



High-frequency dynamics of Bitcoin futures: An examination of market microstructure

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ABSTRACT

We investigate the high-frequency dynamics of Bitcoin and Ethereum perpetual futures traded on Binance from January 2020 to December 2024. After a thorough discussion of the stylized facts and particularities of Bitcoin perpetual futures, based on previous research in futures markets, we evaluate the fit of two competing models of market microstructure: the Mixture of Distributions Hypothesis (MDH) and the Intraday Trading Invariance Hypothesis (ITIH). Using intraday data at different levels of aggregation, we investigate the relationship between return volatility per transaction and trade size. We find evidence favoring the MDH in the crypto futures market.

1. Introduction

Bitcoin (BTC) is a decentralized electronic currency that enables monetary transfers over the Internet without relying on traditional financial institutions such as banks. It has grown exponentially in popularity since its online emission in 2009 (Eross et al., 2019) and is built on a technology called blockchain, introduced in Nakamoto (2008).

Unlike traditional currencies, Bitcoin is not controlled by any central authority or issued by any central bank. Its decentralized nature makes it resistant to censorship and allows access to financial systems in regions with limited banking infrastructure. Recent research has been conducted on the institutional impact of Bitcoin and challenges for regulation (Berg et al., 2019; Guegan, 2017).

Given its limited supply of 21 million coins (Norland & Putnam, 2019), the resource is scarce and therefore its price is adjusted by market conditions. As such, Bitcoin has been used as a form of payment, a store of value, or an investment (Alfieri et al., 2019). Some authors still question whether it is an asset or a currency, without a clear consensus on this matter (Alfieri et al., 2019; Glaser et al., 2014; Yermack, 2013). In spite of that, there have been periods of increases and shifts in BTC/USD prices, usually related to periods of political, economic or social stress in financial markets.

The first surge in the value of Bitcoin was in 2017, with an increase of more than 21,000% in value from January 12 to March 17, which can be potentially attributed to a decrease in trust in central banks due to financial crises and European bailouts (Eross et al., 2019). More recently, from March 2020 to March 2021, there was an increase of

more than 1000% in the BTC/USD price due to COVID-19 and the subsequent economic crisis. Macroeconomic instability and political uncertainty arising from the invasion of Ukraine in 2016 and again in 2022 and the war in Israel in 2023, as well as other political tensions in Asia, Europe, and the United States, might also have had an effect on the confidence of investors in regulated currencies, leading to BTC/USD all-time highest values in November 2024. For a discussion, see Al-Shboul et al. (2023), Alexakis et al. (2024), Auer et al. (2023), Bouri et al. (2020) and Chen et al. (2024).

Binance is a platform for trading digital assets, including Bitcoin. It was founded in 2017 and has grown to become one of the largest cryptocurrency exchanges worldwide measured by trading volume. In addition, Binance provides intraday historical data on trades since the end of 2019 for the spot and futures market.

Given the increasing prominence of cryptocurrencies and other digital assets on the market, in 2019 Binance launched its first futures market, in particular the BTC/USDT. Bitcoin futures aid in price discovery between the spot and futures markets and allow participants to hedge against Bitcoin price volatility and speculate on price movements. Unlike traditional derivatives, Bitcoin perpetual futures do not have an expiration date, allowing investors to hold positions indefinitely. These contracts use funding rates to align their prices with the spot market and are popular for hedging, leverage trading, or speculating on Bitcoin price movements. For the sake of simplicity, hereinafter we refer to Bitcoin perpetual futures contracts as BTC futures or Bitcoin futures, unless otherwise stated.

After the surge of Bitcoin, other cryptocurrencies and technologies emerged, with Ether (ETH) being one of the most important. Ethereum

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is a decentralized open-source blockchain platform that allows the creation and execution of smart contracts running on the blockchain without the need for intermediaries. It was originally proposed in [Buterin \(2014\)](#), extending the capabilities of Bitcoin by incorporating a built-in programming language, allowing developers to build decentralized applications in several areas, one of which includes finance. Ether is the native cryptocurrency of the platform and is used to pay for computational resources and transaction fees. It is qualitatively different from Bitcoin, as it was designed primarily to be a utility token to pay for the use of the Ethereum Virtual Machine (EVM) ([Bellon & Figuerola-Ferretti, 2022](#)). Ethereum provides a programmable infrastructure, making it one of the most widely adopted blockchain networks. A detailed commentary on Ethereum can be found in [John et al. \(2025\)](#). Following the success of Bitcoin futures, in December 2019 Binance introduced ETH futures to expand its portfolio of products.

In this article, we study the intraday dynamics of Bitcoin futures traded on Binance from January 2020 to December 2024. Following the analysis of [Andersen et al. \(2020\)](#), we aim to understand the dynamics of four variables of interest: volume, trade size, number of trades, and volatility of returns, and in particular to understand whether the Mixture of Distributions Hypothesis (MDH) in the sense of [Clark \(1973\)](#) and [Harris \(1987\)](#) applies to the data or whether the Intraday Trading Invariance Hypothesis (ITIH) in the sense of [Andersen et al. \(2020\)](#) better suits the data.

To provide a complimentary view, we also compare the dynamics of Bitcoin futures and Ether futures. Although these cryptocurrencies are often compared to each other, they differ significantly in qualitative terms as well as in mechanism, scalability, and conceptualization. One of the most important differences is at the protocol level: while Bitcoin operates in proof-of-work, Ethereum has been operating in proof-of-stake since 2022 ([Arslanian, 2022](#)). Therefore, we provide a comparison of two different cryptocurrencies that, although both very liquid, are still structurally and functionally different, with protocol-level characteristics that could affect the microstructure of the market, and that may also have diverse potential traders and investors.

The MDH, in the sense of [Clark \(1973\)](#), describes a theoretical framework in which the return volatility S_t and the trading volume V_t are jointly driven by a latent process or, equivalently, the volatility per number of trades S_t/N_t is related to the trade size Q_t . This essentially means that large spikes in trading volume are related to higher volatility of returns, because price movements are determined by stochastic information arrival. On the other hand, the ITIH proposed by [Andersen et al. \(2020\)](#) and [Kyle and Obizhaeva \(2016\)](#) implies that, all else being equal, the mean trade size Q_t drops if the volatility of the return increases or the trading intensity declines.

Volatility, in the sense of the MDH, is primarily driven by the arrival of new information and macroeconomic factors. In the sense of the ITIH, volatility is a function of risk transfer and market liquidity constraints, which follows the scaling law discussed in [Andersen et al. \(2020\)](#) and [Kyle and Obizhaeva \(2016\)](#). Since these hypotheses have fundamental differences, a test for which model better describes the Bitcoin and Ether futures markets is of interest theoretically and for market players and practitioners.

The MDH of [Clark \(1973\)](#) and [Eross et al. \(2019\)](#) posits that volatility and trading volume are jointly driven by a latent information arrival process. For the MDH, it holds that $S_t/N_t \sim Q_t^\beta$ where $\beta > 0$. Therefore, larger average trade sizes are associated with higher return volatility per trade, supporting the theory that trading activity is not purely noise-driven or liquidity-driven, but responsive to information shocks. If corroborated in our analysis, this would imply that both BTC and ETH futures markets behave like information-diffusion systems, where bursts of new information increase both volatility and trade size. The implications thereof will be discussed in further sections.

Previous studies such as [Eross et al. \(2019\)](#) have analyzed the intraday dynamics of the spot market, but did not employ the methodology

of [Andersen et al. \(2020\)](#). Others, such as [Chou et al. \(2023\)](#), [Patra and Gupta \(2025\)](#) and [Wang et al. \(2019\)](#), have examined the cryptocurrency market at the intraday level or compared it with other settings, and should be considered complementary to this article, as we analyze the BTC and the ETH futures market with the goal of understanding the microstructure of the market. Our study therefore contributes by both investigating the validity of those models applied to a broader class of assets and by studying the cryptocurrency futures market microstructure. To the best of our knowledge, this paper provides a first systematic comparison of these competing frameworks of foundational market microstructure theories using high-frequency data for Bitcoin and Ether derivatives. By evaluating the relationship between volatility per trade and average trade size across an array of time aggregations and an array of distributional quantiles, we provide a detailed overview of information efficiency and order flow dynamics in these specific decentralized derivative markets, which are still underexplored in the literature.

The remainder of the paper is organized as follows. Section 2 presents a review of the literature on cryptocurrencies and high-frequency data. Section 3 provides a thorough discussion of the data and its stylized facts, as well as a description of the methodology of this study. In Section 4 we present and discuss the empirical results. In Section 5 we close with some final remarks.

2. Cryptocurrency literature review

High-frequency time series usually contain stylized facts that should be taken into account when modeling their behavior. Stylized facts can be defined as a set of statistical properties emerging from independent empirical studies of assets ([Cont, 2002](#)) that are partly due to the formation of prices and market microstructure. They are usually shadowed in lower frequencies and other types of time series, but also depend on the liquidity of the financial markets in which assets are traded ([Dacorogna et al., 2001](#); [Zivot & Wang, 2005](#)).

