

Further Study of BRST-Symmetry Breaking on the Lattice

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We evaluate the so-called Bose-ghost propagator $Q(p^2)$ for SU(2) gauge theory in minimal Landau gauge, considering lattice volumes up to 120^4 and physical lattice extents up to $13.5 fm$. In particular, we investigate discretization effects, as well as the infinite-volume and continuum limits. We recall that a nonzero value for this quantity provides direct evidence of BRST-symmetry breaking, related to the restriction of the functional measure to the first Gribov region. Our results show that the prediction (from cluster decomposition) for $Q(p^2)$ in terms of gluon and ghost propagators is better satisfied as the continuum limit is approached.

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1. BRST-Symmetry Breaking

The minimal Landau gauge in Yang-Mills theories [1] is obtained by restricting the functional integral to the set of transverse gauge configurations for which the Faddeev-Popov (FP) matrix $\mathcal{M}$ is non-negative, the so-called first Gribov region Ω . On the lattice, this gauge condition is imposed by considering a minimization procedure. On the contrary, in the Gribov-Zwanziger (GZ) approach in the continuum [2], this restriction is forced by adding a nonlocal horizon-function term $\gamma^4 S_h$ to the usual (Landau-gauge) action. The resulting (nonlocal) GZ action may be localized by introducing the auxiliary fields $\phi_\mu^{ab}(x)$ and $\bar{\omega}_\nu^{cd}(y)$, yielding $S_{GZ} = S_{YM} + S_{gf} + S_{aux} + S_\gamma$. Here, S_{YM} is the usual four-dimensional Yang-Mills action, S_{gf} is the covariant-gauge-fixing term, S_{aux} is defined as

$$S_{aux} = \int d^4x \left[\bar{\phi}_\mu^{ac} \partial_\nu \left(D_\nu^{ab} \phi_\mu^{bc} \right) - \bar{\omega}_\mu^{ac} \partial_\nu \left(D_\nu^{ab} \omega_\mu^{bc} \right) - g_0 \left(\partial_\nu \bar{\omega}_\mu^{ac} \right) f^{abd} D_\nu^{be} \eta^e \phi_\mu^{dc} \right]$$

and is necessary to localize the horizon function, and S_γ , given by

$$S_\gamma = \int d^4x \left[\gamma^2 D_\mu^{ab} \left(\phi_\mu^{ab} + \bar{\phi}_\mu^{ab} \right) - 4 \left(N_c^2 - 1 \right) \gamma^4 \right],$$

allows one to fix the γ parameter through the so-called horizon condition. Also, one can define [3] for these fields a nilpotent BRST transformation s that is a simple extension of the usual (perturbative) BRST transformation leaving $S_{YM} + S_{gf}$ invariant. However, in the GZ case, the BRST symmetry s is broken by terms proportional to a power of the Gribov parameter γ . Since a nonzero value of γ is related to the restriction of the functional integration to Ω , it is somewhat natural to expect a breaking of the (extended) BRST symmetry s , as a direct consequence of the nonperturbative gauge-fixing.¹ More precisely —as nicely explained in Ref. [7]— an infinitesimal gauge transformation is formally equivalent to a (perturbative) BRST transformation. Since the region Ω is free of infinitesimal gauge copies, applying s to a configuration in Ω should result in a configuration outside Ω . The breaking of the BRST symmetry in minimal Landau gauge is then inevitable, since the functional integration is limited to the region Ω . This interpretation is supported by the introduction [6] of a nilpotent nonperturbative BRST transformation s_γ that leaves the local GZ action invariant. The new symmetry is a simple modification of the extended BRST transformation s , by adding (for some of the fields) a nonlocal term proportional to a power of the Gribov parameter γ .

