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Isomorphic Group (and Loop) Algebras

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E.G. Goodaire [4] defined *R.A. loops* as those loops whose loop algebra over any ring with no 2-torsion is alternative and O. Chain and E.G. Goodaire [1] gave a description of such loops. If L is an R.A. loop then it contains a group G with $[L:G]=2$, such that $G'=\{1,e\}=L'$ is a group of order 2 and $Z(G)=Z(L)$, where $G', L', Z(G)$ and $Z(L)$ denote the commutators and centers of G and L respectively. Also, $L/Z(L)\cong C_2\times C_2\times C_2$, where C_2 denotes a cyclic group of order 2, and hence $G/Z(G)\cong C_2\times C_2$.

In this paper we consider the class of all groups G such that $G/Z(G)\cong C_p\times C_p$, where p is a rational prime. In §1 we show that these groups satisfy property (p) in the sense of D.B. Coleman [2] and give a full description in

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terms of direct factors. In §2 we study the isomorphism problem for rational group algebras of groups of this kind, i.e., if G is one such group and $\mathbb{Q}G \cong \mathbb{Q}H$ what can be said about G and H? Finally, in §3 we consider the isomorphic problem for alternative loop algebras over the field of rational numbers.

§1 - Basic Facts.

We wish to show that the groups under consideration have irreducible complex representations of a very special kind. In order to do this, we need to compute their commutators.

(1.1) Lemma. Let G be a group such that $G/Z(G) \cong C_p \times C_p$. Then the commutator subgroup G' is cyclic of order p.

Proof. Set $x, y \in G$. Then

(i) $[x,y] = [y,x]^{-1}$

(ii) $[xy,z] = [x,z]^y[y,z]$ and $[x,yz] = [x,z][x,y]^z$.

Since $G/Z(G)$ is abelian, we know that $G' \subset Z(G)$ and hence, in our case, we have:

(ii)' $[xy,z] = [x,z][y,z]$ and $[x,yz] = [x,z][x,y]$.

If $G/Z(G) \cong C_p \times C_p$, there exist elements $x, y \in G$ such that $G = \langle x, y, Z(G) \rangle$, with $x^p, y^p \in Z(G)$. Then:

$$G' = \langle [x^n y^m, x^r y^s] \mid 0 \le n, m, r, s < p \rangle .$$

Now, (i) and (ii)' readily give that:

$$[x^n y^m, x^r y^s] = [x,y]^{(n+m)(r+s)},$$

hence G' is cyclic, generated by $[x,y]$.

Also, since $y^p \in Z(G)$ we see that:

$$1 = [x, y^p] = [x,y]^p,$$

so $|G'| = p$. □

(1.2) Theorem. Let G be a group. Then $G/Z(G) \cong C_p \times C_p$ if and only if $G = D \times A$ where A is an abelian group and D is an indecomposable p-group such that $D = < x, y, Z(D) >$ where $x^p, y^p \in Z(D)$ and $Z(D)$ can be written in the form $Z(D) = C_{p^{m_1}} \times C_{p^{m_2}} \times C_{p^{m_3}}$ with $m_1 \geq 1$ and $m_2, m_3 \geq 0$.

Proof. Since $G/Z(G) \cong C_p \times C_p$ there exist elements $x_1', y_1' \in G$ such that $x_1'^p, y_1'^p \in Z(G)$ and $G = < x_1', y_1', Z(G) >$. Set $Z(G) = B \times C$ where B is p-group and $p \nmid |C|$. We can write:

$$x_1'^p = b.c \qquad \text{where} \qquad b \in B,\ c \in C.$$

Let $n = o(c)$. Since the map $x \mapsto x^P$ is surjective in C, we can find an element $\gamma \in C$ such that $\gamma^p = c^{n-1}$. If we set $x' = \gamma x_1'$ we see that $x_1^p = \gamma^P x_1'^p = c^{n-1}bc = b \in B$ and, clearly, $G = < x_1, y_1', Z(G) >$. In a similar way we can find y_1 such that $G = < x_1, y_1, Z(G) >$ and $x_1^p, y_1^p \in Z(G)$.