Typical stylized facts for prices are long-range dependence on the conditional mean and conditional variance, intraday jumps, volatility clusters, fat-tailedness, non-normality, and skewness ([Cont, 2002](#); [Dacorogna et al., 2001](#)). In terms of structure, the data are available in the “tick-by-tick” format, which means that the quantities of interest are not usually available in homogeneously spaced time, but rather are irregularly spaced. Therefore, a regularizing procedure is necessary to make the time series homogeneous, so that quantities can be computed from the trades and quotes databases.

Cryptocurrencies are not exempt from these stylized facts. In fact, [Scaillet et al. \(2018\)](#) discusses the impacts of jumps on market activities, for example, with jumps commonly occurring at the intraday level. However, because cryptocurrency markets operate in a continuous 24/7 setting and do not have trading hours as other usual markets do, [Pinto et al. \(2023\)](#) notes that the magnitude of intraday jumps or spikes in prices is less noticeable compared to assets traded in regulated markets with regular trading hours.

The presence of price clusters in Bitcoin is discussed in [Urquhart \(2017\)](#). It is also possible to identify jump clusters in other cryptocurrencies, [Pinto et al. \(2023\)](#), [Scaillet et al. \(2018\)](#). In [Conlon et al. \(2024\)](#) the authors study the relationship between volume and volatility of Bitcoin in futures and spot markets, using estimated realized volatility and a metric based on the Chicago Mercantile Exchange (CME) Bitcoin Reference Rate.

In [Bariviera et al. \(2017\)](#), the authors investigate other stylized facts of the Bitcoin market, assessing the presence of long-range dependence to infer whether it is generated by a self-similar stochastic process.

A study of the cointegration of implied and nominal Bitcoin exchange rates is provided in [Smith \(2016\)](#). [Auer et al. \(2023\)](#) explores how rising Bitcoin prices drive the entry of new retail users, particularly younger and risk-seeking investors, and the effect of exogenous shocks

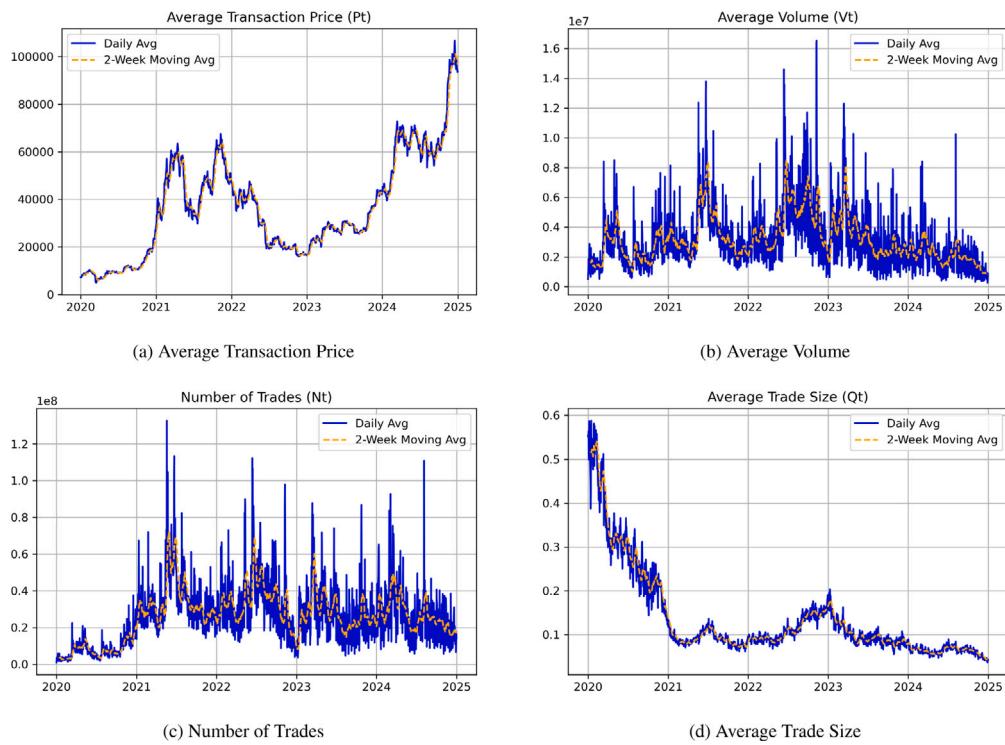


Fig. 1. Daily averages and 2-week moving averages of price, volume, number of trades, and trade size of Bitcoin futures from January 2020 to December 2024.

in the trading volume. In a recent analysis of the liquidity of Bitcoin, [Loi \(2018\)](#) concludes that, on average, stocks are more liquid than Bitcoin.

On the matter of the nature of Bitcoin, [Alfieri et al. \(2019\)](#) argues that Bitcoin behaves like a common stock, evaluating its performance with risk adjusted return models and finding that it offers opportunities for diversification due to its low correlation with market indexes and replicating portfolios. Even though central authorities are usually against recognizing Bitcoin as a medium for exchange, central banks are considering introducing cryptocurrencies and digital currencies of their own ([Del Tedesco Lins & Ribeiro Hoffmann, 2024](#); [Hairudin & Mohamad, 2024](#)). Ether is rather different, as it exists on a smart contract platform used for decentralized applications (DeFi, NFTs, etc.) and Ether functions as the utility token to pay for the EVMs, therefore attracting not only speculators but also developers, DeFi traders and NFT users, for instance ([Arslanian, 2022](#); [John et al., 2025](#)).

An analysis by [Kajtazi and Moro \(2019\)](#) discusses the role of Bitcoin in optimal portfolios across different constraints using conditional value at risk (CVaR), suggesting that it remains a speculative asset with potential diversification benefits.

A recent study of [Yi et al. \(2022\)](#) explores the characteristics of Bitcoin as an investment asset by comparing it with other major investment assets. The authors examine Bitcoin's market efficiency using the Hurst exponent and its long-term market equilibrium through Shannon's entropy. Although the findings suggest that the Bitcoin market is less efficient than the other markets the authors compared it with, it does not differ much in terms of long-run market equilibrium.

Regarding studies on the estimation of the volatility of Bitcoin using GARCH models, one can refer to [Charles and Darné \(2019\)](#), [Katsiampa \(2017\)](#) and [Liu and Serletis \(2019\)](#). Still regarding volatility, in [Kim et al. \(2021\)](#) the authors introduce VCRIX, a volatility index for the cryptocurrency market, modeled after the VIX used in traditional financial markets.

In a recent study, [Chen et al. \(2024\)](#) shows that Bitcoin returns and volatility are influenced by political uncertainty indicators, such as geopolitical risk and party conflict indices, especially during financial crises. The authors conclude that some investors use Bitcoin as a

hedge and a safe-haven asset that mitigates political uncertainty. In another sense, [Al-Shboul et al. \(2023\)](#) examined the spillover effects between traditional currencies and cryptocurrencies during the COVID-19 pandemic, concluding that cryptocurrencies acted as “safe havens” during this period of market uncertainty. Other studies on this matter include ([Alexakis et al., 2024](#); [Bouri et al., 2020](#)).

3. Methodology and data

3.1. The data

Our dataset comprises intraday trades of Bitcoin and Ether futures executed on Binance from January 2020 to December 2024.¹ We therefore observe all transactions executed across years and, from this information, we aggregate it into several levels of aggregation: 1-min, 5-min, 10-min, 15-min, 30-min, 60-min levels for intraday aggregations and a daily level of aggregation of four financial metrics: average transaction price (denoted hereinafter P_t), transaction rate (trades per time unit, N_t), average percentage return variance (S_t), average (unsigned) number of contracts per transaction (Q_t) and cumulative trading volume (V_t). We denote the lowercase $p_t = \log(P_t)$, $n_t = \log(N_t)$, $s_t = \log(S_t)$, $q_t = \log(Q_t)$, and $v_t = \log(V_t)$. Following [Andersen et al. \(2020\)](#), the intraday and daily aggregations are calculated by averaging and summing over the respective interval of the 1-min level aggregation. The choice of using progressively increasing frequencies for the analysis diminishes the effects of microstructure noise at very high frequencies and reveals whether the hypothesized law persists across time scales. This provides a full array of results, analyzing their consistency and stability across frequencies.

It is important to stress that, although we include the 1-min aggregation in the analysis, we acknowledge that the data might be too noisy and lead to distortions at such a high frequency, since the series can be subject to market microstructure effects (e.g. bid–ask bounce, discrete

¹ Data can be obtained from Binance's Public Market Data webpage.

