Accuracy analysis and improvement of a semi-Lagrangian exponential integration method for the shallow-water equations on the rotating sphere

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Abstract

Governing equations modeling atmospheric circulation are characterized by nonlinear advective processes and stiff linear terms. Semi-Lagrangian exponential schemes, proposed by Peixoto and Schreiber (2019), have improved stability and accuracy properties when tackling this type of problem, by combining a stable semi-Lagrangian treatment of the advection and an accurate integration of the linear terms using exponential integration. However, their method was limited to first-order accuracy. In this work, we conduct a detailed truncation error analysis to understand this limited accuracy, and we demonstrate that it is related to the approximation of the integration factor in the discretization of the linear term. Based on this study, we propose an alternative discretization, which effectively leads to a second-order scheme. The stability properties of the schemes are also discussed. Numerical simulations of the shallow-water equations on the rotating sphere, considering challenging test cases, are conducted to validate the analytical results and to compare the proposed method, in terms of accuracy, stability, and computational cost, with other Eulerian and Semi-Lagrangian exponential schemes, as well as with the popular SL-SI-SETTLS, used in operational atmospheric circulation models.

Keywords: semi Lagrangian, exponential integration, accuracy analysis, shallow water equations on the rotating sphere, atmospheric circulation modeling

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