Since $< x_1, y_1, B >$ is a p-group, it follows easily that

$$G = < x_1, y_1, B > \times C.$$

Also, $G' \subset Z(G)$ is a p-group so $G' \subset B$. We wish to show that we can find a decomposition of B in cyclic factors $B = C_{p^{m_1}} \times \ldots \times C_{p^{m_k}}$ such that $G' \subset C_{p^{m_1}}$.

In fact, let $G' = < e \mid e^p = 1 >$ and let $C_{p^{m_i}} = < t_i >$ be subgroups of a decomposition of B in which e is written as a product with minimal number of factors. We can choose the generators t_i, $i \leq i \leq k$ in such a way that

$$e = t_1^{p^{m_1 - 1}} \ldots t_\ell^{m_\ell - 1} \quad \text{with} \quad \ell \leq k.$$

We wish to show that $\ell = 1$ so, assume that $\ell > 1$ and $m_1 \geq m_2$. Clearly

$$< t_1 > \times < t_1^{p^{m_1 - m_2}} . t_2 > = C_{p^{m_1}} \times C_{p^{m_2}}$$

so

$$B = C_{p^{m_1}} \times < t_1^{p^{m_1 - m_2}} . t_2 > \times \ldots \times C_{p^{m_\ell}}.$$

Since $(t_1^{p^{m_1 - m_2}} . t_2)^{m_2 - 1} = t_1^{m_1 - 1} . t_2^{m_2 - 1}$ it follows that $e \in < t_1^{p^{m_1 - m_2}} . t_2 > \times \ldots \times C_{p^{m_\ell}}$, a contradiction.

Like $x_1^p \in B$ so we can write $x_1^p = \alpha_1 \ldots \alpha_k$ with $\alpha_i \in C_{p^{m_i}}$. Assume that, for a given index i the corresponding element α_i is not a generator of $C_{p^{m_i}}$. Then $\alpha_i^{o(\alpha_i) - 1}$ is not a generator also and we can write $\alpha_i^{o(\alpha_i) - 1} = t_i^{v_i}$ where $p \mid v_i$.

Now, the element $t_i^{\frac{v_i}{p}} . x_i$ is such that $(t^{\frac{v_i}{p}} . x_1)^p \in B$ and has no component in $C_{p^{m_i}}$. Repeating this process, if necessary we can find an element x such that $< x, y_1, B > = < x_1, y_1, B >$ and $x^p = \alpha_{i_1}\alpha_{i_2}\ldots\alpha_{i_\ell}$ were α_{i_j} is a generator of $C_{p^{m_{i_j}}}$, $1 \leq j \leq \ell$. Assume that none of these elements belongs to $C_{p^{m_1}}$. We can write

$$C_{p^{m_{i_1}}} \times \ldots \times C_{p^{m_{i_\ell}}} = < \alpha_{i_1} \ldots \alpha_{i_\ell} > \times < \alpha_{i_2} > \times \ldots \times < \alpha_{i_\ell} >$$

$$x^p \in < \alpha_{i_1} \times \ldots \times \alpha_{i_\ell} >$$

so, in any case, we can write B in the form $B = C_{p^{m_1}} \times C_{p^{m_2}} \times \ldots \times C_{p^{m_k}}$ where

$G' \subset C_{p^{m_1}}$ and $x^p \in C_{p^{m_1}} \times C_{p^{m_2}}$.

In a similar way, we can find an expression for B and elements $x, y \in G$ such that:

$$G = < x, y, B >$$

$$B = C_{p^{m_1}} \times \ldots \times C_{p^{m_\ell}}$$

and

$$G' \subset C_{p^{m_1}}, \qquad x^p \in C_{p^{m_1}} \times C_{p^{m_2}} \qquad \textit{and} \quad y^p \in C_{p^{m_1}} \times C_{p^{m_2}} \times C_{p^{m_3}}.$$

Notice that actually we have shown that x^p and y^p belong to a product of *at most* three of these factors.