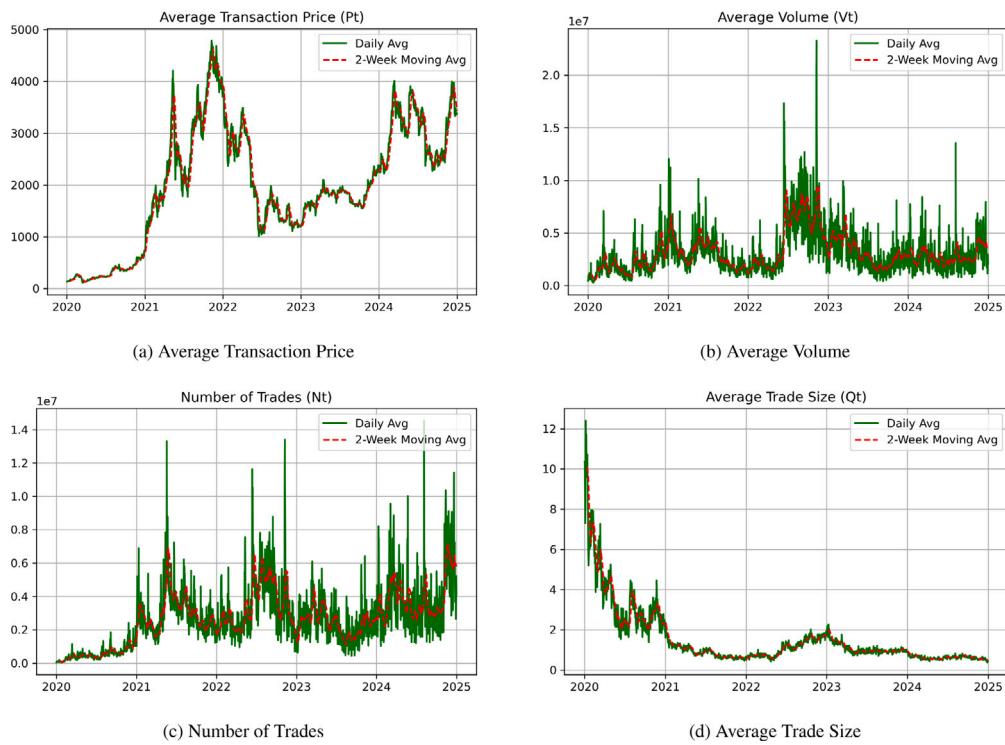


Fig. 2. Daily averages and 2 week moving average of prices, volume, number of trades and trade size of Ether futures from January 2020 to December 2024.

price movements, and order book frictions), which have the potential to distort volatility and therefore our results. The 5-min interval has been shown to generate the most accurate results and forecasts when realized volatility is used in microstructure studies of the market (Liu et al., 2015). A comprehensive discussion on the impact of frequency choice can be found in the works of Andersen et al. (2001), Liu et al. (2015), Liu and Serletis (2019) and Ngene and Wang (2024).

The logarithms of each quantity (the lowercase aggregations) are straightforward. We recompute the annualized volatility according to its corresponding time aggregation and transform it by the square-root rule: $S_t^{\text{Annual}} = S_t \times \sqrt{T}$, where T represents the number of time intervals per year at each respective frequency. Since Bitcoin has no trading hours, to annualize 1-min volatility, we use $T = 525,600$, and use the same logic to annualize other frequencies (e.g. $T = 52,560$ for 10-min and $T = 365$ for daily data).

Following Andersen et al. (2020), we will also analyze the data from the perspective of trading hours, which we hereinafter refer to as “regimes”. The construction of the trading regimes is adapted and based on the segmentation of 24-h central time (CT) to reflect global trading activity across major financial markets. They are defined as follows.

1. Asia: from 19:00 to 2:00 CT.
2. Europe: from 2:00 to 8:30 CT.
3. Americas: from 8:30 to 15:15 CT.
4. Transition Zone: between 15:15 and 19:00 CT.

The Transition Zone represents a lower liquidity period when the US markets have closed and Asian markets are not yet fully open.

Segmentation into these regimes allows for the examination of market behavior, such as volatility and trading volume, during each trading regime and highlights the impact of overlaps and transitions between major global markets.

Figs. 1 and 2 depict the daily aggregation of the four market metrics in both datasets. The presence of significant price swings aligns with the aforementioned price cycles—particularly the sharp appreciations in 2021 and 2024—even though it is more pronounced for BTC than for

ETH. The two-week moving average reveals that, while price volatility remains high, the long-term growth trajectory follows a structured pattern.

From Figs. 1 and 2 one can also note that surges in trading activity often coincide with price fluctuations. Peaks in volume during speculative phases could indicate that market participation intensifies during periods of high volatility.

The number of trades appears to exhibit a cyclical pattern with increased activity during bullish market periods. The increasing frequency of transactions over time is indicative of an evolving market structure.

Finally, the bottom right panel of both Figs. 1 and 2 indicates a noticeable negative trend in the average trade size. This suggests a structural shift in the microstructure of the crypto market, which would merit attention and warrant further investigation, as it may stem from the growing influence of retail trading or increased order fragmentation by institutional investors.

Figs. 3 and 4 depict one of the aggregations (daily) of the data that will be used as a test in the methodology and empirical results sections, displaying the log transform of the following quantities: $s_t - n_t$, s_t , n_t , and q_t . There has been a negative trend in the difference between log-variance and log-trade-count over the years. One can infer that, as the number of trades increases, the return variance per trade decreases, an observation consistent with increasing market efficiency.

With the exception of some episodic spikes in volatility, there was also a decrease in return volatility over time (s_t). The increase in the trade count of both Bitcoin and Ether futures, n_t , suggests that the depth of the crypto futures market has increased over time, leading to greater price efficiency. There is a noticeable decrease in trade size over the years, following a pattern similar to that of the difference between log variance and log-trade-count.

Figs. 5 and 6 show intraday variations in trading behavior, analyzing trade count, trade size, return variance, and volume across the different market regimes previously defined and over different years. The plots show the average value at the yearly level of v_t , s_t , n_t and q_t for each trading hour for the years 2020 to 2024. It is clear from the

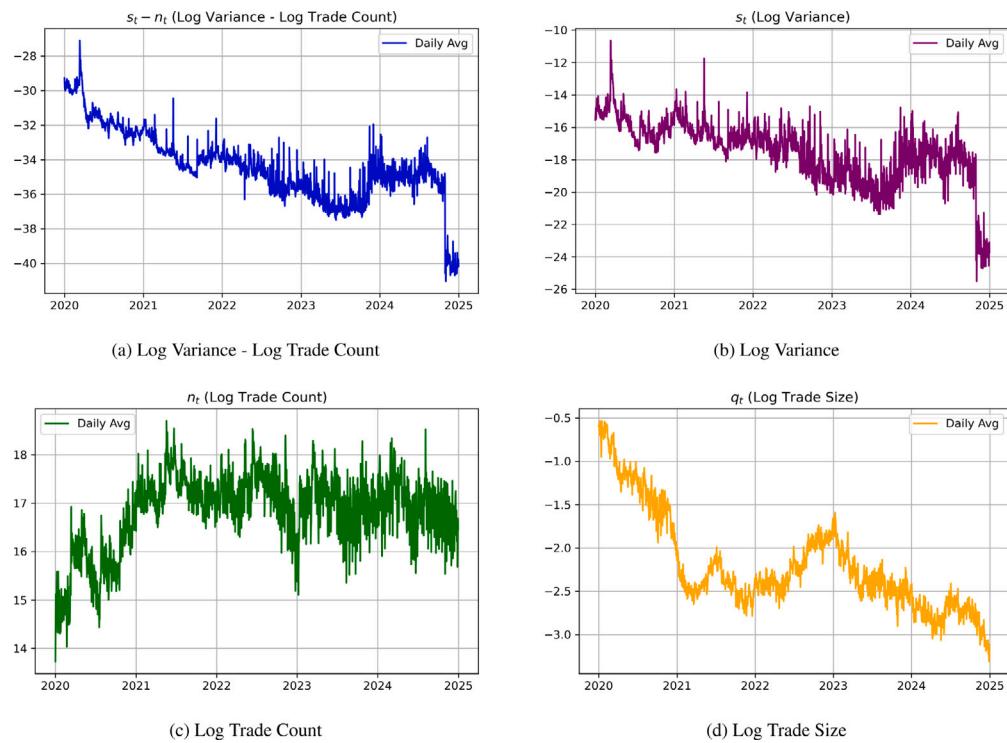


Fig. 3. Daily aggregation of market metrics for BTC futures used for testing the MDH vs. the ITIH.

