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Torsion Units in Integral Group Rings II

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Abstract

Special cases of Bovdi's conjecture are proved. In particular the conjecture is proved for supersolvable and Frobenius groups. We also prove that if $exp(G/Z)$ is finite, $\alpha \in V\mathbb{Z}G$ a torsion unit and m the smallest positive integer such that $\alpha^m \in G$ then m divides $exp(G/Z)$.

Let G be a group and let $V\mathbb{Z}G$ be the group of units of augmentation one of the integral group ring $\mathbb{Z}G$. Given an element $x = \sum x(g)g \in \mathbb{Z}G$ we set

$$T^{(k)}(x) = \sum_{g \in G(k)} x(g) \ ,$$

called the k-generalized trace of x. Here $G(k) = \{g \in G : o(g) = k\}$. We also set

$$\tilde{x}(g) = \sum_{h \sim g} x(h) \ .$$

A.A.Bovdi proved the following [1] [1]:

[1]AMS Subject Classifiation: Primary 20C05, 20C07, 16S34. Secundary 16U60.
Key words and frases: Group Rings, torsion units, generalized trace.

Lemma 1: *If p is a prime, $x \in V\mathbb{Z}G$ and $o(x) = p^n$, then $T^{(p^n)}(x) \equiv 1 (\mathrm{mod}\ p)$ and $T^{(p^j)}(x) \equiv 0 (\mathrm{mod}\ p)$ for $j < n$. In particular there is an element $g \in G$ such that $o(x) = o(g)$.*

Considering these statements he conjectured that if x is as in Lemma 1 then

BC1: $T^{(p^n)}(x) = 1$ *and* $T^{(p^j)}(x) = 0$ *for* $j < n$.

In [2] BC1 is proved for metabelian nilpotent groups. Bovdi also conjectured the following:

BC2: *Let $n = \exp(G/Z(G))$ be finite, where $Z(G)$ denotes the center of G. If $\alpha \in V\mathbb{Z}G$ is a torsion unit and m is the smallest positive integer such that $\alpha^m \in G$, then m divides n.*

We recall that J.H.Zassenhaus had conjectured the following:

ZC1: *Let $\alpha \in V\mathbb{Z}G$ be a torsion unit then α is conjugated in QG, to an element of G.*

Lemma 1.1 below shows that ZC1 implies BC1. In this paper we deal with the conjectures BC1 and BC2 and show that BC1 holds for Frobenius groups and polycyclic groups whose commutator subgroup is nilpotent or such that all their Sylow subgroups are abelian. Also, we show that BC2 is true.

In the text we denote by δ_{nj} the function which is 0 if $j \neq n$ and 1 if $j = n$.

I - SOME TECHNICAL LEMMAS

The following results are [8, Theorem 7], [13, 41.12] and [13, 47.5].

Lemma 1.1: *Let G a finite group and $\alpha \in V\mathbb{Z}G$ a unit of finite order. Then $\beta^{-1}\alpha\beta \in G$ for some $\beta \in U(QG)$ if and only if there is an element $g_0 \in G$, unique up to conjugacy, such that $\tilde{\alpha}(g_0) \neq 0$.*

Lemma 1.2: *Let $G = P \rtimes X$ where P is a Sylow p-subgroup of G. Let $H \subseteq U(1+\Delta(G,P))$ be finite. Then there exists $\alpha \in QG$ such that $H^\alpha \subseteq G$.*

Lemma 1.3: *Let G be a noetherian group and $u \in V\mathbb{Z}G$ a torsion element. Let $x \in G$ be of infinite order. Then $\tilde{u}(x) = 0$.*

We now prove some results that will be useful to produce an induction argument in the sequel.

