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Automorphisms of Group Algebras of Dihedral Groups

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1 Introduction

In this paper, we shall give a complete description of the group of automorphisms of the group algebra $\mathbf{Q}D_n$, where D_n denotes the Dihedral group of order $2n$, n odd. Then, we shall use similar methods to give a new proof of the automorphism conjecture for D_n, namely, we shall show that every normalized automorphism σ of $\mathbf{Z}D_n$ can be written in the form $\sigma = \tau_\mu \circ \theta$, where θ is an automorphism induced by an automorphism of the group D_n and τ_μ is a conjugation by a unit $\mu \in \mathbf{Q}D_n$. This fact also follows from a result of G. Peterson [5, theorem 3.6]. Our technique offers a different approach to this problem.

2 Rational Group Algebras

We shall use the following presentation for D_n:

$$D_n = < a, b | a^n = b^2 = 1, bab = a^{-1} > .$$

It follows from [4, theorem 2.2] that the rational group algebra of this group is of the form:

$$\mathbf{Q}D_n \cong \mathbf{Q} \oplus \mathbf{Q} \oplus (\oplus_{\substack{d|n \\ d\neq 1}} M_2(\mathbf{Q}_d))$$

where $\mathbf{Q}_d = \mathbf{Q}(\xi_d + \xi_d^{-1})$ and ξ_d denotes a primitive root of unity of order d.

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In order to describe the group af automorphisms of this algebra, which we denote by $Aut(\mathbf{Q}D_n)$, we introduce three subgroups.

First, for each element $\mu \in U(\mathbf{Q}D_n)$, where $U(\mathbf{Q}D_n)$ stands for the set of invertible elements of $\mathbf{Q}D_n$, let us denote by τ_μ the inner automorphism induced by μ and set:

$$Inn(\mathbf{Q}D_n) = \{\tau_\mu \mid \mu \in U(\mathbf{Q}D_n)\},$$

which is a normal subgroup of $Aut(\mathbf{Q}D_n)$.

Also, given an automorphism $\phi_d : \mathbf{Q}_d \to \mathbf{Q}_d$, $d|n, d \neq 1$, we define an automorphism $\overline{\phi_d} : M_2(\mathbf{Q}_d) \to M_2(\mathbf{Q}_d)$ by:

$$\overline{\phi_d}(a_{ij}) = (\phi_d(a_{ij})) \ , \ (a_{ij}) \in M_2(\mathbf{Q}_d).$$

For each family of automorphisms $(\phi_d)_{d|n}$ we shall denote by $\Phi = (\overline{\phi_d})_{\substack{d|n \\ d\neq 1}}$ the automorphism of $\mathbf{Q}D_n$ which is the identity in the first two components and coincides with $\overline{\phi_d}$ in $M_2(\mathbf{Q}_d)$, for each d. We set:

$$\mathcal{M} = \{\Phi = (\overline{\phi_d})_{\substack{d|n \\ d\neq 1}} \mid \phi_d : \mathbf{Q}_d \to \mathbf{Q}_d \text{ is an automorphism, } d|n,\ d \neq 1\}$$

Finally, let e_1, e_2 be the identity elements of the first two components and denote by $\gamma : \mathbf{Q}D_n \to \mathbf{Q}D_n$ the automorphism of $\mathbf{Q}D_n$ such that $\gamma(e_1) = e_2$ and $\gamma(e_2) = e_1$ and is the identity on the other simple components. Set:

$$\Gamma = \{1, \gamma\}.$$

To fully describe $Aut(\mathbf{Q}D_n)$ we shall need some technical results whose proofs are rather simple.

Lemma 2.1 *Let ξ, θ be roots of unity such that $[\mathbf{Q}(\theta) : \mathbf{Q}(\xi)] = 2$ and $o(\xi)$ is odd. Then $o(\theta) = m.o(\xi)$ where m equals 3, 4 or 6.*

Proof. Since $\xi \in \mathbf{Q}(\theta)$ and $o(\xi)$ is odd, we have that $\xi = \theta^j$, for some positive integer j, and hence $o(\xi)|o(\theta)$. Write:

$$\begin{aligned} o(\xi) &= p_1^{b_1} \dots p_t^{b_t} \\ o(\theta) &= p_1^{a_1} \dots p_t^{a_t} q_1^{c_1} \dots q_s^{c_s} \end{aligned}$$

with $b_i \leq a_i$ and p_i, q_i pairwise different prime rational integers, $1 \leq i \leq t$, $1 \leq j \leq s$.

