

Ultra high energy cosmic rays and possible signature of black strings

This content has been downloaded from IOPscience. Please scroll down to see the full text.

JCAP03(2016)014

(<http://iopscience.iop.org/1475-7516/2016/03/014>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 143.107.180.158

This content was downloaded on 11/04/2016 at 14:16

Please note that [terms and conditions apply](#).

Ultra high energy cosmic rays and possible signature of black strings

Rita C. dos Anjos,^a Carlos H. Coimbra-Araújo,^a Roldão da Rocha^b
and Vitor de Souza^c

^aDepartamento de Engenharias e Exatas, Universidade Federal do Paraná (UFPR),
Pioneiro, 2153, 85950-000 Palotina, PR, Brazil.

^bCentro de Matemática, Computação e Cognição,
Universidade Federal do ABC, 09210-580, Santo André-SP, Brazil.

^cInstituto de Física de São Carlos, Universidade de São Paulo,
Av. Trabalhador São-carlense 400, São Carlos, Brazil

E-mail: ritacassia@ufpr.br, carlos.coimbra@ufpr.br, roldao.rocha@ufabc.edu.br,
vitor@ifsc.usp.br

Received October 19, 2015

Accepted February 21, 2016

Published March 8, 2016

Abstract. Ultra high energy cosmic rays (UHECRs) probably originate in extreme conditions in which extra dimension effects might be important. In this paper we calculate the correction in black hole accretion mechanisms due to extra dimension effects in the static and rotating cases. A parametrization of the external Kerr horizons in both cases is presented and analysed. We use previous calculations of upper limits on the UHECR flux to set limits on the UHECR production efficiency of nine sources. The upper limit on the UHECR luminosity calculation is based on GeV-TeV gamma-ray measurements. The total luminosity due to the accretion mechanism is compared to the upper limit on UHECRs. The dependence of the UHECR production efficiency upper limit on black hole mass is also presented and discussed.

Keywords: accretion, cosmic ray theory, ultra high energy cosmic rays

ArXiv ePrint: [1510.04992](https://arxiv.org/abs/1510.04992)



Contents

1	Introduction	1
2	Black hole accretion with extra dimension corrections	2
2.1	Static black strings	3
2.2	Rotating black strings	4
2.3	Luminosity of black strings	6
3	Upper limit on the UHECR emission efficiency	7
4	Concluding remarks	7

1 Introduction

The possible presence of extra dimensions in the universe is one of the most astounding features of string theory. Despite the strong theory formalism, extra dimensions still remain inaccessible to experiments [1–6]. Since the presence of ten or more spacetime dimensions is one of the central conditions of string theory and M theory [7–10], it is not unrealistic to say that experimental observation or constraints on the extra dimensions properties would be a major advance in science.

Randall-Sundrum (RS) models are capable of explaining the hierarchy problem [11, 12] and Kaluza-Klein (KK) non compactified theories [13]. The extra dimension implies deviations from Newton’s law of gravity at sub millimetric scales, where objects may be indeed be gravitating in higher dimensions [11, 12]. This main assumption predicts deviations from 4D general relativity in the dynamics of massive astrophysical objects, e.g. black holes (BH). The generalization of a BH in the brane world models usually involve black strings. Within this framework, the total luminosity of Active Galactic Nuclei (AGNs) and radio galaxies have been calculated in references [14–16] for an effective extra dimension size. The general conclusion is that the total luminosity generated by the accretion mechanism is reduced due to possible extra dimension corrections predicted by RS and KK models.

Ultra high energy cosmic rays (UHECRs — $E > 10^{18}$ eV) are produced in the same environment in which extra dimension effects might be important. The recent results of the Pierre Auger Observatory limiting drastically the flux of gamma rays [17] and neutrinos [18] with energy above 10^{18} eV have minimised the importance of exotic non-acceleration production of UHECR known as top-down mechanisms. Moreover, the study of the arrival direction of the events measured by the Pierre Auger Observatory with energy above 58 EeV cannot reject, on a 2 sigma level, a correlation with local mass and a source distribution compatible with the known AGN spacial distribution [19].

From the theory point of view, the potential of a source to accelerate particles up to the extreme energies is summarized in the Hillas plot [20]. Following his argument, a short list of the most probably acceleration sites can be selected by requiring that during acceleration the particles must be confined in the source, in other words, the size of the source must be larger than the Larmor radius of the particle in the magnetic field present in the acceleration region. Further arguments have been developed in order to pinpoint source candidates concerning the acceleration time scale [21, 22] and jet power [23]. The convergence of such studies

means that the leading contenders for accelerating particles to the highest energies are the most powerful radio galaxies, AGNs, gamma-ray bursts, fast spinning newborn pulsars and interacting galaxies. For details about the acceleration see references [24–31].

The accretion mechanism in BH is known to be the main source of energy which powers all sorts of emission. The luminosity in each channel (X) is usually written as a fraction of the total power of the accretion mechanism (L_{acc}), i.e. $L_X = \eta_X L_{\text{acc}}$. The linear correlation between the accretion mechanism and power of the compact radio core was studied in reference [32]. References [33, 34] suggests that $\eta_{\text{X-ray}} \sim 10^{-5}$. A recent publication [35] has investigated the acceleration of UHECRs in low-luminosity AGNs suggesting that $\eta_{\text{CR}} \sim 1/3$. However the role of UHECR in the energy balance of the accretion mechanism is yet to be carefully studied.

Recently, a method has been developed to calculate upper limits of UHECR luminosities of individual sources [36]. The method is based on the measurement of an upper limit of the integral flux of GeV-TeV gamma rays. In reference [37], this method was used to set the upper limit on the UHECR luminosity of twenty nine sources. The calculated upper limits on the proton and the total UHECR luminosity are of the order of $\sim 10^{45}$ erg/s.

In the present paper, we calculate the luminosity of nine of these sources taking into account deviations from general relativity due to RS bulk and compare the results. Considering the extra dimension corrections, the predicted total luminosity goes from the fundamental Eddington limit ($\sim 10^{47}$ erg/s) to $\sim 10^{45}$ erg/s which is the same order of the UHECR luminosity calculated in reference [37]. We studied the dependence of the black string radius as a function of the total mass and a parametrization is proposed.

The bolometric luminosities measured for AGNs and radio galaxies are roughly $\sim 10^{42} - 10^{43}$ erg/s [38] which is orders of magnitude below the values predicted by accretion mechanisms ($\sim 10^{45} - 10^{47}$ erg/s) and therefore do not to constrain the models. On the other hand, we show here that the upper limits of the UHECR are of the same order as the flux predicted by accretion mechanisms with extra dimension effects in AGNs and radio galaxies. For seven out of nine sources studied here, we manage to set an upper limit on $\eta_{\text{CR/pr}}$. Extreme cases of pure proton and pure iron nuclei emission are considered. The case of pure iron nuclei limits the total UHECR flux as explained in reference [37].

We also investigate the dependence of η_{pr} on the mass of the BH with and without extra dimension corrections. We show a hint of decreasing η_{CR} for increasing BH mass.