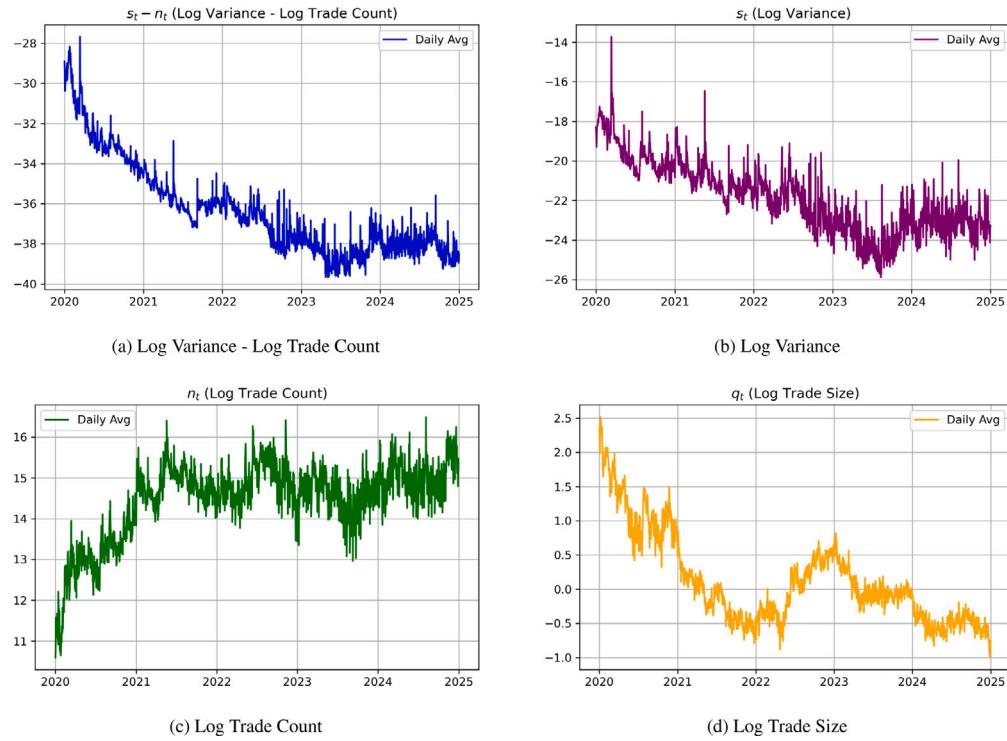


Fig. 4. Daily aggregation of market metrics for ETH futures used for testing the MDH x the ITIH.

plots that the year 2020 is largely different for both derivatives from the remaining years of 2021 to 2024 in terms of volatility, trading volume, trade size, and trade counts, which could be related to the COVID-19 pandemic and its effects on cryptocurrency markets. It could also stand out because it was the first trading year of the BTC and ETH futures and

the financial instrument may not yet have reached maturity during that period.

Analyzing yearly averages of log volatility s_t per time of day, one can note that volatility tends to increase in trading hours close to the transitions between regimes, particularly during the Asia-to-Europe and

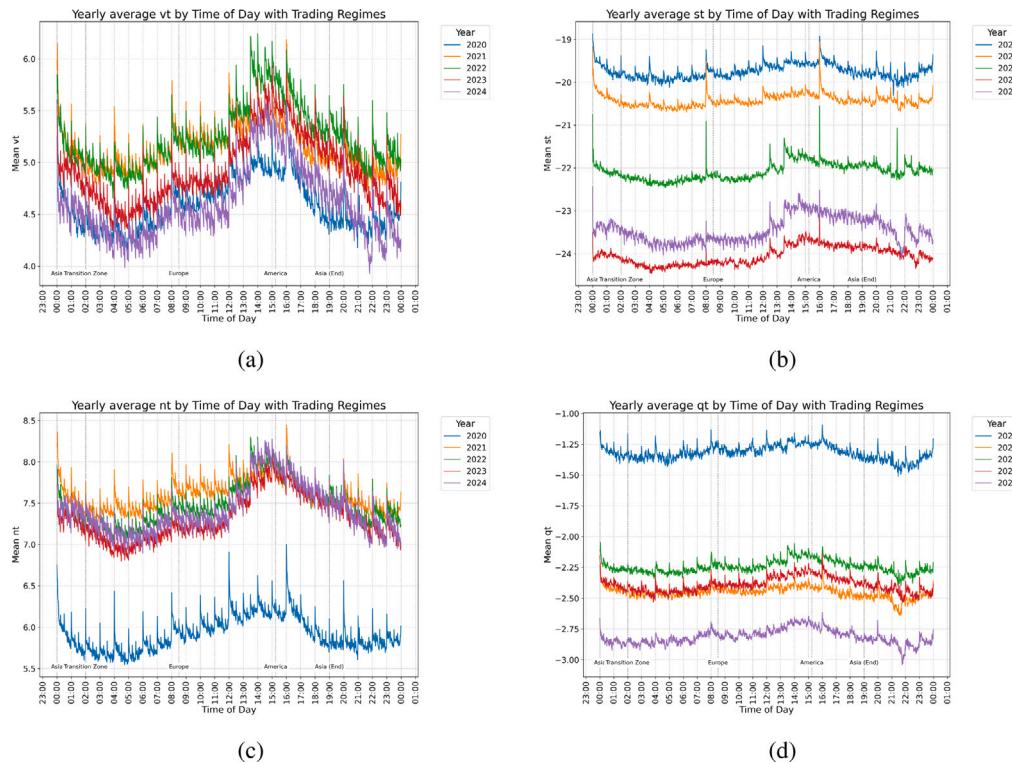


Fig. 5. Average BTC futures metrics per time of day — yearly aggregation.

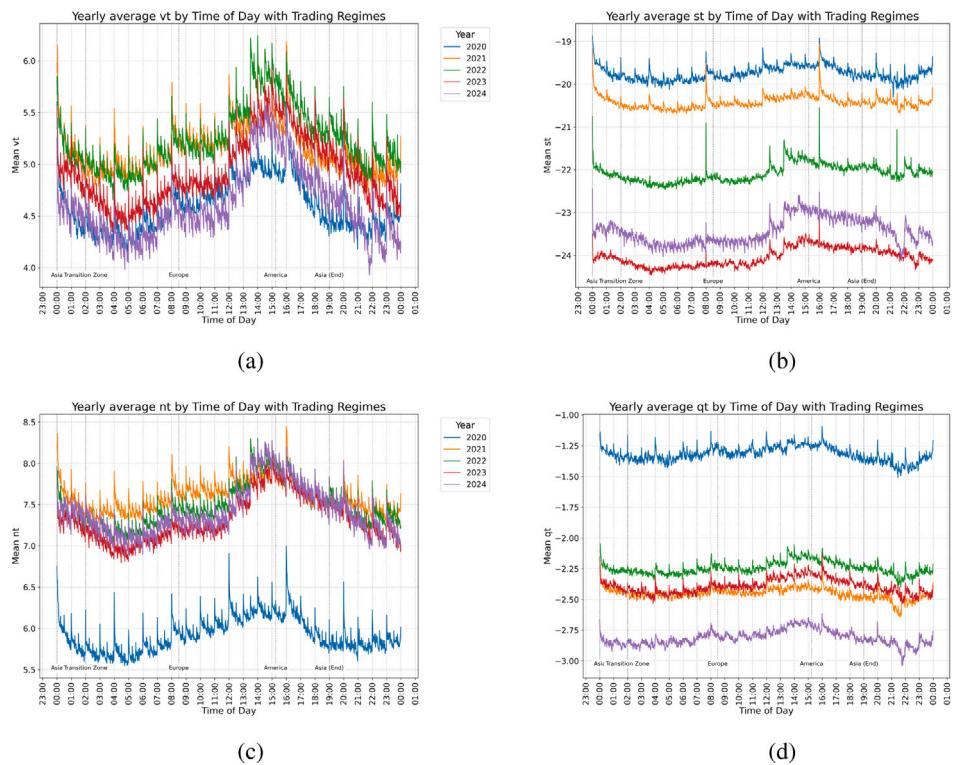


Fig. 6. Average ETH futures metrics per time of day — yearly aggregation.

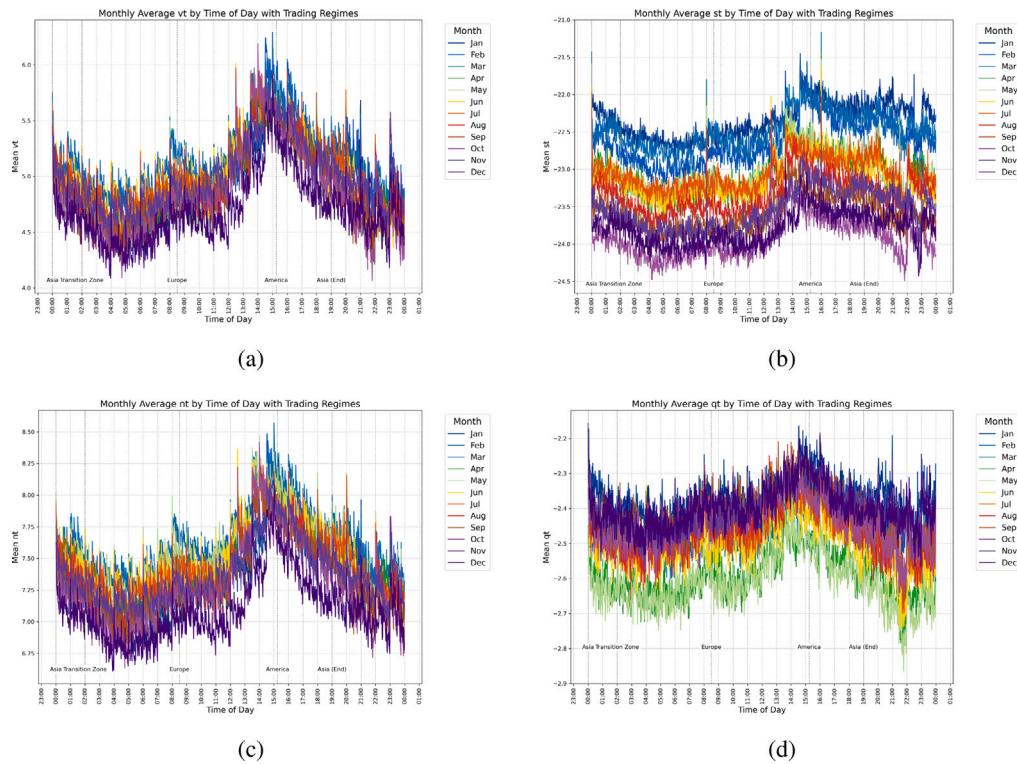


Fig. 7. Average BTC futures metrics per time of day — monthly aggregation for 2022–2024. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Europe-to-Americas regime changes, which is an interesting stylized fact of BTC futures. Years 2020 and 2021 exhibit a clear pattern of increased volatility compared to the subsequent years, with less pronounced spikes and a lower volatility level. American and Asian regimes are associated with higher volatility in the years 2021–2024. The transition zone is associated with a lower volatility than the others.

