De Souza [2] simplified this theory based on the notion of quasi-satisfaction and proposed a model for approximating truth, where the underlying logic is classical. Among other recent developments, Coniglio and Silvestrini [3] used the Tarskian notion of satisfaction to propose a three-valued paraconsistent model of quasi-truth.

This work aims to extend the concept of quasi-truth by leveraging bilattices – a mathematical structure introduced by Ginsberg and further developed by Fitting [4]. Bilattices are characterized by their dual ordering systems for truth and information, provide a unified framework to represent both the degrees of truth and the availability of information. Classical and three-valued models lack a unified treatment of truth degrees and information dynamics, which bilattices uniquely provide Our objective is to demonstrate how this duality can refine the modeling of approximate truth, enabling the integration of semantic accuracy with epistemic progress during the scientific investigation process.

Our strategy is twofold: first, we will characterize the theory of quasi-truth using Belnap's four-valued propositional logic [1] based on the method of Coniglio and Silvestrini [3]; second, we will analyze this theory within a distributive bilattice structure – i.e., a bilattice where the distributive equivalences are valid. This structure also incorporates negation and conflation, which refers to the merging or blending of conflicting information to accommodate inconsistencies.

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The underlying bilattice for Belnap's four-valued logic (bilattice $\mathcal{B}_4$), is presented in Figure 1. The four values – undefined (u), false (f), true (t), and overdefined/conflict (c) – directly address incompleteness (modeled by 'u' for lack of information) and inconsistency (captured by 'c' for contradictory evidence), while 't' and 'f' retain classical truth-values under sufficient information. This structure formalizes how scientific theories navigate gaps and conflicts in evolving knowledge while approximating truth.

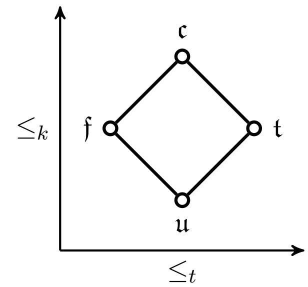

Figure 1: The bilattice $\mathcal{B}_4$ with four values: ($\mathfrak{u}$) undefined, ($\mathfrak{f}$) false, ($\mathfrak{t}$) true, and ($\mathfrak{c}$) overdefined/conflict.

The next step will be to generalize this framework to predicate logic. We will demonstrate that our proposal offers several notable advantages. First, it integrates two key aspects of the theory of quasi-truth: the semantic and epistemic, capturing the intuition that the more information available about the model, the closer it is to the truth. Second, it facilitates the representation of inconsistency and incompleteness in scientific theories, providing a way to understand how the concept of (quasi) truth can function in contexts of scientific fallibilism and contradictions.

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Contradiction in Hegel's Logic and Hegel's Semantics of Negation

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Keywords: contradiction, negation, *ex falso quod libet*

For Hegel, contradiction is the omnipresent fact of reality and of the thinking reflection (*die denkende Reflexion*). He observes the maximum that has been given to the contradiction in his time, for reality or for reflection, was the status of abnormality. In his *Doctrine of Essence*, Hegel is in total opposition with that position: contradiction for him is not abnormality, but negativity in its essencial determination. In the essay, it is discussed the semantical and logical consequences of his position. It is also shown why *ex falso* principle does not apply to contradiction in Hegel's thought.

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Exploring Mathematical Methodology

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Keywords: Mathematical methodology, Curry-Howard, Computational Paths

We present a record of our explorations on the development of some kind of methodology of mathematics – at least the approach to mathematics that's called "theory-building" [4]. We do so via the exploration of a case study, which is transporting a proof across mathematical theories. We start with the observation of a similarity between a well-established result – the Curry-Howard Correspondence (CHC) –, and a new contribution in the form of a particular formal system – [6]'s computational paths. Throughout the process, we introduce several concepts relating to formal systems such as those under the CHC, plus a CHC-like theorem explaining some features of computational paths, and the start of what Christopher Alexander calls a "pattern language", but for mathematics. Finally, we relate the methods that we found with one account of how Grothendieck approached doing mathematics [5].

In our exploration, we chose to engage in what could be called praxis: performing the kind of goal-oriented activity we want to schematize, and then, by recording our actions while doing so, we can later reflect about our choices (which will inform our future actions). Faced with such a description, we analyze it, synthesize our findings, and come up with a first contender for a method. Following this, we may critique, suggest improvements, and test it all again. So, to develop our methods, we must half-paradoxically put them to practice.

We started with the observation of a similarity between two situations. On the one hand, is the traditional, pedestrian Curry-Howard Correspondence between intuitionistic propositional logic and the simply typed λ-calculus (with products, sums, etc). On the other, there's what [6] call "computational paths", which are term-like objects that describe changes require to transform a term into another. Intuitively, they looked not only analogous but actual instances of the same general pattern.

To investigate this matter, keeping in mind the need of maintaining records of it all, we started by writing down the statement and proof in a semi-formalized way (just plain language, but capturing as many details as possible). By performing what we later understood to be a form of presupposition analysis, we came up with a graph representing the structure of the theory, together with dependency relations between its components, the statement and each step of the proof of the theorem.

This graph displayed a remarkable amount of redundancy. This means that concepts can be factored out, and this is in practice a form of compression of the theory. But this is a generalizable pattern: we use our intuition to identify in the current domain of discourse some latent structure; we named it, factored it out, and connect it to the original context via some kind of implementation of that structure. The next crucial observation is that there is a relatively low-effort almost-trivial generalization of the CHC that could be pulled along this loose thread: "given two logics (seen as a form of fibration of types), a morphism between them, and a function between their total spaces of proofs which commutes with the rest of the arrows, we get a form of CHC".

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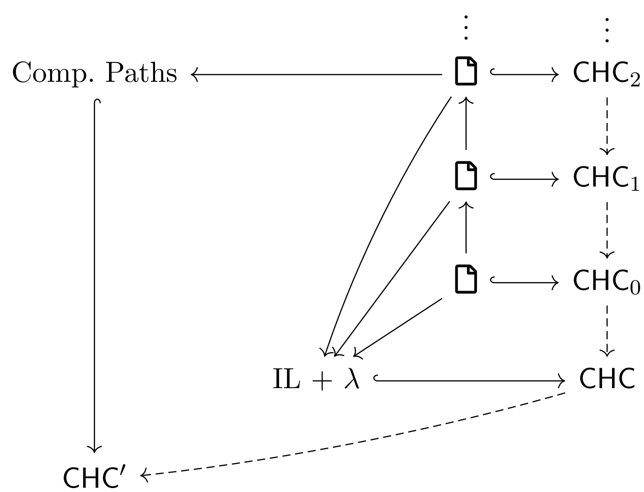
By repeating this process, each time seeking to explain the structures uncovered in the previous step, we engage in a process we call unraveling. And it gives us, besides a methodological guide, a particular form in which to record our results. That is, we go through this process, all the while writing down the generated artifacts in a structured way. We end up with a sequence of theories, each explaining the one below, but still being present in the original situation and even allowing for a new version of the original theorem.

All throughout the process of unraveling, we extract conclusions from each of the steps, such as that the relation between proofs and terms and their types is one of evaluation (maybe call it "splitting"); that weakening corresponds to variable renaming[1]; or that assumption introduction corresponds to variable declaration.

With the unraveling done, we turn to the original observation: the similarity of this situation with [6]'s computational paths. By this point the situation was clear enough. We just chose an appropriate level of abstraction whose structures were easily seen to be present in that other situation. By proving this, we effectively perform a sort of push-forward of the original theorem, thus creating a new theorem in this new context: that computational paths arise from intuitionistic type theory through a form of CHC-like transformation.

The resulting web of theories can be illustrated with the following graph:



We chose to present these methods by using Christopher Alexander's pattern languages [1]. Despite originating in architecture, this concept found ample adoption in the world of software development (cf. [3]), and can even be found in other areas (cf. [2]). In short, this means we can visualize the methods we found as a pattern, which conceivably could become part of a larger language of patterns. Besides, during our exploration we found several new concepts and constructions, not to mention a series of alternative readings of a well-established theorem relating to these structures. Finally, we also proved a theorem reconstructing [6]'s computational paths as an instance of a CHC-like construction.

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UTLFI, a Universal Three-Valued Paraconsistent Logic

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Keywords: conditionals, logics of formal inconsistency, many-valued logics

A well-known problem in the philosophy of logic is the issue of conditionals with false antecedents. In classical logic, when the antecedent is false, the conditional is considered true, but this does not align with how we understand conditionals in natural language. This problem has been analyzed since Peirce, according to Belikov in [2], but it was in the works of Adams in [1] and Stalnaker in [9] that the problem started to be intensely discussed, as their proposals lead to D. Lewis's triviality results (cf. [6] and [7]).

To address this issue, Égré and his collaborators in [4] and [5] propose using a three-valued logic to approximate conditionals with false antecedents to those in natural language. In this context, the conditional with a false antecedent is taken to assume a third value, which, in their terminology, is the undeterminate value.

With a different purpose, Olkhovikov in [8] proposes a three-valued paraconsistent logic called *LImp*. This logic also exhibits characteristics of modal logic and relevant logic. Olkhovikov proves that *LImp* is functionally complete but does not extensively explore the paraconsistent aspect of this logic.

My aim is to show that *LImp* can be expressed in the language of Logics of Formal Inconsistency (LFIs), as in [3]. We name this system **UTLFI** and show that it has a kind of universal property, in the sense that any other three-valued logic can be derived from it. Based on **UTLFI** I intend to analyze whether it would be possible to develop alternative systems to Égré's *et allia* system to address the problem of conditionals.

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Homology & Archetype, Analogy & Prototype, Hypothesis & Truth as Sources of Knowledge

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Keywords: Logical and statistical inference 1, Analogy 2, Homology 3.

A *Structural archetype* is defined in this article as follows, see also Owen (1866, p.146):
(H1) A *Schematic plan*, or a topological map characterizing the structure of a complex system or organism by its layout of organization from constituent parts, or by its pattern of construction from basic components. (H2) Moreover, two distinct organs or organism described by the same or similar structural archetypes are said to be *homologous*. (H3) For example, the following forelimbs of distinct (Mammalia) animals share homologous skeleton structures, although having different functions: Bats' wings, for flying; Whales' flippers, for swimming; Moles' arms, for digging; Human arms, for object manipulation. (H4) Furthermore, in the context of modern theories of biological evolution, homology may be a useful tool for investigating and tracing distinct paths of phylogenetic development diverging from a common ancestor, as it is the case in the last example.

A *Functional prototype* is defined in this article as follows, see also Owen (1854, p.263):
(A1) A *Humanly invented machine*, or a *Proof-of-concept model*, or a *Proof-of-principle device* demonstrating the key functional aspects of a system, that provides an analogy explaining how something else, be it a natural organism or a constructed artifact, actually works or can possibly work in order to achieve its purpose. (A2) Moreover, two distinct organs or organism explained by the same or similar functional prototypes are said to be *analogous*. (A3) For example, humans (Chordata) and octopi (Mollusca) have organs designated by the same name, eyes, for they are used for a similar purpose, namely, vision. The key functional aspects of all these organs can be explained by analogy to a photographic camera, its lenses and focus mechanisms. (A4) Furthermore, in the context of modern theories of biological evolution, analogy may be a useful tool for investigating convergent paths of phylogenetic development, as it is the case in the last example.

Richard Owen (1804-1892) developed the concepts of homology and analogy and used them in his studies of comparative anatomy of animals. After Charles Darwin and Alfred Russel Wallace published their versions of the theory of biological evolution, in 1859, homology and analogy became important tools for tracing divergence and convergence relations in phylogenetic evolution. Karl von Frisch and Konrad Zacharias Lorenz shared a Nobel prize (1973) for their discoveries concerning the organization and elicitation of individual and social behavioral patterns. In their work, homology and analogy are fundamental tools of investigation, see Lorenz (1935, 1974, 1978) and Frisch (1954, 1974). Nevertheless, in spite of (or because of) the ever expanding use of these conceptual tools, their use has been plagued by pernicious and persistent logical and statistical fallacies.

According to Boyden (1943): *Owen distinguished two chief kinds of resemblance in corresponding organs or parts of the bodies of different animals: (1) essential* structural agreements *relating particularly to relative position and connections; (2)* similarities in the function *or use to the organism. These are really different qualities and they have* no necessary dependency *upon each other.*

In spite of Boyden's clear warnings, logical fallacies are often engendered by introducing spurious logical dependencies between the concepts of homology, analogy, divergent evolution from a

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common ancestor, and convergent evolution to a common objective, see Hall (1994). For example: (F1) Although homology is often used to test for common ancestry, some authors include common ancestry in the definition of homology, a classical fallacy engendered by inverting an implication; or (F2) Although, in some situations, homology and analogy offer valid alternative or complementary explanations for issues of interest, some authors define homology and analogy as mutually exclusive, a classical fallacy engendered by having the full relational possibilities contemplated by an hexagon of oppositions reduced to a square, see Stern et al. (2018, 2024, 2025) for related comments.

Furthermore, homology refers to similarities in structure often described by discrete coincidences between interconnection diagrams, as in example H3, while, in contrast, analogy refers to similarities in function often described by continuous mathematical models, as in example A3. This situation leads to statistical fallacies that are even more prevalent than their logical counterparts, for example: (F3) Some authors use discrete statistical models, that are appropriate to test homology, in contexts where analogy could best be tested using continuous statistical models, see Haldane (1954) and Stern et al. (2014, 2017, 2020, 2025). The study of the aforementioned logical and statistical fallacies and appropriate ways and methods to correct them is the main goal of this article.

Finally, this article extends the discussion of aforementioned topics to the field of Ethology, by investigating how homology and analogy may be used to characterize the (phylogenetically) *inherited symbolic* (i.e. analogical) (communication structured as a) *language* of the honeybees.

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Linear (Multi) Algebra I: Linear Systems, Matrices and Vector Spaces over Superfields

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Keywords: multioperations, superrings, Linear Algebra

Motivated by recent advances in abstract theories of quadratic forms, this work embarks on a discussion of matrices, linear systems, and vector spaces over superfields. The aim is to produce an expansion of Linear Algebra into the realm of multivalued structures. Specifically, we introduce and analyze matrices and determinants within the framework of commutative superrings. Additionally, we investigate linear systems and vector spaces over superfields, providing definitions that align with the contextual requirements of abstract theories of quadratic forms and broader semantic studies over multialgebras, as in [3]. We finish with an application of our theoretical developments, establishing an isotropy interpolation principle applicable to both algebraic and abstract theories of quadratic forms.

A k-ary multioperation on a set A is just a function $A^k \to P(A) \setminus \{\emptyset\}$. The data of k-ary multioperation on A is equivalent to a $k+1$-ary relation on A, satisfying a convenient $\forall\exists$ axiom. Therefore, a multialgebraic structure is just a certain kind of first-order structure.

The concept of a multialgebraic structure – an algebraic-like entity endowed with multiple-valued operations – has been under investigation since the 1930s. Notably, in the 1950s, Krasner introduced the notion of hyperrings, which are essentially rings with a multivalued addition. Since the middle of the 2000s decade, the notion of multiring, as discussed in Marshall's work [6], has obtained more attention: a multiring is a lax hyperring, satisfying a weak distributive law, but hyperfields and multifields coincide. Additionally, superrings, as recentlty considered by Ameri et al. [1], are characterized by both their multivalued operations of sum and product. Extensive algebraic inquiries into multialgebras have been conducted, as evidenced by studies such as those by Golzio [4] and Pelea [7].

Multirings have been studied for applications in many areas: in abstract quadratic forms theory, tropical geometry, algebraic geometry , valuation theory, Hopf algebras, etc. A more detailed account of variants of the concept of polynomials over hyperrings is even more recent: see [5], [1], [2], and [8].

There are numerous significant distinctions among rings, hyper/multirings, and superrings. However, the analogical extensions of concepts from the algebraic realm to the multi-algebraic

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domain yield surprisingly profound implications in other theoretical frameworks. For instance, the utilization of polynomials over superfields and their evaluation, coupled with an appropriate semantics, yields a quantifier elimination procedure as demonstrated in [8]. Additionally, Marshall's quotient over a suitable analogue of quadratic extensions leads to the derivation of the Arason-Pfister Hauptsatz for special groups, as elucidated in [9].

The present work embarks on a discussion of matrices, linear systems, and vector spaces over superfields. The aim is to produce an expansion of Linear Algebra into the realm of multivalued structures. We have achieved our aim with a considerable degree of success: the development of the theory proceeds relatively smoothly. Specifically, we have established that matrices $M_{mn}(R)$ over a superring R form a non-commutative superring if R is full and scalation of linear systems exhibits relatively well-behaved properties. Moreover, natural examples such as F^n, polynomials, matrices, extensions, etc., are readily available for (multi) vector spaces over superfields. Additionally, for hyperfields, we have obtained the:

Theorem. *Let F be a hyperfield and V be a finitely generated (multi) F-vector. If V is full then V has a basis.*

With two conditions over the ground superfield F, linearly closeness (every homogeneous linear system with more variable than equations has a non-trivial solution), and the full vector space with rigid basis (essentially, $\lambda_1 v_1 + ... + \lambda_n v_n$ is a singleton), we have obtained the following important Theorems:

Theorem. *Let F be a linearly closed superfield and V be a full and finitely generated (multi) F-vector space with $V = \langle v_1, ..., v_n \rangle$ $(v_1, ..., v_n \in V)$. If V is rigidly generated by $\{v_1, ..., v_n\}$ then every linear independent subset of V has at most n elements.*

Theorem. *Let F be a linearly closed superfield and V be a (multi) F-vector space. If B_1 and B_2 are rigid basis of V then $|B_1| = |B_2|$.*

The two preceding theorems suggest that the generalization of linear algebra methods for superfields will proceed smoothly if we confine ourselves to full (multi) vector spaces over a linearly closed superfield F. Indeed, this abstract framework are effective when applied to multi-structures arising from abstract theories of quadratic forms. We have established the following two general structural theorems. The first one offers many examples of linearly closed superfields, beginning with a hyperbolic hyperfield (i.e., a hyperfield F where $1 - 1 = F$):

Theorem. *Every hyperbolic hyperfield is a linearly closed superfield.*

And the second providing an Example where we can calculate the dimension of a (multi)-vector space in this general setting:

Theorem. *Let F be a linearly closed superfield and $p \in F[X]$ be an irreducible polynomial with* $\deg p = n + 1$. *Then $F(p)$ is also linearly closed full (multi) F-vector space and* $\dim(F(p)) = n + 1$.

Surprisingly, we have obtained a nice application in the context of classical algebraic theory of quadratic forms over fields:

Theorem (Isotropy Interpolation)**.** *Let $K = M(F) := F/_m(F^2 \setminus \{0\})$ for a field F (of characteristic not 2) or $K = G \sqcup \{0\}$, for a formally real special group G. Consider a matrix $A \in M_{n\times m}(K)$, saying $A = (a_{ij})$. If $m > n$, there exists $d_1, ..., d_n \in F$, not all zero, such that all the forms $\{\varphi_1, ..., \varphi_n\}$ with*

$$\varphi_i := \langle a_{i1}d_1, a_{i2}d_2, ..., a_{im}d_m \rangle$$

are isotropic.

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Syllogistic Diagrams and Existential Import: A Study in the History of Traditional Logic

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Keywords: diagrams, syllogistic, existential import, ontological assumptions, traditional logic

Logical diagrams were widely used to represent syllogistic inferences throughout the centuries. The purpose of this talk is to give an account of how the logical diagrams historically used for diagrammatically rendering syllogistic inferences might have affected the historical evolution of the debate on existential import, empty terms, and their use in syllogistic. We propose to view the evolution of syllogistic diagrams and the evolution of answers to the problem of existential import in parallel, and consequently draw lines of possible mutual influences between these two areas.

The problem of empty terms and existential import (i.e., broadly speaking, the question of whether the denotations of syllogistic terms need to exist for the syllogisms to be logically valid) was debated throughout history, although with different levels of engagement. It was not yet stated by Aristotle, and was first explicitly addressed in the Arabic world, starting with Al-Farabi. In Europe, it was not debated in detail until the rise of nominalistic philosophy, to be thereafter resurrected by Leibniz and receive a full attention only after the development of the Boolean algebra and modern formal systems (for a more elaborated account see [10]).

To better understand these sudden historical shifts in attention, we propose to investigate the parallel evolution of syllogistic diagrams. We note that:

1. In the Late Antiquity and, most notably, in the Byzantine Empire, a sophisticated system of diagrams was gradually developed [1]. These diagrams were constructed so as to allow for rendering both all the moods from respective syllogistic figures as well as the reductions of third- and second-figure moods to first-figure ones (see [9]). However, they did not allow for indicating whether or not the terms are (not)empty.
2. In the Arabic world, Al-Barakāt has proposed a system of linear logical diagrams. Although at first he allowed for empty terms, his diagrammatic method had no way of representing them, and he later renounced them altogether [5].
3. Leibniz himself and, following him, Lambert [6], have created systems of linear diagrams for syllogistic as well. These diagrams were developed so as to explicitly display the distribution of terms in statements [3], and, in case of Lambert, treated a single dot as indicating an existing individual term [4, p. 31].
4. Leibniz also developed a system of spacial diagrams for representing syllogistic moods, similar to the later system of Euler circles, which allowed for representing the set of individuals constituting the extension of the term. Euler, later, introduced an asterisk to explicitly indicate that a part of diagram is non-empty [4, p. 33].

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5. Venn diagrams allowed for indicating the emptiness of a term by shading its relevant area [4, p. 37]. Later, Peirce added a method for explicitly indicating the existence of a term by placing an "x" in the area of its extension [2] (see also [8]).

Building on this historical background, we discuss several mutual influences that can be observed. First, as the existence of terms in syllogistic was not explicitly addressed by Aristotle, it was not explicitly addressed in the Greek diagrams either, which further prompted their neglect in Europe. Second, in the Arabic world, where the issue was discussed explicitly, it was also discussed with respect to diagrams and prompted the creation of diagrams which are able to note solely the existing subjects. Third, in the West, diagrams which allowed for explicitly marking the existence of term-subjects were not developed until the time of Leibniz, who has himself discussed the problem of existential import and hence was aware of it. Moreover, the linear diagrams of Leibniz and Lambert also seem to have no way of indicating the emptiness of terms, which, in case of Leibniz, aligns with his intensional interpretation of terms and his view on existential import as presented in his *Difficultates quaedam logicae* [7, p. 115–121]. Lastly, the Venn and Venn-Peirce diagrams, devised to map Boole's algebraical logic and thus allowing to explicitly mark the existence of terms, were further applied to syllogistic, making the issue of existential import in syllogistic more explicit and provoking further discussions on the topic.

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Non-Alethic Logic: An Ecumenical System or a Fragmented Approach to Negation?

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Keywords: Non-Classical Logic, Negation, Paraconsistency, Paracompleteness

The concept of negation has been central to logic, yet remains subject to ongoing philosophical and technical debates. Newton da Costa's Non-Alethic Logic ($\mathscr{N}_n$) introduces a hierarchy of logical systems capable of combining classical, paraconsistent, and paracomplete negations [1]. The non-alethic approach aims to encompass the behaviors of these negations in a unified system. This study critically examines whether the $\mathscr{N}_n$ systems can truly manage such diversity in negations, particularly when different types of negations interact within the same logical context.

Da Costa's work on paraconsistent and paracomplete logics has established key methodologies to tolerate contradictions without trivialization [2], and to allow simultaneous falsity of a formula and its negation without collapsing into incoherence [3, 4]. The $\mathscr{N}_1$ system extends this framework, introducing operators for "paraconsistent well-behavior" (α°) and "paracomplete well-behavior" ($\alpha^{\bullet}$), enabling a nuanced treatment of negations. By doing so, $\mathscr{N}_1$ claims to preserve theorems of classical logic ($\mathcal{CPL}$) while accommodating both paraconsistency ($\mathscr{C}_1$) and paracompleteness ($\mathscr{P}_1$).

This paper analyzes two key challenges within $\mathscr{N}_1$: (1) whether formulas with iterated negations can adequately represent interactions among classical, paraconsistent, and paracomplete contexts; and (2) whether $\mathscr{N}_1$ aligns with theoretical insights from the Theory of Oppositions, such as the Aristotelian Square [5]. For instance, cases of iterated negations like $\neg_p\neg_q\alpha$ raise questions about whether $\mathscr{N}_1$ can consistently handle such combinations without collapsing into either classicality or incoherence.

Key results demonstrate that while $\mathscr{N}_1$ provides a robust framework for isolated contexts of paraconsistency or paracompleteness, it struggles with mixed contexts involving multiple negation types. Iterations of negations reveal limitations: formulas such as $\neg_c\neg_p\neg_q\neg_c\alpha \to \alpha$ lead to contradictions when analyzed under $\mathscr{N}_1$, challenging its ecumenical aspirations. Additionally, comparisons with the Theory of Oppositions suggest that the behavior of negations in $\mathscr{N}_1$ may not fully align with classical interpretations of contrariety and subcontrariety [6].

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Sobre Extremidades de Grau $\aleph_1$

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Palavras-chave: Conjectura de Halin, Extremidades, Árvores de Aronszajn

Em um grafo infinitos $G = (V, E)$ um **raio** é um caminho infinito em uma direção, cujos subgrafos conexos infinitos são chamados **caudas**. Dizemos que dois raios em um grafo G são **equivalentes** se nenhum conjunto finito de vértices os separa; as classes de equivalência correspondentes dos raios são as **extremidades** de G, definidos em [3]. O conjunto de extremidades de um grafo G é denotado por $\Omega(G)$. O **grau** $\deg(\varepsilon)$ de uma extremidade ε é o supremo dos tamanhos das coleções de raios dois a dois disjuntos em ε. O supremo é sempre atingido (veja, por exemplo, [4]).

Dizemos que um grafo S é uma κ-**estrela de raios** se ele é formado por um raio central R e κ raios vizinhos $\{R_i : i < \kappa\}$, todos disjuntos entre si, e cada R_i envia uma família de infinitos caminhos disjuntos para R, de modo que caminhos de famílias distintas só se encontram em R. Em [5], Halin conjecturou o seguinte:

Conjectura de Halin para $\aleph_1$ - $HC(\aleph_1)$: Toda extremidade de grau $\aleph_1$ admite uma subdivisão de uma $\aleph_1$-estrela de raios.

Em [2], foi exibido um contra-exemplo para $HC(\aleph_1)$ utilizando árvores de Aronszajn especiais. Em [2], também é perguntado se todo contra-exemplo seria uma subdivisão do grafo construído por eles. Em [1], apresentamos uma resposta negativa que é consistente com ZFC para essa pergunta, esse novo contra-exemplo é baseado em árvores de Aronszajn semi-especiais. Nesta apresentação iremos falar sobre a Conjectura de Halin e sobre o novo contra-exemplo exibido em [1].

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Conjecturas de Coberturas por F-limites de Ciclos

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Palavras-chave: coberturas, ciclos, submodelos elementares, grafos

A conjectura da 2-cobertura por ciclos foi proposta na década de 70, e segue em aberto desde então. Aqui, iremos estudar alguns resultados relativos a essa conjectura e também a sua generalização, a conjectura da cobertura fiel por ciclos.

Conjectura 1. *(Conjectura da 2-cobertura por ciclos) Todo grafo sem pontes possui uma 2-cobertura por ciclos, ou seja, uma família de ciclos tal que cada aresta do grafo aparece em exatamente duas arestas do ciclo.*

Definição 1. *Dado G grafo e $p : E(G) \to \mathbb{N}$, uma cobertura fiel por ciclos (faithful cycle cover) de (G, p) é uma família de ciclos tal que dado $e \in E(G)$, e aparece em exatamente $p(e)$ ciclos da cobertura.*

Definição 2. *Um mapa p é dito admissível se satisfaz:*

1. *$p(F) = \sum_{f \in F} p(f)$ é par para todo F corte finito.*
2. *$p(e) \leq p(F)/2$ para todo F corte e para toda aresta $e \in F$.*

O mapa p é par se $p(e)$ é par para todo $e \in E(G)$.