Lemma 1.4: *Let G be a finite group and $H \lhd G$ a normal subgroup of G. Let $\psi : \mathbb{Z}G \to \mathbb{Z}(G/H)$ be the natural projection and $\alpha \in V\mathbb{Z}G$ such that $(o(\alpha), |H|) = 1$. If $\beta = \psi(\alpha)$ then $T^{(k)}(\alpha) = T^{(k)}(\beta)$ for every k such that $(k, |H|) = 1$ and $T^{(k)}(\alpha) = 0$ if $(k, |H|) \neq 1$.*

Proof: The second part follows by [8, Theorem 7] and the fact that $G(k)$ is a normal subset of G. So suppose $(k, |H|) = 1$. Set

$$\begin{aligned} S &= \{g \in G : o(g) = km, (k, m) = 1, g^k \in T\} \\ S_1 &= \{g \in S : o(g) = km, m \neq 1\} \end{aligned}$$

Note that if $g \in G$ is such that $(o(g), |H|) = 1$ then $o(g) = o(\psi(g))$. Also if $(o(g), |H|) \neq 1$ then $\tilde{\alpha}(g) = 0$ by [8, Theorem 7]. Hence, $\tilde{\alpha}(g) = 0$ for all $g \in S_1$. Since S_1 is a normal subset of G we have that $\sum_{g\in S_1} \alpha(g) = 0$. Using these facts we have that:

$$\begin{aligned} T^{(k)}(\beta) &= \sum_{o(\psi(g))=k} \alpha(g) = \sum_{g\in S} \alpha(g) = \sum_{o(g)=k} \alpha(g) + \sum_{g\in S_1} \alpha(g) \\ &= \sum_{o(g)=k} \alpha(g) = T^{(k)}(\alpha) \end{aligned}$$

□

Lemma 1.5: *Let p be a prime and G a finite group. Suppose that G has unique subgroup H of order p. Let $\alpha \in V\mathbb{Z}G$ be such that $o(\alpha) = p^n$. Then, with the notation of Lemma 1.4, we have that $T^{(p^j+1)}(\alpha) = T^{(p^j)}(\beta)$.*

In particular if BC1 holds for G/H then BC1 holds for G.

Proof: Let $g \in G$ be an element of order p^j. Then $g^{p^{j-1}} \in H$, by the uniqueness of H. Hence $o(\psi(g)) = p^{j-1}$. Also if $o(\psi(g)) = p^j$ then $p^j \in H - \{1\}$. Hence $o(g) = p^{j+1}$. Using these facts we have that

$$T^{(p^j)}(\beta) = \sum_{o(\psi(g))=p^j} \alpha(g) = \sum_{o(g)=p^{j+1}} \alpha(g) = T^{(p^{j+1})}(\alpha)\,.$$

The second statement is a consequence of the first part and Lemma 1.4 □

Lemma 1.6: *Let G be a noetherian group containing $H \triangleleft G$ with H torsion free. If $\alpha \in V\mathbb{Z}G$ is a torsion element then, with the notation of Lemma 1.4, we have that $T^{(k)}(\alpha) = T^{(k)}(\beta)$. In particular BC1 holds for G if it holds for G/H.*

Proof: Let $g \in G$. We set $\overline{g} = \psi(g)$ and $\overline{G} = \psi(G)$. Let $g \in G$ be an element of finite order. Then, since H is torsion free , we have that $o(\overline{g}) = o(g)$. Hence we have that $\psi^{-1}(\overline{G}(k)) = G(k) \cup \{g \in G : o(g) = \infty, o(\overline{g}) = k\}$. Now $S = \{g \in G : o(g) = \infty, o(\overline{g}) = k\}$

is a normal subset of G and hence it is a disjoint union of conjugacy classes. So, by Lemma 1.3, $\sum_{g\in S}\alpha(g)=0$ and hence we have that $T^{(k)}(\beta = T^{(k)}(\alpha)$. □

Let G be a group and m,n positive integers. We shall say that G is (m,n)-*absorvent* if the subgroup $\langle g\in G : o(g)|m^n\rangle$ has expoent less or equal to m^n. If G is (m,n)-absorvent for all pairs (m,n) then G is called *absorvent.* Clearly abelian groups, regular p-groups and K_8 are absorvent. Here K_8 denotes the quaternion group of order eight.