From the annual mean number of trades, n_t , it is clear that trading activity follows a strong cyclical pattern, with lower participation during the early Asian regime and the Transition Zone, and peak activity occurring during the American regime. This has been consistent over several years and both cryptocurrencies, maintaining this U-shaped pattern of number of trades across the trading day. This observation is consistent over years, although 2020 clearly differs from other years, indicating a persistent intraday liquidity cycle. Peaks in n_t apparently occur in trading hours associated with regime transitions.

There is a clear trend of a decrease in q_t as time passes, which is consistent with the negative slope noticed in Figs. 1 and 2. Trade sizes tend to be larger at regime start and end times, which could be caused by liquidity-driven rebalancing at key market intervals, but this matter merits further analysis and investigation.

The yearly average trading volume v_t by time of day exhibits an interesting U-shaped pattern. Volumes tend to be higher following the European and American trading regimes, peaking in the transition between them. During the Transition Zone regime, as expected, trading volume is lower for all year averages, which suggests that it is associated with lower liquidity in markets.

We also explore the intraday cycle with monthly aggregation of the data. Since we realized that years 2020 and 2021 were behaving differently than the years 2022 onward, we separate out those years and compute the monthly average across 2022–2024.

Figs. 7 (BTC futures) and 8 (ETH futures) show the four metrics v_t , n_t , s_t and q_t with their monthly averages over years, excluding the years 2020 and 2021. The color palette is adjusted to correspond to the seasons of the year in the Northern Hemisphere, thus making visualization of seasonal patterns straightforward.

As expected, Figs. 7 and 8 follow the same hourly pattern seen in Figs. 5 and 6, since the latter is an annual aggregation and the former is a monthly aggregation. Over the months, the annual pattern identified in Figs. 5 and 6 remains stable: trading activity is higher in the American regime and during regime transitions, trading activity is cyclical and v_t and n_t are U-shaped, peaking in the American regime and reaching their lowest values in the Transition Zone.

However, a few aspects are worth noting, particularly seasonal behavior. For both BTC and ETH futures, the trade volume v_t appears to remain unaffected or less affected by the month of the year, following the pattern identified in the annual aggregation. The other variables seem to exhibit a more seasonal pattern, with the months of the year clearly distinguishable in the plot, even if for n_t it is to a lesser extent.

The average of the log number of contracts per trade q_t from 2022–2024 is lower in April and May, then increasing in June, July, and August. In Fig. 7 (BTC futures) it shifts to its higher phase from September to March. It appears to follow a clear seasonal pattern, with a lower q_t in months associated with spring and summer months in the Northern Hemisphere, and a higher q_t in months associated with fall and winter in the Northern Hemisphere. This could be the results of portfolio adjustments before the end of the year and preparations for the market trends for Q1. Although the difference between spring/summer and fall/winter is less distinct in the case of ETH futures, a discernible differentiation remains, following the same overall trend.

A similar but reversed pattern is identified in n_t , even though the months are less distinguishable in Figs. 7 and 8: for the years 2022–2024, there was a higher number of trades in the first semester, with n_t decreasing from January onward. This phenomenon is less noticeable in the plot of ETH futures.

Log volatility, s_t , is another variable with clear, distinguishable seasonal patterns over the months observed in 2022–2024 for both ETH and BTC. January, February, and March have higher volatility, followed by a medium level of volatility from April to August and lower volatility from September to December. This pattern of volatility is associated with new-year and end-year positioning, with risk-off

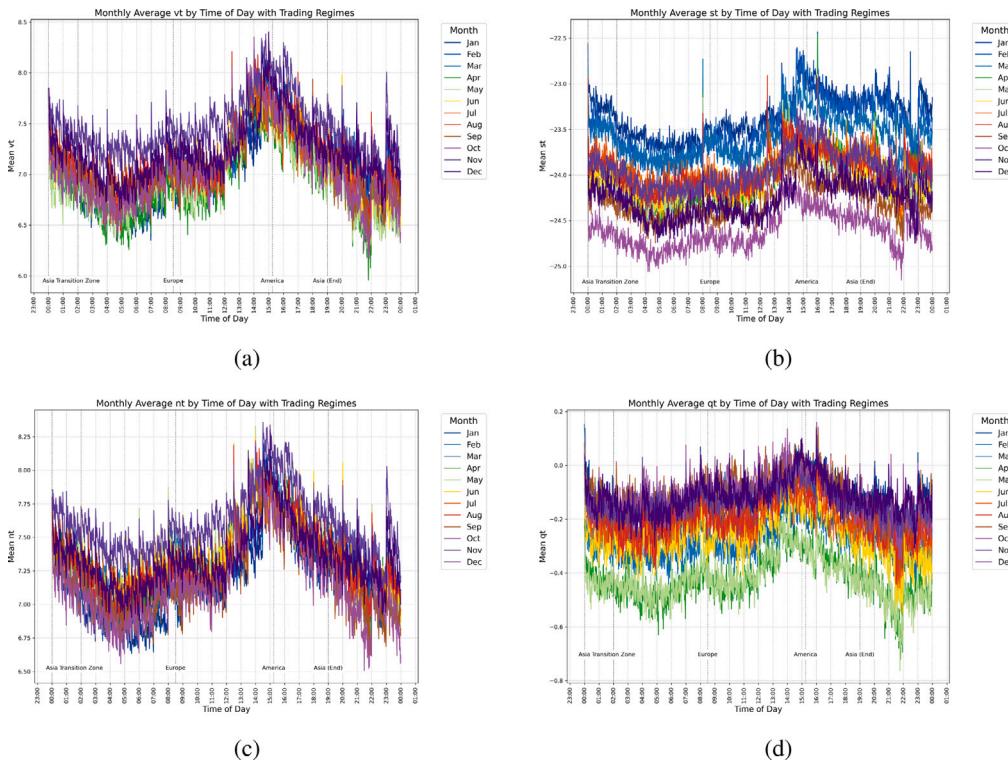


Fig. 8. Average ETH futures metrics per time of day — monthly aggregation for 2022–2024. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

behavior for the latter and more speculation or hedging in the former. This suggests trading behavior that is most likely associated with institutional investors, and is consistent in both derivatives.

The high activity levels in certain months could be due to fiscal reporting cycles, risk management adjustments, and portfolio rebalancing. The lower activity levels in summer in the Northern Hemisphere is apparent in both figures.

3.2. Methodology

Our analysis is based mainly on [Andersen et al. \(2020\)](#). The paper explores an invariance relationship in high-frequency trading data, specifically in the E-mini S&P 500 futures market. The authors established that the volatility of the return per transaction is inversely proportional to the square of the expected trade size. This relationship holds across both time series and intraday trading cycles, which challenges existing models in market microstructure with a trading invariance hypothesis.

The Mixture of Distributions Hypothesis (MDH) was presented in the seminal paper of [Clark \(1973\)](#) and was further discussed in several articles, including those of [Harris \(1987\)](#) and [Tauchen and Pitts \(1983\)](#). It offers an explanation for the (positive) linear relationship between trading volume and the volatility of returns, assuming that both are driven by the same underlying information flow. Before [Clark \(1973\)](#), in an initial analysis, [Osborne \(1962\)](#) observed that the trading volume was proportional to the number of transactions, implying that the volatility of the return is proportional to the volume $S \sim V$. In [Andersen et al. \(2020\)](#), the authors describe three formulations for MDH: MDH-V, MDH-N and generalized MDH, corresponding respectively to Eqs. (1), (2) and (3) as stated below.

$$s_t = c + v_t + u_t, \quad (1)$$

$$s_t = c + n_t + u_t, \quad (2)$$

$$s_t - n_t = c + \beta q_t + u_t, \quad (3)$$

where $s_t = \log(S_t)$ is the log-volatility of returns, $v_t = \log(V_t)$ is the log-volume, $n_t = \log(N_t)$ is the log trade size, c is a constant and u_t is a zero-mean noise process for all three equations. Moreover, $\beta \geq 0$ in (3). Eqs. (1) and (2) describe a stochastic process evolving as in [Mandelbrot and Taylor \(1967\)](#), whereas Eq. (3) encompasses the two previous models, with β to assume any non-negative value. In [Epps and Epps \(1976\)](#), the authors find evidence of $\beta > 1$ to be consistent, while a value of $\beta \in (0, 1)$ would be an effect that falls between the predictions of MDH-V and MDH-N. More details of the theoretical framework can be found in [Clark \(1973\)](#) and [Epps and Epps \(1976\)](#).