Since $[\mathbf{Q}(\theta) : (\mathbf{Q}(\xi)] = 2$ we readily see that $\phi(o(\theta)) = 2\phi(o(\xi))$, where ϕ denotes Euler's Totient function. Thus, $a_i = b_i$. $1 \leq i \leq t$ and $q_1^{c_1 - 1} \ldots q_s^{c_s - 1}(q_1 - 1) \ldots (q_s - 1) = 2$.

Hence, we have the following possibilities: either $q_1 = 2$ with $c_1 = 2$ or $q_1 = 2$ with $c_1 = 1$ and $q_2 = 3$ with $c_2 = 1$, or $q_1 = 3$ with $c_1 = 1$. The result follows □

Lemma 2.2 *Assume there exists a* $\mathbf{Q}$*-isomorphism* $\phi : M_2(\mathbf{Q}_d) \to M_2(\mathbf{Q}_m)$, *where* d *and* m *are divisors of* n. *Then* $d = m$.

Proof. Such an isomorphism maps the center of $M_2(\mathbf{Q}_d)$ onto the center of $M_2(\mathbf{Q}_m)$, so we obtain, by restriction, a $\mathbf{Q}$-isomorphism $\phi : \mathbf{Q}_d \to \mathbf{Q}_m$.

Since $\mathbf{Q}_d$ is a normal extension of $\mathbf{Q}$, we have that ϕ is actually an automorphism and thus $\mathbf{Q}_d = \mathbf{Q}_m$.

Let θ be a root of unity such that $\mathbf{Q}(\theta) = \mathbf{Q}(\xi_d, \xi_m)$. We claim that $[\mathbf{Q}(\theta) : \mathbf{Q}(\xi_d)] = 1$ or 2. In fact, since ξ_m is a root of

$$X^2 - (\xi_m + \xi_m^{-1})X + 1 \in \mathbf{Q}(\xi_m + \xi_m^{-1})[X] = \mathbf{Q}(\xi_d + \xi_d^{-1})[X] \subset \mathbf{Q}(\xi_d)[X],$$

it follows that $[\mathbf{Q}(\theta) : \mathbf{Q}(\xi_d)] = [\mathbf{Q}(\xi_d, \xi_m) : \mathbf{Q}(\xi_d)] \leq 2$. Also, note that $[\mathbf{Q}(\theta) : \mathbf{Q}(\xi_d + \xi_d^{-1})] = [\mathbf{Q}(\theta) : \mathbf{Q}(\xi_m + \xi_m^{-1})]$ so $[\mathbf{Q}(\theta) : \mathbf{Q}(\xi_d)] = [\mathbf{Q}(\theta) : \mathbf{Q}(\xi_m)]$.

If this dimension is 1, it follows readily that $\mathbf{Q}(\theta) = \mathbf{Q}(\xi_m) = \mathbf{Q}(\xi_d)$ and, since m and n are odd, [6, III.2.14] shows that $m = d$.

If $[\mathbf{Q}(\theta); \mathbf{Q}(\xi_m)] = 2$, taking into account lemma 2.1 and the fact that m and d are both odd we obtain again that $m = d$. □

We are now ready to prove:

Theorem 2.3 $Aut(\mathbf{Q}D_n) = (Inn(\mathbf{Q}D_n) \rtimes \mathcal{M}) \times \Gamma$

Proof. Since the elements in $Inn(\mathbf{Q}D_n)$ act trivially on $Z(\mathbf{Q}D_n)$, the center of $\mathbf{Q}D_n$, it is clear that $Inn(\mathbf{Q}D_n) \cap \mathcal{M} = \{1\}$. Furthermore, an easy computation shows that $\sigma^{-1}\tau_\mu\sigma \in Inn(\mathbf{Q}D_n)$, for all $\sigma \in \mathcal{M}$ and $\tau_\mu \in Inn(\mathbf{Q}D_n)$. Also, $(Inn(\mathbf{Q}D_n) \rtimes \mathcal{M}) \cap \Gamma = \{1\}$ and clearly γ commutes with every element in $(Inn(\mathbf{Q}D_n) \rtimes \mathcal{M})$.

Thus, we only need to prove that given an arbitrary element $\psi \in Aut(\mathbf{Q}D_n$ it can be written as a product of elements of the given subgroups. To do so, denote by A_1, A_2 the first two simple components of $\mathbf{Q}D_n$. Since these are the only two components such that $A_1 \cong A_2 \cong \mathbf{Q}$, we have that $\psi(A_1) = A_1$ or $\psi(A_1) = A_2$.

