CariOPT 2025
Book of abstracts

Speaker: **Aaron Defazio (Meta AI)**
Title: *Schedules and Schedule-Free Learning*
Abstract: I will introduce an alternative view of learning rate schedules, where they are considered as a technique for ensuring optimal convergence rates for the last iterate of an optimization procedure. This view leads to highly predictive theory of optimal learning rate schedules, explaining learning rate warmup and annealing procedures used in practice. Going beyond this, I will show how this viewpoint suggests Schedule-Free approaches, where learning rate schedules are replaced by iterate averaging schemes, which yield a number of benefits: no need to specify the stopping time in advance, smoother loss curves and often better eval metrics.

Speaker: **Alejandro Jofré (University of Chile, CMM & DIM)**
Title: *Reaching an equilibrium of prices and holdings of goods through direct buying and selling*
Abstract: The Walras approach to equilibrium focuses on the existence of market prices at which the total demands for goods are matched by the total supplies. Trading activities that might identify such prices by bringing agents together as potential buyers and sellers of a good are characteristically absent, however. Anyway, there is no money to pass from one to the other as ordinarily envisioned in buying and selling. Here a different approach to equilibrium – what it should mean and how it may be achieved – is offered as a constructive alternative. Agents operate in an economic environment where adjustments to holdings have been needed in the past, will be needed again in a changed future, and money is familiar for its role in facilitating that. Marginal utility provides relative values of goods for guidance in making incremental adjustments, and with money incorporated into utility and taken as numèraire, those values give money price thresholds at which an agent will be willing to buy or sell. Agents in pairs can then look at such individualized thresholds to see whether a trade of some amount of a good for some amount of money may be mutually advantageous in leading to higher levels of utility. Iterative bilateral trades in this most basic sense, if they keep bringing all goods and agents into play, are guaranteed in the limit to reach an equilibrium state in which the agents all agree on prices and, under those prices, have no interest in further adjusting their holdings. The results of computer simulations are provided to illustrate how this works.

Jointly with J. Deride and R.T. Rockafellar.

Speaker: **Alfredo Iusem (FGV-EMAp)**
Title: *Univariate representations of solutions to generic polynomial complementarity problems*
Abstract: By using the squared slack variables technique, we show that a general polynomial complementarity problem can be formulated as a system of polynomial equations. Thus, the solution set of such a problem is the image of a real algebraic set under a certain projection. This paper points out that, generically, this polynomial system has finitely many complex zeros. In such a case, we use techniques from symbolic computation to compute a univariate representation of the solution set. Consequently, univariate representations of special solutions, such as least-norm and sparse solutions, are obtained. After that, enumerating solutions boils down to solving problems governed by univariate polynomials. We also provide some experiments on small-scale problems with worst-case scenarios. At the end of the paper, we propose a method for computing approximate solutions to copositive polynomial complementarity problems that may have infinitely many solutions. Joint with Vu Trung Hieu, Paul Hugo Schm lling, Akiko Takeda.

Speaker: **Benar Svaiter (Instituto de Matemática Pura e Aplicada)**
Title: *Weber-Fechner Law and the Optimality of the Logarithmic Scale*
Abstract: Weber-Fechner Law states that the perceived intensity is proportional to the logarithm of the stimulus. Recent experiments suggest that this law also holds true for perception of numerosity. Therefore, the use of a logarithmic scale for the quantification of the perceived intensity may also depend on how the cognitive apparatus processes information. If Weber-Fechner law is the result of natural selection, then the logarithmic scale should be better, in some sense, than other biologically feasible scales. We consider the minimization of the relative error as the target of natural selection and we provide a formal proof that the logarithmic scale minimizes the maximal relative error.

Speaker: **Cristóbal Guzmán (PUC-Chile & Google Research)**
Title: *Differentially-Private Synthetic Data Generation: A Saddle-Point Approach*
Abstract: Generating synthetic data is one of the key problems in private data analysis. In this talk, I will provide a summary of existing work, together with some novel approaches and results. First, we show that the classical Stochastic Entropic Mirror-Descent algorithm can be made differentially private by a vertex-sampling technique, attaining optimal population risk rates for synthetic data generation. Second, for accuracy guarantees based on relative error (i.e, a mixture of additive and multiplicative error), we can show that (counting) error rates can be made poly-logarithmic in the sample size, data universe size, and the number of linear queries; a result which is provably unattainable in the purely-additive counterpart.