Another hypothesis is the one of [Andersen et al. \(2020\)](#) and [Kyle and Obizhaeva \(2016\)](#). While [Kyle and Obizhaeva \(2016\)](#) starts from a market microstructure invariance (MMI) from large speculative bets fragmented into smaller orders, [Andersen et al. \(2020\)](#) proposes an intraday trading invariance hypothesis (ITIH), which extends similar principles to short-horizon transactions while remaining a purely empirical hypothesis, acknowledging the stringent conditions required for invariance relationships to hold in high-frequency settings. The equation associated with the ITIH is:

$$s_t - n_t = c - 2q_t + u_t. \quad (4)$$

The difference between general MDH and ITIH is clear: while an assumption of (3) is that $\beta \geq 0$, (4) requires $\beta = -2$. More details on the formulation of the ITIH and why such a stark difference in β arises can be found in [Andersen et al. \(2020\)](#).

The empirical capacity of the MDH and the ITIH models has been tested in several studies using real data, including [Andersen \(1996\)](#), [Andersen et al. \(2020\)](#), [Benzaquen et al. \(2016\)](#), [Darolles et al. \(2017\)](#), [Richardson and Smith \(1994\)](#) and [Wang et al. \(2019\)](#). However, to the best of our knowledge, the test of the ITIH versus MDH has not been previously tested in cryptocurrency markets, and in particular in the BTC and ETH futures market.

Therefore, in this study, we analyze the intraday and daily dynamics of cryptocurrency futures contracts to assess which hypothesis—ITIH or MDH—better describes the high- and low-frequency behaviors of this market. The implications of each market microstructure model are sharply different, making it interesting both in a theoretical framework and also for market practitioners, investors, players, and risk managers.

If MDH better describes the data, then this means in essence that the crypto futures market is primarily information driven, implying that successful strategies should focus on news, sentiment and the speed of information absorption, as well as macroeconomic and policy effects. If, on the other hand, the ITIH better matches the data, then this means that the crypto futures market is primarily liquidity-driven, with good strategies focusing on market depth, leverage and liquidation risk.

This analysis is important because market microstructure models, in their original formulation, were developed in the context of centralized exchanges with defined trading hours. Other markets, such as global FX markets, also operate continuously, but under different regulatory conditions. Our analysis, on the other hand, is of a decentralized market with no trading hours, thus making it interesting to understand if these theories apply to this market setting.

Furthermore, traditional futures markets (e.g., S&P 500 E-mini) are dominated by institutional players, while cryptocurrency markets involve a large proportion of retail traders. All of these differences make testing the validity of these theories and assessing their limitations to the crypto market intriguing.

To identify which hypothesis best matches the data, we propose a regression model based on (3) and test whether $\beta = 0$, $\beta = 1$, $\beta = -2$ or $\beta > 0$, for daily and intraday minute-by-minute aggregations. Our estimates $\hat{\beta}$ will then be used to assess the validity of either hypothesis in the BTC and ETH futures markets.

Since we have seen from the analysis of Fig. 5 that trading regimes affect our target metrics, we will also perform this regression for each regime, for all time aggregations of the data, and for all four aforementioned hourly trading regimes.

Finally, we implement quantile regressions in order to better measure how the relationship changes as the distributional features of the dependent variable change. Traditional ordinary least squares regression is based on restrictive hypotheses, such as homoscedasticity and linearity across the entire distribution (Seber, 2015). However, as previously discussed, financial data tend to exhibit asymmetries and heavy-tailedness, which become more noticeable as the frequency increases. Although we can use techniques to avoid some of those specification problems, such as robust standard errors, we choose to employ quantile regression to account for potential nonlinearities and stylized facts in the financial data.

Formally, we can write (3) for the quantile regression as:

$$Q_\tau(y_t | q_t) = \beta_{0,\tau} + \beta_{1,\tau} q_t, \quad (5)$$

where $y_t = s_t - n_t$ and $Q_\tau(\cdot | q_t)$ denotes the conditional τ th quantile. In (5), we will have quantile-specific $\beta_{0,\tau}$ and $\beta_{1,\tau}$ capturing the effect of trade size on volatility per trade across different quantiles τ .

This method allows us to examine not only the average effect for each frequency aggregation, but throughout the conditional distribution of the dependent variable, providing a more comprehensive understanding of the dynamics of the cryptocurrency futures market. This is particularly useful for analyzing the tails, where extreme market behavior might occur, and to determine if the response is homogeneous or not. For a complete review of quantile regression and its justifications, see Davino et al. (2013) and Koenker and Bassett (1978).

4. Empirical results

4.1. Linear regression

We fit a regression model for the daily level of aggregation of the data and for the intraday data for the following time aggregations: 1-min, 5-min, 10-min, 15-min, 30-min and 60-min. Since high frequency

minute-by-minute data are noisier, it is expected that the linear regression will be less explanatory at the 1-min level of aggregation. Aggregating data at progressively decreasing levels of frequency (or equivalently, increasing time-intervals) captures the change in the estimated dynamics as frequency varies, and measures how much the effect of the intraday frictions can impact the results.

The results of the standard regression tests can be seen in Table 1 for both BTC and ETH futures. The coefficients are such that $\hat{\beta} > 2$ for all cases and all derivatives. The estimates at 1-min and 5-min levels of aggregation differ slightly from the remaining ones. R^2 was noticeably lower for intraday regression, especially in the case of BTC futures, and improves for aggregation levels larger than 5 min. A performance decrease was expected, as the minute-by-minute data is much noisier than the daily-level data. The explanatory power is relatively strong as data is aggregated over larger time spans, indicating that q_t plays a significant role in the volatility per trade. The BTC data seems much noisier than that of the ETH, as seen in the R^2 value of the 1-min aggregation level. Still, for both currencies, the estimates for β are more or less consistent throughout the whole range of frequencies, with more significant changes in the BTC for daily data than for ETH. In agreement with the previous literature, the 5-min level of aggregation appears to be the highest frequency at which estimates seem reliable, as it is more consistent with other intraday estimates at lower frequencies. After the 10-min level of aggregation, the intraday estimates are practically unchanged.

The Durbin-Watson statistics for all models at all levels of aggregation indicated the presence of autocorrelation in the residuals. For this reason, we report the Newey-West HAC robust standard errors (Newey & West, 1987) in Table 1. There was no statistically significant evidence of multicollinearity.

The results suggest a rejection of the ITIH, since larger trade sizes correspond to higher, not lower, return variance per trade. The point estimates for $\hat{\beta} > 1$ might indicate that large trades tend to occur in more volatile market conditions, rather than stabilizing returns as the ITIH suggests. These results are aligned with the plots in Fig. 3, which indicated a positive relation between $s_t - n_t$ and q_t . Interestingly, despite the lower explanatory power of the regression at the 1-min level of aggregation (especially for BTC), the $\hat{\beta}$ did not disagree much from the point estimates from other frequencies, with a larger difference for BTC compared to ETH.

Since we understood from Figs. 5 and 6 that different trading regimes are associated with different intraday behavior patterns, we calculate the regression for the regime using intraday data in order to assess whether the analysis separating each regime yields different estimates for β . The results are reported in Tables 2 and 3.

Note that not only are the $\hat{\beta}$ for each regime comparable, but the constants also remain practically unchanged, and all of the coefficients are comparable with the intraday regression outputs in Table 1. This suggests that, although different regimes exhibit different intraday patterns, the relationship between trading $s_t - n_t$ and q_t (or equivalently between $S_t/N_t \sim Q_t$) remains practically unchanged across different regimes. The R^2 for each regime for each time aggregation is consistent with that of the regression without regimes, which was expected since the coefficients remained practically unchanged.

Since we noted that the 1-min level of aggregation can result in noisy data and that there are some data points that could distort the estimation, we performed the same regressions considering the Huber loss to estimate β in a more robust approach for all levels of aggregation. The result is practically unchanged, with a $\hat{\beta} > 1$ and confidence intervals indicating $\beta > 2$ for all regressions. For the sake of parsimony, this is not included here, but tables will be made available upon request.

In general, point estimates of β all lie within the interval [2.6; 2.9] for both the ETH and BTC regressions, which might suggest a shared dynamics driving the microstructure of both BTC and ETH futures, and this would merit further attention and investigation. A Wald test

Table 1
Regression output for testing the ITIH \times the MDH in crypto futures.