Conjectura 2. *(Conjectura da cobertura fiel por ciclos) Seja G um grafo finito e p um mapa par admissível. Então (G, p) admite cobertura fiel por ciclos.*

Neste trabalho, iremos trabalhar as conjecturas restritas a grafos infinitos. Em [1], Bruhn, Diestel e Stein mostram que, se a conjectura vale para grafos finitos, então vale para grafos localmente finitos. Em [2], Laviolette mostra que, se a conjectura vale para grafos localmente finitos, então vale para grafos enumeráveis. Em [3], Soukup mostra que, se a conjectura vale para grafos enumeráveis, então vale para grafos não enumeráveis. Dessa forma, parece imediato que, se a conjectura vale para grafos finitos, então vale para todos os grafos. Entretanto, isso não é verdade, pois os artigos em questão trabalham com diferentes definições de ciclos: enquanto no segundo artigo só são considerados ciclos finitos, nos outros dois ciclos infinitos são permitidos - e, no caso do primeiro artigo, são necessários para que o resultado valha.

Aqui, trabalharemos com uma nova conjectura, que busca resolver esse problema:

Conjectura 3. *Seja G um grafo finito e p um mapa par admissível. Então (G, p) admite cobertura fiel por F-limites de ciclos.*

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Onde F-limites são estruturas que podem ser aproximadas por sequências de ciclos finitos. Nosso objetivo é mostrar que, para essa conjectura, é de fato verdade que o caso finito implica o caso geral. Aqui, focaremos principalmente na última implicação - ou seja, que o caso enumerável da conjectura implica o caso não enumerável. O trabalho aqui apresentado é baseado no que foi escrito pelos autores em [4].

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Sobre a Proposta de uma Filosofia Científica e a Função dos Métodos Lógicos em Filosofia a partir de Russell e Newton da Costa

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Palavras-chave: Russell, Da Costa, filosofia científica

Em textos como *Ensaio sobre os fundamentos da lógica* [1], Da Costa parece defender certa concepção de "filosofia científica". Essa concepção de filosofia – que se distanciaria da "filosofia especulativa" sem excluir a possibilidade desta – parece se basear, segundo o próprio Da Costa [1, 25] nos seguintes "métodos principais": "1) a análise semiótica; 2) o recurso às ciências especiais; 3) a exemplificação histórica; 4) elaboração de modelos hipotéticos" [1, 25].

Essa concepção de filosofia parece possuir muitas semelhanças com certa postura que Russell (Bertrand Russell) advoga a partir de *On scientific method in philosophy* [2]: tanto Russell quanto Da Costa parecem entender que o foco da filosofia científica é certa forma de conceber a teoria ou filosofia da ciência. Além disso, ambos os referidos autores (Russell e Da Costa) aparentam entender que o uso de técnicas formais (como os desenvolvimentos mais recentes da lógica ou das lógicas) na constituição da "filosofia científica" é de grande importância.

Sendo assim, o objetivo da presente apresentação – assim como da pesquisa atrelada a ela – consistirá em comparar as propostas (de filosofia científica) de Russell e Da Costa. Essa comparação visará tanto entender as semelhanças e diferenças gerais entre as duas propostas quanto, mais especificamente, analisar o papel das técnicas formais na proposta de cada um dos pensadores analisados; a finalidade dessa comparação consistirá, sobretudo, em verificar a possibilidade de as duas propostas comparadas contribuírem mutuamente no que tange a uma efetivação, cada vez mais rigorosa, de uma filosofia científica em constante progresso.

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Lógica Modal y Complejos Simpliciales

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La lógica modal es un sistema formal que a diferencia del predicativo clásico, estudia expresiones modales de posibilidad y necesidad. Particularmente, en el presente trabajo se dará un enfoque semántico a la lógica epistémica y la lógica espacial por medio de complejos simpliciales, que fue estudiado en [4] y [5].

La lógica epistémica se obtiene al agregar más operadores modales, que dependerán del conocimiento de un conjunto que llamaremos agentes. La lógica epistémica tiene distintas aplicaciones que van desde la filosofía, inteligencia artificial, economía, lingüística, etcétera.

Mientras tanto, la lógica espacial es un sistema formal que es interpretado sobre una clase de estructuras y relaciones geométricas. La lógica espacial tiene aplicaciones en el procesamiento de imágenes, física, medicina, entre otras más.

Por otro lado, un complejo simplicial es un objeto que puede describirse de forma puramente combinatoria, el cual puede caracterizar algebraicamente las propiedades decisivas de ciertos espacios topológicos llamados triangulares.

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On the Observational Equivalence of OPNs and P/T Petri Nets

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Keywords: object systems, petri nets, transition systems

[1] introduces a computational semantics for molecular biology, assigning a specific activity to each biomolecule based on its interactions within an environment. This approach distinguishes activity from biological function, viewing molecular actions (e.g., helicase unwinding DNA) as computational processes. The paper employs object Petri nets [2–5] to formally describe these activities, emphasizing an intensional, non-adhoc semantics grounded in fundamental nucleotide-based interactions. This framework suggests that any biomolecular activity can be expressed as a composition of basic nucleotide-level operations. Furthermore, [6] highlights that the predicted results of biomolecular activity models depend on the chosen level of granularity.

The approach briefly described above provides both a path and motivation to simulate complex biomolecular activities from the behavior of lower-level biological processes. It is worth noting that the activity is formalized on top of the formalism of Object Petri Nets. The implementation and formal analysis of computational behavior of transition systems was proposed to be understood at the OPN's, allowing a modular approach for the behaviour of the system at different abstraction levels. However, the usage of OPNs could remove the vast analytical tooling of vanilla P/T nets [7]. In this work, we propose a way to bridge the gap and transform OPNs into pure Place Transitions (P/T) Petri Nets. We start by exploring the definitions of a particular type of OPN for both structure and dynamics, then we proceed in connecting those definitions with the concept of transition systems. With the connection made explicit, we can build an equivalent vanilla P/T net such that the observed transition system is equivalent in a way that preserves the semantics for the general idea for a Petri net. In this way, we prove the observational equivalence between OPNs of a particular variety and vanilla PNs, such that we achieve the best of both worlds, having the modularity and abstraction of OPNs as well as the rich analytical tools of P/T nets. The main contribution of this work is the proof of this observational equivalence.

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Projeto de Extensão: Lógica, Argumentação e Raciocínio

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Palavras-chave: lógica proposicional clássica, educação básica, extensão universitária

O projeto de extensão Lógica, Argumentação e Raciocínio tem como objetivo elaborar materiais e ações educativas voltadas a uma formação básica em lógica formal, argumentação e construção de raciocínios para alunos do ensino básico público das comunidades de Salvador e Região Metropolitana. Na literatura existente sobre atividades pedagógicas voltadas para o ensino da matemática, constata-se uma escassez de atividades explicitamente focadas no ensino de lógica. Em [1, 2] jogos são explorados como ferramenta pedagógica para o ensino de matemática. Com base nisso, o uso de jogos foi incorporado às propostas do projeto, devido às potencialidades pedagógicas que eles oferecem: o estímulo à curiosidade, a exigência de participação ativa dos estudantes e a exemplificação prática do uso de conceitos teóricos. De acordo com [2], é fundamental que os alunos, além de participarem dos jogos, consigam alcançar uma reflexão mais profunda sobre as estratégias utilizadas e os conceitos envolvidos. Quando os jogos são mal utilizados no contexto educacional, há o risco de que eles sejam tratados de forma superficial, assumindo um caráter puramente recreativo ou aleatório. Nesse cenário, os alunos jogam e se sentem motivados apenas pela dinâmica lúdica, sem compreender os propósitos educacionais que a atividade busca desenvolver. Resolver um jogo por si só não garante, necessariamente, a habilidade de generalizar os métodos para situações semelhantes. Os jogos, embora úteis como ferramentas didáticas, não devem ser vistos como substitutos para o ensino formal da teoria. Sem uma mediação adequada, as resoluções podem se limitar a uma aplicação mecânica de regras, reforçando apenas a memorização de procedimentos, em vez de estimular a compreensão crítica. Portanto, para que os jogos cumpram seu papel pedagógico, é essencial integrá-los a um processo reflexivo que inclua discussões e análises, permitindo aos alunos explorar as conexões entre os desafios propostos e os conceitos teóricos.

Na primeira fase do projeto, foram realizadas a seleção, adaptação e criação de jogos e desafios lógicos, buscando garantir que as atividades escolhidas pudessem ser resolvidas utilizando um raciocínio inteiramente baseado em lógica proposicional clássica. Também buscou-se assegurar que fosse possível descrever detalhadamente o passo a passo das soluções e explicitar as regras lógicas usadas em cada etapa. Além disso, foram conduzidos estudos de caso com diferentes números de participantes e variadas quantidades de incógnitas. No primeiro ano do projeto, foram trabalhados três jogos. Dois deles foram adaptados de jogos disponíveis na internet. O primeiro, batizado de "Jogo das Senhas", é uma adaptação do jogo *Bulls and Cows* (veja [4]) e consiste em descobrir uma senha de três ou quatro dígitos com base em algumas dicas parciais. O segundo jogo, denominado "Jogo de Einstein", é um desafio lógico em que o jogador deve usar pistas fornecidas para associar, de forma única, diversas características - como cor da casa e animal preferido - a um conjunto de pessoas ou itens. O jogo possui diferentes versões e níveis de dificuldade e está disponível em [5]. O terceiro jogo, desenvolvido no projeto e intitulado "Jogo das Princesas", foi baseado em um desafio

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lógico que se encontra em [3]. Neste jogo, os participantes são vendados e recebem aleatoriamente um chapéu vermelho ou verde, retirado de um conjunto previamente conhecido por todos. Um por vez, cada participante retira a venda e tenta deduzir a cor de seu próprio chapéu com base nos chapéus dos outros participantes e nas respostas anteriores. Após dar a sua resposta, o participante se retira e o jogo continua, com as respostas passadas sendo utilizadas como informações para os próximos jogadores.

Em 2024, o projeto de extensão foi apresentado para a gestão e os professores do Colégio Estadual *Thales de Azevedo*, em Salvador. Com a adesão da escola ao projeto, foi criada uma disciplina eletiva chamada "Raciocínio Lógico", na qual os alunos do 3º ano do Ensino Médio puderam se inscrever. Buscando atender ao que foi destacado em [2], as aplicações dos jogos foram intercaladas com outras atividades ao longo do curso. Após cada jogo, os estudantes eram convidados a detalhar por escrito alguns passos dos raciocínios utilizados, com o intuito de desenvolver uma maior consciência sobre seu próprio processo de pensamento e aprimorar sua capacidade de análise. Pois, de acordo com [1], a escrita coloca o educando no centro da aprendizagem, sendo um elemento facilitador na assimilação da linguagem. Os discentes também eram convidados a ir ao quadro e explicar alguma etapa de sua resolução para a turma. A intenção era que, ao se envolverem ativamente, os alunos contribuíssem não apenas para o seu próprio aprendizado, mas também para o desenvolvimento dos colegas, promovendo uma dinâmica colaborativa e enriquecedora dentro da sala de aula.

Após a aplicação dos jogos, iniciou-se uma etapa de formalização de lógica proposicional clássica, através de aulas expositivas dialogadas. Foram introduzidos os conceitos teóricos básicos, como conectivos lógicos, tabelas-verdade, tautologia, contradição, fórmulas contingentes, além de alguns métodos de demonstração, como prova por casos e prova por absurdo. Essa etapa foi desenvolvida ao longo de várias semanas, sempre utilizando exemplos dos jogos previamente aplicados, bem como exemplos relacionados à matemática e ao cotidiano, para tornar os conceitos mais acessíveis e compreensíveis. As aulas teóricas permitiram definir, aprofundar e estruturar os passos dos raciocínios dos jogos de forma progressivamente mais complexa. Nesse processo, enfatizou-se que as ferramentas exploradas, muitas vezes utilizadas de forma intuitiva no dia a dia, podem ser formalizadas e utilizadas para garantir a validade de raciocínios cada vez mais elaborados, permitindo que os alunos realizem seus próprios processos dedutivos de forma rigorosa. Da mesma forma, é possível avaliar e compreender as razões pelas quais um raciocínio pode ser inválido. Ao final do curso, as técnicas aprendidas foram aplicadas na resolução de problemas contextualizados, retirados de concursos públicos, consolidando os conhecimentos adquiridos e demonstrando sua relevância prática para situações concretas. Essa abordagem reforça a importância de todos terem acesso a essas ferramentas, para promover debates fundamentados sobre os mais variados temas de interesse para toda a sociedade.

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Inadequacies of Logic to Address Big Data

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Keywords: Big Data, First-order Logic, Propositional Logic, SAT, Łukasiewicz Logic

Logic and logicians have not been able to address the recent challenges proposed by the Big Data phenomenon. Initially, this might have been discarded as a passing fashion, but the consequential achievements of this field of investigation cannot be discarded.

This work, which may be seen as a position paper, intends to address the inadequacies of Logic**S**, in their several forms, to deal with several problems arising from the need to deal with vast quantities of data. Our exposition covers: efficiency, modelling capacity and explicability. But, we argue, there are unexplored avenues that may be left for logic researchers to explore some important aspects of Big Data modelling.

Efficiency. Simply put, the tools of logic are too complex. First-order logic is undecidable, and something as simple as propositional logic is intractable. There are known tractable fragments of propositional logics [Schaefer, 1978], but modelling real world applications, even when they can be transformed into propositional logics, rarely fits into one of those fragments.

Modelling. Formal logic was developed as a tentative to formalize mathematics. After a bumpy start, first-order logic was consolidated as a tool In which the foundations of mathematics could be formalized, and so set theories could be axiomatized and compared, and now formal proofs may even be automated. But then computer scientist try to apply first order predicates as a model of real world entities and relationships. And for a while it succeeded, by enforcing that the Universe of this course was all contained in a finite database. But this initial View does not transfer to Big Data, which represents fuzzy, contradictory, and mostly inexplicable observations of human behavior. The difference between mathematical concepts and human concepts are that the former is designed to be precise, timeless, immutable, while the latter are anything but. So there is an unbridgeable impedance mismatch between first order predicates and human concepts, a chasm that has been researched and described by recent research in neuroscience [Barrett, 2017].

Explainability. Explainability is a big problem for modern machine learning methods based on neural networks, and logic has been volunteered as the ideal vehicle in which explainable AI should be developed [Darwiche, 2023]. However, the sheer size of models developed with big data makes this approach impractical. A large formula that fits into a 200 MB file, no matter what logic it encodes, is no more satisfying as an explanation as any neural network.

Is there a way out for logic in the context of Big Data? I suggest that yes, logic should be used for what logic is good at: proofs. The idea is to model neural networks, or more generally, any continuous function, using a specific logic. Any continuous function can be densely approximated by piecewise linear functions, and recent work has shown how to represent any such function as a pair of formulas in Łukasiewicz infinitely valued logic [Preto and Finger, 2022b]. The idea is to represent or approximate a neural network as a piecewise linear function, automatically translated

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into a formula and, no matter how big the formula, use logic's proof methods to demonstrate the existence of refutability of some properties of the original neural network [Preto and Finger, 2022a]. For efficiency, approximations of the target logic are being developed [Finger and Preto, 2023] For further technical details referred to [Preto and Finger, 2023, Preto et al., 2023]

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Functional Relations: An Approach through Diagrammatic Reasoning

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Keywords: Diagrammatic reasoning, Basic Graph Logic, Functional relations

We apply the Basic Graph Logic (BGL) in the study of the arithmetical properties of functional relations. BGL, presented in [2], is a diagrammatic system for deriving inclusions between graphs from inclusions between graphs, taken as hypotheses. Graphs provide a natural tool for expressing relations and reasoning about them. Functional relations are relations satisfying the unicity part of the condition for a relation to be a function: each element of the domain may have at most one image through the relation. In this sense, they correspond to partial functions. Therefore, in addition to being interesting in itself, the study of the arithmetic of functional relations can have numerous applications. As evidenced in this work, the use of BGL in the study of functional relations improves our ability to apply diagrammatic reasoning to the demonstration of mathematical results.

The arithmetical properties of functional relations were investigated, at least, by J. Riguet [4], L. Chin and A. Tarski [3], G. Schimdt and T. Ströhlein [5], and R. Bird and O. de Moor [1]. In these works, one can find (1) alternative characterizations of functionality, (2) necessary conditions for a relation to be functional, (3) closure of the class of functional relations under some Boolean and Peircean operations, and (4) generalizations of both the notion of functionality and the corresponding properties. In some cases the sufficiency of the given conditions gone unnoticed, in others stronger conditions could have been obtained by a more detailed examination of the context in which the result was inserted, and, finally, in a specific case, a result stated as false could have been better investigated to undo the mistake. We carry out our research, filling all these gaps and offering complete diagrammatic proofs of all the assertions. In particular, we show how attempting to produce diagrammatic proofs leads us to obtain the mentioned sufficiency proofs, stronger conditions, and a corrected result.

We treat this study as an exercise in applying BGL. To do this, we follow two paths. First, we provide intuitive diagrammatical proofs of the known results on functional relations. Second, we use the facilities provided by BGL to obtain generalizations of some of these results. Moreover, we start investigating the transformation of diagrammatic proofs into equational proofs. Our main example of the improvement that BGL can bring to the study of the calculus of binary relations is to obtain, from a diagrammatic proof, an equational proof that the dual composition of the functional relations is functional. A result that A. Tarski himself thought it was false (see [3], page 365). In the end, we have a comprehensive study of functional relations—which, as far as we know, has no counterpart in the bibliography on relational calculi—and we exemplify how the use of BGL in the study of basic mathematical concepts can be pleasant and fruitful.

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Towards Efficient Fragment of PDL

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Keywords: Dynamic logic, KAT, GKAT, GPDL, Natural Deduction, Complexity

Propositional dynamic logic (PDL) [3] plays an important role in formal specification and reasoning about sequential programs and systems. PDL is essentially a multi-modal logic with one modality $\langle\pi\rangle$ for each program π, which is built upon regular languages, from a set of atomic programs and the operators of sequential composition $(\pi_1;\pi_2)$, iteration (π^*), nondeterministic choice $(\pi_1+\pi_2)$, and the test operator $(\varphi?)$, where φ is a formula of PDL. The latter is used to model assertions about programs and, together with the other operators, allow the encoding of the typical imperative programming constructs **if-then-else** and **while-do** in PDL. Such expressiveness makes PDL a standard language to reason about properties of imperative programs.

As such, PDL has been used to describe and verify properties and the behaviour of sequential programs and systems. Correctness, termination, fairness, liveness and equivalence of programs are among the properties that one usually wants to verify. A Kripke structure is the commonly provided semantics for this logic. It is based on a frame $\mathcal{F} = \langle W, (R_\pi)_{\pi\in\Pi}\rangle$, where W is a non-empty set of possible program states and $(R_\pi)_{\pi\in\Pi}$ is a family of binary relations on W such that $(w, w') \in R_\pi$ if and only if there is a computation of π starting in w and terminating in w'.

Many variants of PDL have been introduced with different purposes, from which two are relevant for this work. One is Deterministic PDL (DPDL), which has the same syntax as PDL, but its semantics is defined via a restriction of relation R_{π_0}, for any atomic program π_0, to a partial function. The other, called Strict Deterministic PDL (SDPDL), adds to the previous restriction the limited use of the operators $+$, ? and * to the context of **if-then-else** and **while-do** programs. It is well-known that the validity problem for PDL and DPDL is EXPTIME-complete, while for SDPDL it is PSPACE-complete [4].

Kleene Algebra with Tests (KAT) was introduced in [7] and it is the algebraic counterpart of PDL. Both programs and tests belong to the same structure, which comes embedded with a set of axioms allowing a quasi-equational way to reason about programs. Recently, Guarded Kleene Algebra with Tests (GKAT) has drawn attention due to its equational theory being decidable in nearly linear time [8]. While in KAT one can construct an arbitrary non-deterministic or iterative program, in GKAT such is not possible. However, it is expressive enough to encode the imperative programming constructs **if-then-else** and **while-do**, standard commands in imperative programming languages, by embedding tests into some operators as guards. This restriction in the language, compared to KAT, together with a proper automata-based semantics constitute the basis to reason about equality of GKAT terms in almost linear time [8]. The motivation behind our work is to provide an efficient algorithm, based on the one proposed for GKAT [8], to verify logic equivalence of programs in a fragment of PDL whose programs are GKAT terms. Such a fragment, that we introduce in this document, is *guarded propositional dynamic logic (GPDL).*

One of the most important benefits of reducing the expressiveness of KAT, and of its logic counterpart PDL, is the reduction of computational complexity in verifying the equivalence of two

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programs. This problem, which is PSPACE-Complete in Kleene algebra with tests, becomes almost linearly decidable when proving the equivalence of two GKAT programs, as reported in [8]. The main result of such work is *the equivalence of GPDL porgrams e and f can be established in almost linear time* $O(n \times \alpha(n))$, where α denotes the inverse Ackermann function and $n = (\mid e \mid + \mid f \mid)$.

The authors propose an algorithm to compute the equivalence of GKAT terms that runs in almost linear time $O(n \times \alpha(n))$. We prove a theorem which establishes that two GKAT terms are equivalent if and only if the two correspondent GPDL programs are logically equivalent, i.e., $|[e]| = |[f]| \Leftrightarrow \ \models \langle e \rangle p \leftrightarrow \langle f \rangle p$. Since GPDL programs are built using the same syntax as GKAT terms, this result allows us to use their algorithm to compute equivalence between GPDL programs. The algorithm consists in converting the expressions in Thompson automatas (which computes in $O(n)$ by Proposition 4.2, from [8]), normalize them (which computes in $O(n)$), and check bisimilarity by an adapted version of Hopcroft-Karp algorithm (which computes in $O(n \times \alpha(n))$ by [9]).

It is important to notice that such automata construction can be used to verify properties of programs on a GPDL model (or on a class of models), similarly as to how tree automata are used in Vardi and Sockmeyer [10] and Emerson and Jutla [2] for satisfiability in μ-calculus, delta-PDL and many modal logics of programs.

This work opens up some possibilities of future work. We would like to implement a program to verify the equivalence of GPDL programs based on the algorithm proposed in [8]. In [1] a Dynamic logic is presented in which the programs are terms of a process algebra. It would be interesting to investigate fragments of these logics with a better computational complexity for establishing program equivalence based on [8]. Finally, it is also our purpose as future work to investigate the normalization of the Natural Deduction system presented in this paper.

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Uma Abordagem Topológica para Mudança de Crenças AGM

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Palavras-chave: mudança de crenças, teoria AGM, teoria abstrata de modelos

A teoria de mudança de crenças AGM é um dos principais paradigmas no estudo de dinâmica epistêmica, com importante influência em áreas como a Inteligência Artificial e a Epistemologia Formal. Apesar de sua popularidade, é bem reconhecido que tal abordagem depende de uma forte idealização das capacidades do agente e da lógica de suporte, sendo aplicável somente a agentes logicamente oniscientes raciocinando sobre uma lógica supraclássica, compacta e contendo os operadores booleanos clássicos.

Para lidar com as restrições impostas pela teoria AGM, permitindo sua aplicação a um conjunto maior de lógicas e sistemas epistêmicos, autores como Hansson e Wassermann [3], Flouris [1], Ribeiro et al. [5], Souza e Wassermann [6], entre outros, estudaram modificações no arcabouço de AGM de forma a estabelecer condições gerais que devem ser satisfeitas por uma lógica para que as operações de mudança AGM sejam definíveis ou propondo operações de mudança de crença que sejam definíveis a uma ampla classe de lógicas.

Tais resultados de definibilidade de operações de mudança epistêmica em lógicas abstratas apontam como as propriedades de uma lógica podem ser alavancadas para construir essas operações. Tais abordagens se baseiam, entretanto, em grande parte, em um arcabouço sintático para construir e estudar tais operações. Abordagens semânticas, como perseguidas por Grove [2], Ribeiro et al. [5] e Souza [7], apontam que, apesar de provadamente diferentes, tais operações propostas na literatura derivam de uma estrutura similar mais geral e que sua definibilidade e construção em lógicas específicas surgem da interação de tal estrutura com características topológicas do espaço de modelos/teorias definidos pela lógica.

Explorando conexões bem conhecidas entre Lógica, Álgebra e Topologia, com base no arcabouço de uma teoria abstrata de modelos tal qual estudada por Lewitzka [4], nosso trabalho explora uma noção geral de contração de crenças, que pode ser conectada a operações bem estudadas na literatura, como contrações racionais da AGM, contrações de encontro parcial e contrações múltiplas. Mostramos que as contrações racionais da AGM podem ser caracterizadas em nossa estrutura e podemos estabelecer conexões com diferentes resultados na literatura.

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On the Evaluation of LLM's Understanding of Logical Validity Through Minimal Pairs

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Keywords: large language models, reasoning benchmark, minimal pairs, logical validity

Linguists use minimal pairs to specify testing correct grammatical forms against incorrect variants. For each correct sentence, an incorrect sentence in which only a single aspect of the original is changed to an incorrect form. Minimal pairs are used to test speakers' intuitions regarding grammar competence. The same general idea can be carried over, with some caveats, to the evaluation of Large Language Models (LLMs) [2, 3, 7], as it is done in the benchmark named "BLiMP" [5].

Take this example from BLiMP:

Correct:	Many girls insulted themselves.
Incorrect:	Many girls insulted herself.

This minimal pair is meant to test the following phenomenon: "the requirement that reflexive pronouns like himself (a.k.a. anaphora) agree with their antecedents in person, number, gender and animacy". We propose an extension of this methodology to apply it to valid forms of reasoning. This is partially motivated by the role in teaching of the opposition between valid forms and invalid arguments. For instance, the deceptive similarity between Modus Ponens and the affirming the consequent fallacy (or Modus Tollens and denying the antecedent).

We can understand a Reasoning Minimal Pair (RMP) as a pair of arguments (either in formal renderings, as in [8], or natural language instances, as in [6]) in which only one differences contrast the two and only one is valid while the other is not. It is not needed that an RMP involves famous analogues of valid forms. That is, we do not need to focus in famous valid forms like Modus Ponens or Modus Tollens and their analogue formal fallacies, for instance.

We here present the core of an extendable benchmark for the evaluation of LLM reasoning capabilities and a methodology for extending it. The idea is to test the understanding of validity (as codified classically, at least initially) that a model has through Reasoning Minimal Pairs. Presented with the pairs, the model will need to select one option and it will be scored accordingly. Its accuracy ranging a series of RMPs will provide the overall score of the model, providing a straightforward comparison between models to help ascertain their basic reasoning capabilities.

The present benchmark intends to measure basic reasoning capabilities of models to help the research community understand what kinds of low level capacities are involved in high level tasks, for instance: summarization or general language understanding (as in [1, § 5.3]). The tasks, initially, should be readily solvable by humans with basic logical training. This approach contrasts with less controlled evaluations, where capacities of different type and level of complexity are involved, such as [4].

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As a way of example, we will present RMPs that tackle the evaluation of the models competence in the usage of classical logical connectives, i.e. negation, conjunction, disjunction, material conditional and quantifiers. We construe a hierarchy of increasing complexity that has as its basic cases instances of introduction and elimination of connectives up until evaluating arguments with equivalent schema to derived rules in natural deduction calculi.

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An Application of Algebraic Geometry Tools to Intuitionistic Logic

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Keywords: Logic, Category Theory, Algebraic Geometry

Makkai has established a few theorems extending Stone Duality to full First-Order logic (FOL) using categorical tools (see, for instance, [2] and [3]). More specifically, for any theory $\mathbb{T}$ in Classical FOL, we can construct its classifying Boolean pretopos $\mathcal{T}$. Models of this theory in any other Boolean pretopos $\mathcal{R}$ will be then given by Boolean pretopos functors $\mathcal{T} \to \mathcal{R}$. In this way, we can prove an adjunction

$$\mathbf{BP}^{*\mathrm{op}} \underset{F}{\overset{G}{\rightleftarrows}} \mathbf{UG} \quad (\top)$$

between the 2-category $\mathbf{BP}^*$ of Boolean pretopoi and natural isomorphisms between them and the 2-category $\mathbf{UG}$ of ultragroupoids. An ultracategory is a category with an additional "ultraproduct structure", the main example being the category $\mathrm{Mod}(\mathbb{T})$ of set valued models of a FOL theory $\mathbb{T}$ and the usual ultraproduct of models. An ultragroupoid is, then, an ultracategory in which all morphisms are invertible. One side of this adjunction is essentially Gödel's Completeness Theorem.

From the Algebraic Geometry side, the most intriguing highlights in Makkai's theorems are the connections established with the descent theorem for Grothendieck Toposes of Joyal and Tierney [1], as it was shown by Zawadowski [5]. This fact was first glimpsed by Pitts [4] when he proved in a sequence of papers that Heyting pretoposes satisfy the interpolation property. Zawadowski was able to show a more general theorem in respect to all pretoposes and make an explicit link between Makkai's Stone Duality, the interpolation property, and the proof of a (lax) descent theorem. It is worth noting that descent theory is at the heart of scheme theory, one of the first milestones of Algebraic Geometry.