Speaker: **Daiana Oliveira dos Santos (UNIFESP)**
Title: *Strong global convergence properties of algorithms for nonlinear symmetric cone programming*
Abstract: Sequential optimality conditions have played a major role in proving strong global convergence properties of numerical algorithms for many classes of optimization problems. In particular, the way complementarity is dealt is fundamental to achieve a strong condition. Typically, one uses the inner product structure to measure complementarity, which gives a very general approach to a general conic optimization problem, even in the infinite dimensional case. In this paper we exploit the Jordan algebraic structure of symmetric cones in order to measure complementarity, which gives rise to a stronger sequential optimality condition. Our results improve some known results in the setting of semidefinite programming and second-order cone programming in a unified manner. In particular, we obtain global convergence results which are stronger than the ones known for augmented Lagrangian algorithms and interior point methods for general symmetric cones.

Speaker: **Douglas Gonçalves (UFSC)**
Title: *A Levenberg–Marquardt algorithm for completely positive matrix factorization*
Abstract: We will present a projected inexact Levenberg–Marquardt (LM) method for solving completely positive matrix factorization problems. Aimed to reduce the computational cost, and based on a symmetry condition/assumption, we introduce an inexact LM direction which converges to the exact one as the residue approaches zero. Global convergence results are established for the proposed method endowed with a non-monotone line search. A series of numerical experiments on different classes of the problem are carried out and indicate that the new algorithm outperforms well-established alternatives such as algorithms based on proximal gradient or alternating minimization methods. Joint work with Roger Behling, Hugo Lara and Harry Oviedo.

Speaker: **Ernesto Birgin (USP)**
Title: *Regularized block coordinate descent methods: Complexity and applications*
Abstract: In this work, we propose block coordinate descent methods for bound-constrained and nonconvex constrained minimization problems. Our approach relies on solving regularized models. For bound-constrained problems, we introduce methods based on models of order p, which exhibit asymptotic convergence to pth-order stationary points. Moreover, first-order stationarity with precision ϵ with respect to the variables of each block is achieved in $O(\epsilon^{-(p+1)/p})$; while first-order stationarity with precision ϵ with respect to all the variables is achieved in $O(\epsilon^{-(p+1)})$. For nonconvex constrained minimization, we consider models with quadratic regularization. Given feasibility/complementarity and optimality tolerances $\delta > 0$ and $\epsilon > 0$ for feasibility/complementarity and optimality, respectively, it is shown that a measure of $(\delta, 0)$-criticality tends to zero; and the number of iterations and functional evaluations required to achieve

(δ, ϵ)-criticality is $O(\epsilon^{-2})$. Numerical experiments illustrate the effectiveness of our methods. We apply the first method to solve the Molecular Distance Geometry Problem, while the second method is used to enhance heuristic approaches for the Traveling Salesman Problem (TSP) with neighbors, a variant of the classical TSP problem where regions in the plane must be visited instead of cities. The case where regions are described by arbitrary (nonconvex) polygons is considered.