The article is organized as follows. In section 2 we review the calculation of the BH radius in the static and rotating case with and without extra dimension corrections. A simple parametrization of the solutions is presented. In this section, the accretion mechanism powering the BH luminosity is also summarized. In section 3 the calculation of the upper limit on the UHECR luminosity is presented. Section 4 concludes this study.

2 Black hole accretion with extra dimension corrections

The approach in this section is to summarize corrections in black hole horizons due to brane-world effects, consisting of the black string framework. Static and rotating black holes are calculated both for Randall-Sundrum (RS) and Casadio-Fabbri-Mazzacurati metrics in references [14] and [15], respectively. In this paper, we use the RS formalism in which volcano barrier-shaped effective potentials for gravitons are induced around the brane [39]. Such bulk modes introduce small corrections at short distances. Compact dimensions do not affect the localization of matter fields, however true localization is achieved merely for mass-

less fields [40]. In the massive case, the bound state is metastable and may leak into the extra space.

The RS metric is in general expressed as

$$^{(5)}ds^2 = e^{-2k|y|} g_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2.1)$$

where $k^2 = 3/2\ell^2$ and ℓ denotes the AdS_5 bulk curvature radius. Moreover, the term $e^{-2k|y|}$ is the so called *the warp factor* [11, 12, 41], which confines gravity in the brane at low energies [5, 11, 12, 41]. It is worth to mention that in the original RS formalism $g_{\mu\nu}$ is the Minkowski metric [5, 11, 12], however a more general setup will be considered here. The Gaussian coordinate y denotes the normal direction out of the brane into the bulk.

In a braneworld framework embedded in AdS_5 bulk, the Einstein field equations are expressed as

$$G_{\mu\nu} = -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{1}{4}\kappa_5^4 \left[\frac{1}{2}g_{\mu\nu}(T^2 - T_{\rho\sigma}T^{\rho\sigma}) - T_\nu{}^\rho T_{\mu\rho} + TT_{\mu\nu} \right] - E_{\mu\nu}, \quad (2.2)$$

where Λ_5 is the 5D cosmological constant, $T = T_\rho{}^\rho$ denotes the momentum-energy tensor trace, $\kappa_5^2 = 8\pi G_5$ (with G_5 the 5D Newton constant) and $E_{\mu\nu}$ denotes the components of the electric part of the Weyl tensor, that describes high energy effects originating Kaluza-Klein (KK) modes and information regarding collapse processes of BHs as well. In particular, the gravitational collapse induces deviations from the 4D general relativity [5]. RS KK modes provide a deviation of the Newtonian potential as [11, 12]

$$V(r) = \frac{GM}{c^2 r} \left(1 + \frac{2\ell^2}{3r^2} + \mathcal{O}\left(\frac{\ell^4}{r^4}\right) \right). \quad (2.3)$$

Some useful equations can be derived forthwith from Bianchi identities $\nabla_\mu G^{\mu\nu} = 0$ and vacuum on the brane $T_{\mu\nu} = 0$. For instance we have $\nabla_\nu E^{\mu\nu} = 0$ (see e.g. [14, 41, 42]). Moreover a perturbative procedure is then brought into play, to study the bulk metric near the brane, which in particular further inherits the black string warped horizon along the extra dimension as well. It is accomplished by a Taylor expansion to probe properties of a static BH on the brane [43]. Such expansion, for a vacuum brane metric, up to terms of order $\mathcal{O}(y^5)$, reads

$$\begin{aligned} g_{\mu\nu}(x, y) = & g_{\mu\nu}(x, 0) - E_{\mu\nu}(x, 0) \frac{y^2}{2!} - \frac{12}{\ell} E_{\mu\nu}(x, 0) \frac{|y|^3}{3!} \\ & + 2 \left[6E_\mu{}^\alpha E_{\alpha\nu} + 2E^{\alpha\beta} R_{\mu\alpha\nu\beta} + \left(\square - \frac{32}{\ell^2} \right) E_{\mu\nu} \right]_{y=0} \frac{y^4}{4!} + \dots \end{aligned}$$

where $\square$ denotes the standard d'Alembertian, showing that the propagating effect of 5D gravity is merely evinced at the fourth order. A more general expression was derived in [44].

2.1 Static black strings

For a static spherical metric on the brane

$$g_{\mu\nu} dx^\mu dx^\nu = -F(r) dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2, \quad (2.4)$$

where $d\Omega^2$ denotes the spherical 2-volume element in the 3-brane. Moreover, the electric part of the Weyl tensor on the brane has the following components:

$$\begin{aligned} E_{00} &= \frac{F}{r} \left(\frac{H-1}{r} + H' \right), & E_{\theta\theta} &= H-1 + \frac{rH}{2} \left(\frac{H'}{H} + \frac{F'}{F} \right), \\ E_{rr} &= \frac{1}{r} \left(\frac{F'}{FH} + \frac{H^{-1}-1}{r} \right) \end{aligned} \quad (2.5)$$

By denoting $\psi(r)$ a deviation from the usual Schwarzschild metric term [41, 45–51]

$$H(r) = 1 - \frac{2GM}{c^2 r} + \psi(r), \quad (2.6)$$

where M is usually identified to the effective mass and, given the expansion of the warped horizon area (determined by $g_{\theta\theta}$),

$$g_{\theta\theta}(r, y) = r^2 - \psi' \left(1 + \frac{2}{\ell} |y| \right) y^2 + \left[\psi' + \frac{1}{2} (1 + \psi') (r\psi' - \psi)' \right] \frac{y^4}{6r^2} + \dots, \quad (2.7)$$

the function ψ can be shown to determine the modification of the warped horizon areal radius along the extra dimension [41]. For a black hole with horizon $r \gg \ell$, eq. (2.3) yields

$$\psi(r) \approx -\frac{4GM}{3c^2 r} \left(\frac{\ell^2}{r^2} \right). \quad (2.8)$$

The corrected horizon R_{Sbrane} for the static BH including extra dimension corrections comes by fixing $H(r) = 0$, i.e. by solving equation $1 - \frac{2GM}{c^2 R_{\text{Sbrane}}} + \psi(R_{\text{Sbrane}}) = 0$ in eq. (2.6), resulting in [14–16]

$$R_{\text{Sbrane}} = \frac{2GM}{3c^2} + (g_2 - \sqrt{g_1})^{1/3} + (g_2 + \sqrt{g_1})^{1/3}, \quad (2.9)$$

where

$$g_1 = \frac{4G^2 M^2 \ell^2}{9c^4} \left(\frac{8G^2 M^2}{9c^4} + \ell^2 \right) \quad (2.10)$$

$$g_2 = \frac{2GM}{3c^2} \left(\frac{4G^2 M^2}{9c^4} + \ell^2 \right), \quad (2.11)$$