However, something is missing in the literature. The Stone Duality was (explicitly) proven for Boolean pretoposes (Classical FOL), Zawadowski's main descent theorem was proven for general pretoposes (Coherent Logic) and Pitt's Interpolation Theorem was proven for Heyting pretoposes (Intuitionistic FOL). Therefore, our aim is to fill the gaps and prove Stone Duality and the Descent Theorem, but now for Heyting pretoposes. In the meantime, we wish to establish the explicit connections with several other pieces of work that somehow orbit this result.

Finally, the proposal of this talk is to present some the basic ingredients of the constructions we mentioned above, as well as give a conceptual map that could possibly illustrate how those ideas provide a link between the most of the category theoretical advances we have had in the last half of a century.

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On Connections between Forcing Relations

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Keywords: Forcing, model theory, Boolean-valued models, sheaf theory

Forcing was first introduced by Paul J. Cohen in his work on the independence of the Continuum Hypothesis, see [1] and [3]. Inspired by the ideas of Cohen, other formulations of forcing appeared using Model Theory [7], Boolean-valued Models [2], and Topos Theory [4]. There is a well-known claim that these three approaches are the *same*, at least at the level of their *mathematical content*[1]. In this talk, we will present two results not found in the literature toward establishing connections between these versions of forcing.

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[1] As Mac Lane and Moerdijk mention in [4] page 283.

Disputas Lógicas e Desacordos Profundos

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Palavras-chave: Disputas Lógicas, Framework Propositions, Desacordos Profundos.

Fogelin, em The Logic of Deep Disagreements (2005), argumenta que os desacordos profundos decorrem de choques entre framework propositions (proposições estruturantes ou certezas fulcrais). Esses desacordos envolvem conflitos sobre as crenças mais fundamentais dos disputantes, refletindo visões de mundo completamente ou quase completamente distintas.Nessas situações, abandonar essas certezas não está em jogo, pois os interlocutores carecem de um solo comum para dialogar racionalmente. Fogelin identifica debates sobre temas como aborto e ações afirmativas como exemplos de desacordos profundos, os quais não podem ser resolvidos por meio da razão (resistência racional) devido á ausência de solo comum.

Mas o que ocorre quando o desacordo se dá no âmbito da lógica? Quando diferentes lógicas entram em conflito, configura-se um desacordo profundo? Acreditamos que sim: algumas disputas lógicas são desacordos profundos, e proposições lógicas funcionam como as framework propositions descritas por Fogelin. Por exemplo, de acordo com Martin (2019), a proposição "nenhuma contradição é verdadeira" se comporta dessa maneira para os lógicos clássicos; entretanto, para dialéticos, não.

Diante do exposto, a presente pesquisa tem como objetivo examinar essas questões a fim de desafiar a tese central de Fogelin, a de resistência racional. Além disso, será defendido que a lógica pode ser um terreno fértil para desacordos profundos, e a investigação dos desacordos nesse campo revela-se, portanto, uma perspectiva promissora.

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Listing All 2-Ary Sheffer Functions for Propositional S5

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Keywords: functional completeness, sheffer functions, S5 modal logic.

We investigate the interplay between two semantics for propositional S5. One of them is specially suitable to (1) [list all modal functions of a given arity], while the other is the framework of a result that gives us (2) [a functional completeness criterion for modal functions]. We define a couple of notions that allow us to bridge these semantics, and to combine the results (1) and (2) to determine list of all 2-ary Sheffer functions.

The semantics that is suitable for (1) is essentially the restriction to the propositional fragment of the semantics presented in [1]. This semantics, along with its tabular representation, is due to [2]. This tabular representation makes it very easy to see that every formula of propositional S5 has an equivalent disjunctive normal formula (DNF, for short).

We then turn to the framework used in the determination of (2). Here we consider the interpretation of S5 formulas as operations on n–dimensional cubes (we will focus on $n \leq 4$); then we define the relation expressed by a formula. Next, we define the notion of polymorphism of a relation, and give a list of relations $R_0 - R_{25}$ whose polymorphisms are maximal clones of modal operations. All these notions and results can be found in [4].

We present an expedite way, already found in [3], to express all DNFs of 2-ary S5 formulas in terms of an octuples of truth-functions. We call these octuples *moody truth-functions*. We then introduce to notion of a *modal profile* that bridges the two semantics we are considering, and we show how to evaluate if a moody truth-function is a polymorphism of a given relation.

Once we have the list of all or binary modal functions and a functional completeness criterion, the task of filtering the functions which are Sheffer is purely mechanical. The size of the lists, however, are prohibitive for human calculation. Therefore, if we want to determine the complete list we must evoke a computer.

We built a program that filtered from the 2^{32} binary modal functions, the 42100768 which are not included in any of the maximal clones of modal operations and are, therefore, funcionally complete by themselves - i.e. *Sheffer functions*.

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Success Conditions of Imperatives and the Semantics of Deontic Logic

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Keywords: deontic logic, paradoxes, imperatives, formal semantics

Deontic logic — the logic of the notions of obligation and permission — is notoriously riddled with puzzles that have long resisted theoretical resolution, as the most well-known proposed system of deontic logic — so-called *Standard Deontic Logic* (SDL) — validates many inferences that seem intuitively unacceptable. Relating to the notion of obligation, perhaps most famous among them are Chisholm's paradox [2], Ross's paradox [3] and Prior's good Samaritan paradox [4].

Despite SDL's longstanding failure to properly account for those and many other puzzles, no consensus on what might be the adequate system of deontic logic has ever been reached. This, it can be argued, is due to the fact that no system so far proposed in the literature has managed to solve SDL's puzzles in an uniform fashion from intuitively appealing principles. In particular, it would be desirable from a system of deontic logic that it be compatible with a formal semantics that has at least as much explanatory appeal as Kripke's possible worlds semantics seems to have in regards to SDL.

In this talk, I propose semantics for deontic logic extending classical propositional logic that, I argue, at the very least gets closer to doing the trick than other proposals. Its is based on the simplified version of situation theory presented in [1], and it takes its cue for interpreting the notion of obligation from pragmatic considerations on the success conditions of imperatives.

Due to time constraints, in this talk I'll exclusively address the obligation-only fragment of the language of deontic logic. Nevertheless, it will be shown that the proposed semantics suffices to bar at least the problematic inferences involved in Ross's paradox and Prior's good Samaritan paradox. Perhaps more interestingly, although Chisholm's set of sentences remains inconsistent according to this interpretation of the language of deontic logic, the proposed semantics provides us with a compelling explanation of why, contrary to unaided intuition, Chisholm's set of sentences ought, indeed, to be regarded as unsatisfiable.

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Large Cardinals Behind Uncountable Generalizations of Deligne Theorem

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Keywords: Deligne theorem, large cardinals, infinitary categorical logic.

One of the most known conjectures in non-elementary model theory is Shelah's conjecture on categoricity transfer theorem in Abstract Elementary Classes, which has some partial answers by R. Grossberg and M. VanDieren (by assuming tameness, see [3]) and a set-theoretical consistency proof by W. Boney (see [1]) and by S. Shelah and S. Vasey (see [6]), by assuming large cardinals axioms.

C. Espíndola claimed to prove this conjecture without assuming any large cardinals axioms but just GCH, by using categorical logic tools (see [2]).

In this talk, we will focus on the way that one of the key results in Espíndola's argument -an uncountable version of the Deligne theorem- is quite related to large cardinals.

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Quantified Bilateral Systems for Arbitrary Terms

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Keywords: arbitrary objects; bilateralism; arbitrary terms; weak rejection; negation

An arbitrary object is an object which is representative of a certain class of individuals, insofar its properties reflect those shared in the class. This distinguishing characteristic of theirs is known as the *Principle of Generic Attribution* (shortly, the *PGA*), and is tantamount to their very concept (with the exception of Leon Horsten's view [4]). Arbitrary objects have recently resurfaced in the philosophical literature, from Leon Horsten's novel theory [4] and Kit Fine's comments on it [7], to applications to diverse areas, such as the philosophy of logic [3] and the philosophy of mathematics [5]. However, as simple and intuitive as their concept seems to be, problems rise when one takes them at face value.

Many counterarguments have been advanced against arbitrary objects by showing how the PGA clashes with classical logic, the most famous of which are Berkeley's [1] anf Leśniewski's [9] problem. As a result, authors who work on the subject usually take arbitrary objects to behave according to some non-classical logic, such as supervaluationism [6] or intuitionistic [11]. However, as [14] shows, this clash between the PGA and classical logic stems from a *unilateral* setting with respect to assertion.

Unilateral views of assertion – such as, famously, Frege's [2] – take rejection to be reducible to assertion. Rejecting A would be nothing more than asserting *it is not the case that* A. Bilateral views [12] [10], however, take the two acts to be on equal footing. More recently, Incurvati and Schlöder in [8] have argued assertion and rejection, in a sense, are not interdefinable nor interchangeable, and offered a propositional natural deduction system reflecting that called *weak rejectivist logic*.

In that context, [14] shows how, when formulated in the weak rejectivist logic, the problems rising from the interaction between the PGA and classical operators – such as the two previously mentioned – are easily dismissed, with the upshot that the fragment of asserted formulas of the system is classical. The result, arguably, shows that a bilateral setting is the right one for reasoning about arbitrary objects, if one wants to retain classicality.

Following that work, in this paper we offer two quantified natural deduction bilateral systems, which may be seen as first-order extensions of the weak rejectivist logic, to work directly with arbitrary objects. Similarly to Fine [6], who shows how his models of arbitrary objects provide a semantics for natural deduction systems such as Gentzen's and Copi's, we show how a different sort of model of arbitrary objects, akin to those offered in [13], provide an adequate semantics for the simplest extension of the weak rejectivist logic to first-order. In that way, it may be seen as a bilateral counterpart to Fine's work. We then offer a different system, which introduces term-forming operators that allows us to directly talk about arbitrary objects. These new terms may be seen as counterparts to ε-terms. The new system lacks quantifiers, as the new terms play their role, and thus act as objectual quantifiers. We then further show how some semantic aspects of Fine's theory of arbitrary objects, such as his concept of dependence, may be more intuitively elaborated in our system, and in a syntactic manner.

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Horn Semantics and Craig Interpolation

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Keywords: Horn Semantics, Craig Interpolation, Filter Pairs

In this work we consider semantics of a logic in a class of first order structures axiomatized by universal Horn sentences, a *Horn class*. We give conditions on such a semantics which ensure that an amalgamation property for the Horn class implies Craig interpolation for the logic. This generalizes the well-known result that for an algebraizable logic the amalgamation property for the associated class of algebras implies Craig interpolation.

Let κ be a regular cardinal. Let Σ be a signature consisting of function symbols and Σ^+ an expansion of Σ by relation symbols. A κ-Horn theory is a theory axiomatized by universal strict basic κ-Horn sentences (sentences of the form $\forall \vec{x}\colon \bigwedge_{i\in I} P_i(\vec{x}) \to P(\vec{x})$, with P_i, P atomic formulas over Σ^+ not equivalent to $\perp$ and $|I| < \kappa$). A κ-Horn class is a class of Σ^+-structures axiomatized by a κ-Horn theory.

Let L be a logic over a signature Σ. Recall that an algebraic semantics for a logic L is a translation from formulas of L to sets of equations over the signature of L (i.e. atomic formulas of the first order language associated to Σ), commuting with substitution, and such that inference in the logic under this translation corresponds exactly to inference in the equational logic in a quasivariety $\mathbf{K}$.

This situation has been abstracted into the notion of *filter pair* in [1], [2], [3]: A filter pair is a functor $G\colon \Sigma\text{-Str} \to \kappa\text{-AlgLat}$ together with a natural transformation to the power set functor $i\colon G \to \wp$, which objectwise preserves infima and κ-directed suprema. In the case of algebraic semantics for the functor one takes $G := Co_{\mathbf{K}}(\mathrm{Fm}) := \{\theta \mid \mathrm{Fm}/\theta \in \mathbf{K}\}$.

For a κ-Horn theory $\mathbb{T}$, we define a lattice of atomic Horn formulas that will replace the congruence lattice from algebraic semantics: For a Σ-structure A let

- $G^{\mathbb{T}}(A) := \{(\theta, S) \mid \theta$ is a Σ-congruence on A and S an interpretation
  . of $\mathfrak{R}$ on A/θ s.t. the resulting Σ^+-structure on A/θ is a $\mathbb{T}$-model$\}$
- We define an order on $G^{\mathbb{T}}(A)$ by declaring $(\theta, S) \leq (\theta', S')$ iff $\theta \subseteq \theta'$ and the induced quotient map $q_{\theta\theta'} : A/\theta \twoheadrightarrow A/\theta'$ is a homomorphism of Σ^+-structures for the interpretations S, S'.

Horn Semantics arises by replacing the congruence lattice with the above lattice of atomic formulas of an expansion Σ^+ of Σ.

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Theorem Let τ be a set of *atomic* Σ^+-formulas with at most one free variable, such that $|\tau| < \kappa$. The collection of maps $i^\tau = (i^\tau_A)_{A\in\Sigma-Str}$, defined by

$$\begin{array}{rcl} i^\tau_A : G^{\mathbb{T}}(A) & \to & (\mathcal{P}(A), \subseteq) \\ (\theta, S) & \mapsto & \{a \in A \mid \forall\varphi(x) \in \tau : \; A/(\theta, S) \vDash \varphi(a)\} \end{array}$$

is a natural transformation and for any $A \in \Sigma - Str$, i^τ_A preserves arbitrary infima and κ-directed suprema. In other words, $(G^{\mathbb{T}}, i^\tau)$ is a κ-filter pair.

Such a filter pair is called *Horn filter pair* and a *κ-Horn Semantics* for a logic L is a Horn filter pair whose image over the formula algebra is the lattice of theories of L. It is an *equivalent Horn Semantics* if the natural transformations are injective.

Some examples that demonstrate the potential of the approach are:

- For $\Sigma^+ = \Sigma$ a Horn semantics is precisely an algebraic semantics, and an equivalent Horn semantics corresponds precisely to an algebraizable logic.
- For $\Sigma^+ = \Sigma \cup \{F\}$ an expansion of the signature with a unary relation symbol one can define an equivalent Horn semantics corresponding to matrix semantics.
- For $\Sigma^+ = \Sigma \cup \{\leq\}$ an expansion of the signature with an inequality symbol, and a Horn theory demanding that this be an order relation, a Horn semantics is precisely an order algebraic semantics, and an equivalent Horn semantics corresponds precisely to an order algebraizable logic in the sense of [4]

We shall say that a class $\mathbf{K}$ of Σ^+-structures has the *atomic amalgamation property* if given $A, B, C \in \mathbf{K}$ and maps $i_B \colon A \to B$, $i_C \colon A \to C$ that preserve and reflect the validity of atomic formulas (atomic embeddings), there exist a Σ^+-structure $D \in \mathbf{K}$ and atomic embeddings $e_B : B \to D$, $e_C : C \to D$ such that $e_B \circ i_B = e_C \circ i_C$.

Using the formalism of filter pairs, we can prove a general Craig Interpolation result:
Theorem Let $(G^{\mathbb{T}}, i^\tau)$ be a Horn semantics for a logic L. Suppose that the filter pair $(G^{\mathbb{T}}, i^\tau)$ has the "theory lifting property". If $\mathbf{K} := \mathrm{Mod}(\mathbb{T})$ has the atomic amalgamation property, then the logic L associated to $(G^{\mathbb{T}}, i^\tau)$ has the Craig entailment property.

The "theory lifting property" is a technical condition, satisfied by every filter pair presenting an equivalent Horn Semantics, but also in other cases.

In the talk we will review the notion of filter pair and explain the above results with examples.

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Frege's Informal Notion of the Deduction Theorem

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Keywords: Begriffsschrift, Inference, Deduction Theorem

From a modern point of view, one of the most perplexing features of Frege's philosophy of logic is his "notorious doctrine" ([1], 132) that it is not possible to infer anything from mere assumptions. Already implicit in the formalization of inference as a relation between judgments (Cf. [2] paragraphs 2, 5, 6), it becomes explicit in his late reflections about logic (Cf. [3] 147, [4] 47/48). It is one of the few points on which even Michael Dummett deviates from his apparent maxim that Frege was always right: (Cf. [9], 59) "Frege's account of inference allows no place for a[n ...] act of supposition. Gentzen later had the highly successful idea of formalizing inference so as to leave a place for the introduction of hypotheses". ([8], 309) As I showed on other occasions, it turns out that Frege was conscious about the possibility, later to be proven by Gentzen ([6], Section V), of rendering the mathematician's proof from assumptions into a proof from axioms in conditional form. (Cf. [7] 5–6, [5] 245-246, [9] Section 2, [11], 117) This sets Frege's notorious doctrine into the context of the deduction theorem ($\Gamma, A \vdash B$ iff $\Gamma \vdash A \implies B$).

The recent literature discusses the hypothesis that Frege's remarks on inference and deductions bear witness of one of the first articulations of the deduction theorem. ([1], 144, [9], section 2) On first sight, Frege simply makes the epistemological distinction between valid and correct arguments. (Cf. [7], 12) He does not negate the possibility to deduce consequences from mere assumptions, such that in all cases in which the premises were true, the conclusion would be true, also. But he reserves the term "inference" for the case in which the premises are actually recognized as true, a crucial demand for his aim to reduce arithmetic to logic. (Cf. [5], 3, [1], 133) He points out that a formal deduction from assumptions does not prove the truth of the conclusion, but merely the truth of a conditional with the assumptions as antecedent and the conclusion as consequent. ([1], 133, [9], 57-58) Thus, he implicitly claims without a proof that to any valid deduction from hypotheses corresponds a conditional proof from axioms. (Cf. [1], 134, [9], section 9) There is, moreover, some circumstantial evidence that Frege was aware of the logical structure of the deduction theorem. The first two axioms of his formal system almost seem to be "reverse-engineered to accommodate the proof" ([1], 137) later given by Herbrand and Bernays. (Cf. [9], 74-78) This led to the development of the calculi of natural deduction which take the rule of introducing implication by a derivation from an assumption as the other side of the coin of modus ponens. ([1], 156, [7], 7-8) Already in Begriffsschrift, (but overlooked by [1], 157) Frege said that modus ponens is a consequence of the definition of implication and that his system is thereby complete in the sense that any other valid deduction rule could be reduced as an antecedent of a conditional to modus ponens. ([2], paragraphs 6, 13) Therefore, modus ponens could be seen as a definition of the conditional, a possibility to be made explicit by Gentzen. (Cf. [6], paragraph II 5.13, [1], 141, [7], 8) Moreover, the status of the deduction theorem in Frege in connection with his corresponding completeness claim remains to be discussed. (Cf. [1], 157) Nevertheless, there remains doubt whether Frege fully appreciated the value of his insight. His comments on formal derivations from assumptions are mainly negative (Cf. [1], 138) leading to the evaluation that the deduction theorem was anathema to him. (Cf. [9], 58)

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In this talk, I want to explore this tension by investigating the logical significance of Frege's distinction between inference and deduction. I will exegetically set out the relevant passages in Frege's writings and systematically work out their significance for the understanding of the deduction theorem.

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Frege's Philosophy of Language: On the Way to Pragmatics?

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Keywords: Gottlob Frege, Paul Grice, semantics, pragmatics, conventional implicatures, presuppositions, colouring, sense, meaning

In the year 2025, it marks the centenary of the demise of the classicist and founder of analytical philosophy, Gottlob Frege. The philosophy of language, mathematical and logical studies, as well as the personal life of the philosopher himself, continue to command considerable interest among scholars. This is evidenced by the substantial number of works published annually. One of the most pertinent themes for researchers of Frege is the question of the presence of pragmatic elements in his philosophy of language. This issue, on one hand, arose from the critique of Michael Dummett's interpretation of Gottlob Frege [12]; on the other hand, it revolves around the neo-Gricean dispute between Lawrence Horn and Christopher Potts concerning the nature of conventional implicatures and presuppositions in linguistics.

Lawrence R. Horn, in his articles ([4], [5], [6]), analyses how Frege anticipated Grice's understanding of conventional implicatures. Frege engaged in the analysis of meanings that do not affect the truth or falsity of a statement, which was later developed into Grice's conception of implicatures. Horn discusses Frege's views on the differences in meaning introduced by words such as *although*, *but*, and *still*, which add layers of meaning without altering the truth value of the sentence. Furthermore, Frege adhered to a strict interpretation of the term presupposition ($Voraussetzung$), limiting its application to proper names. Frege's principal idea was that presuppositions are associated with potential truth-value gaps that arise when proper names fail to refer. This creates situations where a statement cannot be true or false.

Following Horn, Torsten Sander [10] examines how Frege's ideas on presuppositions influenced subsequent philosophical and linguistic discussions, particularly concerning the distinction between presuppositions and conventional implicatures. In another work, Sander [11] begins with a discussion of universal statements in the context of Frege's logic, highlighting how Frege used the concept of side-thoughts ($Nebengedanken$) to explain phenomena that appeared incompatible with classical predicate logic. Furthermore, Sander discusses terms such as colouring (*Färbung*) and illumination (*Beleuchtung*), which Frege used to describe the contribution of linguistic means, such as but (*aber*), to expressed thoughts. In yet another work, Sander [9] starts with a critique of two erroneous interpretations of Frege's theory of colouring. The first interpretation perceives this theory as radical subjectivism or emotivism, while the second views it as a precursor to Grice's theory of conventional implicature.

Thus, Horn and Sander consider the claim of pragmatics in Frege's philosophy of language to be valid because of Frege's interpretation of conceptions such as presuppositions, colouring, and tone. On the other hand, a number of theorists, particularly Stephen Neale [7], Eva Picardi [8], and Joan Weiner [13], have critiqued the position that sees elements of pragmatics in Frege's philosophy of language. Their stance can be characterized as strictly semantic (following Dummett [1]).

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The purpose of this report is to establish a criterion for demarcation between pragmatics and semantics in Gottlob Frege's philosophy of language and to address two key questions:

1) How justified are the arguments for the presence of pragmatics in Frege's philosophy of language?

2) If so, how is the boundary between semantics and pragmatics delineated in Frege's philosophy?

The report will highlight the main arguments for pragmatics put forth by Horn and Sander, as well as the semantic arguments by Neal, Picardi, and Weiner. It will also address the relevance of Frege's interpretation by Michael Dummett. In our position, we will rely on the criteria for semantics and pragmatics proposed directly by their founders, Frege [2] and Grice [3].

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More on the Logic and Ontology of Quantum Potentialities

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Keywords: Contradiction; Contrariety; Paraconsistency; Potentiality; Quantum metaphysical indeterminacy.

'Superposition' is a crucial concept in quantum mechanics, yet its proper understanding remains in dispute. This work compares the Paraconsistent Approach to Quantum Superpositions (PAQS), as developed by Newton da Costa and Christian de Ronde [4, 5], and the Contrariety Approach to Quantum Superpositions (CAQS), as developed by Jonas Arenhart and Décio Krause [1, 2] with recent glutty and gappy accounts of quantum metaphysical indeterminacy [3]. We conclude that both PAQS and CAQS fail to accommodate experimental situations.

A textbook approach to explaining why superpositions are crucial is via the famous double-slit experimental setup. In this experiment, quantum entities might pass through slit A or slit B. The most empirically adequate description of the system inside the experimental setup is that its state is a superposition of having passed through slit A and having passed through slit B. In other words, the system is described as existing in a superposition of different spatial regions delimited by the slits. In Dirac notation:

$$|\psi\rangle = |\psi_A\rangle + |\psi_B\rangle \tag{1}$$

One of the consequences of the standard quantum-mechanical dynamical laws is that this situation should evolve to the superposition of macroscopically distinguishable states, e.g., the state of a measuring apparatus' pointer pointing to the pointer $|A\rangle$ in virtue of having detected the quantum system passing through slit A plus analogously pointing to $|B\rangle$. That's a dire consequence, as we don't experience superposed macroscopic states. They're thus (somewhat) standardly interpreted as tendencies to something, or more specifically as *potentialities* [6].

Let us assume, with Werner Heisenberg and others that superposed states are understood as a representation of the states potentially available for the system, that is, as a kind non actualized physical state, which becomes actual as a measurement takes place. There are different ways to cash this out in logical and ontological terms. Here we're interested in two, *viz.*, the PAQS and the CAQS.

The PAQS is articulated by pointing out that the inference "having passed through slit A" is logically equivalent to "not having passed through slit B". Call them α and $\neg\alpha$ respectively, whenever the system is in a superposition, you will find a contradiction: $\alpha\wedge\neg\alpha$. Quantum mechanics, however, is anything but trivial, so we'd better come up with some reason for that. Paraconsistent logical systems fit perfectly since they stop you from jumping straight from a contradiction to triviality. PAQS then accommodates the situation, arguing that we might live with contradictions in superpositions. And equation (1) would thus imply that the quantum system passed both through

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spatial regions A and B, at least potentially. Insofar as the system possesses both the properties of being located in spacial regions A and B, it would count as a glutty metaphysical indeterminacy.

The CAQS questions just that. Drawing on Aristotle's Square of Opposition, it is argued that quantum superpositions cannot count as contradictory. This is because, according to the square, contradictory negations cannot both be true nor both false. Such a situation would only arise when the system is no longer described by a superposition or by potentialities. Instead, it would be described by actual, unique measurement results, which yield exclusively the states $|\psi_A\rangle$ or $|\psi_B\rangle$. Instead, during superposition situations, quantum-mechanical systems should be cashed out in terms of contrarieties—which, according to the Square—cannot be simultaneously true but can be simultaneously false. As it is false that the system *is* in state $|\psi_A\rangle$ and it is false that the system *is* in state $|\psi_B\rangle$, contrariety, not contradiction, best describes the situation. Insofar as the system possesses neither the properties of being located in spacial regions A nor B, it would count as a gappy metaphysical indeterminacy.

The case against PAQS, we think, is straightforward. Only actual states would count as contradictory, and such are not observed. The glutty account of metaphysical indeterminacy it entails, moreover, has its own conceptual difficulties with regard to multiplication. While such difficulties are grist to the mill for the CAQS approach, the gappy account of metaphysical indeterminacy it entails is also problematic. While it is fair that the quantum-mechanical system is located neither in the spatial region A nor in the spatial region B, it's surely located *somewhere*, *viz.*, in the apparatus, in the lab, in the city, and so on.

By bringing to light such difficulties with each kind of approach, we shall also investigate the connection of the logical approaches with their respective metaphysical descriptions of superpositions: the glutty metaphysical indeterminacy for the PAQS, and the gappy metaphysical indeterminacy for the CAQS (see [3]). This connection was not investigated in the previous versions of the debate between the PAQS and the CAQS, and, by adding such a metaphysical layer to the formal debate, we expect to investigate whether the formalisms offered by the PAQS and the CAQS are up to the task of correctly representing the corresponding metaphysical views associated with paraconsistency and paracompleteness. This procedure will shed further light on the features of each of the approaches.

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Combinatorics of Ideals: Weakly Selective vs Cohen-Indestructible

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Keywords: Borel ideals, Cohen-indestructible ideals, selective ideals

An *ideal* on a set X is a collection of subsets of X that is closed under finite unions and taking subsets of its elements. Ideals are a very useful notion in topology and set theory and have been studied for a long time. For ideals on a countable set X, several questions naturally arise:

- How definable is the ideal in terms of descriptive set theory? For instance, is it F_σ, Borel, analytic, etc., with respect to the topology of the Cantor space 2^X?
- Does the ideal satisfy any combinatorial properties, such as being *selective*, *Ramsey*, or *weakly selective*?
- What happens to the ideal in a generic extension after a forcing iteration?

The *Category Dichotomy* (see [4, Theorem 3.1]) provides a framework for addressing these questions in the context of Cohen forcing. Specifically, the dichotomy states that within the class of Borel ideals, there are no tall ideals that are both Cohen-indestructible and weakly selective. Counterexamples (no Borel) to this dichotomy exist under additional set-theoretic assumptions beyond ZFC. For instance:

- Assuming $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, one can construct selective ultrafilters (see [2, Proposition 13.9]) whose duals are selective Cohen-indestructible ideals.
- Assuming $\mathfrak{a} < \mathrm{cov}(\mathcal{M})$, or $\mathfrak{b} = \mathfrak{c}$ (see [3, Proposition 6]), one can construct Cohen-indestructible MAD families, whose generated ideal are also selective Cohen-indestructible.