2. The Bose-Ghost Propagator

As implied above, the Gribov parameter γ is not introduced explicitly on the lattice, since in this case the restriction of gauge-configuration space to the region Ω is achieved by numerical minimization. Nevertheless, the breaking of the BRST symmetry s induced by the GZ action may be investigated by the lattice computation of suitable observables, such as the so-called Bose-ghost propagator

$$Q_{\mu\nu}^{abcd}(x,y) = \langle s(\phi_\mu^{ab}(x) \bar{\omega}_\nu^{cd}(y)) \rangle = \langle \omega_\mu^{ab}(x) \bar{\omega}_\nu^{cd}(y) + \phi_\mu^{ab}(x) \bar{\phi}_\nu^{cd}(y) \rangle.$$

¹This issue has been investigated in several works (see e.g. [4, 5, 6, 7] and references therein).

Since this quantity is BRST-exact with respect to the (extended) BRST transformation s , it should be zero for a BRST-invariant theory, but it does not necessarily vanish if the symmetry s is broken. On the lattice, however, one does not have direct access to the auxiliary fields $(\bar{\phi}_\mu^{ac}, \phi_\mu^{ac})$ and $(\bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$. Nevertheless, these fields enter the continuum action at most quadratically and they can be integrated out exactly, giving for the Bose-ghost propagator an expression that is suitable for lattice simulations. This yields

$$Q_{\mu\nu}^{abcd}(x-y) = \gamma^4 \left\langle R_\mu^{ab}(x) R_\nu^{cd}(y) \right\rangle, \quad (2.1)$$

where

$$R_\mu^{ac}(x) = \int d^4z (\mathcal{M}^{-1})^{ae}(x,z) B_\mu^{ec}(z) \quad (2.2)$$

and $B_\mu^{ec}(z)$ is given by the covariant derivative $D_\mu^{ec}(z)$. One can also note that, at the classical level, the total derivatives $\partial_\mu(\phi_\mu^{aa} + \bar{\phi}_\mu^{aa})$ in the action S_γ can be neglected [3, 4]. In this case the expression for $B_\mu^{ec}(z)$ simplifies to

$$B_\mu^{ec}(z) = g_0 f^{ebc} A_\mu^b(z), \quad (2.3)$$

as in Ref. [4]. Let us stress that, in both cases, the expression for $Q_{\mu\nu}^{abcd}(x-y)$ in Eq. (2.1) depends only on the gauge field $A_\mu^b(z)$ and can be evaluated on the lattice.

3. Numerical Simulations and Results

The first numerical evaluation of the Bose-ghost propagator in minimal Landau gauge was presented—for the SU(2) case in four space-time dimensions—in Refs. [8, 9]. In particular, we evaluated the scalar function $Q(k^2)$ defined [for the SU(N_c) gauge group] through the relation

$$Q^{ac}(k) \equiv Q_{\mu\mu}^{abcb}(k) \equiv \delta^{ac} N_c P_{\mu\mu}(k) Q(k^2),$$

where $P_{\mu\nu}(k)$ is the usual transverse projector and k is the wave vector with components $k_\mu = 0, 1, \dots, N-1$, for a lattice of N points per directions. The lattice momentum $p^2(k)$ is obtained using the improved definition (see Ref. [8]). This calculation has been recently extended in Ref. [10], where we have investigated the approach to the infinite-volume and continuum limits by considering four different values of the lattice coupling β and different lattice volumes $V = N^4$, with physical volumes ranging from about $(3.366 fm)^4$ to $(13.462 fm)^4$. We find no significant finite-volume effects in the data. As for discretization effects, we observe small such effects for the coarser lattices, especially in the IR region. We also tested three different discretizations² for the sources $B_\mu^{bc}(x)$, used in the inversion of the FP matrix $\mathcal{M}$, and find that the data are fairly independent of the chosen lattice discretization of these sources.

Our results concerning the BRST symmetry-breaking and the form of the Bose-ghost propagator are similar to the previous analysis [8, 9], i.e. we find a $1/p^6$ behavior at large momenta and a double-pole singularity at small momenta,³ in agreement with the one-loop analysis carried out

²See Eqs. (30), (31) and (32) in Ref. [10].