Lemma 1.7: *Let G be group and $\alpha\in V\mathbb{Z}G$ an element such that $o(\alpha)=p^n$, p a prime. If G is (p,k)-absorvent for all $k\leq n$ then $T^{(p^j)}(\alpha)=\delta_{nj}$.*

Proof: Since G is (p,k)-absorvent we have that $H_k=\{g\in G : o(g)|p^k\}$ is a normal subgroup of G. Consider the projection $\psi:\mathbb{Z}G\to\mathbb{Z}(G/H_k)$. Since α is a torsion unit we have, by [12, III 1.3], that $\sum_{g\in H_k}\alpha(g)\in\{0,1\}$. Since $\sum_{g\in H_k}\alpha(g)=\sum_{0\leq j\leq k}T^{(p^j)}(\alpha)$ it follows that $\sum_{0\leq j\leq k}T^{(p^j)}(\alpha)\in\{0,1\}$ for all $0\leq k\leq n$. Since [12, III 1.3] shows that $\alpha(1)\in\{0,1\}$ we have, inductively, that $T^{(p^j)}(\alpha)\in\{0,1\}$ for all $0\leq j\leq n$. Lemma 1 now gives us the desired result. □

The following result is well-known but we give the proof for the sake of completeness.

Lemma 1.8: *Let H be an abelian Sylow p-subgroup of a finite solvable group G. Then one of the following holds:*

i) $H\lhd G$

ii) $O_{p'}(G)\neq 1$.

Proof: Denote by F the Fitting subgroup of G. We discuss separately two cases.

Case 1: F is not a p-group.

In this case we choose a prime $q \neq p$ and let H be a Sylow q-subgroup of F. Since H is normal in G we are done.

Case 2: F is a p-group. Since G is solvable and a Sylow p-subgroup of G is abelian we have, by [9,5.4.4], that F is a Sylow p-subgroup of G. The result is proved. □

Lemma 1.9: *Let G be a group such that $\exp(G/Z(G))$ is finite. Let $\alpha \in V\mathbb{Z}G$ be a torsion unit and $\psi : \mathbb{Z}G \to \mathbb{Z}(G/Z(G))$ the natural projection. Set $\beta = \psi(\alpha)$ and let m be the smallest positive integer such that $\alpha^m \in G$. If there exists an element $g \in G$ such that $o(\beta) = o(\psi(g))$ then m is a divisor of $\exp(G/Z(G))$.*

Proof: Let $k = o(\beta)$. Then by hypothesi we have that $k \mid \exp(G/Z(G))$. Also, $\alpha^k - 1 \in \Delta(G, Z(G))$. Since α is a torsion unit we have by [13, 47.3] that $\alpha^k = g \in G$. By the minimality of m we must have that $m|k$ and hence $m|\exp(G/Z(G))$. □

Lemma 1.10: *Let α and β be torsion units in the integral group ring $\mathbb{Z}G$ such that $(o(\alpha), o(\beta)) = 1$. Let n, m be the smallest positive integers such that α^n, $\beta^m \in G$. Then $(n, m) = 1$.*

Proof: Note that $n|o(\alpha)$ and $m|o(\beta)$. Hence $(n, m) = 1$. □

II - BC1

The following two results appeared in [5].

Theorem 2.1: *BC1 holds for any finite solvable group such that every Sylow subgroup of*

G is abelian.

Theorem 2.2: *Let G be a finite sovable group such that, for every prime p, if $p \mid |G|$ then $p^4 \nmid |G|$. Then BC1 holds for G.*

In this this section we shall prove the folling:

Theorem 2.3: *BC1 holds for supersolvable groups.*

Theorem 2.4: *BC1 holds for finite Frobenius groups.*

The following result also appeared in [5].