The main objective of this talk is to affirmatively answer the following question: *Is there a tall, weakly selective, Cohen-indestructible ideal in ZFC without additional assumptions?*

In particular, we present sufficient conditions on a countable topological space X that ensure the ideal of nowhere dense subsets of X, denoted $\mathrm{nwd}(X)$, is both Cohen-indestructible and weakly selective. Furthermore, building on ideas introduced by A. Dow in [1], we reformulate a topology τ on $\omega^{<\omega}$ that satisfies these conditions. This construction provides the first known counterexample to the Category Dichotomy in ZFC.

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Contractions Based on Optimal Repairs

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Removing unwanted consequences from a knowledge base has been investigated in belief change under the name contraction and is called repair in ontology engineering. Simple repair and contraction approaches based on removing statements from the knowledge base (respectively called belief base contractions and classical repairs) have the disadvantage that they are syntax-dependent and may remove more consequences than necessary. Belief set contractions do not have these problems, but may result in belief sets that have no finite representation if one works with logics that are not fragments of propositional logic. Similarly, optimal repairs, which are syntax-independent and maximize the retained consequences, may not exist. In this paper, we want to leverage advances in characterizing and computing optimal repairs of ontologies based on the description logics EL to obtain contraction operations that combine the advantages of belief set and belief base contractions. The basic idea is to employ, in the partial meet contraction approach, optimal repairs instead of optimal classical repairs as remainders. We introduce this new approach in a very general setting, and prove a characterization theorem that relates the obtained contractions with well-known postulates. Then, we consider several interesting instances, not only in the standard repair/contraction setting where one wants to get rid of a consequence, but also in other settings such as variants of forgetting in propositional and description logic. We also show that classical belief set contraction is an instance of our approach.

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Truth Tables for Ecumenical Logic

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Keywords: Ecumenical systems, RNMatrices, semantics

In logic, the term *ecumenical* denotes systems where multiple logics coexist within a unified framework. One of the most prominent examples is Prawitz's ecumenical logic [7], which harmoniously integrates classical and intuitionistic logic. In this system, classical and intuitionistic logicians share the universal quantifier, conjunction, negation, and the constant for absurdity (neutral connectives: $\forall, \wedge, \neg, \bot$) while maintaining distinct existential quantifiers, disjunctions, and implications (*e.g.* $\rightarrow_c$ and $\rightarrow_i$ for classical and intuitionistic implications resp.), each with its own specific meaning. Prawitz's key contribution was to provide these distinct meanings through a framework that is acceptable to both classical and intuitionistic logicians. While proof-theoretic aspects were also explored, his work primarily focused on the philosophical significance of translating classical logic into intuitionistic logic.

In [5], a Kripke-style sound and complete semantics for Prawitz's ecumenical system was proposed. This semantics builds on intuitionistic Kripke semantics for first-order intuitionistic logic, with the classical operators defined solely in terms of negation and neutral connectives, grounded by logical equivalences. For example, $A \rightarrow_c B \equiv \neg(A \wedge \neg B)$.

More recently, a novel ecumenical approach was introduced [4] that shifts the focus from connectives to *proofs*. By analyzing ecumenism through the lens of *proof-theoretic semantics* [8], the authors clarified the meaning of logical proofs and established proofs as foundational for semantic analysis. This approach highlights fundamental issues, such as the implications of accepting or rejecting *reductio ad absurdum* as a valid method of proof.

In this work, we aim to further investigate the meaning of proofs by providing a semantic perspective on ecumenism using Restricted Non-Deterministic Matrices (RNMatrices) [2]. Non-Deterministic Matrices (Nmatrices [1]) allow for the non-deterministic computation of truth values, while RNMatrices impose a restricted set of valuations. Since finite-valued non-deterministic truth tables, combined with an algorithm to eliminate inadequate rows, constitute a decision procedure, our objective is hence to develop a truth-table-based decision procedure tailored to ecumenical systems.

Our starting point is a common question that arises when presenting Prawitz's system: if there are two implications, one classical and the other intuitionistic, why is there only one negation? This is a fair and important question. The explanation given in, for instance, [6], is far from satisfying and, arguably, philosophically debatable. It claims that the reason lies in the equivalence $A \rightarrow_c \bot \equiv A \rightarrow_i \bot$, meaning that when the target formula is $\bot$, the two implications collapse. However, as discussed earlier, $A \rightarrow_c B \equiv \neg(A \wedge \neg B)$, and this equivalence does not justify the

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assertion that both formulas have the same intended meaning, far from it! This highlights the need for a better explanation or a new perspective on ecumenical negation. In our work, we tackle this issue from a Kripke-semantics perspective, proposing new forms of ecumenical negation alongside an ecumenical notion of monotonicity. Additionally, we introduce a novel ecumenical neutral operator @, which captures the property of "being a classical tautology" or "having a classical proof." Finally, we define the resulting ecumenical logic and propose a related RNmatrix, built upon the RNmatrix for propositional intuitionistic logic [3].

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The Problem of Hallucinations of Chatbots Based in Large Language Models from a Tarskian Perspective

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Keywords: Large Language Models, Hallucinations, Truth, Tarski, Wittgenstein

Based on Tarski's semantic theory of truth and on the Wittgensteinian notion of nonsense (meaning an expression that is not a well-formed formula and, therefore, may not assume any truth-value), we will determine in what sense could a text generated by a chatbot based on a Large Language Model, such as ChatGPT, be considered truth, false, or nonsensical.

Considering that a Large Language Model establishes statistical relations between tokens (that is, words, parts of words, or even characters) present in the textual data in which it is trained or with which it is later fed, in the Tarskian sense, the generated sentences that establish relations that are in fact present in those data would be considered true, while the sentences that establish relations that are not present in those data would be considered false. The production of nonsensical results by such models, by its turn, would occur only if they were capable of, for instance, creating new tokens that do not exist in the training data, or of establishing new relations between tokens not already established during training.

Once established such criteria, we will elucidate the problem of the so-called "hallucinations" (defined as false or nonsensical results generated by Large Language Models, characterized by unfaithfulness to the data and/or by non-factuality), concluding that some kinds of results that are considered hallucinations (for instance, a result that is faithful to the training data, but not to the facts) are in fact true in the Tarskian sense and, therefore, that some types of hallucinations are inherent to Large Language Models.

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The Contradictory God Thesis: A Logical-Conceptual Assessment

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Keywords: Contradictory God, Paraconsistent dialetheic theism, Concept of God, Law of Non-Contradiction

When faced with the charge that a given concept of God is contradictory, the standard move among philosophers and theologians has been to try to explain away the contradiction and show that the concept of God in question is consistent. This has to do, of course, with the Law of Non-Contradiction (LNC). Another option, which has recently generated interest among logicians and analytic philosophers of religion, is to reject such a move as unnecessary and defend what might be called the contradictory God thesis [7] [1] [6] [8] [5] [4] [3] [2] [9].

To be sure, something close to that can be found in philosophers such as Pseudo-Dionysius, Thomas Aquinas and Nicolaus de Cusa. However, it is only recently that this approach has gained momentum, certainly driven by the contemporary advance of dialetheism and glut theoretic approaches in general, and paraconsistent logic. Needless to say, a standard move among defenders of the contradictory God thesis is to challenge the LNC. The argumentation, however, is seldomly framed in conceptual terms. Instead, it is mostly framed in ontological terms, as God being a contradictory entity. From this perspective, the contradictory God thesis is the thesis that God is a contradictory object.

My goal in this paper is to provide a conceptual assessment of the discussion surrounding the contradictory God thesis. To achieve this, I will make use of a general and hopefully non-controversial meta-theory of concepts and adopt a semantic approach rather than a metaphysical one. In addition, I will also pursue the desideratum of operating within a logical-conceptual framework as close as possible to the framework within which the analytic philosophy of religion debate regarding the concept of God takes place, which is broadly in line with what we call classical logic. Against this background, I will address the following questions: What are the different ways we can understand the contradictory God thesis? What grounds are there for rejecting a contradictory concept of God? What standard moves are available to defend oneself from such criticisms and how do they relate to the LNC? What challenges do they present? How do paraconsistency and glut-theoretic approaches stand in relation to them?

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Horizontal Compression of Purely Implicational Proofs

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Keywords: Proof Compression, Purely Implicational Minimal Logic, Directed Acyclic Graphs, Interactive Theorem Proving, Lean Theorem Prover

This work elaborates on the Horizontal Compression Algorithm (HC) and its corresponding set of compression rules. It presents, with examples, the definition of a Dag-Like Derivability Structure (DLDS), a directed acyclic graph representation for proofs and derivations in natural deduction.

It then proves that, for any tree-like natural deduction derivation of minimal purely implicational logic, an analog and compressed derivation, represented by a DLDS, can be obtained via the Moving Upward Edges (MUE) phase of the HC Algorithm. The proof uses the Lean Interactive Theorem Prover. This work also presents an implementation of the HC algorithm done with the Lean Functional Programming Language. Empirical tests showed a compression ratio of almost 88% for this version of the algorithm, with larger and more redundant derivations yielding the best results.

This work is about proof size, representation, and compression in natural deduction for minimal purely implicational logic. It is also a commentary on formal verification with the help of interactive theorem provers. It also argues the properties of Coverage and Soundness for the HC algorithm.

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On Some Notions of Abstract Logic

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Keywords: abstract logic; maximal logics; Post-completeness; structural completeness; structural robustness.

We intend to present some concepts, results and problems in the abstract theory of 'logics', in the 'Polish' sense of 'structural' (or 'substitution-closed') closure relations over a 'propositional' (or 'algebraic') language. The main focus will probably be on the following properties of logics: (i) *maximality* in the lattice of logics over a given language (more precisely, in the subclass excluding the top element, viz. the 'inconsistent' logic); (ii) *Post-completeness* in the sense that the set of valid formulas (i.e. consequences of the empty set) is the one and only substitution-closed consistent theory; (iii) *structural completeness* in the sense that all validity-preserving rules are correct rules of the given logic, or equivalently that if a rule is not correct in the given logic then it has a substitution-instance where all the premisses are valid and the conclusion is not valid; (iv) what we call *structural robustness*, i.e. the property that if a rule is not correct in the given logic then it has a substitution-instance where all the premisses are valid and the conclusion is inconsistent; and (v) a number of '*homogeneity conditions*' modifying in different directions this concept of structural robustness. Various relations between these notions will be presented, as well as some open questions. (Two standard references for material on the concepts (i)–(iii) above are sect. 1.5 of [2] and Ch. 3 of [1].)

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On Categories of o-Minimal Structures

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Keywords: o-minimal structures, first order language, category theory

Our aim in this work is to look at some transfer results in model theory - mainly in the context of o-minimal structures - from the category theory viewpoint. More specifically, at first, we construct a contravariant functor $\mathcal{E}$ from a suitable category of first-order languages to the category of all locally small categories by means of which we can translate, into diagrams of categories, seminal dichotomy theorems for o-minimal structures, namely

Fact 1 (Theorem A, [5]). *Suppose that $\mathcal{R}$ is an o-minimal expansion of an ordered group $(R, <, +)$. Then exactly one of the the following holds: (a) $\mathcal{R}$ is linearly bounded (that is, for each definable function $f\colon R \to R$ there exists a definable endomorphism $\lambda\colon R \to R$ such that $|f(x)| \leq \lambda(x)$ for all sufficiently large positive arguments x); (b) $\mathcal{R}$ defines a binary operation $\cdot$ such that $(R, <, +, \cdot)$ is an ordered real closed field. If $\mathcal{R}$ is linearly bounded, then for every definable $f\colon R \to R$ there exists $c \in R$ and a definable $\lambda \in \{0\} \cup Aut(R, +)$ with* $\lim_{x\to+\infty}[f(x) - \lambda(x)] = c$.

and

Fact 2 (Theorem and Proposition, [4]): *Let $\mathcal{R}$ be an o-minimal expansion of the ordered field of real numbers $(\mathbb{R}, <, +, \cdot, 0, 1)$. If $\mathcal{R}$ is not polynomially bounded, then the exponential function is definable (without parameters) in $\mathcal{R}$. If $\mathcal{R}$ is polynomially bounded, then for every definable function $f\colon \mathbb{R} \to \mathbb{R}$, with f not identically zero for all sufficiently large positive arguments, there exist $c, r \in \mathbb{R}$ with $c \neq 0$ such that $x \mapsto x^r\colon (0, +\infty) \to \mathbb{R}$ is definable in $\mathcal{R}$ and* $\lim_{x\to+\infty} f(x)/x^r = c$.

The dichotomy on o-minimal expansions of ordered groups, asserted in Fact 1, is the analogue of the dichotomy for o-minimal expansions of the real field $\mathbb{R}$ in Fact 2. Facts 1 and 2 can be viewed as implied transfer results of o-minimality property from one structure to another, and they served as our main motivation for this work.

Also, in [1], A. Berarducci and M. Otero point out some transfer results with respect to topological properties from one o-minimal structure to another. Specifically, if $\mathcal{M}$ is an o-minimal expansion of an ordered field and φ is a first order formula in the language of the ordered rings, then the following statements concerning the definable subsets $\varphi^{\mathcal{M}}$ and $\varphi^{\mathbb{R}}$ hold: (1) $\varphi^{\mathcal{M}}$ is definably connected if and only if $\varphi^{\mathbb{R}}$ is connected; (2) $\varphi^{\mathcal{M}}$ is definably compact if and only if $\varphi^{\mathbb{R}}$ is compact; (3) there is a natural isomorphism between the homology groups $H_*^{\mathrm{def}}(\varphi^{\mathcal{M}}) \cong H_*(\varphi^{\mathbb{R}})$; (4) there is a natural isomorphism between the fundamental groups $\pi^{\mathrm{def}}(\varphi^{\mathcal{M}}, x_0) \cong \pi(\varphi^{\mathbb{R}}, x_0)$; and assuming that $\varphi^{\mathbb{R}}$ is compact it follows that (5) if $\varphi^{\mathcal{M}}$ is a definable manifold, then $\varphi^{\mathbb{R}}$ is a (topological) manifold; and (6) if moreover $\varphi^{\mathcal{M}}$ is definably orientable, then $\varphi^{\mathbb{R}}$ is an orientable manifold.

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We then use the Grothendieck construction in order to treat language $\mathcal{L}$ and structure $\mathcal{M}$ as just an object, namely a pair $(\mathcal{L}, \mathcal{M})$, of a larger category. The morphisms in this larger category are pairs (H, α), where H is a morphism of language and α is a morphism of structure. (The concerned dichotomy theorems can also be read in this global context, with α being the identity homomorphism.) Some suggestions of further investigation arise. For instance, we might consider even more general forms of induced functors by changing of languages as in [8] (something in this direction already ocurred in those theorems by C. Miller and S. Starchenko pointed out above). Also, it is natural to ask for examples in the setting of o-minimal structures, where the translation into the global context involves α distinct from the identity homomorphism. A phenomenon like this appears in [7], namely: if $\mathcal{M}$ is any nonstandard model of PA, with $(\mathrm{HF}^{\mathcal{M}}, \in^{\mathcal{M}})$ the corresponding nonstandard hereditary finite sets of $\mathcal{M}$ (by Ackerman coding: the natural numbers of $\mathrm{HF}^{\mathcal{M}}$ are isomorphic to $\mathcal{M}$), then for any consistent computably axiomatized theory T extending ZF in the language of set theory, there is a submodel $\mathcal{N}' \subseteq (\mathrm{HF}^{\mathcal{M}}, \in^{\mathcal{M}})$ such that $\mathcal{N}' \models T$.

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Nonstandard Formulae and Model-Theoretic Paradoxes

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Keywords: inaccessible cardinals, set theory, class theory, nonstandard arithmetic, inner models, model theory.

It is well known in the literature that ZFC+I (ZFC plus the existence of a strongly inaccessible cardinal) proves the consistency of ZFC. This occurs because the κ-th level of the Von Neumann hierarchy V_κ – for a strongly inaccessible cardinal κ – is a model for ZFC. As a consequence, it follows from Gödel's second incompleteness theorem that, if ZFC is consistent, it cannot prove that its own consistency implies the consistency of ZFC+I.

However, there are several gaps – or even inaccuracies – in the proofs of the above results in the literature that can lead us to paradoxes, as incorrect proofs of ZFC's inconsistency. The problem occurs because ZFC is not finitely axiomatizable, and the proofs that V_κ is a model for ZFC is often presented as a theorems' scheme, in which for any axiom φ it is proven that φ holds when relativizated to V_κ. In this way, the quantification yields in the metalanguage, whereas the conclusion about the consistency of ZFC should be stated as a single first-order sentence.

The purpose of this talk is discussing how the literature approaches these results, analyzing the subtlety of working in different language's levels in the process and proposing new and detailed proofs of the following known (but somehow folkloric) results: $V_\kappa \models ZFC$, $V_{\kappa+1} \models KM$ (the Kelley-Morse class theory), for a strongly inaccessible cardinal κ, and $KM \vdash Con(ZFC)$. It follows from these results and Gödel's second incompleteness theorem that the consistency strength of KM is strictly between ZFC and $ZFC + I$. I.e., assuming ZFC is consistent, we cannot prove, using ZFC in metatheory, that $Con(ZFC) \to Con(KM)$ or $Con(KM) \to Con(ZFC + I)$.

Along this abstract, κ always refers to a strongly inaccessible cardinal.

Review of the literature. Some of the most renowned textbooks of Set Theory ([4], [5] and [6]) prove that V_κ is a model of ZFC by proving that each relativization φ^{V_κ} holds, for every axiom φ of ZFC. They all mention that, as a consequence of this fact, $ZFC+I \vdash Con(ZFC)$. However, only [5] points out – with no further details – that this conclusion demands a deeper metamathematical analysis. The problem is that, defining model (or *inner model*, as it frequently appears in the literature) of ZFC in this way is a metalinguistic definition, and the result is presented as a theorems' scheme, since ZFC is not finitely axiomatizable.

A finitary proof that $V_\kappa \models ZFC$ is correctly made in [3], although the author does not deepen in the logical details of the proof.

The main reference to this theorem is [1], where the author proves that $V_{\kappa+1} \models NBG$ (Neumann-Bernays-Gödel class theory). Since it is simple to verify that the elements of $V_{\kappa+1} \setminus V_\kappa$ represent the proper classes, the fact that $V_\kappa \models ZFC$ is seen as an easy consequence of Shepherdson's result. However, he uses a finite axiomatization of NBG, and, hence, considering only the relativizated formulae is correct in this context. On the other hand, the proof that NBG implies ZFC is made again in the metalanguage, as a theorems' scheme (see [7]), which make the conclusion about ZFC compromised.

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The validity of KM in $V_{\kappa+1}$ is mentioned in a review of a Morse's paper for Mathscinet as an "easy" adaptation of Shepherdson's proof. However, again, NBG is finitely axiomatizable, whereas KM is not.

Nonstandard formulae In order to state $V_\kappa \models ZFC$ as a single first-order sentence of ZFC, we need to build a *codified language* (see [2], chapter 9, or [3], chapter 3), where the formulae are finite sequences on a predefined "set of symbols" (which can be taken as ω). However, the domain of some sequences may be *nonstandard* natural numbers (which, by Gödel's completeness and incompleteness theorems, cannot be avoided in any recursive first-order theory which extends arithmetic).

Ignoring nonstandard formulae can lead us to false proofs of the inconsistency of ZFC. In fact, let ZFC^+ be the Set Theory system (defined in [8]) consisting of the language and axioms of ZFC plus the constant M and the following axioms: "M is nonempty and transitive" and φ^M, for each axiom φ of ZFC. It follows from Reflection's Principle that ZFC^+ is relatively consistent to ZFC (see [8]). In any model for ZFC^+, φ^M holds, for every axiom φ of ZFC. Then, by completeness theorem, there is a proof from ZFC^+ that, for every axiom φ of ZFC, we have φ^M and, hence, $M \models \varphi$. Therefore, we prove in ZFC^+ that $M \models ZFC$ and using Gödel's theorem we can deduce that both ZFC^+ and ZFC are inconsistent.

Fortunately, this argument is incorrect, since we can only assume φ^M for the *standard* formulae, which are the codifications of formulae of the metalanguage. In a model where there exists nonstandard integers, M may not be a model for ZFC.

How to fix the proof. Following the steps of [2], used to formalize the definition of Gödel's constructible universe, we must define a set theory language within ZFC, taking the set of formulae $\mathcal{L}_{ST}$ as a subset of $\omega^{<\omega}$, and the relation of satisfatibility between nonempty sets and formulae. A set theory (or class theory) can be defined as a subset of $\mathcal{L}_{ST}$ consisting of all the axioms of such theory.

Going to the metalanguage, when we write $T_1 \vdash Con(T_2)$, for set (or class) theories T_1 and T_2, it means that the following statement can be proven within theory T_1:

$$\exists M((M \neq \emptyset) \wedge \forall \varphi(\varphi \in T_2 \to M \models \varphi)).$$

In our case, T_1 is ZFC+I and theory T_2 can be either ZFC or KM. Model M is V_κ in the first case and $V_{\kappa+1}$ in the second. We need to pay special attention to axioms' scheme of Replacement, in ZFC, and Compreehension, in KM.

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Estendendo Resultado de Correção para LPV^{π}: Em Direção à Completude

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Palavras-chave: Lógica temporal, Modelo algébrico, Lógica da vizinhança.

Neste trabalho, buscamos a adequação entre uma lógica temporal intervalar e uma semântica algébrica. A lógica em questão é a Lógica Proposicional da Vizinhança - LPV [5], o fragmento proposicional e modal da Lógica da Vizinhança [2, 3]; sendo a estrutura de semianel Booleano com subidentidades, munida de operadores de domínio e codomínio (cf. [4, 6, 7]), um possível modelo para a referida lógica.

Axiomas
(Ax0) LPC
(Ax1) $\Box_*(\varphi \to \psi) \to (\Box_*\varphi \to \Box_*\psi)$
(Ax2) $\Box_*\varphi \to \Diamond_*\varphi$
(Ax3) $\varphi \to \Box_r\Diamond_l\varphi$ e $\varphi \to \Box_l\Diamond_r\varphi$
(Ax4) $\Diamond_r\Diamond_l\varphi \to \Box_r\Diamond_l\varphi$ e $\Diamond_l\Diamond_r\varphi \to \Box_l\Diamond_r\varphi$
(Ax5) $\Diamond_*\Diamond_*\Diamond_*\varphi \to \Diamond_*\Diamond_*\varphi$
(Ax6) $\Box_r\Diamond_l\varphi \to \Diamond_l\Diamond_r\Diamond_r\varphi \vee \Diamond_l\Diamond_l\Diamond_r\varphi$ e $\Box_l\Diamond_r\varphi \to \Diamond_r\Diamond_l\Diamond_l\varphi \vee \Diamond_r\Diamond_r\Diamond_l\varphi$
(Ax7) $\Box_*\varphi \wedge \Diamond_*\psi_1 \wedge ... \wedge \Diamond_*\psi_n \to \Diamond_*(\Box_*\varphi \wedge \Diamond_*\psi_1 \wedge ... \wedge \Diamond_*\psi_n)$, para qualquer $1 \leq n$.

Regras de Inferência
(MP) φ, $\varphi \to \psi$ / ψ
(RN) $\vdash \varphi$ / $\vdash \Box_*\varphi$.

Para a adequação, a investigação de metateoremas como Correção e Completude são essenciais. Com o trabalho [4], havíamos conseguido mostrar que o resultado de Correção Forte é possível e, assim, todo axioma, teorema e dedução na axiomática apresentada acima é válida na álgebra modelo. Porém, ao investigar a Completude, alguns percalços emergem.

Uma das etapas constituintes do método de algebrização *à lá Lindenbaum-Tarski* consiste na definição de uma álgebra de Lindenbaum (ou álgebra quociente) construída a partir da álgebra das fórmulas da lógica em questão [1]. Com a álgebra de Lindenbaum bem definida, busca-se demonstrar que esta possuí propriedades da estrutura algébrica que se pensa ser modelo para o sistema lógico.

No entanto, levando em consideração o domínio da assinatura de LPV:

$$\Sigma = \{\neg, \Box_r, \Box_l, \Diamond_r, \Diamond_l, \wedge, \vee, \to\}, \tag{1}$$

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não encontramos conectivos na linguagem para interpretar algumas operações do semianel, como: (i) a composição $\circ$; (ii) a identidade $1'$; e (iii) domínio $\ulcorner$ e codomínio $\urcorner$.

Vislumbramos uma extensão de LPV pelo acréscimo de uma nova constante π na sua linguagem para indicar fórmulas com intervalos degenerados, uma maneira de contornar o problema. Com essa nova constante, entendemos ser possível definir, na linguagem dessa lógica, esses novos conectivos que darão conta das operações algébricas de composição, domínio e codomínio na álgebra das fórmulas.

A saber,

$$\ulcorner\varphi := \Diamond_r\varphi \wedge \pi \tag{2}$$

$$\varphi^\urcorner := \Diamond_l\varphi \wedge \pi \tag{3}$$

$$\varphi;\psi := \neg((\varphi^\urcorner \wedge \ulcorner\psi) \leftrightarrow \bot) \tag{4}$$

O acréscimo da constante reverbera, também, na axiomática da LPV:

$(Ax\pi1)$ $\Diamond_l\pi \wedge \Diamond_r\pi$
$(Ax\pi2)$ $\Diamond_*(\pi \wedge \varphi) \rightarrow \Box_*(\pi \rightarrow \varphi)$
$(Ax\pi3)$ $\Diamond_*\varphi \wedge \Box_*\psi \rightarrow \Diamond_*(\pi \wedge \Diamond_*\varphi \wedge \Box_*\psi)$

A axiomatização de LPV^π - extensão da LPV por π - é introduzida em [5] e é constituída pelo sistema hilbertiano da LPV mais os axiomas mencionados acima. Assim, com o presente trabalho, estendemos o resultado de Correção Forte para LPV^π, mostrando que os três axiomas são válidos na estrutura de semianel Booleano com subidentidades, munida de operadores de domínio e codomínio. O que possibilita o caminhar para o também o almejado resultado de Completude.

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¿Puede Haber una Única Metateoría Correcta?

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Palabras claves: pluralismo, monismo, metateoría, lógicas contradictorias.

El *pluralismo lógico* es la tesis según la cual hay más de una lógica correcta; esta tesis se contrapone al *monismo lógico*, la tesis según la cual hay una única lógica correcta. Griffiths y Paseau plantean por lo menos un problema al que se deben enfrentar los defensores del pluralismo lógico, a quienes nos referiremos como 'pluralistas lógicos'. De acuerdo con ellos, cuando un pluralista lógico trata de destacar algunas de las propiedades que tiene las diferentes lógicas que toman como correctas, el pluralista debe usar de una lógica que le permita validar sus afirmaciones acerca de las diferentes lógicas. Lo anterior es un problema para los pluralistas, ya que si usan una única lógica **L** para hablar de las propiedades de las lógicas, parece que no hay un compromiso genuino con el pluralismo.

En general, para salir de este problema y validar sus afirmaciones acerca de las diferentes lógicas, los pluralistas pueden optar por tomar a todas las lógicas que consideran correctas, como lo sugirieron Beall y Restall, o considerar a los argumentos super válidos como sugiere Shapiro. Los *argumentos super válidos* son argumentos válidos en todas las lógicas que se consideran correctas. Otra vía sugerida por Beall y Restall y por Griffiths y Paseau, es decir que hay más de una lógica correcta, pero que en el dominio donde están llevando la discusión sólo hay una única lógica correcta, en otras palabras, hay monismo en la metateoría.

Griffiths y Paseau argumentan que estas propuestas tienen problemas. Por ejemplo, la propuesta de Shapiro exige que haya reglas super válidas, sin embargo, un defensor del nihilismo lógico puede argumentar que no hay leyes lógicas, por lo que no hay reglas super válidas comunes a todas y cada una de las lógicas. En cuanto a la propuesta de Beall y Restall, es un problema que algunos argumentos son válidos para algunas lógicas e inválidos para otras. Finalmente, para la última propuesta, Griffiths y Paseau consideran que está respuesta puede conducir a la inconsistencia. Nosotros no creemos que la inconsistencia sea necesariamente un problema.