Speaker: **Felipe Lara (Universidade de Tarapacá)**
Title: *Optimal trade characterizations in multi-asset crypto financial markets*
Abstract: Automated market makers are financial markets which involves several research fields such as computer sciences, finance and applied mathematics. One of its most important applications are given by cryptocurrency markets or, more generally, by cryptoassets financial markets such as Uniswap, Balancer, Curve, mStable, and SushiSwap. Initially introduced by Nakamoto with Bitcoin [3], crypto financial markets are Decentralized Exchanges which are usually modeled by trade functions which remain constant after the trade, giving rise to constant function market makers [1, 2]. In this talk, we characterize optimal trades for generalized convex trade functions and generalized convex utility functions, extending previous results that used just convexity. These results also guarantee the robustness against arbitrage of so-designed automatic market makers. Finally, and if the time allow us, we present necessary and sufficient optimality conditions for the optimal routing problem. *References*: [1] G. Angeris, A. Agrawal, A. Evans, T. Chitra, S. Boyd, Constant function market makers: multiasset trades via convex optimization; in: D. A. Tran et al. (eds): "Handbook on Blockchain". Springer Optimization and Its Applications, Vol. 194. Springer. (2022). [2] G. Angeris, T. Chitra, Improved price oracles: Constant function market makers; in Proceedings of the 2nd ACM Conference on Advances in Financial Technologies, ACM, New York. (2020). [3] S. Nakamoto, Bitcoin: A peer-to-peer electronic cash system, https://bitcoin.org/bitcoin.pdf, (2008). [4] C. Escudero, F. Lara, M. Sama, Optimal trade characterizations in multi-asset crypto-financial markets, arXiv: https://arxiv.org/abs/2405.06854, (2024). [5] C. Escudero, F. Lara, M. Sama, Optimality conditions for the optimal routing multi-asset cryptofinancial market problem, In progress, (2025).

Speaker: **Felipe Bueno (UNIFESP)**
Title: *On the optimization methods with inaccurate evaluations of the functions involved*
Abstract: The combination of numerical simulation of physical phenomena and gradient-based optimization has been essential in the design of engineering systems, especially in aerospace engineering, enabling the automated search for optimal configurations. However, since the differential equations governing these systems generally have no analytical solution, the objective function and its gradient must be estimated numerically, often using the discrete adjoint method, which introduces inaccuracies. These inaccuracies can affect the convergence of

optimization methods. Thus, the objective of this work is to investigate how numerical errors propagate in this type of approach, discuss the effect of inexact function and gradient evaluations on optimization convergence, and develop a theoretical framework to formalize this behavior. This work was supported by FAPESP (São Paulo Research Foundation) grants 2013/07375-0, 2021/05007-0, and 2023/08706-1.

Speaker: **Jefferson G. Melo (UFG)**
Title: *An Inexact Proximal Gradient Method with an Explicit Linesearch*
Abstract: n this talk, we present an inexact proximal gradient method for solving composite convex optimization problems, where the objective function consists of a sum of a differentiable and a non-differentiable convex function. We apply an explicit line search to the differentiable component of the objective function, requiring an inexact solution of the proximal subproblem at each iteration. We prove the convergence of the sequence generated by our scheme and analyse its iteration complexity in terms of both the functional values and a residual associated with first-order stationary solutions. Co-authored by Yunier Bello-Cruz and Max L.N. Gonçalves

Speaker: **João Carlos de Oliveira Souza (UFPI)**
Title: *The difference of convex algorithms on Riemannian manifolds*
Abstract: In this talk, we discuss a Riemannian version of the difference of convex algorithm (DCA) to solve a minimization problem involving the difference of convex (DC) function. The equivalence between the classical and simplified Riemannian versions of the DCA is established. We also prove that under mild assumptions the Riemannian version of the DCA is well defined and every cluster point of the sequence generated by the proposed method, if any, is a critical point of the objective DC function. To illustrate the algorithm's effectiveness, some numerical experiments are presented. Joint work with Ronny Bergmann; Orizon P. Ferreira; Elianderson M. Santos.

Speaker: **Gabriel Haeser (USP)**
Title: *Semismooth Newton method for Projection Equations*
Abstract: We study the global and finite convergence of the semi-smooth Newton method for solving a piecewise linear system that arises in cone-constrained quadratic programming problems and absolute value equations. We present some computational experiments designed to illustrate the behavior of the semi-smooth Newton method on sparse large-scale problems and the numerical solution of a discretization of the Boussinesq PDE modeling a two-dimensional flow in a homogeneous phreatic aquifer. The ideas are extended to a general projection equation in finite dimensional spaces which are applied to semidefinite least squares, in particular the nearest correlation matrix problem. This is a joint work with N.F. Armijo and Y. Bello-Cruz.