Theorem 2.5: *Let G be a finite solvable group and $\alpha \in V\mathbb{Z}G$ an element of order p^n. Suppose that a Sylow p-subgroup of G is abelian. Then $T^{(p^j)}(\alpha) = \delta_{nj}$.*

Remark 2.6: Let $\gamma_n(G)$ be the smallest nontrivial term of the lower central series of a group G. Then the quotient $G/\gamma_n(G)$ is nilpotent and a result of A. Weiss [14] shows that ZC1, and hence BC1, holds for $G/\gamma_n(G)$. Thus Lemma 1.4 shows that $T^{(k)}(\alpha) \in \{0, 1\}$ for every $\alpha \in V\mathbb{Z}G$ such that $(o(\alpha), |\gamma_n(G)|) = 1$. In particular, by [2, pg 431-433], there exist an element $g \in G$ such that $o(g) = o(\alpha)$.

So we have the following improvement of Theorem 2.1.

Corollary 2.7: *Let G be a finite solvable group such that if $p \mid |\gamma_n(G)|$ then G contains a Sylow p-subgroup which is abelian. Then BC1 holds for G.*

We now state a slightly more general version of Theorem 2.2 which also appeared in

[5].

Theorem 2.8: *Let G be a finite solvable group and set $L = \gamma_n(G)$ as in the remark above. Furthermore, suppose that if a prime p is such that $p \mid |L|$ then $p^4 \nmid |G|$. Then BC1 holds.*

If G is finite then Theorem 2.3 is a consequence of the following result.

Theorem 2.9: *Let G be a finite group whose commutator subgroup is nilpotent. Then BC1 holds for G.*

Proof: Let G be a least counterexample to our statement and $\alpha \in V\mathbb{Z}G$ an element of order $o(\alpha) = p^n$. We first show that G' has to be a p-group. In fact, since G' is nilpotent we may choose $H \lhd G$, $H \subset G'$, such that p does not divides $|H|$. Since G is a least counterexample we apply Lemma 1.4 to derive a contradiction. Hence G' is a p-group and thus G has a normal Sylow p-subgroup. It follows, by the Theorem of Schur-Zassenhuas [9, 9.1.2], that G is as in Lemma 1.1 and hence Lemma 1.1 and Lemma 1.2 give us that $\tilde{\alpha}(g_0) \neq 0$ for an element $g_0 \in G$ which is unique, up to conjugacy. Hence $T^{(p^j)}(\alpha) = \delta_{nj}$ by Lemma 1.2. So BC1 holds for G, a final contradiction. □

Proof of Theorem 2.3: Since G is supersolvable we have, by [9, 5.4.15], that G has a normal subgroup H, which is torsion free and of finite index in G. Hence G satisfies the condition of Lemma 1.6 . Still by [9,5.4.15], we have that G' is nilpotent. So the result follows from Theorem 2.9. □

We now proceed towards the proof of Theorem 2.4. We shall first handle the case where G is solvable.

Lemma 2.10: *Let G be a finite solvable group such that the Sylow subgroups of G are abelian or generalized quaternion groups. Then BC1 holds for G.*

Proof: If $p \neq 2$ then a Sylow p-subgroup of G is abelian and hence we may apply Theorem 2.1. So we need only to consider the case where $p = 2$. We use induction on $|G|$. Let $\alpha \in V\mathbb{Z}G$, be such that $o(\alpha) = 2^n$. By Theorem 2.1 we may suppose that a Sylow 2-subgroup of G is a generalized quaternion group. Assume first that $Fit(G)$ is not a 2-group. Then, it contains a subgroup H, of odd order, which is normal in G. Consider the projection $\psi : \mathbb{Z}G \to \mathbb{Z}(G/H)$. Since G/H also satisfies the hypotheses of the theorem it follows, by induction, that BC1 holds for G/H and, by Lemma 1.4, we have that $T^{(k)}(\alpha) = T^{(k)}(\beta)$.