La pregunta que queremos responder en esta exposición es si tener una única lógica correcta para la metateoría es un problema para el pluralista lógico. En especial, consideraremos el caso de un pluralista lógico que utiliza una lógica contradictoria en la metateoría. Una lógica **L** es *contradictoria* si y sólo en esa lógica hay un argumento que cumple la propiedad de ser válido y no válido. Argumentamos que el pluralismo lógico resultante tiene las mismas virtudes que el monismo caracterizado por Griffiths y Paseau. Sin embargo, queremos ser claros desde el principio: en este trabajo no pretendemos defender una forma de pluralismo lógico que evite las críticas de Griffiths y Paseau. Por el contrario, queremos señalar una forma en que el pluralismo lógico puede evitar tales críticas.

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Continuous Piecewise Linear Functions in Łukasiewicz Logic

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Keywords: logical representations, continuous piecewise linear functions, Łukasiewicz logic

A widely recognized approach in the study of propositional logical systems consists in considering their formulas as representations of functions. It is well known that formulas of classical propositional logic correspond to Boolean functions. Many applications may follow from such approach.

For instance, when functions are derived from artificial intelligence systems, representing them in logical languages might enable leveraging the interpretative capability of logical frameworks for a deeper understanding of such systems [12]. Interpretability is particularly desirable in models developed through machine learning techniques, which have long been at the forefront of state-of-the-art artificial intelligence. Furthermore, logical representations serve as a first step for certain techniques used in the formal verification of machine learning models [3, 8, 11, 13]. As many models compute functions more general than Boolean functions—usually continuous functions—, there is an interest in logical systems that represent them.

A possibility is to represent rational McNaughton functions in Łukasiewicz logic. These functions are continuous piecewise linear mappings defined on the domain $[0, 1]^n$ with a range contained in $[0, 1]$, where each linear piece has rational coefficients. In the canonical sense of representation, formulas of Łukasiewicz logic just represent (integer) McNaughton functions [4, 5, 9], which are rational McNaughton functions constrained to only allow integer coefficients in their linear pieces. However, rational McNaughton functions have an implicit representation in Łukasiewicz logic called *representation modulo satisfiability* [1, 2, 6, 7], which is performed by pairs $\langle \varphi, \Phi \rangle$, where φ is a formula and Φ is a set of formulas. Thus, only valuations that satisfy Φ are considered to output the function values by means of φ. Finite sets of formulas Φ are enough for representing rational McNaughton functions.

Effectively, applications may compute rational McNaughton functions with linear pieces whose rational coefficients are actually numerical approximations of real (possibly irrational) values. This arises particularly in the context of learning algorithms [10], which ideally may define models using irrational numbers but, in practice, rely on their rational approximations due to numerical computation constraints.

The main goal of this work is to show that the more general continuous piecewise linear functions, defined on the domain $[0, 1]^n$ with a range in $[0, 1]$ and real-valued coefficients in their linear pieces, also have representations modulo satisfiability in Łukasiewicz logic. In these cases, our techniques possibly yield infinite sets of formulas Φ in the pairs $\langle \varphi, \Phi \rangle$.

Although infinitary representations may be of theoretical interest, they cannot be used in practical computations. Then, we also investigate the connection between continuous piecewise linear functions and their approximations by rational McNaughton functions through truncations of linear coefficients. We establish error margins for values of rational McNaughton functions when approximating general continuous piecewise linear functions. As consequence, within given maximum error

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margins, one may determine finite approximate representations modulo satisfiability of continuous piecewise linear functions.

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Preferencias Inconsistentes y Preferencias Imposibles: Nociones de Consistencia en la Lógica de la Justificación

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Palabras claves: preferences, reasoning, justification, modal

En el siguiente trabajo proponemos abordar diferentes nociones de consistencia en la lógica de las preferencias. En el tratamiento estándar que se hace de las preferencias bajo la lógica modal (S4) las preferencias imposibles colapsan con las preferencias inconsistentes. En la lógica de la justificación podemos explicitar las razones que motivan una preferencia permitiéndonos distinguir el caso de una preferencia imposible de una preferencia inconsistente. Además, nos proporciona elementos formales como la especificación de constantes para principios y las operaciones sobre razones. Por medio de la lógica de la justificación presentaremos diferentes principios de consistencia de preferencias y compararemos su fuerza lógica.

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A Six-Valued Semantics for a Self-Referential Sentences

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Keywords: self-reference, Liar, dynamic approximation

This text is a popular introduction to *dynamic approximation* of self-referential sentences, introduced in the [Stepanov, 2022].

Let Sx be a self-referential (s-r) quantifier [Johnstone, 1981]. Then $SxP(x)$ is a s-r sentence satisfying the fixed point axiom: $SxP(x) \leftrightarrow P(SxP(x))$, [Feferman, 1984]. The symbol Sx in the formula $SxP(x)$ connects the free variable x to the $P(x)$. The most famous s-r sentence is the Liar: $Sx\neg Tr(x)$ where $Tr(x)$ obeys the Tarski formula $Tr(\ulcorner x \urcorner) \leftrightarrow x$, where $\ulcorner x \urcorner = x$, by *autonymous*, [Kleene, 1950, §50, 54].

In [Kauffman, 1994] announced the following fact about what sequence of 1s and 0s the sentence Liar generates: Liar: A = $\left\{\begin{smallmatrix} 10101010... \\ 01010101... \end{smallmatrix}\right\}$.
Let us extend this technique to other (atomic) self-referential sentences:
$SxTr(x)$:TruthTeller: V = $\left\{\begin{smallmatrix} 1\ 111111... \\ 0\ 000000... \end{smallmatrix}\right\}$, IdentiT: T = $\left\{\begin{smallmatrix} 1\ 111111... \\ 0\ 111111... \end{smallmatrix}\right\}$, IdentiF: F = $\left\{\begin{smallmatrix} 1\ 000000... \\ 0\ 000000... \end{smallmatrix}\right\}$. These sequences are periodic, with a maximum period of 2. Therefore, we will retain the first three symbols of each of the infinite initial estimates:
Liar: A=$\left\{\begin{smallmatrix} 1\ 01 \\ 0\ 10 \end{smallmatrix}\right\}$=3, TruthTeller: V=$\left\{\begin{smallmatrix} 1\ 11 \\ 0\ 00 \end{smallmatrix}\right\}$=3, T=$\left\{\begin{smallmatrix} 1\ 11 \\ 0\ 11 \end{smallmatrix}\right\}$=5, F= $\left\{\begin{smallmatrix} 1\ 00 \\ 0\ 00 \end{smallmatrix}\right\}$=1.
The numbers on the right indicate the number of units (1) in the formula estimates. They will be useful for constructing the estimate lattice (see below). The negation operation is defined according to the rules of classical logic. For convenience, the result of the operations should be presented in such a way that the first line with index 1 is located at the top:

$\neg$ A=$\neg \left\{\begin{smallmatrix} 1\ 01 \\ 0\ 10 \end{smallmatrix}\right\}$=$\left\{\begin{smallmatrix} \neg(1\ 01) \\ \neg(0\ 10) \end{smallmatrix}\right\}$= $\left\{\begin{smallmatrix} 0\ 10 \\ 1\ 01 \end{smallmatrix}\right\}$=$\left\{\begin{smallmatrix} 1\ 01 \\ 0\ 10 \end{smallmatrix}\right\}$=A.

The binary operations are constructed so that the first indices of the estimates interact with each other according to the principle: one line with index 1 interacts with another line with the same index 1, and vice versa. Example:

A & V = $\left\{\begin{smallmatrix} 1\ 01 \\ 0\ 10 \end{smallmatrix}\right\}$ & $\left\{\begin{smallmatrix} 1\ 11 \\ 0\ 00 \end{smallmatrix}\right\}$ = $\left\{\begin{smallmatrix} (1\ 01)\&(1\ 11) \\ (0\ 10)\&(0\ 00) \end{smallmatrix}\right\}$ = $\left\{\begin{smallmatrix} 1\ 01 \\ 0\ 00 \end{smallmatrix}\right\}$ = av = 2.

A $\vee$ V = $\left\{\begin{smallmatrix} 1\ 01 \\ 0\ 10 \end{smallmatrix}\right\}$ $\vee$ $\left\{\begin{smallmatrix} 1\ 11 \\ 0\ 00 \end{smallmatrix}\right\}$ = $\left\{\begin{smallmatrix} (1\ 01)\vee(1\ 11) \\ (0\ 10)\vee(0\ 00) \end{smallmatrix}\right\}$ = $\left\{\begin{smallmatrix} 1\ 11 \\ 0\ 10 \end{smallmatrix}\right\}$ = va = 4.

The evaluations of self-referential sentences can be represented in the form of a diagram, for Liar:

$$A = \begin{array}{ccc} 1 & — 0 — & 1 \\ | & | & | \\ 0 & — 1 — & 0 \end{array}$$

Such diagrams can be called the "DNA of self-referential sentences".

We declare two-dimensional sequences to be VALUES of self-referential formulas.
Lemma 1: 1. The sentences *Liar* (A) have a tabular model isomorphic to Priest's tabular model for *Liar* (p) [7].
2. The *TruthTeller* (V) have a tabular model isomorphic to Priest's tabular model for *Liar*.

Let's compare Kleene-Priest tables for $\wedge$ of the Liar sentences with the tables obtaind for values A and V:

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Kleene-Priest p

∧	t	p	f
t	t	p	f
p	p	p	f
f	f	f	f

≡

Hypothesis: p = A

∧	T	A	F
T	T	A	F
A	A	A	F
F	F	F	F

≡

Hypothesis: p = V

∧	T	V	F
T	T	V	F
V	V	V	F
F	F	F	F

Lemma 2: When constructing the interaction of V and A, new truth values were obtained: $A\wedge V=\left\{\begin{smallmatrix}1 & 01\\ 0 & 00\end{smallmatrix}\right\}$ = av = ¬(va), $A\vee V=\left\{\begin{smallmatrix}1 & 11\\ 0 & 10\end{smallmatrix}\right\}$ = va =¬(av).

∧	T	A	V	F
T	T	A	V	F
A	A	A	av	F
V	V	av	V	F
F	F	F	F	F

∨	T	A	V	F
T	T	T	T	T
A	T	A	va	A
V	T	va	V	V
F	T	A	V	F

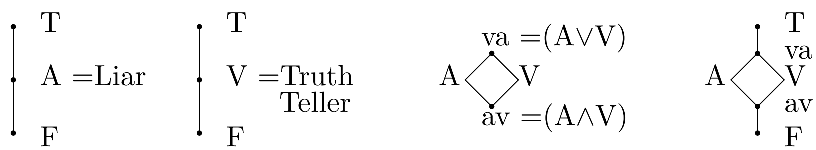
In our case, the tables are not closed: A∨V=va and A∧V=av, which encourages the construction of new, already six-valued ones. Fortunately, they are already closed.

¬	
T	F
va	av
A	A
V	V
av	va
F	T

∨	T	va	A	V	av	F
T	T	T	T	T	T	T
va	T	va	va	va	va	va
A	T	va	A	va	A	A
V	T	va	va	V	V	V
av	T	va	A	V	av	av
F	T	va	A	V	av	F

∧	T	va	A	V	av	F
T	T	va	A	V	av	F
va	va	va	A	V	av	F
A	A	A	A	av	av	F
V	V	V	av	V	av	F
av	av	av	av	av	av	F
F	F	F	F	F	F	F

Lemma 3: The next four lattices are DeMorgan lattices:
(F ≤ av ≤ A ≤ V ≤ va ≤ T) ; (1 ≤ 2 ≤ 3 ≤ 3 ≤ 4 ≤ 5) :



Our manipulations can be presented as a confirmation of Suszko's Thesis (SR):

“Each n-valued Tarski logic can be characterized as two-valued”.

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Study and Completeness Proof for Tableaux in ReLo

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Keywords: Reo; Dynamic Logic; Tableaux; Completeness

The growing importance, criticality, and complexity of software in various fields demands the exploration and development of methods that bring the outcome of software production as close as possible to being correct and secure. Reo [1] emerges as an efficient alternative for software modeling. One of the reasons is that, by representing programs as interactions between components through channels aligns well with the increasing trend of developing complex software in a modular manner. Additionally, Reo has well-established formal semantics, enabling the formalization and application of formal methods [2] to Reo programs. It provides a way to address the growing complexity of software, while ensuring the correctness of a program according to a formalized requirement.

As a formal semantics, the dynamic modal logic **ReLo** [3] provides a formal way to describe and reason about Reo programs.

Definition 1 (Logical Formulae in ReLo)**.**
Logical Formulae in ReLo are defined as: $\phi = p \mid \top \mid \neg\phi \mid \phi \wedge \psi \mid \langle t, \pi \rangle \phi$*, such that* $p \in \Phi$.

We use the standard abbreviations $\top \equiv \neg\bot, \phi \vee \psi \equiv \neg(\neg\phi \wedge \neg\psi), \phi \rightarrow \psi \equiv \neg\phi \vee \psi$ *and* $[t, \pi]\varphi \equiv \neg\langle t, \pi \rangle \neg\phi$*, where* π *is an Reo program and* t *a data sequence.*

The main types of logical formulae in ReLo are $[t, \pi]\varphi$ and $\langle t, \pi \rangle \varphi$, which states that φ is necessarily (or possibly, depending on the modality) true after the execution of the Reo program π upon the data sequence t. The formulae $[t, \pi^*]\varphi$ and $\langle t, \pi^* \rangle \varphi$ introduces the iteration of a Reo program.

ReLo's semantic is based on possible worlds semantic, and the accessibility relation between the existing states in a model is defined by the execution of Reo programs, using auxiliary functions defined to simulate the flow of information between channels and the execution of a Reo program.

Having discussed the semantic of ReLo, we now turn to its deductive system, the Tableau method. This method is similar to tableaux in other modal logics, and the final result of this work is the demonstration of the completeness of this method. The rules for ReLo's Tableaux are similar to those of other modal logics, except that the new states chosen by the modal rules come from states accessible through the execution of the program in question. Another distinction between ReLo and other logics is the iteration operator rules and the auxiliary rules using the token $\mathcal{X}$ to avoid the generation of possibly infinite branches.

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- Some Iteration Operator Rules

$$(\langle t,\pi^\star\rangle - T) \quad \frac{\text{w: } \langle t,\pi^\star\rangle\varphi : \text{T}}{\text{w: } \mathcal{X}_{\langle\rangle} : \text{T}} \qquad\qquad (\mathcal{X}_{\langle\rangle}) \quad \frac{\mathcal{X}_{\langle\rangle} : T}{w : \varphi : \text{T} \qquad w : \varphi : F,\ \text{w} : \langle t,\pi\rangle\mathcal{X}_{\langle\rangle} : T}$$

$$\mathcal{X}_{\langle\rangle} = \langle t,\pi^\star\rangle\varphi$$

The main idea in proving that the method always terminates requires defining a formal procedure for the tableau and adding a mechanism to check, every time a new state is instantiated in the tableau, whether this state is a copy of another state already seen. If so, the branch is ignorable as it will no longer contribute to the proof. This is done by using $\mathcal{X}$ rules to validate whether the same formulae hold in both states. Both this proof and the soundness proof are necessary for the completeness of ReLo's Tableaux and are described in another work [4].

The completeness proof follows the strategy described by Fitting in [5]. The essence of the proof is the idea that, to prove the completeness of this method, we need to show that the formulae that a tableau cannot prove are indeed not true in the context of the logic.

This is achieved by noting that, as a result of the termination proof, for any formula φ that we attempt to prove using a tableau, eventually, after the systematic application of the rules, the tableau will reach one of two states: Either φ is proved, or, is not proved, and there is an open finite branch that provides a counter-model, demonstrating that the given formula is not a tautology in ReLo. The main complexity of the proof lies in showing that a valid ReLo model can always be constructed from an open branch in a finished tableau.

By defining the concept of a Hintikka set in ReLo, we establish a set of formulae that follow specific properties, intuitively describing all formulae and subformulae in a given open tableau branch. With this set, which contains all the necessary formulae and states, we can define a ReLo model. The accessibility relation and other structures necessary for the model follow naturally from the model definition, as well as from the programs and states present in the Hintikka set. The valuation function required for a ReLo model is defined via induction on the structure of the formulae, ensuring that a truth value is assigned to each type of formula that may be present in the Hintikka set. One of the induction cases that is worth mentioning, is the proof for formulae containing the iteration operator (e.g., $[t,\pi^\star]\varphi$), which requires an additional induction on the states accessible by the reflexive transitive closure of π to assign values to the formulae in each of these states.

With these steps, we can now construct a valid model from any open branch in a tableau, and by following the argument provided earlier, we conclude that the method is complete.

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Classes de Lógicas e Lacunas na Formalização da Hierarquia de Leibniz

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Palavras-chave: Lógica Algébrica Abstrata; Hierarquia de Leibniz; Operador de Suszko.

O campo da Lógica Algébrica Abstrata (LAA) estuda e classifica famílias de lógicas proposicionais, isto é, álgebras de fórmulas sobre uma certa linguagem Σ munidas de uma relação de consequência entre fórmulas. Para ilustrar sucintamente, a LAA está para a lógica clássica assim como, em certo sentido, a Teoria de Anéis está para os números reais: no primeiro caso, temos o exemplo da mais célebre lógica do tipo algebrizável, e no segundo, um dos mais importantes exemplos de corpo.

No contexto desse campo, uma das ferramentas de maior destaque é o chamado operador de Leibniz, que relaciona o reticulado dos filtros dedutivos com o reticulado das congruências. Não por acaso, o estudo sistemático das propriedades compartilhadas por diferentes famílias de lógicas em relação a esse operador levou ao estabelecimento da chamada hierarquia de Leibniz. Alguns exemplos de suas principais classes são as lógicas algebrizáveis, fracamente algebrizáveis, equivalenciais, protoalgébricas, *truth-equational*.

Contudo, Czelakowski [4] aponta que no caso mais amplo das lógicas não protoalgébricas, no lugar do operador de Leibniz Ω, o instrumento mais apropriado para tratar dessas classes de lógicas deveria ser o operador de Suszko $\widetilde{\Omega}$. Com efeito, uma das caracterizações de protoalgebricidade é a igualdade entre os operadores de Leibniz e de Suszko, ou seja, desde o início, as propriedades relevantes encontradas para o operador de Leibniz no caso protoalgébrico na verdade diziam respeito ao operador de Suszko.

Essas observações parecem ser corroboradas pela série de artigos de Jansana & Moraschini [7,8], que propõe formalizar a hierarquia de Leibniz como um paralelo da hierarquia de Maltsev da Álgebra Universal, a partir de uma noção de interpretação entre lógicas em função do operador de Suszko, e não de Leibniz. Nesse sentido, em [2, 3], mostramos que existem classes de lógicas definidas por propriedades específicas do operador de Leibniz que não correspondem a classes de Suszko, de acordo com a definição dada por Jansana & Moraschini — referidas confusamente no original como “classes de Leibniz” —, isto é, classes fechadas por expansões compatíveis, termo-equivalências e produtos não indexados. Vejamos aqui uma delas:

Definição. *(Lógica Ω-natural) Dizemos que uma Σ-lógica S é* **Ω-natural** *se, para todo par de Σ-álgebras $\mathbf{A}, \mathbf{B}$, o operador de Leibniz restrito aos S-filtros comuta com homomorfismos inversos — i.e., $\forall\, h \in Hom_\Sigma(\mathbf{A}, \mathbf{B})$ e $\forall\, G \in Fi_S(\mathbf{B})$, vale $(h \times h)^{-1}[\Omega^{\mathbf{B}}(G)] = \Omega^{\mathbf{A}}(h^{-1}[G])$.*

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Tal propriedade necessariamente engloba todas as lógicas equivalenciais (e, portanto, algebrizáveis). Outros dois exemplos de classes de lógicas bastante relevantes e pouco estudadas são a interseção das lógicas *truth-equational* com as Ω-naturais, denominadas ***truth*-naturais**, além dos sistemas unários, ou seja, a classe de todas as lógicas definidas sobre uma linguagem contendo apenas conectivos de aridade no máximo 1.

Demonstramos também em [2,3] que todo sistema unário necessariamente é Ω-natural. Não só isso, mas também mostraremos nesta apresentação o mesmo resultado substituindo o operador de Leibniz pelo de Suszko, o que nos leva a considerar a classe das lógicas **Suszko-naturais**. Nosso resultado principal aqui, estendendo aquele presente em [2,3], pode ser resumido da seguinte forma:

Teorema. *As classes das lógicas Ω-naturais e Suszko-naturais, além de suas respectivas interseções com a classe truth-equational, bem como a classe dos sistemas unários não formam classes de Suszko.*

De fato, indo nessa direção, somos levados a estabelecer um diálogo com o artigo de Albuquerque [1], que traz, como uma das propriedades que caracterizam as lógicas *truth-equational*, a Suszko-naturalidade restrita às substituições (i.e., endomorfismos da álgebra de fórmulas).

Por fim, ressaltamos a necessidade de se rever a hierarquia de Leibniz — ou ainda sua hierarquia irmã, a hierarquia de Suszko —, levando em consideração a vasta quantidade de propriedades já consolidadas relativas ao operador de Leibniz, porém trocando-o pelo operador de Suszko. Por exemplo, tanto Raftery [9] quanto Albuquerque [1] discutem as diferenças entre a injetividade global do operador de Suszko (condição equivalente a ser *truth-equational*) *versus* do operador de Leibniz; ou ainda, conforme [1, Teo. 5.11, Cor. 5.12], também podemos enxergar um paralelismo forte entre a classe das lógicas em que o predicado de “verdade” é equacionalmente definível em $LMod^{Su}$ e a classe das lógicas algebrizáveis, simplesmente intercambiando o operador de Suszko pelo de Leibniz em ambas as caracterizações.

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Logics of Indicative Conditionals

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Keywords: Indicative Conditionals, Connexive Logics, Algebraic Logic

$\wedge_{\mathsf{OL}}$	0	1/2	1
0	0	0	0
1/2	0	1/2	1
1	0	1	1

$\vee_{\mathsf{OL}}$	0	1/2	1
0	0	0	1
1/2	0	1/2	1
1	1	1	1

	$\neg$
0	1
1/2	1/2
1	0

$\wedge_{\mathsf{K}}$	0	1/2	1
0	0	0	0
1/2	0	1/2	1/2
1	0	1/2	1

$\vee_{\mathsf{K}}$	0	1/2	1
0	0	1/2	1
1/2	1/2	1/2	1
1	1	1	1

$\to_{\mathsf{OL}}$	0	1/2	1
0	1/2	1/2	1/2
1/2	0	1/2	1
1	0	1/2	1

$\to_{\mathsf{DF}}$	0	1/2	1
0	1/2	1/2	1/2
1/2	1/2	1/2	1/2
1	0	1/2	1

$\to_{\mathsf{F}}$	0	1/2	1
0	1/2	1/2	1/2
1/2	0	1/2	1/2
1	0	1/2	1

Figure 1: Tables of the three-valued connectives.

Indicative conditionals are the simplest sentences of the *if-then* type that occur in natural language, concerning what could be true – in opposition to counterfactuals, which concern eventualities that are no longer possible. In Boolean propositional logic, an indicative conditional "if φ then ψ" is traditionally formalized as the material implication $\varphi \to \psi$, or equivalently the disjunction $\neg\varphi \vee \psi$. This approach has several limitations that have been remarked early on in the history of modern logic: in particular, a number of authors argued that conditionals having a false antecedent – which are true in Boolean logic independently of the consequent – should instead be regarded as lacking a (classical) truth value. Such a proposal can be traced back at least to Reichenbach (1935), De Finetti (1936), and Quine (1950). "Uttering a conditional amounts to making a *conditional assertion*: the speaker is committed to the truth of the consequent when the antecedent is true, but committed to neither truth nor falsity of the consequent when the antecedent is false" [1, p. 188]; see also [2] and the references cited therein.

Among various possible ways to formalize the above intuition, a very simple one consists in expanding the classical truth values $(\mathbf{0}, \mathbf{1})$ with a third "gap" value (here denoted by $\mathbf{1/2}$) assigned to conditional sentences with a false antecedent; and then extending the truth tables of the propositional connectives in accordance with the above interpretation. In particular, with regard to the implication, one would certainly require $\mathbf{0} \to x = \mathbf{1/2}$, whereas in other cases (e.g. $\mathbf{1/2} \to x$) intuitions may differ (see Figure 1). As for the designated elements to be preserved in derivations, it is natural to include (besides $\mathbf{1}$) also $\mathbf{1/2}$, at least if one wants to retain basic classical tautologies such as the law of identity $(\varphi \to \varphi)$.[1]

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[1] A peculiar consequence of this setup is that there will be valid formulas whose negation is also valid: for instance the formula $\neg\varphi \to (\varphi \to \varphi)$, which turns out to be equivalent (within the systems considered here) to $\mathbf{1/2}$ viewed as a propositional constant. This makes the logics under consideration not only paraconsistent but actually *contradictory* in the sense of Wansing [13].

The above constraints determine a range of three-valued propositional *logics of indicative conditionals* which turn out to be, in general, not subclassical (i.e. weaker than) but rather incomparable with classical two-valued logic. In particular, they may be *connexive* in that they validate the (classically contingent) formulas known as *Aristotle's* thesis $\neg(\varphi \to \neg\varphi)$ and *Boethius'* theses: $(\varphi \to \psi) \to \neg(\varphi \to \neg\psi)$ and $(\varphi \to \neg\psi) \to \neg(\varphi \to \psi)$.

Logics of indicative conditionals are discussed at length in the papers [1–3], which are the main bibliographical source and the starting point for the present research. Here we consider these propositional systems from the standpoint of algebraic logic: in particular, we determine which among them are algebraizable in the sense of Blok and Pigozzi [4], and study the corresponding algebra-based semantics. Besides the ones discussed in [1, 2], we shall also define a few systems obtained by varying the above-mentioned basic parameters (in particular, the designated elements) that do not appear to have been considered in the existing literature; our interest in the latter logics is essentially formal, but future research may prove them to be also relevant to the issues discussed above.

As is well known, a standard way of introducing a propositional logic is to fix an algebra $\mathbf{A}$ together with a subset $D \subseteq A$ of *designated elements* to be preserved in derivations. Such a pair $\langle \mathbf{A}, D\rangle$ is known as a *(logical) matrix*[2], and we may unambiguously denote by $\mathrm{Log}\langle \mathbf{A}, D\rangle$ the propositional consequence relation determined by the matrix $\langle \mathbf{A}, D\rangle$. For the logics of interest here, the universe of the algebra is always going to be the three-element set $A_3 = \{\mathbf{0}, \mathbf{1/2}, \mathbf{1}\}$, with variations only in the algebraic operations considered and possibly the set of designated values. The basic systems are the following (in all cases we fix $D = \{\mathbf{1/2}, \mathbf{1}\}$):

1. $\mathrm{Log}\langle \mathbf{DF_3}, D\rangle$, where $\mathbf{DF_3} = \langle A_3; \wedge_{\mathsf{K}}, \vee_{\mathsf{K}}, \to_{\mathsf{DF}}, \neg\rangle$, which is the logic proposed by De Finetti [5]. We show that, up to definitional equivalence, this system coincides with Priest's *logic of paradox* LP [6] expanded with the propositional constant $\mathbf{1/2}$.

2. $\mathrm{Log}\langle \mathbf{OL_3}, D\rangle$, where $\mathbf{OL_3} = \langle A_3; \wedge_{\mathsf{OL}}, \vee_{\mathsf{OL}}, \to_{\mathsf{OL}}, \neg\rangle$. This is the structural weakening of Cooper's *logic of ordinary discourse* [7], dubbed sOL in the recent papers [8, 9].

3. $\mathrm{Log}\langle \mathbf{CN_3}, D\rangle$, where $\mathbf{CN_3} = \langle A_3; \wedge_{\mathsf{K}}, \vee_{\mathsf{K}}, \to_{\mathsf{OL}}, \neg\rangle$. A system introduced by Cantwell [10] as the *logic of conditional negation* (CN) and independently considered by a number of other authors[3]. We prove that CN may be viewed as a term-definable subsystem of sOL.

4. $\mathrm{Log}\langle \mathbf{F_3}, D\rangle$, where $\mathbf{F_3} = \langle A_3; \wedge_{\mathsf{K}}, \vee_{\mathsf{K}}, \to_{\mathsf{F}}, \neg\rangle$, a logic introduced by Farrell [11]. We show that this system is definitionally equivalent to CN (hence, also to a definable subsystem of sOL).