2.2 Rotating black strings

The spherical rotating BH Kerr metric is represented by the following metric tensor:

$$g_{\mu\nu}^{\text{Kerr}} = \begin{pmatrix} \omega^2 \tau^2 - \alpha^2 & 0 & 0 & -\omega \tau^2 \\ 0 & \rho^2 / \Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\omega \tau^2 & 0 & 0 & \tau^2 \end{pmatrix} \quad (2.12)$$

where the BH presents angular momentum J and mass parameter M , and

$$\begin{aligned} \rho^2 &= \frac{a^2}{c^2} \cos^2 \theta + r^2, & \Delta &= \frac{a^2}{c^2} - \frac{2GM}{c^2} r + r^2, & \tau &= \frac{\Sigma}{\rho} \sin \theta, \\ \alpha &= \sqrt{1 - \frac{2GM}{c^2 r}}, & \omega &= \frac{2aGM r}{c^3 \Sigma^2}, & \Sigma^2 &= \left(\frac{a^2}{c^2} + r^2 \right)^2 - \frac{a^2}{c^2} \Delta \sin^2 \theta. \end{aligned} \quad (2.13)$$

The rotation parameter a is defined by $a = \frac{J}{Mc} \sim \frac{GM}{2c}$. The associated diagonal metric reads

$$\begin{aligned} g^{\text{Kerr}} &= g_{\mu\nu}^{\text{Kerr}} dx^\mu dx^\nu \\ &= \lambda_0 dt'^2 + \frac{\Delta}{\rho^2} dr^2 + \rho^2 d\theta^2 + \lambda_3 d\phi'^2, \end{aligned} \quad (2.14)$$

where

$$\lambda_{0,3} = \frac{1}{2} \left[(\omega^2 \tau^2 - \alpha^2 + \tau^2) \pm ((\omega^2 \tau^2 - \alpha^2 + \tau^2)^2 - 4(\omega \tau^2 + (\omega^2 \tau^2 + \tau^2)^2))^{1/2} \right]. \quad (2.15)$$

The inner and outer Kerr horizons ($R_{\pm\text{Kerr}}$) are achieved when the coefficient $\frac{\rho^2}{\Delta}$ in the denominator of dr^2 in eq. (2.14) is equal to zero. Hence it reads

$$R_{\pm\text{Kerr}} = \frac{GM}{c^2} \pm \sqrt{\left(\frac{GM}{c^2}\right)^2 - \frac{a^2}{c^2}}. \quad (2.16)$$

Now, using the approximation for a we can solve for $R_{+\text{Kerr}}$ and show a straightforward dependency with $M/M_\odot$

$$R_{+\text{Kerr}} = \frac{GM}{c^2} + \frac{\sqrt{3}}{2} \frac{GM}{c^2} \sim 1.38 \times 10^{-27} M \text{ [m/kg]} \sim 2.73 \frac{M}{M_\odot} \text{ [m]}. \quad (2.17)$$

In order to obtain the correction of the Kerr horizons in eq. (2.16) due to braneworld effects ($R_{\text{Kerrbrane}}$), we follow the idea presented in eq. (2.6). Defining $\xi(r)$ as being the deviation from a Kerr standard metric for $\frac{\rho^2}{\Delta}$ (namely the term in the denominator of dr^2 in eq. (2.14)), we provide the *ansatz*

$$\xi(r) = -\frac{4GM\ell^2}{3r(a^2 + c^2 r^2)} \quad (2.18)$$

such that in the static limit where $a \rightarrow 0$ the correction induced in the gravitational potential is consistent with eq. (2.3), and to eq. (2.8), accordingly. The corrections in the Kerr event horizons, as calculated in [14], are achieved through the deformed Kerr metric, as

$$\frac{\rho^2}{\Delta} + \xi(R_{\text{Kerrbrane}}) = 0. \quad (2.19)$$

When the above expression is expanded and using eq. (2.18) it yields

$$\begin{aligned} &c^2 R_{\text{Kerrbrane}}^5 - 2GM R_{\text{Kerrbrane}}^4 + 2a^2 R_{\text{Kerrbrane}}^3 \\ &- GM \left(2\frac{a^2}{c^2} + \frac{4}{3}\ell^2 \right) R_{\text{Kerrbrane}}^2 + \frac{a^4}{c^2} R_{\text{Kerrbrane}} - \frac{4}{3}GM\ell^2 \frac{a^2}{c^2} \cos^2 \theta = 0. \end{aligned} \quad (2.20)$$

See reference [14] for a detailed description about those solutions and other possible groups of $\xi(R_{\text{Kerrbrane}})$ *ansätze*. We have numerically solved equation (2.20) to find $R_{+\text{Kerrbrane}}$. Figure 1 shows the dependency of $R_{+\text{Kerrbrane}}$ with the black hole mass. A straightforward linear relation holds, leading to $R_{+\text{Kerrbrane}} = a + b \times M/M_\odot$ with $a = 5 \times 10^6 \pm 5 \times 10^7$ m and $b = 2954.12 \pm 0.00047$ m.

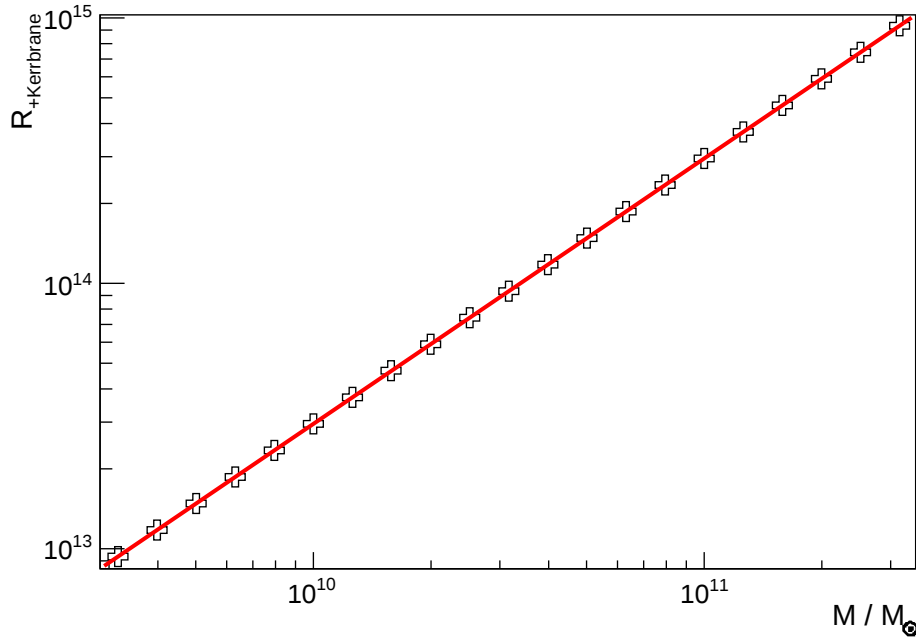


Figure 1. $R_{+Kerrbrane}$ as a function of $M/M_{\odot}$. Crosses show the solution of equation (2.20) for $M/M_{\odot} = [9.5, 11.5]$ in steps of 0.1. Red line is a linear fit to the points $R_{+Kerrbrane} = a + b \times M/M_{\odot}$ with $a = 5 \times 10^6 \pm 5 \times 10^7$ m and $b = 2954.12 \pm 0.00047$ m.