So, we may suppose that Fit(G) is a 2-group. Since a Sylow 2-subgroup of G is a generalized quaternion group we have that either Fit(G) is cyclic or it is also a generalized quaternion group. Hence, by [9, p. 141], we have that either Aut(Fit(G)) is a 2-group or it is isomorphic to S_4, where the last case occurs only if Fit$(G) \cong K_8$. Recall that if H is a subgroup of G then the quotient group $N_G(H)/C_G(H)$ has a monomorfic image in Aut(H). Also, since G is solvable, it follows by [8,5.4.4] that the centralizer of Fit(G) equals its centre. So if Aut(Fit(G)) is a 2-group then, G is a 2-group and hence A.Weiss' result [13] applies. If Fit$(G) \cong K_8$ then, $|G| = 48$. Set $H = Z(\text{Fit}(G))$; then H is the unique subgroup of order 2 of G. By Theororem 2.2 the quotient group G/H satisfies BC1. Hence we may apply Lemma 1.5 to conclude that G satisfies BC1. □

If G is a finite solvable Frobenius group in Theorem 2.4, then the following result proves BC1 for G.

Lemma 2.11: *Let $G = A \rtimes X$, where A is nilpotent and $(|A|, |X|) = 1$. Suppose that BC1 holds for X; then BC1 also holds for G.*

Proof: Let G be a least counterexample to the statement and $\alpha \in V\mathbb{Z}G$ an element of order $o(\alpha) = p^n$, p a prime. We first show that A has prime power order. In fact, suppose that two distinct primes divide $|A|$. Then we may choose a prime $q \neq p$ such that $q \mid |A|$. Let H be a Sylow q-subgroup of A; then $H \triangleleft G$. Consider the projection $\psi : \mathbb{Z}G \to \mathbb{Z}(G/H)$ and set $\beta = \psi(\alpha)$. Then, by Lemma 1.4, we have that $T^{(p^j)}(\alpha) = T^{(p^j)}(\beta)$. Now, by the minimality of G, G/H satisfies BC1 and hence we have a contradiction.

Now we shall show that the prime envolved in $|A|$ is not p. In fact, if A is a p-group then, by our hipotese, A is a Sylow p-subgroup of G and hence, by Lemma 1.2, ZC1, and hence BC1, holds, a contradiction.

So we must have that p divides $|X|$. In this case consider the projection $\psi : \mathbb{Z}G \to \mathbb{Z}(G/A)$. Then, with the notation of Lemma 1.4, we have that $T^{(p^j)}(\alpha) = T^{(p^j)}(\beta)$. Since BC1 holds for X, by our hypothesis, we have a final contradiction. □

Proof of Theorem 2.4 (solvable case) : By the results of Thompson and Burnside on finite Frobenius groups, [9, 10.5.6], we have that G is as in Lemma 2.11 and the Sylow subgroups of X are cyclic or generalized quaternion groups; hence, by Lemma 2.10, X satisfies BC1. The result then follows once more from Lemma 2.11. □

Lemma 2.11 tells us that in order to prove the non-solvable case we only have to prove BC1 for non-solvable Frobenius complements.

Remarks

1: In Lemma 2.10 we may change generalized quaternion by dihedral. The proof is the same if we use the classification of these groups [4,pg 462].

2: Let G be a group and p a prime such the Sylow p-subgroups of G are elementary abelian. Suppose that $\alpha \in \mathbb{Z}(G)$ is an element whose order is a power of p, say $o(\alpha) = p^n$. By [8, Theorem 7] we have that $\tilde{\alpha}(g) = 0$ if g is not a p-element. Hence, since $\alpha(1) \in \{0, 1\}$ by [12,III.1.3], we have that $T^p \in \{0, 1\}$.

Lemma 2.12: *BC1 holds for $G = SL(2,5)$.*

Proof: Let $G = SL(2,5)$. By [15,18.6] we have that G is a Frobenius complement and hence a Sylow 2-subgroup of G is isomorfic to the quaternion group of order 8. Observe that $|G| = 120 = 2^3.3.5$. Hence, by item 2 of the remarks above, we may consider units $\alpha \in V\mathbb{Z}G$ such that $o(\alpha) = 2^n$. By the Theorem of Brauer-Suzuki [6, pg 102 Theorem 7.8], G has a unique subgroup H of order 2 and hence G/H has elementary abelian Sylows 2-subgroups. So Lemma 1.5 applies. □

We are now ready to prove:

Lemma 2.13: *Let G be a non-solvable Frobenius complement. Then BC1 holds for G.*

Proof: By [15, 18.6] G has a normal subgroup H such that $H = SL(2,5) \times H_0$ where 2, 3 and 5 do not divide $|H_0|$ and hence all Sylow subgroup of H_0 must be cyclic, so H_0 satisfies BC1. Moreover we have that either:

i) $G = H$ or ii) $[G : H] = 2$.