Besides the above systems, we consider a few related ones that, as far as we are aware, have not yet appeared in the literature. These are obtained by:

5. Varying the set D of designated elements on A_3: for instance, logics that result from taking $D = \{\mathbf{1/2}\}$, which is a natural choice at least from a formal standpoint.

6. Considering a set of matrices based on the same algebra. In this way we study *degree-preserving logics* associated to the above-mentioned algebras (see e.g. [12]).

In each case we determine whether the system is algebraizable, thereby settling some issues on the algebraization of logics of indicative conditionals that were raised but left unsolved in [2]. Algebraizable logics are well-behaved in many ways, and in particular one may easily obtain a presentation of the algebraic semantics from an axiomatization of the logic, and vice-versa. In these cases we produce such axiomatizations, and also introduce *twist representations* (akin to that in [9]) that provide further insight into the algebraic semantics; in all the other cases we nevertheless employ algebraic logic techniques to try and obtain some understanding of the models of the logic under consideration.

[2] See, e.g., [14] for further background on the theory of logical matrices.

[3] As pointed out in [17], this logic – or equivalent systems, with slight variations in the choice of primitive connectives – seems to have been introduced independently in a number of papers from the 1980s to the 2000s (see, e.g., [15, 16]).

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A New Perspective on Intuitionistic and Non-Classical Modal Logics

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Keywords: Intuitionistic Modal Logic, Non-classical Modal Logics, Kripke Semantics

The formalization of model-theoretic semantics for non-classical modal logics is often uniquely challenging. In the classical setting, propositional models are defined through functions that assign truth values to propositions, whereas modal semantics are defined through Kripke models that contain a set of propositional models (its "worlds") plus some relation between them. As such, classical modal semantics are obtained through a generalization of the propositional semantics. The problem with the unrestricted use of this approach is that many non-classical logics already require use of Kripke models in their propositional semantics, so it is not clear (neither from a technical nor from a conceptual point of view) what their modal semantics should look like.

In the particular case of intuitionistic logic, the answer traditionally provided by the literature takes the shape of a birelational model, which is a Kripke model with two distinct relations between its classical propositional models – the first used to induce the desired intuitionistic propositional behavior, the second to play role of a proper modal relation [1] [2]. This answer is not entirely satisfactory because, in order to provide semantics for some interesting intuitionistic modal logics, we need to restrict our models by requiring satisfaction of some frame conditions between the two relations [3], which bring about some practical complications. Although technically sound, this approach is also not easy to justify from a conceptual viewpoint, since it does not provide principled answers for the acceptance or rejection of possible frame conditions.

Our work promotes a very simple change in perspective that leads to a new kind of modal semantics for intuitionistic logic in particular and non-classical logics in general. First, we show that it is indeed possible to define an intuitionistic modal model as a set of intuitionistic propositional models plus some relation between them, which leads to a natural generalization of the very notion of a Kripke model. Second, we argue that the same temporal intuitions used to philosophically justify propositional intuitionistic models also justify the new modal semantics. Third, we prove that this framework is mathematically equivalent to the birelational one by providing mappings from one class of models into the other. Fourth, we show that the frame conditions externally imposed on birelational models emerge naturally from the new framework and don't need to be explicitly stipulated, since different frame conditions are induced depending on what kind of intuitionistic propositional models we allow in the modal model. Finally, we show that the new approach leads to modular definition capable of generating a reasonable modal version (more specifically, the strongest possible modal version) of each and every non-classical logic semantically characterized by Kripke models. The approach is also modular in the sense that it works for the whole spectrum of modal logics, so even though we deal only with K it is straightforward to show that stronger logics may be obtained through the usual conditions on accessibility relations.

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Dynamic Epistemic Logic of Resource Bounded Information Mining Agents

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Keywords: Resource Bounded Agents, Dynamic Epistemic Logic, Common Knowledge

Logics for resource-bounded agents have been getting more and more attention in recent years since they provide us with more realistic tools for modelling and reasoning about multi-agent systems. While many existing approaches are based on the idea of agents as imperfect reasoners, who must spend their resources to perform logical inference, this is not the only way to introduce resource constraints into logical settings. In this paper we study agents as perfect reasoners, who may purchase a new piece of information from a trustworthy source. For this purpose we propose dynamic epistemic logic for semi-public queries for resource-bounded agents. In this logic (groups of) agents can perform a query (ask a question) about whether some formula is true and receive a correct answer. These queries are called semi-public, because the very fact of the query is public, while the answer is private. We also assume that every query has a cost and every agent has a budget constraint. Finally, our framework allows us to reason about group queries, in which agents may share resources to obtain a new piece of information together. We demonstrate that our logic is complete, decidable and has an efficient model checking procedure.

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Geometric Constructions: Algorithms and Games

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Keywords: Algorithms, Geometric constructions, Impossibility proofs

The notion of geometric construction has been widely discussed in recent studies (cf. e.g. [3, 5, 6, 9]). Of the many approaches that have been developed, we would like to draw attention to two lines of investigation. The first was to understand in what sense geometry could be considered constructive, according to contemporary constructivist principles, such as those of logic and mathematics [1]. This is a complex task that involves both deep philosophical discussion and complex historical analysis. Two notable logical-historical analyses of the structure of *ancient geometry* and its constructions are [4], which rejects the so-called historical-algebraic approach to the Elements–mainly that of Zeuthen–and [6], which analyzes the emergence of deductive practices in geometry, pointing out the essential role that diagrams and their relations to constructions played in this practice. The second is the problem of giving a precise formal definition of what is to be understood by a construction in ancient geometry. Upenskiy and Shen [10], among others, are concerned with the proper notion of construction and/or algorithm that should be formally adopted for Euclidean geometry in order to determine when and how impossibility results should be reformulated in terms of such definitions.

Notably, important impossibility results, such as the trisection of an angle and the duplication of a cube, took a long time to solve—two millennia—and their solution made essential use of algebraic apparatus. But until very recently, even after Hilbert's reformulation of Euclid's Geometry, it was still not clear which notion of construction would correctly correspond to plane geometry, our authors say. They focus, more precisely, on books I-VI of The Elements. What is interesting in their presentation is that they introduce the discussion presenting other attempted definitions and point what were they failures from their perspective. Finally, they give a formal definition in which a construction is characterized by means of a procedural game, therefore, not exactly a simple algorithm.

The authors call attention to a very interesting fact concerning the impossibility results. The lack of a precise concept may have led us to accept some mistaken proofs of impossibility. They exemplify the case with a proof–attributed to Hilbert–showing the impossibility of obtaining plane geometry using only a ruler. But the alleged proof, often reproduced in many different textbooks, contains a misstep in its final development. And this, they say, occurs because even after Hilbert, we still did not have a precise formal definition of what a plane geometric construction is.

Interestingly, mathematical problems that proved insoluble in the 19th century with restricted means, i.e. ruler and compass, were formulated with sufficient clarity to allow us to definitively accept the results. However, just as there are still doubts about what exactly it means to be non-constructible, in geometric terms, there are also doubts about how to deal with issues related to this concept, which implies that the subject still contains characteristics that need to be investigated.

In [7, 8], we set out to approach the plane geometry of *The Elements* [2] through the concept of

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Euclidean Machine. Now, our aim is to confront *vis-à-vis* the interpretation underlying Euclidean Machines, both with the definitions that Upenskiy and Shen review and with the one they propose. More precisely, in each case, the focus is on the notion of construction and our aim is to reveal what they mean and, possibly, confront their extensions. It seems that some surprises may still appear if we look more closely at the formal definitions of geometric construction.

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Pensando sobre a Lógica e o Ensino na Educação Básica

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Palavras-chave: Lógica Formal, Lógica Informal, Ensino

O objetivo deste trabalho é propor que a lógica formal e lógica informal podem ser complementares no ensino de lógica no nível básico. Tomando o ensino de lógica como desenvolvimento de certas habilidades argumentativas (BNCC: Base Nacional Comum Curricular (Brasil, 2018 [1]), será argumentado que a lógica deve ser vista não como uma área isolada, mas sim enquanto uma metodologia possível de ser relacionada com outras áreas do saber no ensino. Com base em diferentes demarcações entre lógica formal (Gensler 2016 [2], Mortari 2016 [5]) e lógica informal (Johnson 1999 [3], 2000 [4]; Woods 2004 [6]), será argumentado que a lógica formal e a lógica informal podem ser complementares no desenvolvimento dessas das habilidades citadas na BNCC. Desse modo, o presente trabalho visa: (1) destacar as habilidades lógicas argumentativas que aparecem na BNCC; (2) definir lógica informal e lógica formal a partir das demarcações apresentadas nos manuais de lógica; (3) explorar em que medida as "duas lógicas" tando a lógica formal quanto a lógica informal, entendidas enquanto complementares, podem ajudar no alcance das habilidades apresentadas.

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Non-Quineian Unless

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Keywords: Convergency, Divergency, Linguistic, Quine, Temporal propositions, Unless

In Quine's prospect [4], *unless* has been interpreted as "inclusive or". Quineian *unless* as a functional connective induces a contradiction due to lack of equivalency between $p \vee q \vee r$ and $(p \vee q) \wedge (p \vee r)$ [1]. The proper analysis of unless lies at the intersection of logic and linguistics [3]. This proposal presents two linguistic interpretation for *unless* according to its dictionary meaning, which are functional and communicative connectives. "Exclusive or" as the first interpretation is inferred, below:

p unless q ≡ p except if q ≡ p holds true except when q ≡ p holds as long as q does not hold and p stops holding as soon as q holds ≡ $(\neg q \rightarrow p) \wedge (q \rightarrow \neg p) \equiv (p \longleftrightarrow \neg q)$ ≡ p XOR q ≡ p Non-Quineian *unless* q ≡ q XOR p ≡ q unless p.

The following argument presents the second interpretation as "not both":

p unless q ≡ p only if not q ≡ $p \rightarrow \neg q \equiv \neg p \vee \neg q \equiv \neg(p \wedge q)$ ≡ p Non-Quineian *unless* q ≡ $\neg(q \wedge p)$ ≡ q unless p.

Quine recognized that *unless* might not be commutative [2]. Although not successful, he tried to modify the phrases containing *unless* to defend his interpretation as commutative truth-functional connective. Unlike Quine, I didn't change the tense of propositions and yet preserved commutativity:

$$(\texttt{p at t}_1)\ \texttt{unless}\ (\texttt{q at t}_2) \equiv (\texttt{q at t}_2)\ \texttt{unless}\ (\texttt{p at t}_1)$$

Holding temporal propositions in their own tenses remove divergency from commutative parts. Following inference represents a convergent commutative *unless* on Quine's divergent example:

Smith will sell unless he hears from you ≡ Smith will sell, only if he doesn't hear from you ≡ Selling by Smith in the future, implies that he has not heard from you yet ≡ Hearing from you now implies that Smith won't sell in the future ≡ Smith hears from you only if he won't sell ≡ Smith hears from you unless he will sell.

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Posters
Pôsteres

Paraconsistent Decision Making: An Overview on Ways of Avoiding Medical Iatrogenesis

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Keywords: Paraconsistent Logic, Medical Diagnostics, Medical Iatrogenesis, Decision-Making, Healthcare Data Analysis

This work proposes the application of paraconsistent logic to enhance the diagnostic process and reduce medical iatrogenesis. Paraconsistent logic offers a robust framework for managing conflicting and incomplete information ([4], [2], [1]), which are common challenges in medical diagnostics, particularly among elderly patients with multiple comorbidities [6]. By addressing scenarios where classical logic fails, such as handling contradictions without requiring both α and $\sim \alpha$ to be true simultaneously, paraconsistent logic enables more nuanced decision-making ([7], [5]).

Medical diagnostics are inherently complex, often involving vast and conflicting data, where similar symptoms can lead to distinct diagnoses. This complexity is further compounded by medical iatrogenesis, where treatments inadvertently cause harm. Despite advancements in medical science, diagnostic errors – a significant form of iatrogenesis – persist and demand innovative approaches to improve accuracy ([3]).

By leveraging paraconsistent logic, this work aims to explore the potential of non-classical logics to mitigate diagnostic errors and enhance patient outcomes, offering a promising framework for improving medical decision-making in complex scenarios.

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Fuzzy Logic in Biomedical and Healthcare Applications: An Overview

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Keywords: Fuzzy Logic, Fuzzy Decision-Making, Healthcare Data Analysis

During medical diagnoses and procedures, there are imperfections in the information acquired from patients, such as inaccuracy, randomness, lack of completeness of data or whether the data is considered valid or not ([7]). Due to such a situation, this leads health professionals to make a decision that is as close to what is true regarding the patient's condition as possible, even though there may be such problems mentioned. However, in 1965, a nonclassical multivalued logic, studied and made official by Lofti A. Zadeh, called Fuzzy Logic, began to have applications in several areas, as it is a type of logic that does not only deal with true or false values and premises, but with partial truths, which vary according to the uncertainty of the data and information, trying to reach an exact approximation of what is most real and legitimate ([2]).

Knowing this, this concept and definition could be expanded to the healthcare industry, enabling more precise and less uncertain decision-making when taking into account, for example, patients' mental health, image inaccuracy, randomness of diagnostic data, and many other applications ([4]).

Therefore, the objective of this research is twofold: first, provide an introduction to Logic and its applications in biomedical situation (solely based on [6], [3] and [1]). Second, inspired in [5] and [2], understand how Fuzzy Logic can help with the various inaccuracies that the medical field faces, by understanding its definition more deeply and how it would apply exactly in these cases.

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A Presença da Lógica no Pensamento Cristão Medieval

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Keywords: Lógica, Pensamento medieval, Pensamento cristão.

A divisão usual da História consiste em cinco grandes períodos: Pré-História, Antiguidade Clássica, Idade Média, Idade Moderna e Idade Comtemporânea. Cada período possui características específicas. A Antiguidade, por exemplo, é marcada pela grande influência dos pensadores gregos, percurssores das ciências, como a Matemática, a Física, a Filosofia, entre outras. É neste cenário que surge, a partir dos textos de Aristóteles, a Lógica Proposicional Clássica, a qual continua a ser estudada e discutida até os dias atuais. Posteriormente à Antiguidade, inicia-se a Idade Média, cuja principal característica é a difusão do pensamento cristão. Com o clero assumindo o papel de autoridade central, a academia e o conhecimento centralizaram-se nas mãos dos padres, bispos, monges e demais autoridades religiosas e, por este motivo, por muito tempo acreditou-se que o período medieval representava um retrocesso para a Ciência, carecendo de desenvolvimento intelectual e dos saberes. Contudo, ao estudar a história da Lógica, torna-se perceptível que este pensameto sempre esteve equivocado, uma vez que vários dos pensadores cristãos que viveram naquele contexto dedicaram-se a estudar, aprofundar ou inovar os conhecimentos lógicos existentes até aquele momento. O conjunto de conteúdos lógicos desenvolvidos no decorrer da Idade Média, nomeamos como Lógica Medieval, pode ser dividido em três períodos, os quais recebem os nomes de *Logica Vetus* (Lógica Velha), *Logica Nova* (Lógica Nova) e *Logica Modernorum* (Lógica Moderna). Analisemos os principais trabalhos lógicos desenvolvidos dentro de cada um destes. Atribui-se o título de principal autoridade da *Logica Vetus* a Boécio. Ele exerceu importante influência sobre seus sucessores, tendo em vista que foi responsável pelas traduções do grego para o latim do Organon de Aristóteles e do Isagoge de Porfírio, acrescentando seus próprios comentários a muitas destas traduções. Além disso, mesmo em seus textos que não tratavam especificamente de Lógica, é notório que ele aplicava fortemente a argumentação tópica, ou então apoiava-se no uso de predicáveis e conceitos lógicos para argumentar a respeito de temas metafísicos ou de fé. Dentro do período da Logica Nova, destacamos Anselmo de Cantuária e Pedro Abelardo como os principais nomes. Vivendo em uma época onde a igreja vinha perdendo forças, Anselmo se propõe a estudar a relação entre fé e razão, concluindo que, tendo um caminho a ser conhecido e trilhado, a razão é o que nos permite conhecê-lo, enquanto a fé é o que nos permite trilhá-lo sem desistir. Logo, ambas são necessárias e atuam de forma colaborativa entre si. Ademais, ele também adentra na área da argumentação, visando utilizar exclusivamente a razão em busca de provas da existência de Deus, pois considerava incompreensível um ser racional como o ser humano crer sem compreender os motivos da crença. Pedro Abelardo, por sua vez, é conhecido por sua célebre obra Logica Ingredientibus ("Lógica para principiantes"), em que discute o problema dos universais proposto por Aristóteles.

Por fim, adentramos no período da Logica Modernorum, no qual podemos citar os trabalhos de Tomás de Aquino, Guilherme de Ockham e Jean Buridan. Neste período, a tentativa de relacionar

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ciência e religião se fortalece. Tomás de Aquino, por exemplo, em seu comentário aos "Segundos Analíticos" de Aristóteles, estabelece uma corresponência entre a razão e o espírito humano, diferenciando três operações do espírito, sendo elas respectivamente: a apreensão simples, o juízo e o raciocínio. Partindo para Ockham, nos deparamos com sua Summa Logicae, a qual é dividida em três livros, o primeiro tratando da lingugem e dos termos; o segundo, das proposições formadas pelos termos explorados no primeiro livro, além de apresentar uma teoria sistemática a respeito do "Quadrado das Oposições"; e o terceiro, subdividido em outros dois livros, tratando dos silosgismos e dando início ao que conhecemos como Lógica Modal. Buridan, por sua vez, é conhecido pelo caso do "Asno de Buridan", no qual o autor utiliza o exemplo de um asno faminto que, estando entre dois feixes idênticos de feno, morre de fome por não ser capaz de decidir qual comer. Este caso é utilizado pelo autor para discorrer a respeito da vontade e do juízo humano, concluindo que o juízo é seguido pela vontade e, quando aquele não percebe diferenças entre dois bens, esta não é capaz de decidir entre eles. Contudo o ser humano possui a habilidade de suspender o juízo e, por conseguinte, não morreria de fome estando na mesma situação em que se encontrava o asno. Mediante o exposto, o período medieval não é sinonimo de retrocesso para a Ciência, para a Lógica ou para o conhecimento. Pelo contrário, houve grandes avanços na área da Lógica que devem ser conhecidos e divulgados.

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Type Systems for Concurrent Programming Based in Message Passing

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Keywords: Pi-calculus, Timed session types, Concurrent Programming

Communication in concurrent systems can occur in two ways: access to shared memory or via message passing. The Go programming language encourages using its channels and goroutines (lightweight threads) to communicate by message passing to facilitate the understanding of the flow of execution of a program [2, 4]. However, Go programs can still have common concurrency problems, like mismatch in communication and deadlock – even though a global deadlock detector is present, it is inefficient in complex programs [2]. Go has a type system for its procedural programs, but when it comes to concurrent programs the language relies mainly on the user expertise [5].

To address this kind of problem in communication, we can model concurrent programs in Go using typed process calculi [2, 6], that is, we identify the channels involved in communication, the recursive and/or parallel processes, and the time constraints (if the calculus is sensitive to time). After the specification, we can use a type system to verify whether the processes are well-typed and if their interactions will occur as intended. This work aims to investigate whether concurrent programs written in Go can be translated to the time-sensitive calculus of [1], allowing a static type checking with the time-sensitive session type system.

The work in [2] is an early attempt at creating a framework to detect communication errors and partial deadlocks in concurrent programs in Go. They introduced a process calculus called *MiGo* and presented two examples of concurrent systems modeled in the calculus: one for a Prime Sieve and one that generates the Fibonacci sequence. This calculus is not a real-time system, and it represents the communication patterns of Go programs as behavioural types that guarantee the lack of communication errors [6].

The Pi-calculus is a process calculus that models processes and their interaction through communication channels [3, 9]. In [6], a system composed of a client, a server, and a Raspberry Pi was specified in a variation of the Pi-calculus not sensitive to time [7]. It models a distributed system in which the client can communicate with the server that can communicate with the selected Raspberry Pi, which receives a file sent by the client.

In our previous work [10], we have translated the two examples in [2] and the system in [6] into the untyped asynchronous calculus specified in [1]. As seen in [6], the simple translation of its system to the $s\pi$ calculus (introduced in the work) does not prevent deadlocks or locks from happening: it was necessary to develop a model checker in Maude to detect that [7].

The verification that these three translated examples can be properly typed using the typing rules of [1] is still a work in progress. In the ongoing work, we expect to type more relevant concurrent

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programs in Go modeled with the asynchronous variation of the Pi-calculus to understand what kind of concurrent programs can be statically typed.

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Formalizing the Legendre Symbol in Coq

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Keywords: *Legendre symbol*, *Number Theory*, *Coq*.

In Number Theory, a quadratic residue modulo p is a number a for which there is a number x such that $x^2 \equiv a \pmod{p}$. From this concept can be defined the Legendre symbol, a function described as follows for all $p, a \in \mathbb{Z}$:

$$\left(\frac{a}{p}\right) = \begin{cases} 0, \text{ if } p \mid a \\ 1, \text{ if } a \text{ is a quadratic residue modulo } p \\ -1 \text{ otherwise} \end{cases}$$

Quadratic congruences, i.e., congruences like $x^2 \equiv a \pmod{p}$ (if a is quadratic residue modulo p the equation has a solution), can be solved by algorithms, such as the Tonelli-Shanks algorithm [1]. As many other subjects in Number Theory, those are related to cryptography systems, specially by the use of elipitic curves [2] [3] [4].

Being an alternative for manual proofs, proof assistants are softwares used to verify proofs mechanicaly. The independence of manual verification provided by those softwares turns proofs made with them more reliable. This independence solves the lack of precision existent in manual verification for proofs, what allowed, during history, false theorems to be treated as true [5], as long as nobody found the errors in those proofs.

Coq is a proof assistants known for it's use to proofs of theorems such as the Odd Order Theorem and the Four Color Theorem [6]. This proof assistant relies on being a software simple enough to be verified manually easily. To have this ease Coq is built on what is called Calculus of Inductive Constructions [7].

Naturally, there are a lot of libraries for Coq, containing proofs and other implementations that are commonly used in mathematics. Mathematical Components is one of those libraries, and it is famousily known because it contains the proofs mentioned earlier [6]. However, there are always more content to be added to the library.

The present work brings an implementation for the Legendre Symbol using the Mathematical Components library and following it's patterns for contributions. The motivation to implement this specific content relies in the following reasons: it is not implemented in the library, the content needed for such implementation contains things that are also not implemented and it's implementation opens the possibilty for future works about relevant subjects such as the Tonelli Shanks algorithm. Alongside that this work also brings a discussion about some important implementations, used for it's main purpose, done by Laurent Théry [8].

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Construcción de Lógicas Modales de Orden Superior en Topos

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Palabras claves: Lógica Modal S4, Topos, Álgebra de Heyting, Clasificador de subobjetos, Operador modal.

Desde los trabajos seminales de Lewis en los años 1910, la lógica modal se ha convertido en una herramienta esencial para formalizar el razonamiento sobre necesidad y posibilidad. Siguiendo esta tradición, en el presente trabajo extendemos el análisis al contexto de la teoría de topos, apoyándonos en los aportes de Steve Awodey, Kohei Kishida y Hans-Christoph Kotzsch. Concretamente, introducimos una versión de la lógica modal S4 de orden superior en un topos elemental arbitrario, aprovechando la relación fundamental entre el clasificador de subobjetos, $\Omega_{\mathcal{E}}$, y una álgebra de Heyting completa, $\mathbf{H}$. Mediante la aplicación canónica que une $\mathbf{H}$ con $\Omega_{\mathcal{E}}$, se define un operador modal (comonad) que conserva las propiedades de reflexividad y transitividad características de S4. Esta construcción amplía la lógica intuicionista interna del topos, reemplazando los principios clásicos de extensionalidad por versiones modalizadas. Así, se establece un sólido vínculo entre el marco original y la nueva noción de modalidad, unificando elementos de la lógica modal e intuicionista y creando un puente entre métodos tradicionales y enfoques contemporáneos en el estudio de sistemas lógicos.

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Beyond Classification: The Role of Categorical Logic in Medical Imaging and Pattern Recognition

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Keywords: Category Theory, Machine Learning, Decision Making

Medical imaging and pattern recognition are essential components of modern healthcare, enabling early diagnosis and personalized treatment ([7]). Traditional classification methods often rely on numerical optimization, which can struggle to capture the complex relationships inherent in medical data. In contrast, categorical methods provide a structured and compositional framework for modeling such relationships, offering a novel approach to unify diverse imaging modalities and extract high-level abstractions from datasets ([3], [4], [6]).

Based on [3], [1], [5] and [2], in this work we investigate the intersection of Category Theory, Logic, and Machine Learning to advance decision-making processes in medical imaging and diagnostic scanning. This exploration highlights the power of mathematical abstraction in addressing real-world healthcare challenges, paving the way for novel computational tools in precision medicine. Using the principles of Category Theory, we are investigating and comparing innovative methodologies to address challenges such as data heterogeneity and inconsistencies commonly encountered in medical imaging workflows.

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Modelos Valuados de la Teoría de Conjuntos como Herramienta Filosófica para Comprender la Distinción entre *Verdad* y *Verosimilitud*

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Palabras clave: modelos de Kripke, modelos valuados, retículos residuados, Teeteto, Platón

Dentro de la lógica matemática, un concepto fundamental es el de verdad. Conocer el valor de verdad de una proposición se encuentra en el corazón de esta disciplina. No obstante, ¿conocemos realmente qué es la verdad? Este proyecto busca explorar la respuesta a esta pregunta por la multiplicidad de tal noción a la luz de la lógica intuicionista, y los modelos valuads de la Teoría de Conjuntos. Con el nacimiento de la semántica de Kripke o la dada por un álgebra de Heyting, y al comparar estas semánticas con modelos booleanos, se evidencia en un primer plano el carácter más bien difuso de dicho concepto. Y no es este el único ámbito en el que la noción de verdad tal como la conocemos sufre ataques; podemos rastrear esta problemática a la antigüedad, más precisamente al Teeteto platónico. Allí se yuxtaponen y delimitan la noción platónica de verdad, entendida como aquella que es posible de ser demostrada, y la verosimilitud o la opinión verdadera.

Teniendo en mente la noción de verdad que se gesta al estudiar modelos booleanos de la Teoría de Conjuntos [2], [8]; y su contraste con la construcción de algunos de sus modelos valuados, tal como se realiza en [7], en donde podemos concebir al concepto de verdad como local, dotándolo de plasticidad. Así las cosas, resulta inevitable trazar un paralelo entre ambas situaciones, haciendo más que visible la problemática a tratar. En otros términos, la pregunta que dirigirá este proyecto será: ¿Qué herramientas conceptuales nos puede ofrecer la comprensión de la semántica intuicionista, y los modelos valuados de la Teoría de Conjuntos, para diferenciar los conceptos de verdad y verosimilitud tal como se exponen, por ejemplo, en el Teeteto de Platón?

Nuestra principal inspiración platónica se encuentra repartida en tres diálogos. Cármides, Menón y, especialmente, el Teeteto. Por mor a la claridad de lo que deseamos presentar, resumimos de forma muy somera las consideraciones de verdad y la forma en cómo podemos llegar a ella. La verdad, absoluta e independiente, está fuera de nuestro alcance. Esta verdad no puede, por definición, ser encontrada ni hacer parte de ningún sistema que esté en movimiento o en constante cambio. La verdad, afirma Platón en este diálogo, no podemos conocerla a razón de dos dificultades. Por una parte, la verdad, por definición, no es aprehensible por ningún medio que, al contrario que ella, esté en movimiento y, por tanto, resulte finito y cambiante. Por cuanto somos finitos y no podemos agotar la verdad, sino tan solo dar imágenes móviles o inacabadas de ella, es que decimos que no damos con la verdad. En contraparte, la noción de verosimilitud u opinión verdadera nos sale inevitablemente al paso. La opinión, es justamente una imagen individual que, por definición, admite su dependencia en el sujeto que la formula y además su limitación frente a la verdad. La opinión, decimos puntualmente, se refiere a una interpretación del sujeto. Esta imagen limitada, que pone especial acento en la finitud del sujeto, puede ser verdadera; es decir, puede participar de la verdad

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sin agotarla. Es en este sentido que cabe entender también el término verosimilitud. Tanto la opinión verdadera como la verosimilitud son limitadas, finitas y correspondientes con nuestra naturaleza, sino que, principalmente, si son coherentes con una concepción correspondentista de la verdad; pues, cabe siempre comparar estos casos verosímiles o estas opiniones verdaderas con los hechos que se nos presentan.