2.3 Luminosity of black strings

The luminosity of AGNs are produced essentially by the accretion mechanism, i.e., AGN activity is related to the growth, via accretion, of central supermassive black holes (SMBHs). The accretion efficiency (ϵ), from a simple accretion model, is given by (see, e.g., [52])

$$\epsilon = \frac{GM}{6c^2 R} \quad (2.21)$$

where $R = R_{Sbrane}, R_{+Kerr}$ or $R_{+Kerrbrane}$ are the horizons for a static BH with extra dimension correction, a rotating BH without extra dimension corrections, and a rotating BH with extra dimension corrections, respectively. The luminosity L_{acc} of AGNs due to BH accretion is given by

$$L_{acc}^{Theory} = \epsilon \dot{M} c^2 = \frac{GM \dot{M}}{6R}, \quad (2.22)$$

where $R = R_{Sbrane}, R_{+Kerr}$ or $R_{+Kerrbrane}$. The varying mass term $\dot{M}$ denotes the accretion rate and depends on some specific model of accretion. Several accretion rates are proposed in the literature, for comparison we use here the Bondi mechanism $\dot{M} = \pi \lambda c_S \rho_B r_B$ [53], where λ takes the value of 0.25 for an adiabatic index 5/3, c_S is the sound speed in the medium, r_B denotes the Bondi accretion radius and ρ_B stands for the gas density at that radius. For simplicity we use $\dot{M} = 10^{12} M/M_{\odot} [\text{kg.s}^{-1}]$.

Considering a fraction η_{CR} of the total luminosity goes into UHECR we write

$$L_{CR}^{Theory} = \eta_{CR} L_{acc}^{Theory}. \quad (2.23)$$

3 Upper limit on the UHECR emission efficiency

In reference [36] a method was developed to set upper limits on the UHECR luminosity of individual sources, based on gamma-ray measurements. We summarize this method below.

UHECRs are produced in extragalactic sources and propagate in the background radiation fields. The particles interact with background photons and start a cascade process. Successive interactions of the particles produce large numbers of electrons, positrons, neutrinos and photons. The dependence of the mean free path of the gamma-gamma interaction with energy results in a large abundance of GeV-TeV photons produced in the cascade. The relevant processes and the details of the particle production are explained by references [54–57].

The method connects a measured upper limit on the integral flux of GeV-TeV gamma-rays and the UHECR cosmic-ray luminosity ($L_{\text{CR}}^{\text{UL}}$) of a source by the equation:

$$L_{\text{CR}}^{\text{UL}} = \frac{4\pi D_s^2 (1 + z_s) \langle E \rangle_0}{K_\gamma \int_{E_{\text{th}}}^{\infty} dE_\gamma P_\gamma(E_\gamma)} I_\gamma^{\text{UL}}(> E_\gamma^{\text{th}}), \quad (3.1)$$

where $I_\gamma^{\text{UL}}(> E_\gamma^{\text{th}})$ is the upper limit on the integral gamma-ray flux for a given confidence level and energy threshold, K_γ is the number of gamma-rays generated from the cosmic-ray particles, $P_\gamma(E_\gamma)$ is the energy distribution of the gamma-rays arriving on Earth, E_γ is the energy of gamma-rays, $\langle E \rangle_0$ is the mean energy, D_s is the comoving distance and z_s is the redshift of the source.

The procedure to apply this method to calculate $L_{\text{CR}}^{\text{UL}}$ for a given source is as follows. D_s and z_s must be known for the source from astronomical measurements. $I_\gamma^{\text{UL}}(> E_\gamma^{\text{th}})$ is measured by gamma-ray observatories for each source. E^{th} is given by the gamma-ray observatory and by the data analysis. K_γ and $P_\gamma(E_\gamma)$ are calculated given a primary UHECR energy spectrum. The type of the primary UHECR is needed for the calculation of K_γ and $P_\gamma(E_\gamma)$. We have done calculations for proton and iron nucleus primaries. Since iron nuclei are the particles to produce the largest numbers of GeV-TeV gamma rays, the limit calculated with iron nuclei simulations imposes an upper limit on the total UHECR luminosity. For details about the calculation see references [36, 37].

Twenty nine sources have been studied in references [37] and upper limits on the proton or total UHECR luminosity were set. The sources have been measured by the FERMI-LAT [58], H.E.S.S. [59], MAGIC [60] and VERITAS [61] experiments. We found in the literature the mass estimative of nine of these sources, which are used here. Other UHECR upper limits were calculated in [62] and [63].

The data of each source is shown in tables 1 and 2. Table 1 summarizes the results for sources that only a upper limit on the proton luminosity was set and table 2 shows the same results for sources that the total UHECR luminosity was set. The values of $L_{\text{acc}}^{\text{Theory}}$ for $R = R_{\text{Sbrane}}, R_{+\text{Kerr}}$ or $R_{+\text{Kerrbrane}}$ are shown. We use the upper limits on the UHECR ($L_{\text{CR}}^{\text{UL}}$) to set upper limits on η_{CR} : $\eta_{\text{CR}}^{\text{UL}} = L_{\text{CR}}^{\text{UL}} / L_{\text{acc}}^{\text{Theory}}$. These limits are valid only if $L_{\text{CR}}^{\text{UL}} < L_{\text{acc}}^{\text{Theory}}$. If only a upper limit of the proton UHECR was set, we calculate the corresponding $\eta_{\text{pr}}^{\text{UL}}$. We show the corresponding $\eta_{\text{pr}}^{\text{UL}}$ for $R = R_{+\text{Kerr}}$ and $R_{+\text{Kerrbrane}}$.

4 Concluding remarks

We compare in tables 1 and 2 the luminosity of the BH with and without extra dimension corrections. The luminosity decreases significantly for some cases when extra dimension effects are taken into consideration. On average $\langle L_{+\text{Kerrbrane}} \rangle = 1.08 \langle L_{+\text{Kerr}} \rangle$.

Source name	$\log(M/M_\odot)$	$\mathbf{L}_{\text{acc}}^{\text{Theory}} \left[\text{erg s}^{-1} \times 10^{45}\right]$ $R = R_{\text{Sbrane}} \quad R = R_{+\text{Kerr}} \quad R = R_{+\text{Kerrbrane}}$		$\mathbf{L}_{\text{pr}}^{\text{UL}}$ $\left[\text{erg s}^{-1} \times 10^{45}\right]$	$\eta_{\text{pr}}^{\text{UL}}$ $R = R_{+\text{Kerr}} \quad R = R_{+\text{Kerrbrane}}$	
NGC 5995	9.89	1.16	6.23	0.58	0.90 0.14	—
2MASX J11454045-1827149	10.00	1.50	8.02	0.74	1.30	—
MCG-01-24-012	10.16	2.17	1.16	1.08	0.65	0.60
Mrk 520	10.40	3.78	2.01	1.88	0.98	0.52
2MASX J07595347+2323241	10.57	5.59	2.98	2.78	1.01	0.36
LEDA 170194	10.59	5.85	3.12	2.91	1.48	0.50
NGC 985	10.60	5.99	3.19	2.98	1.03	0.34
CGCG 420-015	10.63	6.42	3.42	3.19	0.95	0.29
NGC 1142	10.70	7.54	4.02	3.75	0.49	0.13

Table 1. Columns show the source name, the mass source according to references [64, 65], the luminosity calculated with equation (2.22) for $R = R_{\text{Sbrane}}$, $R_{+\text{Kerr}}$ or $R_{+\text{Kerrbrane}}$ with $\ell = 10^{-4} m$, the upper limit on the proton UHECR luminosity according to equation (3.1) and the upper limits of η_{pr} for $R = R_{+\text{Kerr}}$ and $R_{+\text{Kerrbrane}}$.

