A numerical modeling of solid waste transport in main drain

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Abstract

The objective of the present work is to investigate the influence of variation of diameter and slope of a horizontal drain pipe on its self-cleaning performance. Specifically, computer simulations of solid transport inside the drain pipe of a water closet with 6 liters per flush are carried out. The fluid-solid interaction phenomena were modeled using a Lagrangean Particle-based computational fluid dynamics method, and four cases, with diameters of 78 and 100 mm and slope of 1 and 2 %, were considered. The computed results show that the dynamic can be divided in two phases. In the first, impulsive hydrodynamic force predominates and the reduction of the diameter seems to be advantageous because it increases the mean flow velocity and, consequently, the thrust on the solid. In the second phase, which predominate the forces due gravitational effect, the final velocity of the solid may be higher for large diameter in longer solid displacement. Moreover, the effects of the slope are more visible in the second phase.

Keywords

Building drainage system, waste transport, fluid-solid interaction, nonlinear hydrodynamics, simulation, particle method.

1 Introduction

Recently, the concerns about the self-cleaning performance of building drainage networks owing to the reductions of flow rate caused by adoption of water saving practices has motivated several experimental studies [1]. On the other hand, analytical or semi-empirical simplified approaches and numerical modeling on the issue have also been published [2]. However, as a highly nonlinear phenomenon that involves multiphase free-surface flows with complex fluid-solid interaction (FSI), more complete modeling for in-deep investigation of the system performance still remains a challenge. In this way, focusing on solid waste transport, the objective of this paper is to present a numerical modeling approach for the FSI problem in a drain pipe. For this purpose, a computational fluid dynamic method denominated Moving Particle Semi-implicit (MPS) is adopted herein. MPS method is a fully Lagrangian meshfree particle-based approach. Proposed by Koshizuka *et al.* [3] for the simulation of incompressible flow with free surface, it solves the governing equations of continuum by replacing the differential operators with algebraic operators derived from a particle interaction model based on a weight function. The method is very effective for the simulation of the flows that involve large deformation of free-surface, fragmentation and merging, moving or deformable solids boundaries, multi-body, multi-phase and multi-physics problems.

In the previous works of the present research, MPS has been used to investigate the transient flows inside a horizontal w.c. drain pipe and the effects of the diameter and slope of the pipe [4], as well as the drainage from a bathroom, located at the second floor, with a water closet, a shower and a wash basin considering typical flow rates of the appliances [5]. After that, a simplified modeling, in which sludge is considered as denser and more viscous fluid so that mixable multi-component flow model is adopted, has been carried out to assess the effects of the geometry of the elbow and slope at a stack's base on the waste transport performance [6].

In the present study, a further study on the complex FSI problem involving solid wastes is performed. For sake of simplicity, the solid wastes are assumed as rigid bodies with free motion and they are taken into account as a cluster of particles of which the relative velocities among them are zero. By integrating hydrodynamic loads on the surface of the solid, the motions of the bodies are determined. Beside the solid fluid interaction, a numerical model for solid-solid contact is also applied. Finally, considering the relatively low flow rate, the air entrapped inside the pipes and the pressure variation due to the entrapment is neglected. As cases of study of solid transport inside the drain pipe of a 6 liters water closet, a homogeneous cylindrical solid is considered and transient flows inside a horizontal pipe with 90^o elbows in its upstream are investigated. The effects of the diameter and slope of the horizontal pipe are also analyzed. As a result of the study, motion behaviors and displacements of the solid are obtained, which provides insights on the hydrodynamics of the waste solid transport.

2 Numerical method

2.1 Moving Particle Semi-implicit

Moving Particle Semi-implicit (MPS) method is a fully Lagrangian meshfree particle-based approach for the simulation of incompressible flow with free surface. It solves the governing equations of continuum by replacing the differential operators with algebraic

operators derived from a particle interaction model based on a weight function.
To solve the incompressible viscous flow, a semi-implicit algorithm is used. At first, predictions of the particle's velocity and position are carried out explicitly by using viscosity and external forces terms of the momentum conservation. The pressure of all particles is calculated by the Poisson equation for the pressure, which is solved implicitly. The RHS term of the Poisson equation is proportional to the deviation of particle number density, which is a parameter that is proportional to the density of the fluid in the vicinity of the particle. Finally, the velocity of the particles is updated by using the pressure gradient term of the momentum conservation and the new positions of the particles are obtained. More detailed description of the MPS method, including the numerical treatment of the boundary conditions, can be found in [3] and in the previous works of the authors [4,5,6].