Speaker: **Gabriel Belém (Unicamp)**
Title: *Optimization of problems involving group sparsity*

Abstract: A key class of optimization problems involves minimizing a differentiable, lower-bounded function over disjoint block-separable sets, subject to penalties and/or constraints that promote block sparsity. We investigate this problem through the lens of recent methodologies and theories introduced by Beck and Hallak [2018], comparing different approaches and providing insights and tools for applications such as graph theory, compressed sensing, and investment portfolio optimization. Our study focuses on proximal mappings and efficient algorithms, particularly nonmonotone variants. We explore two main acceleration strategies: (i) Nesterov acceleration via a FISTA-type inertia strategy Beck and Teboulle [2009a,b], and (ii) one-dimensional quasi-Newton secondorder methods, including spectral or Barzilai-Borwein-Raydan steps Barzilai and Borwein [1988] and their variations. For both strategies, we extend classical results to address the challenges posed by the discontinuity and non-convexity of the ℓ_0 norm and the composite nature of the problem, the latter, in particular, diverging from the origin of traditional spectral methods. We establish new convergence results under alternative hypotheses that accommodate discontinuity and non-monotonicity of the algorithms, reconciling gaps left open by Kanzow and Mehlitz [2021]. This analysis also extends to a novel algorithm inspired by Li and Lin [2015], which integrates FISTA acceleration with a monitoring variable for handling non-convex functions. Preliminary numerical results suggest that these strategies improve performance in our problem setting, motivating further research into their broader applicability. Additionally, we demonstrated a novel generalization of the proximal operator for a class of functions involving the ℓ_0 norm, with potential applications in settings that could benefit from greater control over the sparsity of the solutions.

Speaker: **Luiz-Rafael Santos (UFSC Blumenau)**
Title: *Randomizing Block-wise Circumcenter-Based Methods for Solving Linear Systems*
Abstract: In this talk, I will present a randomized block-wise circumcenter-based approach for solving consistent linear systems of equations. The method builds upon projection-based iterative schemes—such as the classical Kaczmarz method, the Douglas-Rachford method, and more recent advances using circumcenters of reflected points (CRM)—by introducing a randomized selection of hyperplanes at each iteration and computing the circumcenter of reflected points over a block of such hyperplanes. Given a point, this process generates a set of reflected points whose circumcenter approximates the projection onto the solution set. We propose and analyze a method that combines block-wise reflections with circumcenter computation, in which hyperplanes are selected with probability proportional to the squared norm of their normal vectors. We prove that the proposed method converges in expectation, with an explicit rate that depends on the smallest singular value of the system matrix. Numerical experiments demonstrate that our approach outperforms existing randomized methods in both runtime and number of projections required, including on synthetic systems and real-world problems.

Speaker: **Majela Machado (UFBA)**
Title: *An inertial method for solving monotone inclusion problems*
Abstract: In this talk we will present an inertial decomposition method for solving the monotone inclusion problem defined by the sum of 2 maximal monotone operators. We will analyze the convergence and complexity of this method as well as its application to convex optimization problems.

Speaker: **Max Gonçalves (Federal University of Goiás)**
Title: *On FISTA with a relative condition*
Abstract: The fast iterative shrinkage/thresholding algorithm (FISTA) is one of the most popular first-order iterations for minimizing the sum of two convex functions. FISTA is known to improve the complexity of the classical proximal gradient method (PGM) from $O(k^{-1})$ to the optimal complexity $O(k^{-2})$ in terms of the sequence of the functional values. When the evaluation of the proximal operator is hard, inexact versions of FISTA might be used to solve the problem. In this paper, we proposed an inexact version of FISTA by solving the proximal subproblem inexactly using a relative error criterion instead of exogenous and diminishing error rules. The introduced relative error rule in the FISTA iteration is related to the progress of the algorithm at each step and does not increase the computational burden per iteration. Moreover, the proposed algorithm recovers the same optimal convergence rate as FISTA. Some numerical experiments are also reported to illustrate the numerical behavior of the relative inexact method when compared with FISTA under an absolute error criterion. Joint work with: Yunier Bello-Cruz, Northern Illinois University, and Nathan Krislock, Northern Illinois University