Source name	$\log(M/M_\odot)$	$\mathbf{L}_{\text{acc}}^{\text{Theory}} \text{ [erg s}^{-1} \times 10^{45}]$ $R = R_{\text{Sbrane}} \quad R = R_{+Kerr} \quad R = R_{+Kerrbrane}$		$\mathbf{L}_{\text{CR}}^{\text{UL}}$ [erg s ⁻¹ × 10 ⁴⁵]	$\eta_{\text{CR}}^{\text{UL}}$ $R = R_{+Kerr} \quad R = R_{+Kerrbrane}$
2MASX J11454045-1827149	10.0	1.50	8.02	0.74	3.16 — 0.23
LEDA 170194	10.59	5.85	3.12	2.91	3.71 — 0.78
NGC 985	10.6	5.99	3.19	2.98	2.19 0.69 0.73

Table 2. Columns show the source name, the mass source according to references [64, 65], the luminosity calculated with equation (2.22) for $R = R_{\text{Sbrane}}$, $R_{+\text{Kerr}}$ or $R_{+\text{Kerrbrane}}$ with $\ell = 10^{-4} m$, the upper limit on the total UHECR luminosity according to equation (3.1) and the upper limits of η_{CR} for $R = R_{+\text{Kerr}}$ and $R_{+\text{Kerrbrane}}$.

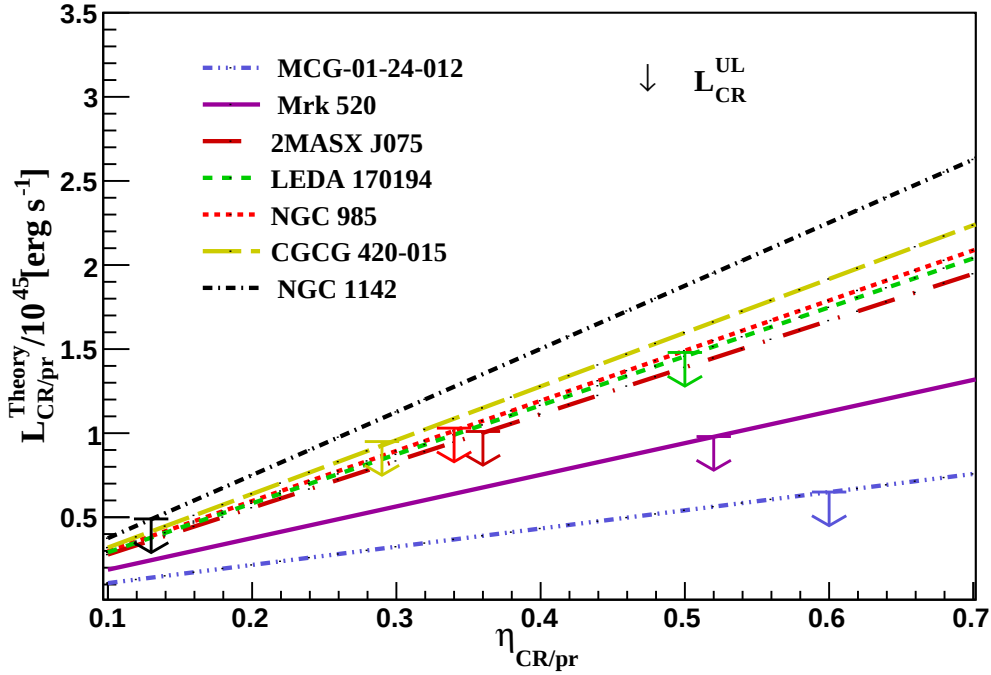


Figure 2. The plots show $L_{\text{CR}}^{\text{Theory}}$ as a function of the parameter η_{CR} . The arrows indicate the values of the luminosity of ultra-high-energy cosmic rays $L_{\text{CR}}^{\text{UL}}$ calculated using the integral flux of GeV-TeV gamma-rays [37].

Figure 2 shows $L_{\text{CR}}^{\text{Theory}}$ as a function of the $\eta_{\text{CR/pr}}^{\text{UL}}$.¹ The arrows indicate $L_{\text{CR/pr}}^{\text{UL}}$ calculated using the integral flux of GeV-TeV gamma-rays [37] and therefore sets $\eta_{\text{CR/pr}}^{\text{UL}}$.

The calculated values of $\eta_{\text{pr}}^{\text{UL}}$ varies from 0.12 (0.13) to 0.69 (0.73) with average 0.36 (0.43) and standard deviation 0.18 (0.18) for the Kerr (Kerr brane) cases. These values, despite being only upper limits are compatible to the estimatives done in reference [32, 35]. Figure 3 shows that $\eta_{\text{pr}}^{\text{UL}}$ decreases with increasing BH mass for $R = R_{\text{Kerrbrane}}$. If this relation reveals a fundamental behaviour of the accretion mechanism or a feature of the methods developed in this paper is a subject to be studied. If $R = R_{\text{Kerr}}$ the picture is not so clear because of the two sources for which an upper limit on $\eta_{\text{pr}}^{\text{UL}}$ was set only for $R = R_{\text{Kerr}}$.

Another important remark is that the theoretical bolometric luminosity $L_{\text{acc}}^{\text{Theory}}$ is sensitive to deviations of Newtonian gravity imposed by the presence of the AdS_5 bulk. This clearly can be seen in equation (2.22), since R comes as solution of equation (2.20) and it is dependent of ℓ parameter, namely, the curvature radius of AdS_5 (sometimes called the warped compactification). Some values of ℓ promote huge gravity deviations and consequently the abrupt drop in luminosity shown in figure 4(a). To recover the upper limits of UHECR luminosities, smaller values of $L_{\text{acc}}^{\text{Theory}}$ give greater values of the conversion fraction $\eta_{\text{pr}}^{\text{UL}}$ (figure 4(b)). The condition $\eta_{\text{pr}}^{\text{UL}} < 1$ gives the allowed values of ℓ and consequently the allowed luminosity region. Since the excluded region only occurs when ℓ is really very large (and the existing limits point that ℓ should be < 1) then we can clearly adopt the $L_{\text{acc}}^{\text{Theory}}$ luminosities calculated in tables 1 and 2.

¹CR for total luminosity or pr for proton only.