2.2 Free solid modeling

For the numerical modeling of solids, three types are considered: fixed solid, solid with forced motion and a free floating solid. The velocity of the fixed and the forced motion solids are imposed as Dirichlet boundary conditions. On the other hand, the motion of the free floating solid are calculated based on forces and moments obtained from the integration of the pressure on the solid surface, i.e., the pressure of the wall particles. In the present study, based on Sueyoshi *et al.* [7], the center of gravity, the mass and the moment of inertia of each free floating solid are input parameters.

2.3 Collision among solids

The collision between two different rigid bodies is identified by monitoring the distance between their wall particles. When the distance between two wall particles, belonging to different solids is smaller than 1.225 l_o, where l_o is the initial distance between particles, collision is computed. The solid repulsion force resulting from the collision is then calculated by summing all the individual components of the force body from each particle. The individual components of the repulsion force are calculated based on a damped harmonic oscillator with linear spring force and damping force proportional to the relative velocity between colliding particles [8].

2.4 Validation of the numerical method

Regarding the validation of the numerical method, its ability to model the dynamics of the transient flow with surface was already been shown in the previous works [4,5,6]. The modeling free floating solids were also extensively validated in other studies such as Tsukamoto *et al.* [9].

3 Descriptions of the case

The configuration of the case consists of a horizontal pipe, which represents a drain pipe of a 6 liters water closet, with 2.5 meters in length. The drain pipe has a 90^o elbow in its upstream end and open downstream end, as shown in Figure 1.

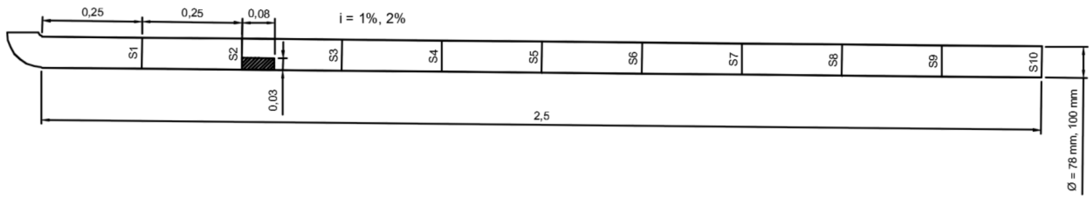


Figure 1 Configuration of the horizontal drain pipe of a 6 liters water closet

For sake of simplicity, the solid waste was modeled as a homogeneous circular cylinder. The cylindrical solid has 0.03 m in diameter and 0.082 m in length, and it is initially in rest inside the pipe with its axis parallel to the axis of the pipe. As the initial conditions for the simulation, the pipe is dry and cylindrical solid is located with its upstream face 0.5 m from the upstream section of the horizontal pipe. The density of the fluid is 1000 kg/m^{3}1010 kg/m^3. The mass of the cylindrical solid is 0.059 kg, homogeneously distributed with density equals to 1010 kg/m^3. As shown in Table 1, four conditions resulting from the combination of two diameters (78 mm and 100 mm) and two slopes (1% and 2%) have been considered in the study.

Table 1 Properties of the sludge

Case	Diameter (mm)	Slope (%)
TS10001	100	1
TS10002	100	2
TS07801	78	1
TS07802	78	2

Figure 2 shows the flow rate discharge as a function of time of the 6 liters water closet provided by Cheng *et al.* [4] and also used and [5].