So, if we set $D = < x, y, C_{p^{m_1}} \times C_{p^{m_2}} \times C_{p^{m_3}} >$ where $m_1 \geq 1$ and $m_2, m_1 \geq 0$ and $A = C_{p^{m_4}} \times \ldots \times C_{p^{m_\ell}} \times C$ we have that $G = D \times A$ as desired.□

(1.3) Corollary. Let G be an indecomposable group such that $G/Z(G) \cong C_p \times C_p$. Then, G is a p–group and $Z(G)$ has rank at most 3.

(1.4) Lemma. Let G be a group. Then $G/Z(G) \cong C_p \times C_p$ if and only if $|G'| = p$ and every irreducible complex representation of G has degree equal to either 1 or p.

Proof. Assume that $G/Z(G) \cong C_p \times C_p$. In this case, we see that $|Z(G)| = |G|/p^2$. Hence, since $|G'| = p$, then every conjugacy class has order equal to 1 or p, so the number of conjugacy classes in G is:

$$|Z(G)| + \frac{|G| - |Z(G)|}{p^2} = \frac{|G|}{p^2} + \frac{|G| - |G|/p^2}{p} = \frac{(p^2+p-1)|G|}{p^3}.$$

We denote by $\Delta_{\mathbf{C}}(G, G')$ the ideal of $\mathbf{C}G$ generated by the set $\{x - 1 \mid x \in G'\}$. We can write [2, lemma (1.1)]:

$$\mathbf{C}G \cong \mathbf{C}(G/G') \oplus \Delta_{\mathbf{C}}(G : G').$$

Since $|G/G'| = |G|/p$ we have that $\mathbf{C}(G/G')$ is isomorphic to a direct sum of $|G|/p$ copies of $\mathbf{C}$. Hence, the number of simple components in the decomposition of $\Delta_{\mathbf{C}}(G : G')$ is:

$$\frac{p^2+p-1}{p^3}|G| - \frac{|G|}{p} = \frac{p-1}{p^3}|G|.$$

Now, we evaluate the dimension of $\Delta_{\mathbf{C}}(G : G')$ in two different ways. On the one hand, we have that:

$$[\Delta_{\mathbf{C}}(G : G') : \mathbf{C}] = |G| - [\mathbf{C}(G/G') : \mathbf{C}] = \frac{p-1}{p}|G|.$$

On the other hand, since all 1-dimensional components of $\mathbf{C}G$ come from $\mathbf{C}(G/G')$. By the Theorem (1.2) if G is indecomposable, therefore a p-group, each component of $\Delta_{\mathbf{C}}(G : G')$ has dimension at least equal to p^2.

If $G \cong D \times A$, $CG \cong CA \otimes CG$ and like A is abelian, again we have that each simple component of $\Delta_C(G : G')$ has dimention at least equal to p^2. Multiplying by the number of components and comparing with the result above, we see that all simple components of $\Delta_C(G : G')$ must have dimension precisely equal to p^2.

Hence:

$$CG \cong \underbrace{C \oplus \ldots \oplus C}_{\frac{|G|}{p} \text{ times}} \oplus \underbrace{M_p(C) \oplus \ldots \oplus M_p(C)}_{\frac{p-1}{p^3}|G| \text{ times}}$$

where $M_p(C)$ denotes the full ring of $p \times p$ matrices over C. Thus, all the irreducible complex representations of G have degree equal to either 1 or p.

Conversely, assume that the representations are as above. Since $CG \cong C(G/G') \oplus \Delta_C(G : G')$ we can compute:

$$[Z(CG) : C] = \frac{|G|}{p} + \frac{|G| - |G|/p}{p^2} = \frac{p^2 + p - 1}{p^3} . |G|.$$

Since $[Z(CG) : C]$ is equal to the number of conjugacy classes in G we get:

$$\frac{p^2 + p - 1}{p^3}|G| = |Z(G)| + \frac{|G| - |Z(G)|}{p}$$

and thus $|G/Z(G)| = p^2$. Also, we known that G is non abelian, hence $G/Z(G)$ cannot be cyclic and, consequently, $G/Z(G) = C_p \times C_p$.□

§2 - Group Algebras.

Our first statement is rather elementary but will be repeatedly needed in the sequel.