Define $\theta \in Aut(\mathbf{Q}D_n)$ equal to ψ or $\psi \circ \gamma$ accordingly. Then $\theta(A_i) = A_i$ for $i = 1, 2$ and lemma 2.2 shows that actually θ fixes all simple components of $\mathbf{Q}D_n$.

It will suffice to show now that $\theta \in Inn(\mathbf{Q}D_n) \rtimes \mathcal{M}$.

For each component $M_2(\mathbf{Q}_d)$, $d|n$, $d \neq 1$, θ induces, by restriction to the respective center, a $\mathbf{Q}$-automorphism $\phi_d : \mathbf{Q}_d \to \mathbf{Q}_d$. Set $\Phi = (\phi_d)_{\substack{d|n \\ d\neq 1}}$.

We claim that $\theta \circ \Phi^{-1} \in Inn(\mathbf{Q}D_n)$. In fact, $\theta \circ \Phi^{-1}$ is the identity mapping on the first two components and is a $\mathbf{Q}_d$-automorphism when restricted to $M_2(\mathbf{Q}_d)$, $d|n$, $d \neq 1$. By the Theorem of Skolem-Noether [3, Theorem 4.3.1], the result follows. □

3 Automorphisms of the Integral Group Ring

We begin this section by showing, in a very elementary way, that $F_n = \mathbf{Q}(\xi + \xi^{-1})$, where ξ is a primitive root of unity of order n, is a minimal splitting field for D_n over $\mathbf{Q}$.

The conjugacy classes os D_n are: $\{1\}$, $bD_n' = b < a >$, and all the classes of the form $\{a^i, a^{-i}\}$, $1 \leq i \leq \frac{n-1}{2}$. Thus, if K is a splitting field for D_n, we know that KD_n has exactly $\frac{n-1}{2} + 2$ simple components.

We denote $\widehat{D_n'} = 1 + a + \cdots + a^{n-1}$ and set:

$$g_1 = \frac{\widehat{D_n'} + b\widehat{D_n'}}{2n}, \quad g_2 = \frac{\widehat{D_n'} - b\widehat{D_n'}}{2n},$$

$$\begin{aligned} e_i &= \tfrac{1}{n}(2 + (\xi^i + \xi^{-i})(a + a^{-1}) + (\xi^{2i} + \xi^{-2i})(a^2 + a^{-2}) + \cdots \\ &\quad + (\xi^{\frac{n-1}{2}i} + \xi^{\frac{n+1}{2}i})(a^{\frac{n-1}{2}} + a^{\frac{n+1}{2}})), \quad 1 \leq i \leq \tfrac{n-1}{2}. \end{aligned}$$

Notice that $E = \{g_1, g_2\} \cup \{e_i\}_{1 \leq i \leq \frac{n-1}{2}}$, is a set of $\frac{n-1}{2} + 2$ elements in the center of FD_n.

Clearly, the elements

$$f_i = \frac{1 + \xi^i a + \cdots + (\xi^i a)^{n-1}}{n}, \quad 1 \leq i \leq n$$

are idempotents in $\mathbf{C} < a >$ such that $\sum_{i=1}^n f_i = 1$. Since $\mathbf{C} < a >$ has n simple components, these are the principal idempotents of $\mathbf{C} < a >$. Hence, $e_i = f_i + f_{n-i}$ is also an idempotent, $1 \leq i \leq \frac{n-1}{2}$ and:

$$\sum_{i=1}^{\frac{n-1}{2}} e_i = \sum_{i=1}^{n-1} f_i = 1 - f_n = 1 - \frac{\widehat{D'_n}}{n}.$$

Thus

$$g_1 + g_2 + \sum_{i=1}^{\frac{n-1}{2}} e_i = 1$$

and E is the set of principal idempotents of FD_n. Consequently, F is the minimal splitting field of D_n over $\mathbf{Q}$.

Since $g_1 + g_2 = \frac{\widehat{D'_n}}{n}$, it follows from [1, lemma1] that

$$FD_n = FD_n \frac{\widehat{D'_n}}{n} \oplus FD_n(1 - \frac{\widehat{D'_n}}{n}) \cong F(D_n/D'_n) \oplus FD_n(1 - \frac{\widehat{D'_n}}{n}),$$

where $FD_n(1 - \frac{\widehat{D'_n}}{n})$ is the sum of all the non commutative simple components of FD_n.