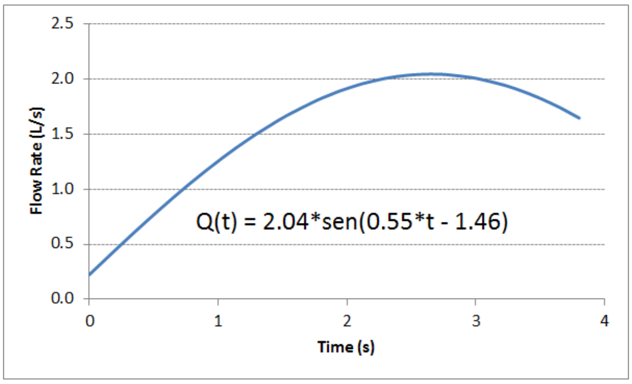


Figure 2 Water closet flush profile [4]

For all the cases simulated, the initial distance between particles (l_0) is 0.003 m, which leads to models with about 532 K particles for 78 mm diameter and 620 K particles for 100 mm diameter. The interaction between the solid and the pipe wall was models as a spring-mass system (harmonic oscillation without damping) with spring constant of 3809.8 N/m. A numerical friction coefficient of 0.22 Ns/m was also introduced to simulate the dynamic friction coefficient of 0.26. The time step adopted was 3 x 10^{-4} s, which leads to processing time of about 15 hours in the cases with 78 mm diameter and 16 hours for the cases with 100 mm diameter, for 5 seconds of simulation.

4 Results and discussions

Figure 3 gives a sequence of images obtained from the computational simulations of the solid transport by a discharge of 6 liters. In order to make easy the visualization, only section views of the initial part of the horizontal pipe and the dynamics of the first few seconds are shown. For the comparison of the results, some instants of time of the cases TS07801 and TS10001 are given, respectively, in the left and right columns of Figure 3.