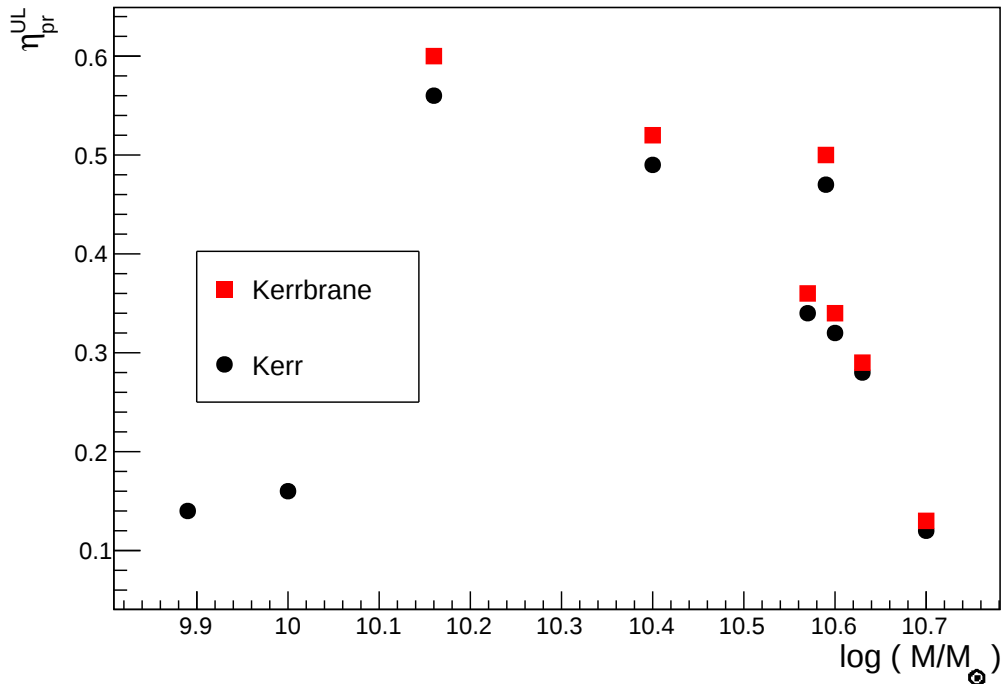


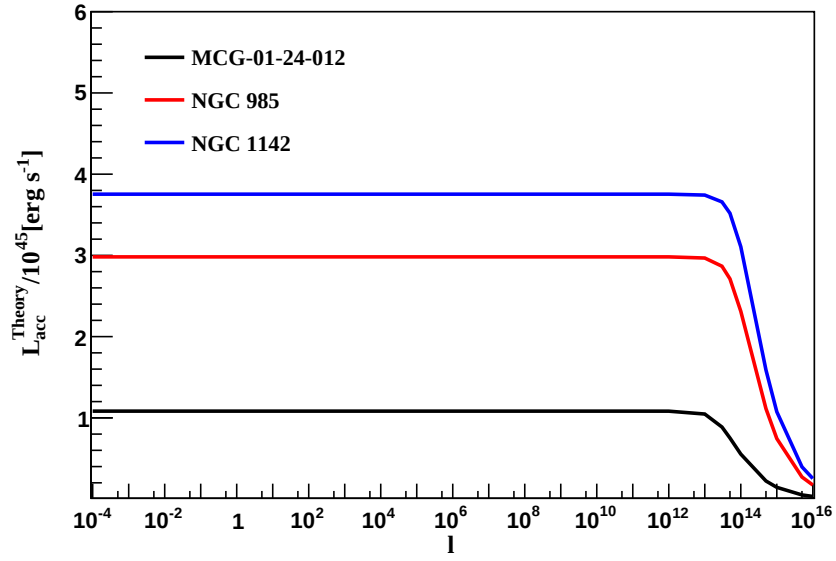
Figure 3. $\eta_{\text{pr}}^{\text{UL}}$ as a function of $\log(M/M_{\odot})$. The graphics show the results the calculation of $\eta_{\text{pr}}^{\text{UL}}$ for the standant Kerr BH (Kerr — black circles) and for Kerr BH with extra dimension correction (Kerrbrane — red squares).

In this paper we calculated the luminosity due to accretion of nine point sources. These calculations lead to an parametrization of the radius of a BH with extra dimension corrections as a function of its mass. The luminosities calculated for nine specific source based on theory were compared to an upper limit on the UHECR luminosity. The comparison resulted in the determination of upper limits on the energy conversion from accretion to UHECR. The calculations and theoretical estimations of this conversion efficiency are rarely found in the literature and represent an important piece of information in the understanding of the energy balance in BH.

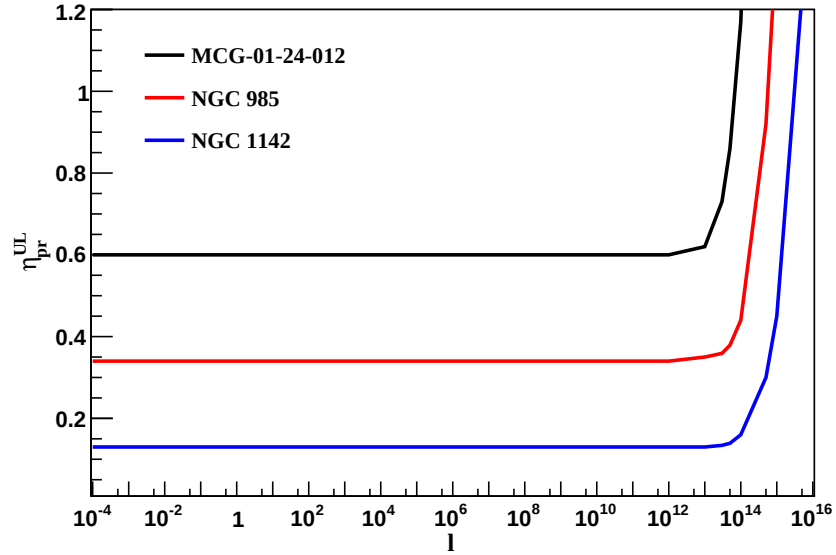
Given the upper limit on the proton or total UHECR luminosity used here, we did not manage to use these values to constrain extra dimension corrections. Such a limitation could be imposed if the proton or total UHECR luminosity were actually measured, this is currently beyond the experimental possibilities. Nevertheless, our studies show which sources could be more interesting to measure if this were the target. In principal one should look for the largest BH mass and the lowest upper limit on UHECR that could be set. Tables 1 and 2 summarize the findings of this paper.

Acknowledgments

RdR is grateful to CNPq (grants No. 303027/2012-6, No. 451682/2015-7 and No. 473326/2013-2), and to FAPESP (grant No. 2015/10270-0) for partial financial support. VdS thanks the support of the Brazilian population via CNPq and FAPESP (2014/19946-4). RCA and VdS thank the Pierre Auger Collaboration.



(a) $L_{\text{acc}}^{\text{Theory}} \times \ell$ study.



(b) $\eta_{\text{pr}}^{\text{UL}} \times \ell$ study.

Figure 4. The plots show the excluded regions with the values of ℓ to sources MCG-01-24-012, NGC 985 and NGC 1142. Figure (a) shows the variation of $L_{\text{acc}}^{\text{Theory}}$ as a function ℓ . Figure (b) shows the variation of $\eta_{\text{pr}}^{\text{UL}}$ as function of ℓ .