Freq.	BTC					ETH				
	Const.	SE	q_t	SE	R^2	Const.	SE	q_t	SE	R^2
Daily	−28.549	0.266	2.648	0.131	0.508	−36.690	0.182	2.761	0.238	0.601
60 min	−25.197	0.057	2.855	0.027	0.516	−33.686	0.098	2.796	0.131	0.567
30 min	−24.490	0.041	2.888	0.020	0.514	−33.025	0.071	2.793	0.094	0.557
15 min	−23.827	0.030	2.900	0.014	0.508	−32.359	0.051	2.772	0.068	0.544
10 min	−23.466	0.024	2.893	0.012	0.502	−31.964	0.042	2.746	0.056	0.533
5 min	−22.908	0.018	2.853	0.008	0.487	−31.280	0.031	2.676	0.040	0.510
1 min	−23.842	0.005	2.312	0.002	0.170	−29.632	0.015	2.272	0.019	0.404

Table 2
Regime-wise regression for different frequency aggregations for BTC futures.

(a) Asia					(b) Europe						
	Const.	SE	q_t	SE	R^2		Const.	SE	q_t	SE	R^2
60 m	−25.0547	0.505	2.8292	0.242	0.516	60 m	−24.8525	0.502	2.9897	0.248	0.554
30 m	−24.3387	0.372	2.8677	0.178	0.515	30 m	−24.1993	0.359	2.9997	0.177	0.548
15 m	−23.6757	0.270	2.8796	0.128	0.510	15 m	−23.5582	0.262	2.9954	0.128	0.540
10 m	−23.3157	0.224	2.8726	0.106	0.504	10 m	−23.2097	0.217	2.9801	0.105	0.532
5 m	−22.7599	0.162	2.8323	0.076	0.489	5 m	−22.6788	0.158	2.9233	0.076	0.513
1 m	−23.7103	0.052	2.2930	0.022	0.171	1 m	−23.7098	0.053	2.3366	0.022	0.178
(c) America					(d) Transition Zone						
	Const.	SE	q_t	SE	R^2		Const.	SE	q_t	SE	R^2
60 m	−25.3691	0.510	2.8588	0.254	0.512	60 m	−25.4975	0.666	2.7628	0.326	0.485
30 m	−24.6698	0.391	2.8924	0.193	0.510	30 m	−24.7747	0.500	2.8073	0.244	0.484
15 m	−24.0049	0.279	2.9100	0.137	0.504	15 m	−24.0773	0.375	2.8314	0.183	0.482
10 m	−23.6353	0.233	2.9076	0.114	0.499	10 m	−23.7072	0.309	2.8331	0.150	0.477
5 m	−23.0634	0.167	2.8754	0.081	0.485	5 m	−23.1189	0.227	2.8079	0.109	0.464
1 m	−23.9450	0.053	2.3538	0.023	0.171	1 m	−23.9673	0.072	2.3041	0.031	0.163

Table 3
Regime-wise regression for different frequency aggregations for ETH futures.

(a) Asia					(b) Europe						
	Const.	SE	q_t	SE	R^2		Const.	SE	q_t	SE	R^2
60 m	−33.4842	0.1780	2.7843	0.2270	0.5690	60 m	−33.6285	0.1940	2.8653	0.2420	0.5850
30 m	−32.8260	0.1290	2.7831	0.1670	0.5590	30 m	−32.9766	0.1350	2.8453	0.1740	0.5730
15 m	−32.1595	0.0930	2.7595	0.1220	0.5460	15 m	−32.3023	0.0970	2.8158	0.1260	0.5580
10 m	−31.7653	0.0770	2.7329	0.1010	0.5350	10 m	−31.9035	0.0800	2.7851	0.1040	0.5470
5 m	−31.0832	0.0560	2.6607	0.0730	0.5110	5 m	−31.2150	0.0580	2.7056	0.0750	0.5220
1 m	−29.4453	0.0280	2.2348	0.0340	0.4000	1 m	−29.5604	0.0290	2.2680	0.0350	0.4080
(c) America					(d) Transition Zone						
	Const.	SE	q_t	SE	R^2		Const.	SE	q_t	SE	R^2
60 m	−33.8712	0.1790	2.8103	0.2300	0.5730	60 m	−33.8047	0.2370	2.7557	0.3020	0.5450
30 m	−33.2148	0.1340	2.8100	0.1760	0.5640	30 m	−33.1510	0.1720	2.7618	0.2270	0.5370
15 m	−32.5584	0.0950	2.7934	0.1260	0.5490	15 m	−32.4761	0.1290	2.7491	0.1710	0.5270
10 m	−32.1635	0.0790	2.7708	0.1050	0.5400	10 m	−32.0871	0.1060	2.7287	0.1400	0.5170
5 m	−31.4830	0.0570	2.7076	0.0750	0.5170	5 m	−31.4045	0.0770	2.6690	0.1020	0.4970
1 m	−29.8324	0.0280	2.3378	0.0350	0.4180	1 m	−29.7552	0.0380	2.2992	0.0480	0.3990

statistic for the ITIH, which means $W = (\hat{\beta}+2)^2/(\text{se}(\hat{\beta}))^2$, strongly rejects the hypothesis of $\beta = -2$ for all models. This indicates that volatility per trade increases more than proportionally with trade size, which means that trades tend to occur in more volatile market conditions rather than when prices are more stable. Therefore, the high-frequency dynamics of cryptocurrency futures on Binance resembles traditional speculative markets where volatility increases with trading volume, which could be due to the absence of a stabilizing mechanism.

4.2. Quantile regression

Following the analysis of Section 4.1, we perform quantile regressions for the BTC and ETH futures data. We consider the regression over the following quantiles: $\tau = 0.1, \tau = 0.25, \tau = 0.5, \tau = 0.75$, and $\tau = 0.9$ for the same frequency aggregations as before. The resulting estimates of $\beta_{1,\tau}$ are reported in Figs. 9 (BTC futures) and 10 (ETH futures).

The quantile regressions reinforce the findings from the previous section, as they agree with the standard regressions in Table 1. For

both BTC and ETH, the positive and statistically significant slope coefficients across both the quantiles and frequencies support the MDH, which suggests that the return volatility per trade increases with trade size. However, the quantile-specific patterns diverge notably. Bitcoin futures display a monotonically decreasing slope profile, suggesting a stronger sensitivity of volatility to trade size in low-volatility regimes. In contrast, Ether futures exhibit a hump-shaped pattern, with the strongest effect near the median quantile, indicating a more symmetric and stable volatility response. These findings are theoretically relevant and warrant further investigation: they indicate that, while MDH might broadly hold, the impact of trade size could be shaped by distributional regimes and might also be instrument-specific.

It is also important to stress that the results at the 1-min level of aggregation confirm the presence of microstructure noise, especially for BTC, as indicated by lower R^2 and flatter quantile profiles, but microstructure noise also appears in ETH data. This is consistent with the previous Section 4.1 and with previous literature statements that 5-min

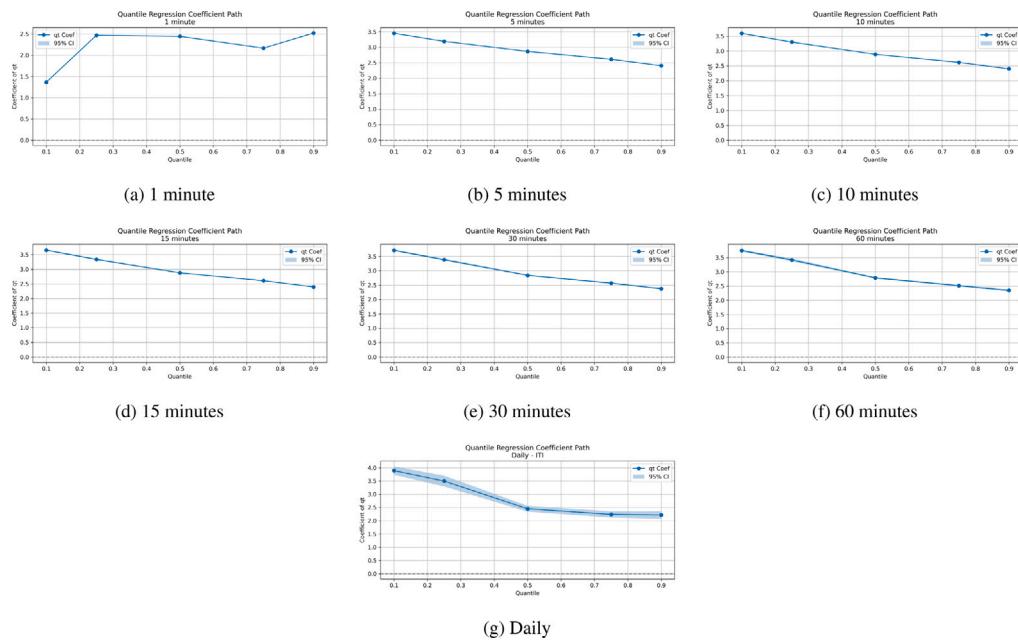


Fig. 9. Estimates of $\beta_{1,tau}$ in quantile regression for BTC futures under different levels of aggregation.