Case TS07801 Case TS10001

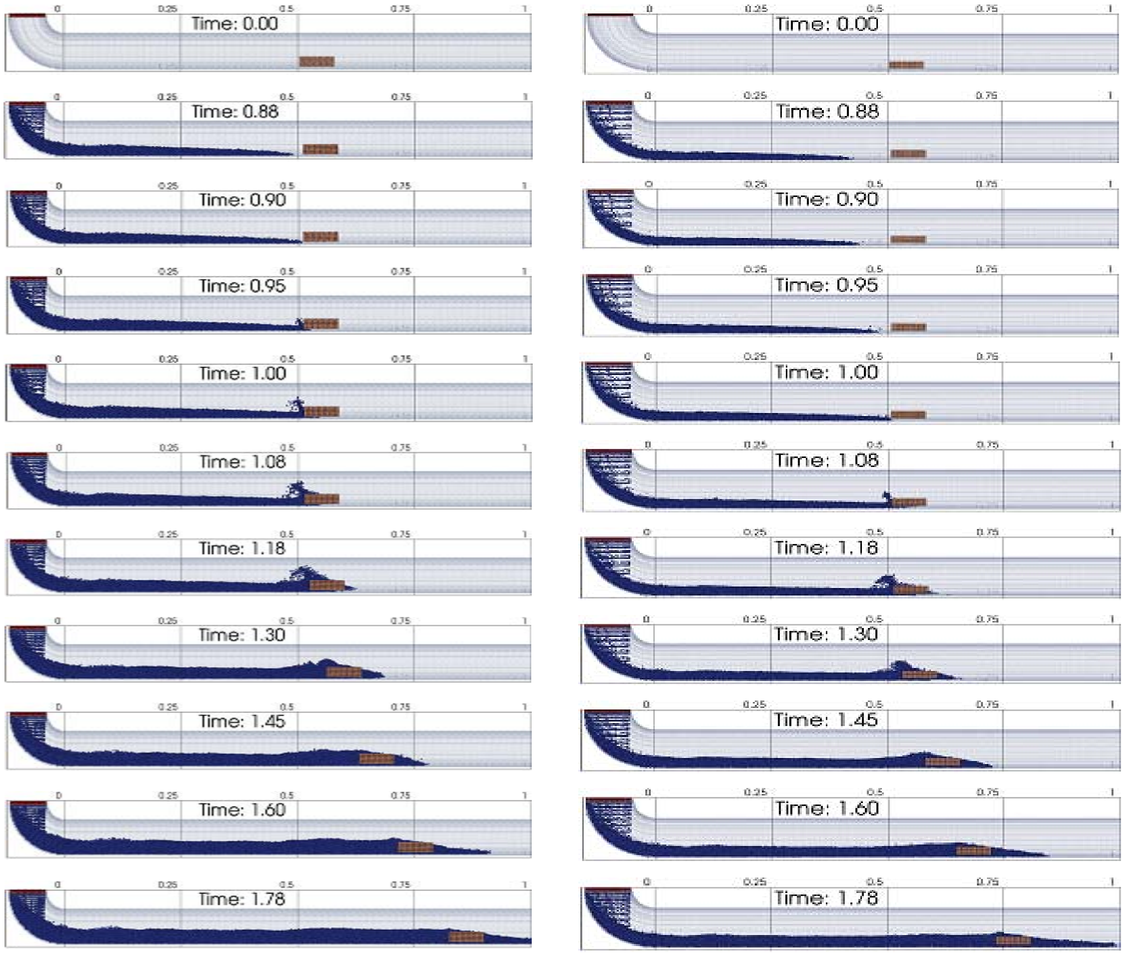


Figure 3 Sequence of images obtained from the simulations of the cases T07801 (left column) and T10001 (right column)

As mentioned before, at instant t=0.00 s, when the flush starts, the upstream face of the solid is 0.5 m from the initial section of the horizontal pipe. As the diameter of TS07801 is smaller than that of TS10001, for the same incoming flow rate, its average flow velocity is higher. Thus, while the incoming flow hits the solid around t=0.90 s in TS07801, in the case of TS10001 this occurs after t=0.95 s. In addition to this, the higher flow velocity of the case TS07801 also produced a larger hydrodynamic load on the solid, as a consequence, at least in the initial instants for the impact shown in Figure 3, a faster displacement of the solid in relation to TS10001 is observed. In other words, the computed results show that the reduction of diameter from 100 mm to 78 mm has a positive effect by increasing the flow velocity and, as a consequence, generating a large thrust on the solid so that it displaced faster in the initial phase of the solid transportation process.

From Figure 3, it is possible to observe that the initial phase, in which the impulsive hydrodynamic force is predominant, occurs in a relatively short interval, and it is associated to violent hydrodynamic impact and large deformation of the free surface. Within this interval, whose duration is about 0.5 to 0.7 s, the solid starts the motion. According to Figure 3, after the instant *t*=1.60 s, the geometry of the free surface remained practically unchanged, with the water level in upstream face of the solid higher than the downstream face. In this way, together with the hydrodynamic loads due to relative velocity between the flow and the solid, the thrust acting on the solid may have a significant contribution from the difference of the hydrostatic pressure between upstream and downstream faces of the solid.

The average flow velocities of the case TS07801 computed in the 10 sections S1 to S10 equally spaced in 0.25 m are shown in Figure 4. The time histories of the average velocities in the sections show a characteristic behavior of the fluid flow: after reaching a section, the flow velocity remains practically constant until the peak of flush passes completely trough the sections, when the velocity reduces remarkably, followed by a smooth decay to zero. The duration of the plateau is longer in the first section S1 and decreases gradually for subsequent sections. The energy loss in the fluid can also be observed by gradual reduction of the average velocity along the pipe, in the subsequent sections. In the case TS07801, the average velocity in the region of the plateau is about 0.93 to 0.62 m/s. On the other hand, the case TS10001, which shows the same behavior, the average velocity in the region of the plateau ranges from 0.72 to 0.58 m/s.

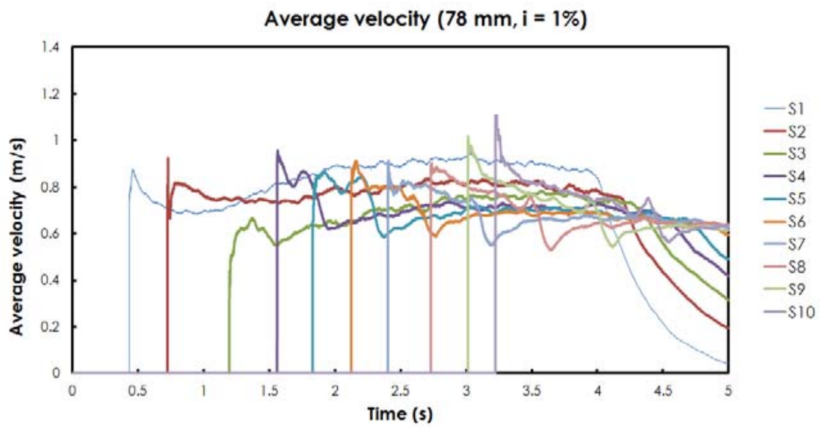


Figure 4 Computed time history of the average mean velocities in the sections of the pipe: diameter 78 mm, slope 1%