References

- [1] C.D. Hoyle et al., *Submillimeter tests of the gravitational inverse square law: a search for ‘large’ extra dimensions*, *Phys. Rev. Lett.* **86** (2001) 1418 [[hep-ph/0011014](#)] [[INSPIRE](#)].
- [2] R.S. Decca et al., *Improved tests of extra dimensional physics and thermal quantum field theory from new Casimir force measurements*, *Phys. Rev. D* **68** (2003) 116003 [[hep-ph/0310157](#)] [[INSPIRE](#)].

- [3] E.G. Adelberger, B.R. Heckel and A.E. Nelson, *Tests of the gravitational inverse-square law*, *Annu. Rev. Nucl. Part. Sci.* **53** (2003) 77 [[hep-ph/0307284](#)].
- [4] J.C. Long et al., *Upper limits to submillimeter-range forces from extra space-time dimensions*, *Nature* **421** (2003) 922 [[INSPIRE](#)].
- [5] R. Maartens and K. Koyama, *Brane-world gravity*, *Living Rev. Rel.* **13** (2010) 5 [[arXiv:1004.3962](#)] [[INSPIRE](#)].
- [6] C.M. Will, *The confrontation between general relativity and experiment*, *Living Rev. Rel.* **17** (2014) 4 [[arXiv:1403.7377](#)] [[INSPIRE](#)].
- [7] M.B Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge U.K. (1987).
- [8] K.R. Dienes, *String theory and the path to unification: A Review of recent developments*, *Phys. Rept.* **287** (1997) 447 [[hep-th/9602045](#)] [[INSPIRE](#)].
- [9] M. Kaku, *Strings, conformal fields and M-theory*, Springer, Germany (2000).
- [10] E. Kiritsis, *Introduction to superstring theory*, Leuven Notes in Mathematics and Theoretical Physics 9, Leuven University Press, Leuven, Belgium [hep-ph/9709062](#).
- [11] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [[hep-th/9906064](#)] [[INSPIRE](#)].
- [12] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)] [[INSPIRE](#)].
- [13] D. Bailin and A. Love, *Kaluza-Klein theories*, *Rept. Prog. Phys.* **50** (1987) 1087.
- [14] R. da Rocha and C.H. Coimbra-Araujo, *Variation in the luminosity of kerr quasars due to extra dimension in brane Randall-Sundrum model*, *JCAP* **12** (2005) 009 [[astro-ph/0510318](#)] [[INSPIRE](#)].
- [15] R. Da Rocha, A. Piloyan, A.M. Kuerten and C.H. Coimbra-Araujo, *Casadio-Fabbri-Mazzacurati black strings and braneworld-induced quasars luminosity corrections*, *Class. Quant. Grav.* **30** (2013) 045014 [[arXiv:1301.4483](#)] [[INSPIRE](#)].
- [16] R. da Rocha and C.H. Coimbra-Araujo, *Extra dimensions in LHC via mini-black holes: effective Kerr-Newman brane-world effects*, *Phys. Rev. D* **74** (2006) 055006 [[hep-ph/0607027](#)] [[INSPIRE](#)].
- [17] PIERRE AUGER collaboration, *Upper limit to the cosmic-ray photon flux above 10^{19} eV using the surface detector of the Pierre Auger Observatory*, *Astropart. Phys.* **29** (2008) 243.
- [18] PIERRE AUGER collaboration, P. Abreu et al., *A search for ultra-high energy neutrinos in highly inclined events at the Pierre Auger Observatory*, *Phys. Rev. D* **84** (2011) 122005 [[arXiv:1202.1493](#)] [[INSPIRE](#)].
- [19] PIERRE AUGER collaboration, P. Abreu et al., *Bounds on the density of sources of ultra-high energy cosmic rays from the Pierre Auger Observatory*, *JCAP* **05** (2013) 009 [[arXiv:1305.1576](#)] [[INSPIRE](#)].
- [20] A.M. Hillas, *The origin of ultrahigh-energy cosmic rays*, *Ann. Rev. Astron. Astrophys.* **22** (1984) 425 [[INSPIRE](#)].
- [21] C.A. Norman, D.B. Melrose and A. Achterberg, *The origin of cosmic rays above 1018.5 eV*, *Astrophys. J.* **454** (1995) 60.
- [22] G. Henri, G. Pelletier, P.O. Petrucci and N. Renaud, *Active galactic nuclei as high-energy engines*, *Astropart. Phys.* **11** (1999) 347 [[astro-ph/9901051](#)] [[INSPIRE](#)].
- [23] R.V.E. Lovelace et al., *Poynting jets from accretion disks*, *Astrophys. J.* **572** (2002) 445.