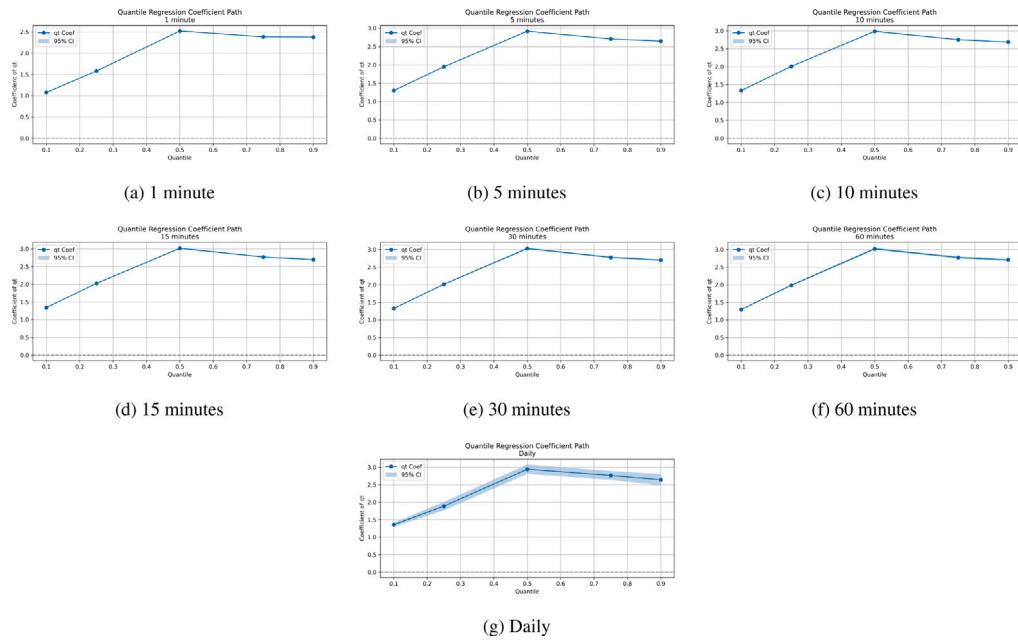


Fig. 10. Estimates of $\beta_{1,tau}$ in quantile regression for ETH futures under different levels of aggregation.

aggregation is the highest frequency at which market microstructure noise is not too distorting.

From Figs. 9 and 10, one can also see that the asymmetry in the β_r slope coefficient across the array of quantiles indicates that market responses to trade size are not totally uniform. The change in estimates of β_r across different quantiles strongly confirms heteroskedasticity in the relationship, which justifies the usage of robust standard errors in Section 4.1.

Overall, for BTC futures, the quantile regressions reveal heterogeneity: volatility per trade responds more strongly to trade size at lower quantiles and less strongly in higher tails. For ETH futures, the hump-shaped plot indicates that the impact of the trade size is strongest at median volatility levels. Irrespective of these differences, all results support the MDH over the ITIH.

4.3. Discussion of results

A consistent finding of $\beta > 0$ across multiple time frequencies for both BTC and ETH futures implies that volatility per trade increases with average trade size, supporting the MDH. This suggests that both markets are information-sensitive and reflect latent information flows, with implications for risk management, price formation, and strategic trade execution in cryptocurrency futures.

This might indicate that there is limited depth in the market, since larger trades tend to be executed during volatile periods, suggesting that the market reacts disproportionately to large trades. Reasons for this might vary: for instance, it could be due to a thin liquidity pool, meaning that large orders are not absorbed efficiently and suggesting the potential for order book imbalance, especially in fast-moving settings.

Since a value of $\beta > 0$ goes against the proposition of Andersen et al. (2020), this suggests that the risk-transfer mechanism is not scale-invariant in BTC and ETH futures. As such, this “invariance efficiency” could be a characteristic of a very mature institutional futures market, like the S&P 500 E-Mini studied in Andersen et al. (2020), while the crypto market still carries frictions such as asymmetric information, shallow order books, or retail-driven businesses in market settings, or it could be market/instrument specific.

A positive value of β reinforces the idea that the arrival of information drives both volume and volatility, agreeing with the MDH as in Clark (1973) and Epps and Epps (1976). A value of $\beta > 2$ might also indicate that crypto futures exhibit liquidity fragmentation, where large trades disproportionately impact market volatility. This could also indicate that market order imbalances play a critical role in price fluctuations. These results align with the evolving nature of the Bitcoin and Ether futures microstructures, where market depth and liquidity may still be maturing compared to traditional futures markets, which merits further analysis.

Nevertheless, a value of $\beta > 0$ is also important for risk management. Under this assumption, especially the case of $\beta > 2$, monitoring trade size becomes predictive of volatility, as larger trade sizes can signal incoming volatility spikes. This is useful not only for market participants but also for the exchange’s risk management and internal controls. For example, it might be interesting to incorporate trade-size covariates to enhance GARCH-type models for volatility forecasting.

When analyzing the quantile regression, BTC futures’ declining β coefficient implies that larger trades lose explanatory power under extreme conditions. This has important trading implications because, at the tails, trade size no longer predicts volatility as effectively. For risk management, this stresses the importance of nonlinear modeling in Value-at-Risk (VaR) or tail risk settings with extreme value theory.

Since larger trades are associated with a much higher return variance, investors may experience significant price slippage and market impact. This suggests that liquidity in the market may not be sufficient to absorb large trades. Furthermore, the strong link between trade size and volatility creates opportunities for volatility-based trading strategies, such as market-making and statistical arbitrage arising from low information diffusion.

There are a few particularities of the BTC and ETH futures market that could explain why the MDH matches the data better than the ITIH. First, since the crypto market operates in a continuous 24/7 regime, with no trading hours, information arrives asynchronously across different time zones. The different reactions of traders to these information arrivals might lead to heterogeneous returns distributions. Moreover, the greater proportion of retail traders might make the market more sensitive to the arrival of information, perhaps due to emotional trading and sentiment-driven moves. If the latter applies, it would be possible to see a change in β over time as the proportion of institutional traders increases. It is, nonetheless interesting that a foundational microstructure model originally developed for equity and FX markets extends well to decentralized crypto derivatives.

5. Final remarks

Figs. 1 to 4 give the impression that Bitcoin and Ether futures markets are maturing, with increasing participation and reduced return variance. The intraday trading patterns observed in Figs. 5 and 6 reveal the strong effects of the market structure, with clear regime-dependent liquidity cycles throughout the trading day.

Figs. 7 and 8 might suggest seasonal patterns are a stylized fact in crypto futures, since it was clear in the years 2022–2024. This is a potential topic of investigation for researchers interested in the behavior of cryptocurrencies markets and, in particular, of futures.

Therefore, these preliminary analyses of stylized facts could spur further research related to structural breaks, nonlinear dependencies, and market microstructure models to further dissect the complexities

of cryptocurrency trading dynamics and, in particular, to theoretically explain some of the stylized facts and to investigate whether they are noticeable in other cryptocurrencies and trading platforms.

This result is important for both investors and policymakers. For market participants, evidence favoring MDH implies that volatility is primarily information driven, which has direct implications for trading strategies, execution timing, and volatility forecasting, as well as for risk management strategies. For trading platforms, regulators, and portfolio managers, a rejection of the ITIH suggests that the market invariance conditions observed in Andersen et al. (2020) do not hold for this particular dataset, highlighting potential idiosyncrasies in crypto futures markets. By investigating how the different derivatives conform to or deviate from theoretical predictions, our aim is to provide another framework for assessing the maturity and structure of crypto derivative markets, thus filling the gap between theoretical models and practical market behavior.

Our analysis finds consistent evidence favoring the MDH over the ITIH—for both BTC and ETH futures—to empirically explain the behavior of these futures markets, with more explanatory power at the daily level of aggregation and at lower frequencies. At the 1-min aggregation level, we notice the strong effects of microstructure noise, distorting the results and the explanatory power of the regression. This effect is stronger for BTC futures, but is also seen in ETH futures.

We recognize that aggregation issues may affect our analysis primarily due to data availability constraints. Specifically, it is not always possible to report all contracts traded at an identical price against an incoming order as a single combined transaction quantity, as was done in Andersen et al. (2020). If these data limitations were overcome, future research could beneficially explore the MDH vs. ITIH test under the conditions outlined above, provided sufficient data are available.

Further advances in the theory of market microstructure could be beneficial in explaining this non-linear relationship. This could involve a test assuming other market microstructure models, such as the Order Flow Toxicity of Easley et al. (2012), which lies beyond the scope of this paper.

Further research would be beneficial to associate macroeconomic and corporate events with months linked to higher volatility and increased market activity, and the effect of institutional behavior on the microstructure of the cryptocurrency futures market. A systematic analysis encompassing other liquid futures traded on the same platform and on other platforms would be interesting. Finally, in order to assess whether retail traders are more sensitive to the arrival of information, as discussed in the previous section, further studies could test this using the VIX index, which measures sentiment-driven trading by retail and institutional investors.

Finally, our study focuses exclusively on data from Binance. Although it is a major exchange, it would be interesting to investigate the coherence of the identified behavior on other trading platforms and exchanges. Cross-exchange comparisons of this type would be an worthwhile continuation of this line of research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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