Note that if $p \in \{3, 5\}$ than a Sylow p-subgroup of G is elementary abelian and hence, as remarked above, we need only to consider units whose orders are powers of a prime p, with p distinct from 3 or 5. Recall also that a Sylow 2-subgroup of G is a generalized quaternion group so the Theorem of Brauer-Suzuki, [6, pg 102 Th 7.8], applies. We now discuss the two cases, mentioned above, separately.

Case 1: $G = H$.

In this case we may apply the Lemmas 1.4, 2.12 and 2.13 to obtain the result.

Case 2:$[G : H] = 2$.

Let $\alpha \in V\mathbb{Z}G$ be a torsion element such that $o(\alpha) = p^n$. We discuss two sub-cases.

Case (i): $p \neq 2$. Note that $SL(2,5) \lhd G$, hence we may apply Lemma 1.4, with $H = SL(2,5)$, and then Lemma 2.10 to obtain that $T^{(p^j)}(\alpha) = \delta_{nj}$.

Case (ii): $p = 2$. Note that $H_0 \lhd G$. Consider the quotient group $\overline{G}=G/H_0$. Then $|\overline{G}| = 240$. Now $\overline{G}$ has a unique subgroup of order 2, say H_1. So we may apply Lemma 1.5 for $\overline{G}$ and H_1. The quotient group. $\overline{G}/H_1$ must be non-solvable, of order 125 and hence must be S_5, for which ZC1 holds, (see [7]). So, by Lemma 1.5, we have that BC1 holds for G/H_0. Hence, appying Lemma 1.4 for G and H_0, we obtain that $T^{(2^j)}(\alpha) = \delta_{nj}$. □

Proof of Theorem 2.4 (non-solvable case) : The proof is the same as in the solvable case, using Lemma 2.13 instead of Lemma 2.10. □

The same proof of Lemma 2.10 together with Lemma 1.4 and the remark 2.6 give us the following result:

Theorem 2.14: Let G be a finite solvable group such that if a prime p divides $|\gamma_n(G)|$ then a Sylow p-subgroup of G is cyclic or a generalized quaternion group.. Then BC1 holds for G.

Remark

1. In the statement of Theorem 2 we may change policyclic by the assumption that G contains a normal torsion free subgroup of finite index an is noetherian.

III - BC2

In this section we shall prove that BC2 holds.

Theorem 3.1: *Let $n = \exp(G/Z(G))$ be finite, where $Z(G)$ denotes the center of G. If $\alpha \in V\mathbb{Z}G$ is a torsion unit and m is the smallest positive integer such that $\alpha^m \in G$, then m divides n, i.e, BC2 holds.*

Proof: Let $\alpha \in V\mathbb{Z}G$ be a torsion unit. Write $o(\alpha) = p_1^{r_1} \cdots p_n^{r_n}$. Let $m_i = \prod_{j \neq i} p_j^{r_j}$ and set $\alpha_i = \alpha^{m_i}$. Then $o(\alpha_i) = p_i^{r_i}$. Denote by k_i the smallest positive integer such that $\alpha_i^{k_i} \in G$. Then, by Lemma 1 and Lemma 1.9, we have that $k_i | \exp(G/Z(G))$. Hence, by Lemma 1.10, $k = \prod k_i$ divides $\exp(G/Z(G))$. Since $(m_1, \ldots\ldots, m_n) = 1$ we may choose integers $c_1, \ldots\ldots, c_n \in \mathbb{Z}$ such that $c_1 m_1 + \cdots\cdots + c_n m_n = 1$. So we have that $\alpha = \prod (\alpha_i)^{c_i}$. Thus $\alpha^k \in G$ and hence $m|k$. Consequently we have that $m | \exp(G/Z(G))$. □

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