- [24] F. Takahara, *On the origin of highest energy cosmic rays*, *Prog. Theor. Phys.* **83** (1990) 1071 [[INSPIRE](#)].
- [25] J.P. Rachen and P.L. Biermann, *Extragalactic ultrahigh-energy cosmic rays. 1. Contribution from hot spots in FR-II radio galaxies*, *Astron. Astrophys.* **272** (1993) 161 [[astro-ph/9301010](#)] [[INSPIRE](#)].
- [26] M. Nagano and A.A. Watson, *Observations and implications of the ultrahigh-energy cosmic rays*, *Rev. Mod. Phys.* **72** (2000) 689 [[INSPIRE](#)].
- [27] J. Blumer, R. Engel and J.R. Horandel, *Cosmic rays from the knee to the highest energies*, *Prog. Part. Nucl. Phys.* **63** (2009) 293 [[arXiv:0904.0725](#)] [[INSPIRE](#)].
- [28] A. Letessier-Selvon and T. Stanev, *Ultrahigh energy cosmic rays*, *Rev. Mod. Phys.* **83** (2011) 907 [[arXiv:1103.0031](#)] [[INSPIRE](#)].
- [29] A.V. Olinto and K. Kotera, *The astrophysics of ultrahigh-energy cosmic rays*, *Annu. Rev. Astron. Astrophys.* **49** (2011) 119 [[arXiv:1101.4256](#)].
- [30] M. Lemoine, *Acceleration and propagation of ultrahigh energy cosmic rays*, *J. Phys. Conf. Ser.* **409** (2013) 012007 [[arXiv:1209.6442](#)].
- [31] A.A. Watson, *High-energy cosmic rays and the Greisen-Zatsepin-Kuz'min effect*, *Rep. Prog. Phys.* **77** (2014) 036901 [[arXiv:1310.0325](#)].
- [32] H. Falcke and P.L. Biermann, *The jet-disk symbiosis. 1. Radio to X-ray emission models for quasars*, *Astron. Astrophys.* **293** (1995) 665 [[astro-ph/9411096](#)] [[INSPIRE](#)].
- [33] P. Saikia, E. Körding and H. Falcke, *The fundamental plane of black hole activity in the optical band*, *Mon. Not. Roy. Astron. Soc.* **450** (2015) 2317 [[arXiv:1504.00363](#)].
- [34] J.M. Miller, M. Nowak, S. Markoff, M. Rupen and D. Maitra, *Exploring accretion and disk-jet connections in the LLAGN M81**, *Astrophys. J.* **720** (2010) 1033 [[arXiv:1006.5755](#)] [[INSPIRE](#)].
- [35] I. Dutan and L.I. Caramete, *Ultra-high-energy cosmic rays from low-luminosity active galactic nuclei*, *Astropart. Phys.* **62** (2015) 206 [[arXiv:1409.8162](#)] [[INSPIRE](#)].
- [36] A.D. Supanitsky and V.d. Souza, *An upper limit on the cosmic-ray luminosity of individual sources from gamma-ray observations*, *JCAP* **12** (2013) 023 [[arXiv:1311.4820](#)] [[INSPIRE](#)].
- [37] R.C. Anjos, V. de Souza and A.D. Supanitsky, *Upper limits on the total cosmic-ray luminosity of individual sources*, *JCAP* **07** (2014) 049 [[arXiv:1405.3937](#)] [[INSPIRE](#)].
- [38] M. Sikora et al., *Radio loudness of active galactic nuclei: observational facts and theoretical implications*, *Astrophys. J.* **658** (2007) 815 [[astro-ph/0604095](#)] [[INSPIRE](#)].
- [39] J.D. Lykken and L. Randall, *The shape of gravity*, *JHEP* **06** (2000) 014 [[hep-th/9908076](#)] [[INSPIRE](#)].
- [40] R. Gregory, V.A. Rubakov and S.M. Sibiryakov, *Brane worlds: the gravity of escaping matter*, *Class. Quant. Grav.* **17** (2000) 4437 [[hep-th/0003109](#)] [[INSPIRE](#)].
- [41] A. Herrera-Aguilar, A.M. Kuerten and R. da Rocha, *Regular bulk solutions in brane-worlds with inhomogeneous dust and generalized dark radiation*, *Adv. High Energy Phys.* **2015** (2015).
- [42] T. Shiromizu, K.-i. Maeda and M. Sasaki, *The Einstein equation on the 3-brane world*, *Phys. Rev. D* **62** (2000) 024012 [[gr-qc/9910076](#)] [[INSPIRE](#)].
- [43] N. Dadhich, R. Maartens, P. Papadopoulos and V. Rezanian, *Black holes on the brane*, *Phys. Lett. B* **487** (2000) 1 [[hep-th/0003061](#)] [[INSPIRE](#)].
- [44] R. da Rocha and J.M. Hoff da Silva, *Black string corrections in variable tension braneworld scenarios*, *Phys. Rev. D* **85** (2012) 046009 [[arXiv:1202.1256](#)] [[INSPIRE](#)].
- [45] P. Kanti and K. Tamvakis, *Quest for localized 4D black holes in brane worlds*, *Phys. Rev. D* **65** (2002) 084010 [[hep-th/0110298](#)] [[INSPIRE](#)].

- [46] R. Casadio, A. Fabbri and L. Mazzacurati, *New black holes in the brane world?*, *Phys. Rev. D* **65** (2002) 084040 [[gr-qc/0111072](#)] [[INSPIRE](#)].
- [47] P. Kanti, I. Olasagasti and K. Tamvakis, *Quest for localized 4D black holes in brane worlds. 2. Removing the bulk singularities*, *Phys. Rev. D* **68** (2003) 124001 [[hep-th/0307201](#)] [[INSPIRE](#)].
- [48] C. Germani and R. Maartens, *Stars in the brane world*, *Phys. Rev. D* **64** (2001) 124010 [[hep-th/0107011](#)] [[INSPIRE](#)].
- [49] N. Deruelle, *Stars on branes: the view from the brane*, [gr-qc/0111065](#) [[INSPIRE](#)].
- [50] M. Visser and D.L. Wiltshire, *On brane data for brane world stars*, *Phys. Rev. D* **67** (2003) 104004 [[hep-th/0212333](#)] [[INSPIRE](#)].
- [51] I. Giannakis and H.-c. Ren, *Possible extensions of the 4D Schwarzschild horizon in the brane world*, *Phys. Rev. D* **63** (2001) 125017 [[hep-th/0010183](#)] [[INSPIRE](#)].
- [52] S.L. Shapiro and S.A. Teukolski, *Black holes, white dwarfs, and neutron stars*, Wiley-Interscience, New York U.S.A. (1983).
- [53] H. Bondi, *On spherically symmetrical accretion*, *Mon. Not. Roy. Astron. Soc.* **112** (1952) 195.
- [54] J.L. Puget, F.W. Stecker and J.H. Bredekamp, *Photonuclear interactions of ultrahigh-energy cosmic rays and their astrophysical consequences*, *Astrophys. J.* **205** (1976) 638 [[INSPIRE](#)].
- [55] G.R. Blumenthal, *Energy loss of high-energy cosmic rays in pair-producing collisions with ambient photons*, *Phys. Rev. D* **1** (1970) 1596 [[INSPIRE](#)].
- [56] S.R. Kelner and F.A. Aharonian, *Energy spectra of gamma-rays, electrons and neutrinos produced at interactions of relativistic protons with low energy radiation*, *Phys. Rev. D* **78** (2008) 034013 [Erratum *ibid.* **D 82** (2010) 099901] [[arXiv:0803.0688](#)] [[INSPIRE](#)].
- [57] A. De Angelis, G. Galanti and M. Roncadelli, *Transparency of the Universe to γ rays*, *Mon. Not. Roy. Astron. Soc.* **432** (2013) 3245 [[arXiv:1302.6460](#)].
- [58] FERMI-LAT collaboration, A.A. Abdo, *The first catalog of active galactic nuclei detected by the Fermi Large Area Telescope*, *Astrophys. J.* **715** (2010) 429 [[arXiv:1002.0150](#)] [[INSPIRE](#)].
- [59] HESS collaboration, . F. Aharonian, *Upper limits from HESS AGN observations in 2005-2007*, *Astron. Astrophys.* **478** (2008) 387 [[arXiv:0711.3196](#)] [[INSPIRE](#)].
- [60] MAGIC collaboration, J. Aleksic et al., *Detection of very high energy gamma-ray emission from the Perseus cluster head-tail galaxy IC 310 by the MAGIC telescopes*, *Astrophys. J.* **723** (2010) L207 [[arXiv:1009.2155](#)] [[INSPIRE](#)].
- [61] VERITAS collaboration, N. Galante, *VERITAS observations of radio galaxies*, [arXiv:0912.3850](#) [[INSPIRE](#)].
- [62] M. Unger, G.R. Farrar and L.A. Anchordoqui, *Origin of the ankle in the ultrahigh energy cosmic ray spectrum and of the extragalactic protons below it*, *Phys. Rev. D* **92** (2015) 123001 [[arXiv:1505.02153](#)] [[INSPIRE](#)].
- [63] E. Waxman, *Cosmological gamma-ray bursts and the highest energy cosmic rays*, *Phys. Rev. Lett.* **75** (1995) 3 [[astro-ph/9505082](#)].
- [64] M.W. Lisa, *Extragalactic X-ray Surveys of ULXs and AGNs*, dissertation, University of Maryland, U.S.A. (2008).
- [65] M. Koss et al., *Host galaxy properties of the swift bat ultra hard X-ray selected active galactic nucleus*, *Astrophys. J.* **739** (2011) 57.