We have that $F(D_n/D'_n) \cong F \oplus F$, so:

$$2n = [FD_n : F] = 2 + \sum_{i=1}^{\frac{n-1}{2}} [(FD_n)e_i : F] \geq 2 + 4\frac{n-1}{2},$$

and thus $[(FD_n)e_i : F] = 4,\ 1 \leq i \leq \frac{n-1}{2}$.

Hence, we can write:

$$FD_n \cong F \oplus F \oplus (\bigoplus_{i-1}^{\frac{n-1}{2}} M_2(F)).$$

Now, let $\psi : \mathbf{Z}D_n \to \mathbf{Z}D_n$ be a normalized automorphism. We shall use our knowledge of the structure of FD_n to prove that ψ can be written in the form $\psi = \tau_\mu \circ \theta$, where θ is an automorphism induced by an automorphism of D_n and τ_μ is a conjugation by a unit $\mu \in \mathbf{Q}D_n$.

We shall denote also by ψ the natural extension of this map to an automorphism of FD_n. Notice that ψ must permute the principal idempotents, preserving the dimensions of the respective simple components. Also, since g_1 is the only principal idempotent whose augmentation is 1, we must have that $\psi(g_1) = g_1$; hence, $\psi(g_2) = g_2$.

Set $e = g_1 + g_2 = \frac{\widehat{D'}}{n}$ and let W be the F-linear subspace with basis $B_1 = \{1, a+a^{-1}, \ldots, a^{\frac{n-1}{2}} + a^{\frac{n+1}{2}}\}$. Notice that the set $B_2 = \{e, e_1, \ldots, e_{\frac{n-1}{2}}\} \subset W$ is linearly independent; thus, it is also a basis of W.

Since $\psi(B_2) = B_2$, ψ gives, by restriction, an automorphism of W. Denote by P the matrix of ψ relatively to the basis B_2; then, $P = (x_{ij})$ is a permutation matrix with $x_{11} = 1$ and $x_{i1} = 0$ if $i \neq 1$.

Let Q be the matrix of ψ relatively to the basis B_1 and set:

$$A = \frac{1}{n} \begin{bmatrix} 1 & 2 & 2 & \cdots & 2 \\ 1 & \xi + \xi^{-1} & \xi^2 + \xi^{-2} & \cdots & \xi^{\frac{n-1}{2}} + \xi^{\frac{n+1}{2}} \\ & & & \cdots & \\ 1 & \xi^{\frac{n-1}{2}} + \xi^{\frac{n+1}{2}} & \xi^{n-1} + \xi^{n+1} & \cdots & \xi^{(\frac{n-1}{2})^2} + \xi^{\frac{n^2-1}{4}} \end{bmatrix}$$

Then it is easy to see that $Q = APA^{-1}$.

The entries in the j^{th} column, $j \neq 1$, are the coefficients of e_{j-1} when written as a linear combination of the elements in B_1. Taking into account [2, Theorem 33.8], if we denote by χ_j the irreducible character of D_n afforded by the simple component $FD_n e_j$, $1 \leq j \leq \frac{n-1}{2}$, we can write:

$$A = \frac{1}{n} \begin{bmatrix} 1 & \chi_1(1) & \cdots & \chi_{\frac{n-1}{2}}(1) \\ 1 & \chi_1(a) & \cdots & \chi_{\frac{n-1}{2}}(a) \\ & & \cdots & \\ 1 & \chi_1(a^{\frac{n-1}{2}}) & \cdots & \chi_{\frac{n-1}{2}}(a^{\frac{n-1}{2}}) \end{bmatrix}.$$

Lemma 3.1 *The matrix Q is a permutation matrix.*

Proof. Since ψ is a normalized automorphism of $\mathbf{Z}D_n$, it follows that Q is a matrix with integral entries and it will suffice to show that $Q^tQ = I$, where Q^t denotes the transpose of Q.