Speaker: **Manoel Jardim (IMPA)**
Title: *A Dantzig-Wolfe Decomposition Method for Quasi-Variational Inequalities*
Abstract: We propose an algorithm to solve quasi-variational inequality problems, based on the Dantzig-Wolfe decomposition paradigm. Our approach solves in the subproblems variational inequalities, which is a simpler problem, while restricting quasi-variational inequalities in the master subproblems, making them generally (much) smaller in size when the original problem is large-scale. We prove global convergence of our algorithm, assuming that the mapping of the quasi-variational inequality is either single-valued and continuous or it is set-valued maximally monotone. Quasi-variational inequalities serve as a framework for several equilibrium problems, and we illustrate our algorithm on an important example in the field of economics, namely the Walrasian equilibrium problem formulated as a generalized Nash equilibrium problem. We show that the proposed method demonstrates good performance for the large-scale cases. Our numerical section tackles big problems in the theory of abstract economy, as well as some academic examples that have been previously employed in the literature. Joint work with Claudia Sagastizábal and Mikhail Solodov.

Speaker: **Orizon Ferreira (Federal University of Goiás)**

Title: *The Busemann Subdifferential and Difference of Convex Algorithms on Hadamard Manifolds*

Abstract: This paper explores the properties of the Busemann function on Hadamard manifolds and its application in optimization algorithms within Riemannian settings. We introduce a new subgradient definition, crucial for nonsmooth optimization, leveraging the Busemann function. Compared to traditional subgradients, this novel concept is more suitable for Riemannian optimization In Hadamard manifolds, a subgradient typically ensures that a convex function is lower-bounded by an expression involving the subgradient and the inverse exponential mapping. However, this bound often lacks convexity and concavity. Our proposed subgradient definition leads to a concave bounding function by utilizing key properties of the Busemann function. This concavity plays a fundamental role in solving difference of convex (DC) optimization problems on Hadamard manifolds. We reformulate and analyze both the classical Difference of Convex Algorithm (DCA) and its proximal variant (pDCA), extending them from Euclidean spaces to Riemannian contexts.

Speaker: **Paulo J.S. Silva (Unicamp)**

Title: *Parallel Newton methods for the continuous quadratic knapsack problem: A Jacobi and Gauss-Seidel tale*

Abstract: The continuous knapsack problem minimizes a diagonal convex quadratic function subject to box constraints and a single linear equality constraint. It has numerous applications in areas such as resource allocation, multicommodity flow problems, machine learning, and classical optimization applications like Lagrangian relaxation and quasi-Newton updates. The fastest algorithms for solving it combine Newtonian ideas with variable fixing (active constraint identification), enabling the solution of large instances in seconds. Among these methods, the most effective was introduced by Condat, which employs a Gauss-Seidel approach to the variable fixing strategy. Although very fast, this variant is difficult to parallelize properly, similar to the challenges faced when parallelizing the Gauss-Seidel algorithm compared to its Jacobi counterpart. In this talk, we will present our efforts to parallelize such methods. Our main tool uses a Newtonian approach to guide the method, ensuring a small number of iterations. Concurrently, we employ opportunistic variable fixing strategies within each worker. We will present both CPU and GPU implementations, discussing the limitations and advantages of each computational model. Joint with Leonardo D. Secchin.

Speaker: **Roger Behling (UFSC Blumenau)**

Title: *On Levenberg-Marquardt methods and error bound conditions*

Abstract: The Levenberg-Marquardt method is a famous newtonian tool developed in the forties and fifties for solving systems of nonlinear equations. In the last two decades, it has been proven to be able to track nonisolated solutions under assumptions referred to as error bound conditions. In this talk, we are going to present advances on Levenberg-Marquardt methods concerning both unconstrained equations as well as constrained ones.

Speaker: **Wilfredo Sosa (Catholic University of Brasilia)**
Title: *From Euclidean segment to premonotonicity*
Abstract: In 2009, Iusem-Kassay-Sosa introduced the concept of premonotony as a generalization of the concept of monotony using a non-negative exogenous function, which must be defined in the effective domain of the operator. My objective is to introduce functions that generate premonotonic operators, which we will call preconvex functions, starting from the Euclidean segment.