The computed positions of the solid as a function of time for the four cases computed in the present study are shown in Figure 5. From comparison among the results, it is clear that the motion patterns of the solid are significantly different for the pipes of 78 mm and 100 mm. For cases with 78 mm diameter (TS07801 and TS07802), in the initial phase of the solid motion, between 0.9 to 1.5 s, where the impulsive load is predominant, the solid starts to move and is accelerated to an almost constant velocity around 0.6 m/s. Then

from t=2.0 s, the effects of the pipe slope can be visualized. The motion of the solid becomes slightly faster when the slope is increased from 1% to 2%. This small but significant difference in velocity shows the effects of the gravitational force on the motion of the solid after the initial phase of impulsive loads. As a result, by t=4.5 s, the solid in the case TS07802, with 2% slope, reaches the downstream end of the pipe of 2.5 m in length shortly before the TS07801 case (1% slope).
On the other hand, for the cases with 100 mm diameter (TS10001 and TS10002), the curves of the position as function of time can be easily divided in two nearly linear segments: in the first, up to t=2.70 s, the gradient of the curve, i.e., the solid velocity, is slightly lower than that computed for 78 mm diameter. Nevertheless, by t=2.70 s, a small increase of the solid velocity occurs, and surpasses that computed for 78 mm diameter, so that the solid reaches earlier the downstream end of the pipe. Regarding the gravitational force due the variation of pipe slope, its effect is quite similar to the former two cases except they can be visualized in early stage of the solid motion.

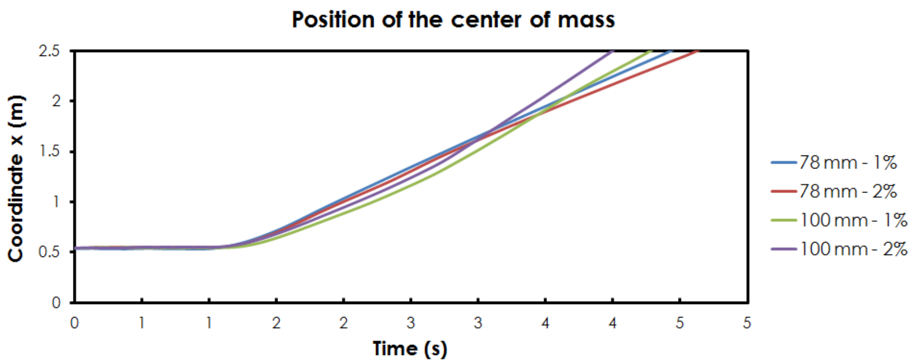
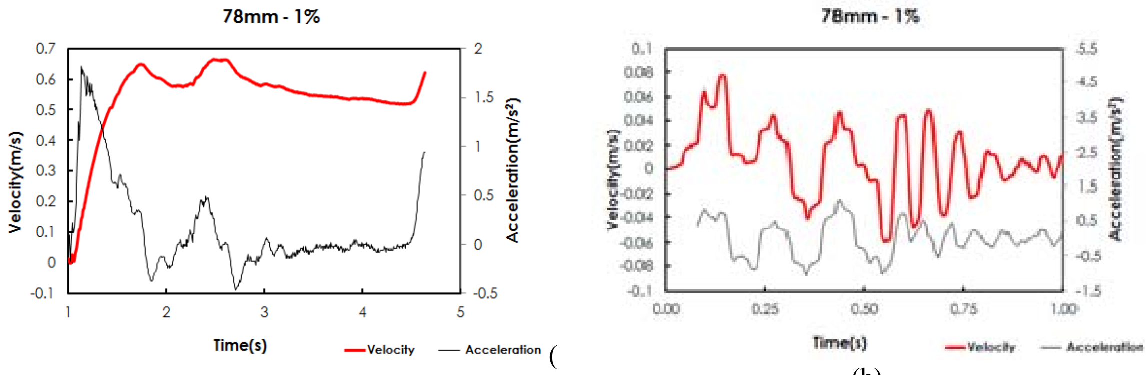