Now, $Q^tQ = (A^{-1})^t P^t A^t A P A^{-1}$. Since $P^tP = I$, it is enough to show that A^tA commutes with P. To this end, set $A^tA = (y_{ij})$; then clearly, $y_{11} = \frac{n+1}{2}$. Also,

$$\epsilon(e_j) = 2 + \sum_{i=1}^{\frac{n-1}{2}} 2(\xi^{ij} + \xi^{-ij}) = 0 \ , \ 1 \leq j \leq \frac{n-1}{2},$$

where ϵ denotes the augmentation mapping of $\mathbf{Z}D_n$. Thus,

$$\sum_{i=1}^{\frac{n-1}{2}} \xi^{ij} + \xi^{-ij} = -1, \quad ; 2 \leq j \leq \frac{n-1}{2};$$

hence:

$$y_{1j} = 2 + \sum_{j=1}^{\frac{n-1}{2}} (\xi^{ij} + \xi^{-ij}) = 1, \quad \text{for all } j.$$

Since $A^t A$ is symmmetric, we also have that $y_{i1} = 1, \ , 2 \leq i \leq \frac{n-1}{2}$, One of the orthogonality relations (see [2, 31.11]) gives, in our case, that:

$$\chi_k(1)\chi_l(1) + \sum_{i=1}^{\frac{n-1}{2}} 2\chi_k(a^i)\chi_l(a^{-i}) = \delta_{kl} 2n.$$

So:

$$\chi_k(1)\chi_l(1) + \sum_{i=1}^{\frac{n-1}{2}} \chi_k(a^i)\chi_l(a^i) = \frac{\delta_{kl} 2n + \chi_k(1)\chi_l(1)}{2},$$

thus:

$$n^2 A^t A = \begin{bmatrix} \frac{n+1}{2} & 1 & 1 & \cdots & 1 \\ 1 & n+2 & 2 & \cdots & 2 \\ 1 & 2 & n+2 & \cdots & 2 \\ & & & \cdots & \\ 1 & 2 & 2 & \cdots & n+2 \end{bmatrix} = (y_{ij}).$$

Set $n^2 P^t A^t A P = (z_{ij})$ and let $\sigma \in S_{\frac{n+1}{2}}$ be the permutation such that $P = (\delta_{\sigma(j)j})$.

Then $z_{ij} = y_{\sigma(i)\sigma(j)}$. Since $\psi(e) = e$, we see that $z_{i1} = y_{\sigma(i)1} = 1$, for all $i > 1$ and thus, also $z_{1i} = 1$, for all $i > 1$.

If $i, j \neq 1$ and $i \neq j$, we have that $\sigma(i) \neq \sigma(j)$. Thus, $z_{ij} = y_{\sigma(i)\sigma(j)} = 2$ and, when $i = j \neq 1$, we have that $z_{ii} = y_{\sigma(i)\sigma(i)} = n + 2$. Hence, $PA^tAP^t = A^tA$ as desired. □

We are now ready to prove:

Theorem 3.2 *There exists a unit $\mu \in \mathbf{Q}D_n$ and an automorphism θ of $\mathbf{Z}D_n$ induced from an automorphism od D_n such that $\psi = \tau_\mu \circ \theta$.*

Proof. Since ψ is a normalized automorphism and permutes class sums, we have that $\psi(a + a^{-1}) = a^j + a^{-j}$, for some index j, $1 \leq j \leq \frac{n-1}{2}$. We claim that $gcd(j, n) = 1$. To see this, we compute:

$$(a+a^{-1})^n = 2 + \sum_{i=1}^{\frac{n-1}{2}} \binom{n}{i} (a^{n-2i} + a^{n+2i}).$$

since ψ permutes class sums, we see that the coefficient of 1 in $\psi((a+a^{-1})^n)$ is equal to 2. On the other hand, we have that:

$$\psi(a+a^{-1})^n = (a^j + a^{-j})^n = 2 + \sum_{i=1}^{\frac{n-1}{2}} \binom{n}{i} (a^{j(-2i)} + a^{j(2i)}).$$

Thus, $a^{2ij} \neq 1$ for $1 \leq i \leq \frac{n-1}{2}$; consequently $gcd(j,n) = 1$ and thus $o(a^j) = o(a)$.

Finally, let $\theta : D_n \rightarrow D_n$ be the automorphism defined by $\theta(a) = a^j$ and $\theta(b) = b$ and denote also by θ its natural extension to $\mathbf{Z}D_n$.

Then, $\theta^{-1} \circ \psi$ extended to $\mathbf{Q}D_n$, fixes all conjugacy classes of D_n and thus the center of $\mathbf{Q}D_n$. By the Theorem of Skolem-Noether, working componentwise, we can find an inner automorphism τ_μ of $\mathbf{Q}D_n$ such that $\theta^{-1} \circ \psi = \tau_\mu$. Hence $\psi = \theta \circ \tau_\mu$. □

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