A lo anterior cabe tan solo agregar que la concepción de opinión verdadera dispone de un especial aspecto interpretativo que, en último término, no es más que un claro énfasis a la finitud del intérprete: el sujeto. A, su vez, lo verosímil pretende resaltar la idea de concordancia y correspondencia con una verdad que no es del todo agotable o aprehensible, pero que, en medio de su finitud, no resulta una mera bagatela. Lo verosímil es la mayor expresión de verdad a la que podemos aspirar en tanto finitos y en tanto incapaces de agitar la verdad.

En suma este proyecto busca mostrar cómo construcciones lógicas, tales como las subyacentes a modelos valuados de la Teoría de Conjuntos, permiten servir como herramientas para explorar respuestas a preguntas fundamentales de filosofía de la lógica, en especial a las relacionadas con la naturaleza del concepto de verdad.

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A Computer-Assisted Formalization of the Modal Interpretation of Intuitionistic Logic

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Keywords: Translations between logics, Modal Logic, Intuitionistic Logic, Roqc

Translations between logic systems are functions that map sentences from one system to another while preserving certain predefined properties. These translations serve as a means of interpreting one system in terms of another, with the preserved properties varying in strength across the literature. The first known translation between logic systems was an embedding of classical logic into intuitionistic logic, introduced by Kolmogorov [1]. This translation, often referred to as the double-negation translation, aimed to show that the principle of the excluded middle does not lead to contradiction [2].

Another well-known translation is the interpretation of intuitionistic logic within the modal system S4. This embedding was first presented by Gödel [3], motivated by the idea that the modality of possibility could be understood as a modality of constructivity [4]. This interpretation will be the focus of this work, specifically the two equivalent translations presented by Troelstra and Schwichtenberg [4], along with their proofs of soundness and faithfulness.

Modal logic and intuitionistic logic are two distinct branches of formal logic. Intuitionistic logic—developed based on the work of Brouwer—focuses on constructive proof methods, where the truth of a proposition is tied to the existence of a proof. Thus, unlike classical logic, intuitionistic logic does not accept the law of excluded middle as universally valid, emphasizing a more constructive notion of truth. On the other hand, modal logic extends classical logic by introducing modal operators, typically *necessity* and *possibility*, to express statements about what is necessarily or possibly true [4].

To ensure a rigorous formalization of the aforementioned proofs, this work will utilize the Rocq theorem prover—formally known as Coq—to mechanize the reasoning behind the discussed translations. The use of a proof assistant such as Coq not only guarantees correctness but also enables automated verification of key lemmas and theorems [5]. These proofs will make extensive use of several metatheoretical results, among which the deduction metatheorem for modal logics plays a central role in simplifying derivations and making them more intuitive.

Furthermore, this work will build upon the modal logic library initially developed by Silveira et al. [6] and later expanded by Nunes, Roggia, and Torrens [7]. By extending this existing formalization, the goal is to provide a robust framework for reasoning about modal embeddings and their computational interpretations. This contribution aims to facilitate further research in modal logic, constructive mathematics, and their applications in proof theory and programming language semantics.

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Comprehending the Completeness of Grove's Systems of Spheres for AGM

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Keywords: AGM, Grove Spheres, Formal Epistemology

The Poster presents the first outcome of my ongoing scientific initiation research financed by FAPESP. The research is about means by which modal logic may model AGM axioms (also known as 'Belief Revision' axioms). Those axioms seek to represent 3 dynamic belief processes done by belief agents: Expansion, a process in which one comes to believe in something that doesn't contradict any previous belief; Contraction, a process in which one comes to disbelief in something previously believed and Revision, a processes in which one comes to believe something that may contradict a previous belief (revisions also involve giving up some beliefs in order to add the new one). On the other hand, epistemic logics, especially dynamic ones, use modal logic to represent similar phenomena. It is known that they can be used to model AGM. My research studies how it is done and to what extent it is possible. It also seeks to present some formal contributions to the field.

The first result is a comprehensive, detailed, and rigorous demonstration of the completeness of Systems of Spheres for the modeling of AGM-like revisions. Spheres are sets of maximally consistent theories. Systems of spheres are sets of spheres that are totally ordered by the $\subseteq$ relation. Revision operations can be captured by set-theoretic operations between spheres of a system. The similarity between maximally consistent theories and possible worlds makes Systems of Spheres the first approximation between modal logic and AGM (the first I know, at least). Systems of Spheres were first presented by Adam Grove on [2]. Sometimes those systems are called by referring to their creator ('Grove Systems', 'Grove Spheres', etc.).

It is a known fact that Systems of Spheres are complete relative to AGM-like revisions. But the 3 main sources used for studying those systems ([1–3]) it is either not proved ([3]), partially proved by taking some nontrivial steps as obvious ([2]) or it is properly proved but with some reduction to absurd and by referring to previous theorems ([1]). I, therefore, consider that it would be helpful to present a proof based on the one found in [1, pp. 296-299] but without referencing previous theorems (only axioms) and changing the proofs by absurd to direct proofs. In this way, I wish to make the completeness of Grove Spheres as clear and comprehensible as possible.

My Poster, then, will have:

1. The 8 AGM axioms that define what is an AGM-like revision as it is presented in [3].
2. The definition of a System of Spheres and a revision on Systems of Spheres as presented in [2].
3. My proof of the completeness of revisions on Systems of Spheres relative to AGM-like revisions.
4. One or more schemata for the sake of elucidation.
5. One example of a revision operation on Systems of Spheres.

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Uma Abordagem *Fuzzy* via Sistema Especialista

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Palavras-chave: Conjuntos Fuzzy; Sistemas especialistas; Sistema Baseado em Regras Fuzzy.

Na década de 1960, Loft A. Zadeh [4] introduziu a teoria dos conjuntos *fuzzy*, também conhecidos como conjuntos nebulosos ou difusos. Essa teoria se apresenta como uma alternativa à teoria tradicional dos conjuntos, sendo mais flexível e menos rígida. Nela, a transição de pertinência para não pertinência de um elemento em um conjunto *fuzzy* ocorre de forma gradual, ao contrário das abordagens convencionais.

Entende-se um conjunto *fuzzy* como uma função em um domínio V, em um universo de discurso, no intervalo real [0,1]. Assim, possibilita-se a passagem da pertinência para a não pertinência de maneira não abrupta, como ocorre nos conjuntos usuais.

Um conjunto *fuzzy* A_f é dado através de uma função $f_A : V \to [0,1]$, em que o conjunto V é o conjunto universo ou domínio do conjunto *fuzzy*, $[0,1]$ é um intervalo de números reais e f_A é a função de pertinência de A_f. Denota-se:

$$\begin{aligned} f_A : \quad & V \to [0,1] \\ & x \to f_A(x) \end{aligned}$$

Um conjunto *fuzzy* A_f é denotado por um conjunto de pares ordenados, em que o primeiro elemento pertence a V e o segundo elemento indica o seu grau de pertinência em A_f.

$$A_f = \{(a, \mu)/f_A(a) = \mu \text{ e } \mu \in [0,1]\}.$$

De maneira semelhante à extensão do conceito de conjunto usual, Zadeh também propôs uma adaptação da lógica clássica, a lógica *fuzzy*, que nos permite presumir soluções e resoluções de problemas, mesmo quando encontradas imprecisões nos dados e/ou informações e que nos permite um tipo de raciocínio aproximado diante de termos vagos ou ambíguos.

A lógica *fuzzy* foi construída, inicialmente, através de conceitos já estabelecidos na lógica clássica, mas de modo a ampliá-la e permitir raciocínios aproximados. Segundo Tákacs [3], o raciocínio sob termos imprecisos da linguagem é fundamental na lógica *fuzzy*, pois é um sistema baseado em regras. O raciocínio aproximado pode ser descrito por meio da lógica *fuzzy*, pois permite estabelecer o controle do sistema com base na representação do conhecimento através de regras do tipo "se..., então".

Para Shaw e Simões [2], a característica principal da lógica *fuzzy* é oferecer aos pesquisadores uma nova maneira de trabalhar com informações imprecisas. A lógica *fuzzy* nos permite traduzir em valores numéricos as expressões verbais, vagas, imprecisas e qualitativas, encontradas na

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comunicação humana, o que possibilita a conversão da experiência humana em uma linguagem decodificável por computador.

O estudo de problemas e situações reais utilizando a linguagem matemática para compreendê-los, simplificá-los e solucioná-los, em busca de uma possível revisão e modificação do objeto em estudo, é parte do processo da modelagem matemática. Devido a possibilidade de manipulação de informações incertas e de seu respectivo armazenamento em computadores, o tratamento *fuzzy* de variáveis linguísticas subjetivas, referentes ao sujeito, ganhou um espaço substancial na modelagem matemática. Assim, toda tecnologia baseada no "enfoque *fuzzy*" ganha aplicabilidade prática, permitindo que a experiência e a intuição de controladores humanos sejam incorporadas em sistemas de controle computadorizado.

Para Castillo e Melin [1], sistemas especialistas que utilizam lógica *fuzzy* vêm sendo aplicados com sucesso nos problemas de decisão, controle diagnóstico e classificação, pois esses sistemas possuem a capacidade de gerenciar o raciocínio complexo presente nas áreas de aplicação. Os sistemas especialistas são compostos por uma base de dados, ou base de conhecimento, sendo um banco de informações retiradas de um domínio em estudo por especialistas, onde é representado o conhecimento possuído por eles sobre o domínio do problema, contendo os dados e as formas de condução para identificação e solução de um determinado problema e um mecanismo de inferência (raciocínio), que atua como um processador e trabalha com as informações fornecidas pela base de dados em função dos dados do problema abordado. Além disso, utilizam-se regras (*fuzzy*) através do mecanismo de inferência para lidar com a base de dados.

Desse modo, esta pesquisa em andamento possui como objetivo geral a análise do desempenho acadêmico dos estudantes da UNESP, investigando as diferenças entre aqueles que ingressaram através do sistema universal e os que ingressaram por meio dos sistemas de cotas, a partir de um banco de dados fornecido pela universidade. Tal análise será realizada a partir de uma modelagem via Sistema Baseado em Regras *Fuzzy* (SBRF) no qual foi adotado o método Mamdani.

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Modelo Canônico Enumerável para a Lógica FOS4

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Palavras-chave: lógica **FOS4**, modelo canônico, topologia

A lógica **FOS4** é axiomatizada com os esquemas de axiomas da lógica quantificada de primeira ordem com identidade, suas regras de inferência e os axiomas da lógica **S4**, com suas regras de inferência aplicadas às fórmulas da linguagem de primeira ordem estendida com a introdução do(s) símbolo(s) modal(is). Em [1] demonstra-se a completude de **FOS4** em relação à classe de feixe-interpretações (interpretações fibradas com estrutura topológica). Tal demonstração utiliza uma adaptação do método de Henkin (henkinização preguiçosa) e das propriedades categoriais do *topos espacial* (categoria de feixes construída a partir de uma slice categoria de SET).

São conhecidos alguns resultados de completude desta lógica para certas estruturas, como nas álgebras Lebesgue-mensuráveis - ver [3] . Contudo, procuro neste trabalho exibir o modelo canônico enumerável para esta lógica, técnica que pode ser utilizada para investigar demonstrações de completude para outras lógicas modais ou avaliar as propriedades topológicas do modelo, o que pode iluminar certas propriedades da lógica a partir de suas propriedades semânticas. Para isso, consideremos os seguintes resultados publicados na tese [1].

1. Se $\mathcal{L}$ for uma linguagem (finitária e enumerável) de primeira ordem com operadores não clássicos, serão chamadas de **estruturas de relação de satisfação clássicas no sentido estendido** aquelas estruturas para esta linguagem que obedecem as mesmas propriedades clássicas de satisfação para as fórmulas fechadas, abertas e substituição de termos para todas as fórmulas, inclusive aquelas com operadores não clássicos.

2. A noção de interpretação para as fórmulas da linguagem $\mathcal{L}$ a partir destas estruturas de satisfação clássica no sentido estendido satisfaz:

a. Se $\mathcal{L}^\theta$ for uma tradução da linguagem não clássica para uma linguagem clássica, adicionando símbolos de predicado para cada classe de equivalência de fórmulas da linguagem em que um operador não clássico é o operador principal;

b. Então esta interpretação das fórmulas não clássicas é igual à união disjunta (fibrada) das interpretações das fórmulas da linguagem clássica traduzida de $\mathcal{L}$ por θ;

c. Se $\Box$ for um dos operadores não clássicos da linguagem $\mathcal{L}$, então este operador será interpretado com o operador topológico de interior. Nesta situação, temos uma feixe interpretação para as fórmulas da linguagem não clássica.

Teorema 1. [Teorema Awodey-Kishida] seja Γ uma teoria $FOS4$-consistente em $\mathcal{L}$. Existe X espaço topológico e $(\pi, [\![-]\!])$ uma feixe interpretação tal que para toda fórmula $\varphi \in For(\mathcal{L})$:

$$\Gamma \vdash_{FOS4} \varphi \text{ se e somente se } [\![\bar{x} \mid \varphi]\!] = D^n$$

Construção do Modelo Canônico Enumerável

Seja $\mathcal{L}$ linguagem de primeira ordem em que $\Box$ é o único operador não clássico. Nesta situação simplificada, proponho a seguinte construção (para mais detalhes, consultar [2]):

1. Para cada conjunto de sentenças classicamente consistente Γ de fórmulas de $\mathcal{L}^\theta$, existe Δ um conjunto consistente e maximal que estende Γ e, para tal Δ, existe M uma $\mathcal{L}^\theta$-estrutura que

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satisfaz as propriedades clássicas de satisfação. Usamos o teorema de Lowenhein-skolem para obter uma estrutura, a partir dela, de cardinalidade no máximo enumerável.

2. Seja $\Phi = \{\Delta \subset sent(\mathcal{L}^{\theta}_{\aleph_0}) \mid \Delta$ é consistente maximal $\}$, para $\mathcal{L}^{\theta}_{\aleph_0}$ a linguagem estendida de $\mathcal{L}^{\theta}$ pelo processo de henkinização preguiçosa. Definimos para cada Δ o conjunto quociente D_{Δ} das constantes da linguagem por meio de uma relação de equivalência tal que $c_{\gamma_1} \sim c_{\gamma_2}$ se e somente se $c_{\gamma_1} = c_{\gamma_2} \in \Delta$.

3. Para cada $\Delta \in \Phi$ constrói-se o modelo canônico M_{Δ}. Tomando todos estes modelos canônicos, construímos o conjunto M^* (com a noção de satisfação induzida).

Proposição 1. Dadas duas relações de satisfação quaisquer sobre os modelos em M^*, é possível construir duas novas relações de satisfação com domínios disjuntos e isomórficas, cada uma delas, às duas relações originais. Basta tomarmos como novo domínio de M_{Δ} o conjunto dos pares ordenados formados pela classe de equivalência de 2. na primeira posição, e por Δ na segunda posição do par; denominaremos tal conjunto de D^*_{Δ}

4. Seja $\mathcal{M}^{\theta}$ o conjunto de relações de satisfação (clássica) para cada $\Delta \in \Phi$ e de domínios disjuntos, de acordo com a Proposição 1., com domínio no máximo enumeráveis. Em [1] demonstra-se $[\![\varphi]\!] = [\![\theta(\varphi)]\!]^{\theta} = [\![\theta(\varphi)]\!]^{\theta}_{\aleph_0} = \Sigma_{M_{\Delta} \in M^{\theta}} [\![\theta(\varphi)]\!]^{\theta}_{M_{\Delta}}$, e portanto $(\pi, [\![-]\!])$ é feixe interpretação para as fórmulas de $\mathcal{L}$.

Proposição 2. Se duas relações de satisfação concordam em todas as fórmulas de $\mathcal{L}^{\theta}_{\aleph_0}$, então elas são construídas sobre o mesmo $\Delta \in \Phi$, ou seja, são idênticas.

Lema 1. Para todo $\Delta \in \Phi$, o cojunto $\theta^{-1}(\Delta)$ é conjunto $FOS4$-consistente e maximal de sentenąs de $\mathcal{L}_{\aleph_0}$.

Definição 1. Seja Ξ a coleção dos conjuntos de sentenças $FOS4$-consistente e maximal de $\mathcal{L}_{\aleph_0}$.

Lema 2. Para todo $\Delta \in \Xi$, o cojunto $\theta(\Delta)$ é conjunto consistente e maximal de sentenąs de $\mathcal{L}^{\theta}_{\aleph_0}$.

Teorema 2. Existe uma bijeção entre Ξ e Φ.

Proposição 3. Se duas relações de satisfação cássicas no sentido estendido concordam em todas as fórmulas de $\mathcal{L}_{\aleph_0}$, então elas são construídas sobre o mesmo $\Delta \in \Xi$, ou seja, são idênticas.

Definição 2. Seja $W = \{\Delta \mid \Delta \in \Phi\}$ e $\mathcal{D}$ a união (disjunta) dos domínios D^*_{Δ}. Seja $\pi : \mathcal{D} \to W$ tal que $\pi(a) = \Delta$ se e somente se $a \in D^*_{\Delta}$ (lembrando que a é um par ordenado da forma $([c_i], \Delta)$).

Para toda fórmula $\varphi \in For(\mathcal{L})$ temos $[\![\bar{x} \mid \varphi]\!] = \Sigma_{M_{\Delta} \in \mathcal{M}^{\theta}}\ [\![\bar{x} \mid \theta(\varphi)]\!]^{\theta}_{M_{\Delta}}$.

Interpretando $[\![\Box]\!]$ como o operador interior, então:

$$\mathcal{M} = (W, \mathcal{D}, \{\{c^{M_{\Delta}}\}_{\Delta \in \Phi}\}, \{\{f^{M_{\Delta}}\}_{\Delta \in \Phi}\}, \{\{R^{M_{\Delta}}\}_{\Delta \in \Phi}\})$$

é **Modelo topo $FOS4$-canônico enumerável**.

Teorema 3. O espaço base do fibrado deste modelo canônico da Definição 2. é homeomórfico ao *frame* do modelo canônico da lógica **S4**.

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Formalizing the LFI1 Logic in the Rocq Proof Assistant

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Keywords: Rocq, paraconsistent logic, LFI1, logics of formal inconsistency.

Paraconsistent logics are a family of logics in which the presence of contradictions does not necessarily lead to triviality. In other words, they are logical systems that do not always respect the principle of explosion, defined as $\alpha \to (\neg\alpha \to \beta)$ [2]. Traditionally, in classical logics, every inconsistent theory — that is, a theory containing two sentences of the form $\{\alpha, \neg\alpha\}$ — is also a trivial theory (a theory containing every sentence of the language). Thus, paraconsistent logics are regarded as tools that allow the treatment and study of contradictions without trivialization [3].

The logics of formal inconsistency (**LFI**s) are a family of paraconsistent logics that introduce concepts of consistency and inconsistency at object level in their language, as a way of representing the excess of information (evidence for α and evidence for $\neg\alpha$). By doing this, these systems are capable of rescuing triviality in a *controlled* manner. The **LFI1** logic is a logic of formal inconsistency that introduces the concept of consistency by adding the consistency operator (denoted by $\circ$) in its language and establishing an axiom known as the principle of gentle explosion, defined as $\circ\alpha \to (\alpha \to (\neg\alpha \to \beta))$ [3]. In this logic, an information is consistent if it and it's negation are not simultaneously true, that is, given an information α, its consistency $\circ\alpha$ is true if, and only if, α is false or $\neg\alpha$ is false.

The motivations for studying and developing paraconsistent systems can be found across many areas of knowledge [10], such as natural sciences [1], linguistics [8] and computer science. In the context of computer science, the use of logics of formal inconsistency is very useful for modelling and developing evolutionary databases [4].

Proof assistants are tools in the field of formal verification that ensure a program is correct according to a formal specification. This is achieved through the development of proofs using mathematical methods to verify the correctness of a software's property [5]. Traditionally, the validity of proofs is verified manually by evaluators, who follow the author's reasoning and give a verdict based on how convincing the proof is. Proof assistants emerge as alternatives to manual verification, allowing the user to verify proofs as they are developed, making this process easier and more reliable [9].

Proof assistants such as Rocq, Lean, and Isabelle allow users to define and prove properties about mathematical objects with computational value [7]. In the present work, the Rocq proof assistant (a well known proof assistant based on the calculus of inductive constructions, formerly known as Coq) will be used for implementing a library of the **LFI1** logic, as well as for developing proofs of the metatheorems about this logic, such as deduction, soundness, completeness and equivalence between the bivaluation and the matrix semantic systems, similar to what was achieved in [6]. By the time

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of the submission of this work, we've implemented the language, syntax and semantics of **LFI1** and proved the aforementioned metatheorems, with the only pending proof being the existence of a bijection between natural numbers and the language of **LFI1** (which we've postulated as an axiom).

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O Teorema de Ax-Kochen e suas Implicaçõees na Lógica Matemática e Álgebra

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Palavras-chave: Teorema de Ax-Kochen, Teoria dos Modelos, Teoria dos Corpos Valorados.

Este trabalho tem como objetivo apresentar os conceitos fundamentais relativos à compreensãoo do Teorema de Ax-Kochen –também conhecido como Teorema de Ax-Kochen-Ershov– e suas principais aplicações na Lógica Matemática e na Álgebra.

Nesse sentido, analisamos como o Teorema de Ax-Kochen [3], o qual define, para cada grau d, um número limite n_d. Se um primo p é maior ou igual a n_d, então corpo $\mathbb{Q}_p$ satisfaz a condição de ser $C_2(d)$. Basicamente, isso significa que qualquer polinômio homogêneo de grau d, quando tem mais variáveis do que d^2, tem pelo menos uma solução não trivial em $\mathbb{Q}_p$.

No contexto da lógica matemática, a demonstração do Teorema de Ax-Kochen faz uso de técnicas importantes da Teoria dos Modelos como eliminação de quantificadores, decidibilidade e modelo completude no contexto da análise de sentenças que envolvem expressões polinomiais: [4]. Ademais, a noção de corpos $\mathbb{Q}_p$ é central em diversos ramos da lógica aplicada à teoria dos números, já que permite analisar estruturas p-ádicas e comparar fenômenos locais e globais em equações diofantinas.

Do ponto de vista algébrico, a condição $C_2(d)$ exibida por $\mathbb{Q}_p$ (para p suficientemente grande) está em conexão com o estudo de formas quadráticas, conectando-se ao estudo de cohomologia de Galois e propriedades de corpos: [1], [7], [9], [5]. Esses resultados mostram como certas equações podem ou não ter soluções em corpos locais, trazendo implicações profundas para anéis de Witt, formas quadráticas e outras construções na teoria de corpos: [10], [8].

Em síntese, o Teorema de Ax-Kochen se localiza na interseção de vários tópicos e aplicações entre áreas como teoria dos números, teoria dos modelos, geometria algébrica e teoria algébrica de formas quadráticas, servindo de inspiração para potenciais pesquisas em temas emergentes, como a cohomologia de Galois para teorias abstratas de formas quadráticas: [6], [2], [11], [12].

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Exploring Paraconsistent Logic for Clinical Clustering in Healthcare

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Keywords: Paraconsistent Logic, Paraconsistent Annotated Logic, Decision-Making, Clinical Clustering

The integration of advanced computational methods into healthcare data analysis has enabled new opportunities to uncover patterns and improve clinical decision-making. While the use of non-classical logics, particularly Paraconsistent Logic, remains relatively unexplored, it offers significant insights for addressing contradictions and incomplete information, challenges that are ubiquitous in healthcare datasets, as discussed in [8].

By works such as [4], [1], [8] and [6], in the present work we explore the applications of Paraconsistent Logic to clinical cases using real-world datasets, including public health records from the Brazilian healthcare system. The methodology compares classical approaches—such as principal component analysis, discriminant analysis, and clustering methods—with algorithms for annotated Paraconsistent Logic, as discussed in [7] and [1]. This approach has the potential to improve clinical decision making by identifying hidden correlations and inconsistencies that traditional logic-based systems may overlook.

Future research could extend this work by integrating advanced Paraconsistent Logics, including swap structures [5], twist structures [3], and multialgebraic frameworks [2], with modern machine learning systems, further enhancing its applicability to real-world clinical settings.

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GREAT: Gerador Automático de Exercícios de Lógica Proposicional

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Palavras-chave: Fundamentos da Computação, Lógica Proposicional, Ensino de lógica, Geração Automática de Exercícios.

Em uma perspectiva educacional, a resolução de exercícios é essencial para fixação, aprendizado e avaliação dos alunos. No contexto de turmas numerosas e heterogêneas, as ferramentas de *Automatic Question Generation* (AQG) surgem como solução para auxiliar os docentes na produção de tarefas individuais e personalizadas [1], permitindo um melhor acompanhamento pedagógico dos alunos [2], bem como evitando plágios, desonestidade acadêmica e o uso irrefletido de LLMs na resolução de exercícios [3].

Neste trabalho, introduzimos o GREAT, um gerador de exercícios capaz de gerar conjecturas em lógica proposicional para tarefas de demonstração em Dedução Natural, e refutação por contra-exemplo. Os parâmetros configuráveis são o número de átomos proposicionais; quais os conectivos lógicos (negação, conjunção, disjunção, implicação e bi-implicação); qual o intervalo de complexidade das proposições, medido pelo número de ocorrências de conectivos lógicos; e, por fim, o número de premissas para a conjectura. Como forma de acrescentar requisitos de relevância para a geração das conjecturas, nós adicionamos as seguintes configurações opcionais:

- as premissas são relevantes: cada premissa compartilha com pelo menos um átomo proposicional com a conclusão do argumento;
- as premissas são necessárias: a conclusão não será satisfeita sem qualquer uma das premissas;
- a conjunção das premissas é contingente: a conjunção das premissas não são tautológicas, nem contraditórias.
- as conjecturas geradas serão proporcionais, apenas refutáveis, apenas demonstráveis, ou aleatoriamente geradas.

A partir dos parâmetros selecionados, o GREAT consiste na geração aleatória de proposições que atendam ao número de átomos distintos e aos requisitos de relevância. Para cada proposição escolhida como conclusão, o GREAT, inicialmente, gera novas premissas que sigam os parâmetros e requisitos de relevância indicados. A satisfatibilidade dessa conjectura é verificada através de um

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SAT Solver. A conjectura é adicionada ao conjunto de exercícios demonstráveis caso a conjunção de suas premissas implique na sua conclusão e, caso contrário, é adicionada ao conjunto de exercícios refutáveis.

Ao contrário de outra proposta de AQG [4], que se limitam à geração de equivalências proposicionais, o GREAT possibilita a geração de tarefas com premissas e conclusão a serem demonstradas em diferentes formatos, tais como nos assistentes Coq via ProofWeb ou Lean, além dos formatos LaTeX ou Unicode. A ferramenta foi implementada em PHP e JavaScript (disponível em `http://github.com/terrematte/great`). Atualmente, o GREAT é utilizado em disciplinas de fundamentos matemáticos para computação para a geração de conjecturas proposicionais a serem demonstradas ou refutadas no assistente Coq via TryLogic [5].

Como trabalho futuro, vamos implementar a possibilidade dos usuários definir seus próprios templates para distintos assistentes de demonstração de teoremas. Além disso, iremos otimizar o algoritmo de execução do gerador. E também vamos incorporar a implementação de uma estratégia de geração de exercícios similares [6].

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Syllogistic Proofs via Aristotelian Reduction and Isabelle/HOL: A Comparison of Results

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Keywords: syllogistic, Isabelle/HOL, syllogistic reduction, automation, Dictum de omni et nullo

Isabelle/HOL is a generic proof assistant based on higher-order logic which is widely used for formalization of mathematics [6]. The prover has also been used for formalizing both the foundations [2], [3], [4], [5] and historical extensions [5] of Aristotelian syllogistic. The present work is aimed at describing how Isabelle's automation tool Sledgehammer [1] was used to find all alternative direct syllogistic proofs which were reported in the literature [9], additionally showing that a practical application of an interactive theorem prover can bring philosophically interesting results.

Proofs in syllogistic were at the center of attention from the very beginning of its existence. One of the metatheorems proved by Aristotle in his *Prior Analytics* is the later-called *Dictum de omni et nullo*, stating that all syllogistic deductions (moods) can be ultimately reduced to (resp. proved by) the two universal deductions in the First Figure: *Barbara* and *Celarent* (for a more nuanced account see e.g., [7]). Although Aristotle recognizes only one reduction procedure for each mood [8], almost all syllogistic moods can be proved directly (i.e., by applying other mood on a variation of its premises) in more than one way, as it has been showed in [9].