³As proven in Ref. [10], the Fourier transform of the quantity $R_\mu^{ac}(x)$, defined in Eq. (2.2) above, is trivially equal to 0 at zero momentum, i.e. $\sum_x R_\mu^{ac}(x) = 0$. Thus, one needs to consider sufficiently large lattice volumes, in order to have the IR behavior of the Bose-ghost propagator under control.

in Ref. [11]. These behaviors can be clearly seen in Fig. 1, where we fit the data for the Bose-ghost propagator $Q(p^2)$ using the fitting function

$$f(p^2) = \frac{c}{p^4} \frac{p^2 + s}{p^4 + u^2 p^2 + t^2}, \quad (3.1)$$

which can be related [see Eq. (3.2) below] to an IR-free FP ghost propagator $G(p^2) \sim 1/p^2$ in combination with a massive gluon propagator $D(p^2)$.

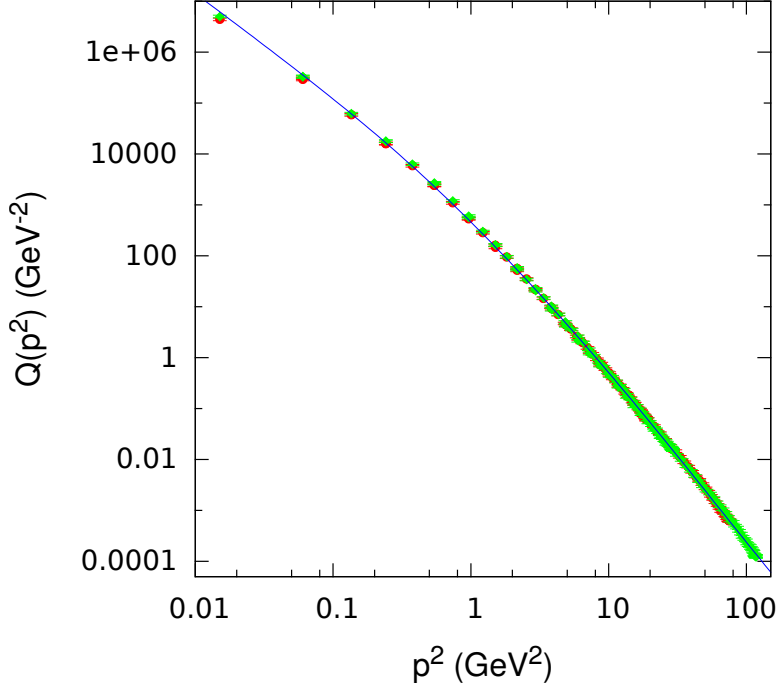


Figure 1: The Bose-ghost propagator $Q(p^2)$ as a function of the (improved) lattice momentum squared p^2 . Here we used as sources $B_\mu^{ec}(z)$ the formula reported in Eq. (32) of Ref. [10]. We plot data for $\beta_2 \approx 2.44$, $V = 96^4$ (●) and $\beta_3 \approx 2.51$, $V = 120^4$ (◆), after applying a matching procedure [12] to the former set of data. We also plot, for $V = 120^4$, a fit using the fitting function (3.1). Note the logarithmic scale on both axes.

In Figs. 2 and 3 we compare the Bose-ghost propagator $Q(p^2)$ to the product $g_0^2 G^2(p^2) D(p^2)$, where g_0 is the bare coupling constant. To this end, the data of the Bose-ghost propagator have been rescaled in order to agree with the data of the product $g_0^2 G^2(p^2) D(p^2)$ at the largest momentum.⁴ This comparison is based on the result

$$Q(p^2) \sim g_0^2 G^2(p^2) D(p^2), \quad (3.2)$$

obtained in Ref. [4] using a cluster decomposition. Even though there is a clear discrepancy between these two quantities we find that this discordance seems to decrease when the continuum limit is considered.

⁴This is equivalent to imposing a given renormalization condition for the propagators at the largest momentum.