Figure 5 Computed time history of the positions of the cylindrical solid

This remarkable difference in the motion pattern due to the variation of the diameter can be explained by the contribution of several loads actuating on the solid. In the initial phase, the pipe with smaller diameter provides higher flow velocity, which results in larger impulsive thrust on the solid. However, this initial advantage is vanished later when the transient impulsive load ceases and other loads, much smaller in magnitude but permanents, such as gravitational force or friction between the solid and pipe wall, become dominants. Thus, due to lower energy loss by friction and other effects, the final velocity in larger diameter pipe may become higher for large displacements of the solid.
The time histories of the velocity and acceleration for the case TS07801 presented in Figure 6 provides a good insight about the complex hydrodynamic process of the solid transport. From the curve of the acceleration in longitudinal direction shown in Figure 6 (a), the abrupt rise up of the thrust due to the violent hydrodynamic impact reaches a peak and decays quickly to approximately zero at about t=2.00 s. After that, oscillations of the force associated to the dynamic variation of the relative velocity between the fluid and the solid marks the transition from impulsive to steady motion dominated by gravitational effects. Figure 6 (b) shows the time histories of velocity and accelerations of the solid in vertical direction. According to the computed results, the motion of the solid in vertical direction is negligible because in this case the density of the solid id slightly higher than the fluid density.



a)   (b)

Figure 6 Computed time history of the velocity (red line) and acceleration (black line) of the cylindrical solid in the longitudinal direction of the pipe (a) and in the vertical direction (b). Case TS07801, diameter 78 mm and slope 1%

Figure 7 illustrates the computed pressure profiles on both upstream and downstream faces of the solid in the case T07801. Figure 7 (a) shows the compute result at *t*=1.00 when the impact just started, and Figure 7 (b) shows the result at *t*=1.85 s. It is important to point out that in both moments the pressure profiles are not linear, which means the presence of the hydrodynamic component. Thus, mainly for the first phase dominated by transient hydrodynamic impact loads, prediction using the simple assumption that consider only difference of the hydrostatic head between upstream and downstream faces of the solid to calculate the thrust will lead to questionable results.

(a)   (b)

Figure 7 Computed pressure profile on the upstream and downstream faces of the solid at *t*=1.00 s (a) and *t*=1.85 s (b). Case TS07801, diameter 78 mm and slope 1%

5 Concluding remarks

In the present work, the influences of the variation in diameter and slope of a drain pipe on its solid waste transport performance was investigated through a particle-based numerical approach. Four cases of transport of the solid inside a drain pipe of a water closet with 6 liters flush considering pipe diameter of 78 mm and 100 mm and slope of 1% and 2% were taken into account. As a result, the computed results provide a very good insight about the complex FSI hydrodynamic process, which, for the duration considered in the present study, can be divided in two phases:

- The first one is dominated by very high impulsive hydrodynamic loads of relatively short duration when the incoming wave front hits the solid. Starting from the rest, the solid is suddenly accelerated in very short interval and very large free surface

deformation may occur. As for a fixed flow rate the reduction of diameter increases the mean flow velocity, in this initial phase the reduction of diameter is advantageous because it will provide higher thrust for the solid.
- In the second phase, when the impulsive load decreased to nearly zero, the effects related to gravity, such as waves, hydrostatic heads, weight, friction between the solid and the pipe wall become relevant. According to the computed results, for longer distance displacements, the solid may reach higher final velocity in case of pipes with larger diameter. Similarly, as additional thrust due to gravity is provided by pipes with larger slope, slope effects are more relevant in the second phase.

As next step, more complex solid geometry, effects of the solid density and other solid proprieties should be considered, as well as the effects of the entrapping air.

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