Building on the formalization of Aristotelian Assertoric Syllogistic done by Angeliki Koutsoukou-Argyraki [2], [3], [4] and on the joint work of A. K.-A. and myself [5], I show how Isabelle was used to find all the direct syllogistic proofs which I have earlier recognized in [9] and thus replicate the results obtained by hand. Moreover, I show how all of the proofs done by Isabelle were found by pure automation, i.e., without me giving any external hints to the prover with respect to the proof-procedure.

Beside presenting the full list of proofs obtained both by myself and by Isabelle and comparing them, I further present some metatheoretical results stemming from the listed proofs and comment on how they correspond with those present in Aristotle. Especially, I show that Isabelle's counterexample tools, Nitpick and Quickcheck, were able to correctly spot all the proof-cases in which additional existential assumptions are needed for a proof to be valid. Those cases, aligning with the principles reported in [10], were once again found by automation.

Finally, I claim that the successful application of Isabelle/HOL to syllogistic can serve as a twofold reassurance: first, that the prover can by itself correctly recognize and provide all the details reported in the literature of the subject; second, that the correctness of the results present in the literature can be further reinforced by the use of the prover. I also briefly comment on the possibility of future research, namely using Isabelle's automation to confirm the correctness of the *indirect* proofs found in [9], as well as of formalising the modal syllogistic.

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Three Proofs for a Theorem by Scheepers and Tall

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Keywords: Set Theoretical Topology, Cardinal Functions, Infinite Games, Elementary Submodels.

In the late 1960's, Arhangel'skii solved a long standing problem proposed by Alexandrov and Urysohn almost 50 years earlier, which was if every first-countable compact Hausdorff topological space had its cardinality limited by $\mathfrak{c}$, the cardinality of the real line. However, Arhangel'skii's results went beyond that, and he actually proved that for every Hausdorff space X, $|X| \leq 2^{L(X)\chi(X)}$, $L(X)$ and $\chi(X)$ being the Lindelöf degree and the character of X, respectively. This result gave a much more complete answer to Alexandrov and Urysohn's problem, but also raised its own questions, such as the one that Arhangel'skii himself asked, if it was possible to obtain a similar bound for the class of Lindelöf spaces with points G_δ. It was proven by Gorelic that such a bound could not be obtained, since he proved that the existence of Lindelöf spaces with points G_δ and arbitrarily large cardinalities was consistent with ZFC.

In this context, Scheeper and Tall's theorem provides a partial answer to Arhangel'skii's question. More specifically, it employs infinite games to define a topological property that, in place of the Lindelöf property, provides a bound to the cardinality of spaces with points G_δ. This result is interesting not only because it points to a way to answer Arhangel'skii's question, but also calls for the investigation of topological properties defined by infinite games and possibly for similar results with weaker assumptions.

Another tool that's been of great use to the theory of cardinal invariants in set theoretical topology is the idea of elementary submodels. It's an application of the Lowenheim-Skolem-Tarski theorem to construct sets that allow us to do "enough set theory" under a certain cardinality and then obtain bounds for the size of topological spaces in different classes. This poster aims to present three different proofs of Scheepers' and Tall's theorem; the original and two more using elementary submodels in order to showcase how topology, set theory and logic work together.

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Um Estudo Diagramático de Regras de Inferências Específicas da Conceitografia

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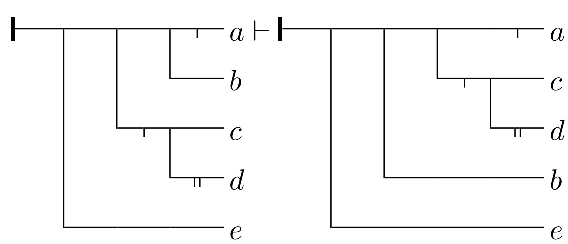
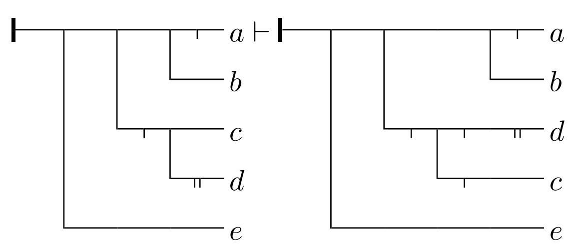
Palavras-chave: Conceitografia, Lógica Diagramática, Frege

No livro *Conceitografia* [2], Frege apresenta e emprega uma notação lógica bidimensional, que ele também apelida conceitografia. É curioso que, apesar do sucesso das obras de Frege, sua notação única foi praticamente esquecida, não parecendo ter sido utilizada por nenhum grande autor posterior a ele. Em parte por causa de dificuldades tipográficas, em parte por causa de sua originalidade excessiva, é fato que hoje em dia particamente não se estuda a conceitografia senão para ler a "escrita antiquada"de Frege e nada mais. Porém, isso não quer dizer que não há características internas dessa linguagem que vale a pena estudar.

Em [1], Schlimm desafia o repúdio pela conceitografia mostrando várias vantagens que ela tem oposição às notações mais usadas. No artigo, entre a falta de parênteses, a disposição clara dos conectivos e a facilidade de traduzir operadores lógicos, uma idiossincrasia da conceitografia salta aos olhos: Frege distribui pelos seus trabalhos regras de inferência facilmente visualizáveis e aplicáveis na linguagem. Schlimm aponta quatro delas. *Troca:* Quaisquer dois termos inferiores de uma fórmula condicional podem ser trocados – Exemplo:



Transposição: Podemos trocar um condicional por seu contrapositivo – Exemplo:



Adição de Dupla Negação e *Subtração de Dupla Negação:* Autoevidentes. Tais regras foram criadas pensando especificamente na notação bidimensional da *Conceitografia*; não é à toa que é no mínimo complexo definir uma regra similar à *Troca* para a escrita linear mais utilizada hoje (mas talvez não mais complexo do que seria efetivamente aplicá-la).

Neste trabalho, pretende-se investigar qual o poder e aplicabilidade das regras de inferência acima, assim como talvez de outras do mesmo tipo. Como tais regras são feitas para serem aplicadas em diagramas bidimensionais, isto será um estudo de lógica diagramática.

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Sobre a Identidade nas Fundamentações Lógicas das Teorias Científicas: Um Olhar através do Desenvolvimento do Método Axiomático

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Palavras-chave: Axiomatização, Fundamentação, Identidade, Lógica, Teorias Científicas

Aqui, pretende-se esboçar um breve excerto em três eixos principais, os quais podemos apontar primeiramente sobre os seguintes termos: i) fundamentação lógica das teorias científicas, ii) método axiomático e seu particular desenvolvimento pelo séc. XX e início do séc. XXI, iii) questões de identidade na Matemática. O objetivo é uma primeira investigação nos passos dados quanto à fundamentação lógica das teorias científicas com atenção e consciência do desenvolvimento recente e ativo do método axiomático, objeto sensível nesta empreitada, acrescendo-se um destaque às questões de identidade na Matemática – tanto enquanto teoria(s), quanto ferramenta a tal empreitada. Neste sentido, será usada de base o livro "*The Logical Foundations of Scientific Theories: Languages, Structures, and Models*" – doravante, *LFST* – de Décio Krause e Jonas R. B. Arenhart [1], somada à leitura selecionada dentro da obra "*Axiomatic Method and Category Theory*" – doravante, AMCT – de A. Rodin [2] acerca dos aspectos históricos e filosóficos do método axiomático e também acerca das questões um tanto desapercebidas da identidade na Matemática.

Assim, em se destacando os principais pontos a serem levantados de *LFST* em interesse do presente trabalho, destacam-se: i) a disputa histórica entre abordagens semânticas e sintáticas no tratamento lógico-formal das teorias científicas – particularmente trazendo as questões de identidade levantadas nelas; ii) críticas, e respectivas respostas, ao método axiomático; iii) a perspectiva adotada pelos autores com respeito a isso – abordagens como *ferramentas filosóficas*–, e, em particular, a apresentação das duas abordagens de axiomatizações exemplares no livro: a) de Da Costa e Chuaqui; b) de Suppes. Claramente, os apontamentos finais dos autores em seu livro serão brevemente trazidos devidamente.

Por outro lado, volvendo-se à obra de Rodin – uma obra muito vasta em discussões ambiciosas acerca da relação entre Matemática, Lógica e as ciências empíricas nesta relação–, selecionar-se-á dois temas principais trazidos em *AMCT*: as mudanças históricas do método axiomático e identidade na Matemática. Com efeito, o autor destrincha as mudanças, ou quase reinvenções, do método axiomático em pontuando três momentos: em Euclides, em Hilbert, e na prática atual. Com o advento da Teoria de Categorias, um forte vento vem alterando a prática matemática e, em particular, atingindo o método (axiomático) pela qual é fundamentalmente exercida atualmente. Outrossim, este advento paralelamente despertam e escancaram fortes considerações acerca da noção e das concepções de identidade na prática matemática. De forma a Rodin apontar, e começar ainda em curtos passos, uma concepção de identidade através da *categorificação*. Todo um terço de sua obra é dedicada a esta questão; enquanto a primeira, ao tratamento histórico e filosófico do método axiomático.

Desta forma, conclusivamente, este trabalho se propõe a trazer de forma sintética o desenvolvimento feito em *LFST* através da leitura cruzada de *AMCT* no tocante ao método axiomático e identidade na Matemática. Apresentar-se-á particularmente, mas não se exaurindo nisso: i)

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a perspectiva geral de Krause e Arenhart em *LFST*; ii) as duas abordagens de axiomatizações exemplares trazidas, com as respectivas preliminares; iii) as considerações acerca das mudanças no método axiomático levantadas por Rodin e seu impacto nas colocações de Krause e Arenhart; iv) a concepção em construção de Rodin e seus possíveis efeitos nestas mesmas colocações. Este é um trabalho em andamento, desejando-se apresentar um resultado parcial e relatorial deste andamento.

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Construtibilidade e Teoria dos Modelos Internos

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Palavras-chave: Teoria do Conjuntos, Contrutíveis, Linguagem, Modelo

modelo dos Construtiveis foi definido por Kurt Gödel em [1] para mostrar a consistência da hipótese do continuo e do axioma da escolha com os axiomas de Teoria dos Conjuntos ZF. Esse modelo é denominado L e, a grosso modo, é utilizado para descrever todos os conjuntos que podem ser definidos a partir de uma fórmula matemática. Essa construção é feita de forma recursiva indexado por ordinais, isto é, definimos $L_\emptyset = \emptyset$ e em seguida definimos $L_{\alpha+1} = Def(L_\alpha)$ para o caso sucessor, e para caso limite $L_\alpha = \bigcup_{\beta<\alpha} Def(L_\beta)$. Finalmente, definimos $L = \bigcup_{\alpha\in\mathbf{On}} L_\alpha$. Para a construção é necessária a formalização da operação Def, que nos fornece os conjuntos definíveis por uma fórmula. Para isso, é necessário um detalhamento crucial na linguagem utilizada.

Com o tempo, surgiram outras aplicações do modelo dos Construtíveis, como os conceitos $\diamondsuit_\kappa$ e $\square_\kappa$ de combinatória infinita que foram primeiramente introduzidos em L por R. Björn Jensen em [3] e também suas mais diversas variações.

No pôster será apresentado um resumo sobre construção e linguagem necessária para a formalização do L, além de suas principais propriedades e aplicações.

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Sistemas Biortogonais em Espaços de Banach

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Palavras-chave: espaços de Banach, sistemas biortogonais, axioma de Martin, forcing.

Uma das maneiras mais eficazes de analisar e interpretar um espaço de Banach é via bases de Schauder [2]. Embora todo espaço vetorial tenha uma base algébrica, isso não é sempre verdade para bases desse tipo. Na verdade, é possível provar que se um espaço de Banach X possui uma base de Schauder então X é separável. Visando definir uma noção similar para espaços não separáveis é que surgem as bases longas de Schauder.

Definição 1. *Uma sequência transfinita de vetores $\{x_\gamma\}_{\gamma<\Gamma} \subseteq X$ é dita uma* ***base longa de Schauder*** *se, para todo $x \in X$, existe uma sequência transfinita única de escalares $(\alpha_\gamma)_{\gamma<\Gamma}$ tal que*

$$x = \sum_{\gamma<\Gamma} \alpha_\gamma x_\gamma.$$

Visto que nem todo espaço de Banach possui base longa de Schauder, muitas vezes é necessário nos restringirmos a subespaços fechados ou quocientes para que possamos desfrutar de tais estruturas. É natural então nos indagarmos se todo espaço de Banach de dimensão infinita possui um quociente de dimensão infinita com base de Schauder. Em caso de resposta afirmativa, qual o maior tamanho possível que uma base desse tipo pode admitir?

Segundo Plichko (1983), a resposta para essa pergunta está intimamente atrelada com a existência de sistemas biortogonais não enumeráveis específicos.

Definição 2. *Sejam X um espaço de Banach e $\Gamma \neq \emptyset$. Uma família $\{(x_\gamma, x^*_\gamma)\}_{\gamma\in\Gamma} \subseteq X \times X^*$ é dita* ***sistema biortogonal*** *se $\langle x_\alpha, x^*_\beta\rangle = \delta_{\alpha,\beta}$ para todo $\alpha, \beta \in \Gamma$.*

Nesse pôster, investigaremos a existência de sistemas biortogonais em espaços de funções $C(K) = \{f : K \longrightarrow \mathbb{R} \ : \ f \text{ é contínua}\}$, onde K é um espaço compacto.

Teorema 1. *Se vale o axioma de Martin, então um espaço compacto K é metrizável se, e somente se, todos os sistemas biortogonais de $C(K)$ são enumeráveis.*

Isso significa que, sob o axioma de Martin, obtemos uma caracterização de quando $C(K)$ possui sistemas biortogonais não enumeráveis. Vários outros resultados podem ser obtidos utilizando hipóteses combinatórias ainda mais fortes como o máximo de Martin ou o axioma do forcing próprio.

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Obtaining Paraconsistent Modal Logics by Means of Combinations of Logics

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Keywords: Paraconsistent Logic, Modal Logic, Combining Logics

Paraconsistent logics are a family of non-classical logics that deny or restrict the principle of explosion. They were initially studied in the first half of the twentieth century by a multitude of logicians and philosophers [8]. However, it was only with the proposal of a hierarchy of paraconsistent logics by da Costa in [9] that these kinds of logics begun being studied intensely.

This intense study of paraconsistent logics resulted in the development of a family of paraconsistent logic known as the Logics of Formal Inconsistency (LFI) by Carnielli and Marcos in [7]. In this family of logics, the concept of consistency is internalized in the language by means of a consistency operator, such that deductive triviality is only obtainable in the presence of a contradiction of consistent formulas.

Almost parallel to the development of paraconsistent logics, modal logics were being formally developed. Although the concept of modes of truth or thought are ancient and widely studied throughout philosophy, these weren't properly formalized in a logical language until the works of Lewis [12] and Kripke [10, 11]. The former defined the language and axiom systems of modal logic and the latter defined the relational semantics of modal logic, this being the most widespread semantical formulation for modal logic, though not the first, this being the formulation of Carnap [5].

The importance of theses families of logic is undeniable, as both have far reaching applications in philosophy, mathematics and computer science. As such, a question naturally arises: Is it possible to combine paraconsistent and modal logics into a unified logical system?

Many have tried to answer this, though this work focuses on the results obtained by Bueno-Soler in [1,3,4]. In this work, the author proposes two families of logics, the anodic logics and the cathodic logics. The former being strictly positive (i.e. without negation or a bottom particle) modal logics and the latter being the result of combining the anodic logics with some LFIs and paraconsistent logics, thus obtaining modal logics with weak negations and consistency operators.

However, these works have a limitation, this being that the logics were combined "manually", that is, they weren't combined by means of a method of combining logics such as fibring (be it algebraic, modulated or by functions), presented in detail in [6]. Instead, the new logics were defined from the ground up, thus requiring rigorous proofs and definitions that are usual when defining new logics, something that is not needed when utilizing methods such as fibring.

This manual method, however cumbersome it may be, does have the positive side effect of allowing for a more in depth analysis of the resulting logical system, something that is not necessarily the case with most methods of combination.

As such, the current work proposes to extend the results obtained in [2] by showing that the cathodic logics already defined may be obtained by means of combinations of logics and that new

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cathodic logics may be obtained as well. The main methods of combinations to be explored are fibring (and it's aforementioned variations) due to it's high expressivity and ability to deal with non-classical logics. Other methods may be explored, although they will not be the focus of this work.

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Teoria de Ramsey: Um Breve Retorno à Lógica Formal através dos Grafos

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Palavras-chave: Teoria de Ramsey, Decidibilidade, Grafos, Sentenças prenexas, Espectro de sentenças

No ano de 1930, o matemático Frank P. Ramsey, com o artigo "*On a Problem of Formal Logic*", deu início ao que, com as contribuições de Van der Waerden, Isaac Schur, Vojtech Rödl e outros, viria a se tornar uma subárea importante da Matemática discreta: a teoria de Ramsey. Como Ramsey afirma no artigo mencionado, em busca de resolver um problema de decidibilidade chegou a resultados de interesse independente da lógica e assim de fato se sucedeu. Nesse trabalho, no entanto, retornaremos à Lógica formal explorando a relação entre a teoria de Ramsey e problemas de decidibilidade por meio da linguagem dos grafos, apesar de o trabalho original ser mais geral. Para isso, estaremos interessados em determinar o espectro de uma sentença da linguagem dos grafos, que consiste no conjunto das quantidades de vértices dos grafos que validam a sentença em questão. Trabalharemos especificamente com as sentenças prenexas do tipo $\exists^*\forall^*$, que são sentenças da forma $\exists x_1 \cdots \exists x_n \forall y_1 \cdots \forall y_m \psi$, em que ψ é uma fórmula sem quantificadores da linguagem. Assim, esse trabalho enfoca no teorema de Ramsey e no seu uso para resolver o problema de decidibilidade do espectro de sentenças do tipo $\exists^*\forall^*$ da linguagem dos grafos.

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Rocq4Sets: Uma Perspectiva Educacional de um Assistente de Demonstração Aplicado a Exercícios Conjuntistas

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Palavras-chave: Fundamentos da Computação, assistente de demonstração, educação matemática.

Um assistente de demonstração é um software que auxilia a construção de demonstrações de teoremas através de regras e axiomas de uma dada teoria matemática. No contexto educacional, assistentes de demonstração garantem que alunos construam demonstrações, diferentemente das escritas em papel e caneta, utilizando apenas passos válidos. Um desafio, contudo, para a incorporação deles no ensino de lógica é o tempo necessário que os alunos precisam despender para aprender a utilizá-los. Superar essa dificuldade, é a motivação do desenvolvimento recente de algumas ferramentas, como Waterproof [1] e Verbose Lean [2], com as quais é possível escrever, em assistentes de demonstrações, demonstrações similares às escritas em papel e caneta.

Neste trabalho, apresentamos o Rocq4sets (disponível em `http://carol.dimap.ufrn.br/jscoq/`), uma biblioteca que possibilita o uso de uma linguagem natural controlada para demonstrar exercícios de teoria dos conjuntos no assistente de demonstração Coq. A implementação do Rocq4sets se deu através da metalinguagem Ltac2 para o Coq [3]. No Coq, cada comando de uma demonstração é denominado tática, e com a Ltac2, foi possível atribuir a uma sequência de táticas uma cadeia de palavras em português. Assim, os alunos podem usar estratégias de demonstração previamente definidas para invocar comandos nativos do Coq.

Como resultado do uso do Rocq4sets com alunos de graduação da disciplina de fundamentos da computação, notamos uma dificuldade em diferenciar entre as justificativas de uma demonstração, de meros comentários explicativos de um código de programação. Através do Rocq4sets, podemos auxiliar os alunos a aplicarem de forma adequada as regras de inferências em uma demonstrações conjuntistas. Em particular, apresentamos as regras de inferências para operações sobre conjuntos, tais como a união, interseção, complemento relativo e diferença simétrica, bem como para as relações de inclusão e conjuntos disjuntos. Ademais, possibilitamos que os alunos realizem exercícios usando

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diversas estratégias de demonstrações, tais como por contrapositiva, por absurdo e por casos.

Como trabalho futuro, vamos aprimorar as mensagens de erro exibidas pelo Rocq4sets, além de expandir as operações e relações conjuntistas nele contempladas. Por fim, integraremos o *Rocq4Sets* à implementação de uma estratégia de geração de exercícios de complexidade similar [4].

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Model Theory of Sheaves of Metric Spaces of Analytic Functions

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Keywords: continuous logic, metric structures, sheaves, analytic functions

We have started to study from a logical perspective a heuristic principle in complex analysis, the so called *Bloch's Principle*, that is, *there is nothing in the infinite that was not previously in the finite; every proposition in whose statement the actual infinity occurs can be always viewed as a corollary, almost immediate, of a proposition where it does not occur, a proposition in finite terms*, proposed by André Bloch in his article [3, p. 84] in 1926, and with a first (non logical) approach obtained by Lawrence Zalcman in [11], following a suggestion of Abraham Robinson, [10, §8, pp. 508–510]. The logical framework we intend to apply is of sheaves of metric structures as described below.

Model Theory of Metric Structures is a recent version of Continuous Logic where the structures are complete metric spaces with bounded diameter, that is, they are endowed with a metric with values in the real interval $[0,1]$, also considered a metric spaces with the usual metric $|y-x|$ (see [1, 2].) This logic generalizes classical logic in the sense that we view classical structures as discrete metric spaces, whose metrics assume values in the set $\{0,1\}$, but with 0 as "truth" and 1 as "false". A metric structure is a complete metric space (M,d), whose metric is the function $d: M\times M\to[0,1]$, n-ary uniformly continuous functions $f: M^n\to M$, and n-ary predicates as uniformly continuous functions $P: M^n\to[0,1]$, together with their "moduli of uniform continuity" $\Delta_f,\Delta_P:[0,1]\to[0,1]$, considered as continuous crescent functions (here we diverge a little from the current literature). The logical part comprises continuous functions $f:[0,1]^n\to[0,1]$ as n-place propositional connectives, and the operators $\sup_x$ and $\inf_x$ as quantifiers.

Sheaves of metric structures were studied in [8, 9]. We propose to study the (pre-)sheaf of analytic functions over $\mathbb{C}$. Choose an enumeration ρ_n for all positive rational numbers and an enumeration ξ_n of $\mathbb{Q}+\boldsymbol{i}\mathbb{Q}\subset\mathbb{C}$. The set of closed disks $B(\xi_m,r_n)=\{z\in\mathbb{C}:|z-\xi_m|\le r_n\}$ generates by unions the topology of $\mathbb{C}$, and we fix an enumeration B_k of such disks. With this we define the metric on the set $\Gamma(U)$ of sections os analytic functions on the open set $U\subseteq\mathbb{C}$ as follows. The chordal distance in $\mathbb{C}$ comes from the stereographical projection of $\mathbb{C}$ into $\mathbb{R}^3$ and reduces to $\chi(z_0,z_1)=\dfrac{|z_0-z_1|}{\sqrt{|z_0|^2+1}\sqrt{|z_1|^2+1}}\in[0,1]$. For each open set $U\subseteq\mathbb{C}$, let $\mathcal{B}_U=\{B_{k_m}:m\in\omega\}$ be the set of all closed balls from $\mathcal{B}$ contained in U, with the enumeration derived from that of $\mathcal{B}$, and let $K_{U,n}=\bigcup_{j=1}^n B_{k_j}$, an exhausting sequence of compact subsets of U. The $[0,1]$-valued metric in $\Gamma(U)$ is $d_U(f,g)=\sum_{j\in\omega}2^{-j-1}\sup\{\chi(f(z),g(z)):z\in K_j\}$. These metrics satisfy $U\subseteq V$ and $f,g\in\Gamma(V)$ imply $d_U(f\restriction_U,g\restriction_U)\le d_V(f,g)$.

We recall that a normal family of analytic functions on an open set $U\subseteq\mathbb{C}$ is a relatively compact subset of $\Gamma(U)$. We intend to formalize this notion in the appropriate language and try to find a common feature of the questions treated in [12], and particularly in [11] [*each property of analytic functions which implies that entyre functions to be constant locally implies that the family with such property is normal*], with the techniques from [4–9].

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Cofibração na Categoria dos Espaços Pseudotopológicos

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Palavras-chave: espaços pseudotopológicos, cofibração, convergência

Espaços de convergência são uma generalização dos espaços topológicos. Na literatura, esses espaços são geralmente descritos por meio da convergência de filtros, como em [1], [2] e [3]. No entanto, neste trabalho, eles são abordados por meio da convergência de redes, seguindo [7], [9] e [10]. Uma subcategoria da categoria de espaços de convergência é a categoria dos espaços pseudotopológicos. Esses espaços são descritos por meio de ultraredes, que são redes cujo filtro induzido é um ultrafiltro. A categoria de espaços pseudotopológicos possui objetos exponenciais, tornando-se, assim, uma categoria conveniente para a Topologia Algébrica. O termo "categoria conveniente de espaços topológicos", introduzido por Brown [5] e popularizado por Steenrod [6], refere-se a qualquer categoria de espaços topológicos suficientemente boa para a Topologia Algébrica. Além disso, com inspiração no trabalho de Rieser [8], mostramos que a categoria dos espaços pseudotopológicos admite uma estrutura de cofibração. Uma estrutura de cofibração em uma categoria é uma estrutura adicional que permite definir e estudar noções de extensão e colagem de objetos de maneira bem comportada. Essa estrutura desempenha um papel central na teoria de homotopia e nas categorias modelo.

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Especificação da Semântica Operacional de Cálculo de Processos Sensível ao Tempo.

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Palavras-chave: Pi-calculus, Lógica de Reescrita, Programação Concorrente

A programação concorrente é um paradigma de programação que permite a execução de múltiplas tarefas simultaneamente, aproveitando a separação delas em processos independentes que podem rodar de maneira isolada ou interagir entre si. Um exemplo comum de comunicação entre tarefas concorrentes é a troca de mensagens, como suportado pelas linguagens *Go* e *Erlang*.

No entanto, o uso desse método de comunicação pode introduzir novos desafios, como falhas no envio ou recebimento de mensagens, duplicidade de envios, problemas de segurança e interrupções de tarefas, tais como *deadlock* e *starvation*. Implementar uma especificação que defina como a comunicação deva ocorrer é uma tarefa que dispensa complexidade em aplicações simples. Porém, à medida que a complexidade e o tamanho da aplicação aumentam, garantir que a implementação siga a especificação torna a tarefa progressivamente mais dispendiosa e propensa a erros.

Portanto, uma forma de se realizar uma verificação formal entre partes concorrentes de uma aplicação garantiria mais segurança e estabilidade. Atualmente têm-se estudado modelos de concorrência que visam garantir essas propriedades, como o π-cálculo [2] e os tipos de sessão [4]. Além disso, a lógica de reescrita tem-se destacado como um framework geral para a especificação de sistemas concorrentes, sendo utilizada em ferramentas como o *Maude System* [1], que permite a especificação executável de sistemas concorrentes.

No artigo [3] foi realizada a especificação do cálculo proposto por [4], assim como a verificação de tipos desse cálculo. A pesquisa de [5] estendeu os tipos de sessão [4] para processos sensíveis ao tempo. Tomando como base o trabalho realizado por [3], o presente estudo propõe a especificação da semântica operacional do cálculo proposto por [5] no *Maude System*.

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Preservation of Topological Properties by Forcing

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Keywords: Forcing, Cohen Reals, Preservation of Topological Properties

Forcing is a technique (or series of techniques) in set theory that gives us an extension of a model of set theory. By adding new functions to the universe, it can break topologies, as they can stop being closed by unions. However, we can use the initial topology as a base and obtain a new topological space. A natural question arises: when does this new space retain the properties of the original. In [1], Watson was one of the first to ask such questions about Cohen forcing, the original form of forcing.

The Lindelof property, a generalization of compactness, is a topological property that is not necessarily preserved by forcing in general. However, it is preserved by Cohen forcing. This was shown in [2], using the technique known as endownments.

Our objective is to provide an overview of recent results, such as attempts to generalize the spaces [3], or the forcing [4], and the techniques used. We aim to offer insight into the current state of research in this area.

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