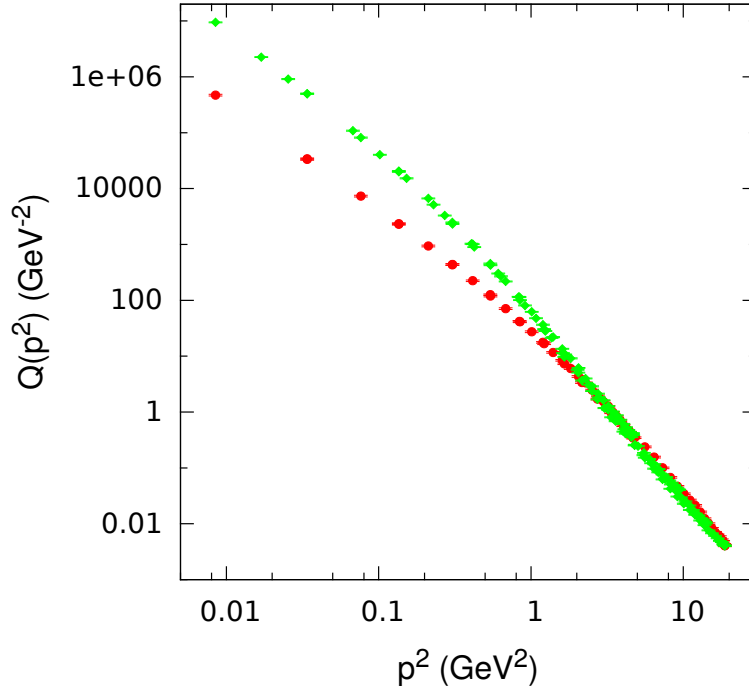


Figure 2: The Bose-ghost propagator $Q(p^2)$ (●) and the product $g_0^2 G^2(p^2) D(p^2)$ (◆) as a function of the (improved) lattice momentum squared p^2 for the lattice volume $V = 64^4$ at β_0 . Here we used as sources $B_\mu^{ec}(z)$ the formula reported in Eq. (32) of Ref. [10]. Note the logarithmic scale on both axes.

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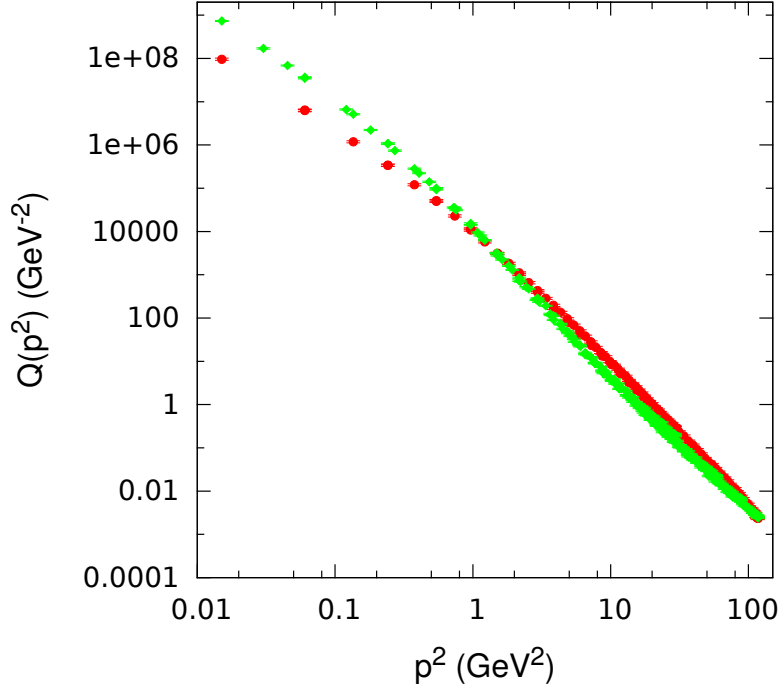


Figure 3: The Bose-ghost propagator $Q(p^2)$ (●) and the product $g_0^2 G^2(p^2) D(p^2)$ (◆) as a function of the (improved) lattice momentum squared p^2 for the lattice volume $V = 120^4$ at $\beta_3 \approx 2.51$. Here we used as sources $B_\mu^{ec}(z)$ the formula reported in Eq. (32) of Ref. [10]. Note the logarithmic scale on both axes.

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