(2.1) Lemma. Let $C_{p^m} = S_m \supset S_{m-1} \supset \ldots \supset S_o = \{1\}$ be the set of all subgroups of the cyclic group of order p^m and set $\widehat{S}_i = \frac{1}{p^i}\sum_{x\in S_i} x$, $0 \leq i \leq m$. Then, the primitive idempotents of the rational group algebra QC_{p^m} are $e_o = \widehat{S}_m$ and $e_i = \widehat{S}_{m_i} - \widehat{S}_{m-i+1}$, $1 \leq i \leq m$.

In particular, we have that

$$QC_{p^m}(1-\widehat{S}_1) = Q(\zeta)$$

where ζ denotes a primitive root of unity of degree $p^{m-1}(p-1)$.

Proof. We have that

$$QC_{p^m} \cong \frac{Q[X]}{(X^{p^m}-1)} \cong \oplus_{i=0}^{m} \frac{Q[X]}{(\Phi_{p^i})} \cong \oplus_{i=0}^{m} Q(\zeta_i),$$

where Φ_{p^i} denotes the cyclotomic polynomial of order p^i and ζ_i is a root of Φ_{p^i}, $0 \leq i \leq m$.

Hence, we see that QC_{p^m} contains precisely $m+1$ primitive idempotents. Clearly $\widehat{S}_{m-i}.\widehat{S}_{m-j} = \widehat{S}_{m-i}$ if $i \leq j$ so the idempotents e_i, $0 \leq i \leq m$ are pairwise orthogonal and $\sum_{i=0}^{m} e_i = 1$, as desired.

Also, $QC_{p^m}.\widehat{S}_1 \cong Q(C_{p^m}/S_1)$ so $[QC_{p^m} : Q] = p^{m-1}$ hence $[QC_{p^m}(1-\widehat{S}_1) : Q] = p^m - p^{m-1}$. Since $QC_{p^m}(1-\widehat{S}_1)$ is a field, we must have $QC_{p^m}(1-\widehat{S}_1) = Q(\zeta)$ where ζ is a primitive root of unity of degree $p^{m-1}(p-1)$. □

(2.2) Lemma. Let D be a group such that $|D'| = p$. Then $Z(QD(1-\widehat{D'})) = Q(Z(D))(1-\widehat{D'})$, where $\widehat{D'} = p^{-1}\sum_{x\in D'} x$.

Proof. Set $g \in G$ and denote by $C\ell(g)$ the conjugacy class of g. Since $|D'| = p$ we have that either $C\ell(g) = \{g\}$ or $C\ell(g) = gD'$. Thus, an element $\alpha \in Z(\mathbf{Q}D)$ can be written in the form:

$$\alpha = \sum_{g \in Z(D)} \alpha_g g + \sum_{g \notin Z(D)} \alpha_g g D', \qquad \alpha_g \in \mathbf{Q}.$$

Since $\mathbf{Q}D(1 - \widehat{D'})$ is a direct summand we have that $Z(\mathbf{Q}D(1 - \widehat{D'})) = Z(\mathbf{Q}D) \cap \mathbf{Q}D(1 - \widehat{D'})$ so, given an element $\alpha \in (\mathbf{Q}D(1 - \widehat{D'}))$ it can be written as above, and also $\alpha = \alpha(1 - \widehat{D'})$. Hence, it can be written as:

$$\alpha = \sum_{g \in Z(D)} \alpha_g g(1 - D') \in \mathbf{Q}(Z(D))(1 - \widehat{D'}).$$

Thus, we see that $Z(\mathbf{Q}D(1 - \widehat{D'})) \subset \mathbf{Q}(Z(D))(1 - \widehat{D'})$. The opposite inclusion is obvious.□

From now on, D will always denote an indecomposable group which can be written in the form $D = \langle x, y, Z(D) \rangle$ as described in Theorem (1.2). Let E be another group such that $\mathbf{Q}D \cong \mathbf{Q}E$. We wish to show that E is also a group of the same kind and that, with a notable exception, the isomorphism occurs if and only if $D/D' \cong E/E'$ and $Z(D) \cong Z(E)$.

So assume that $\mathbf{Q}D \cong \mathbf{Q}E$; then also $\mathbf{C}D \cong \mathbf{C}E$ and we have that:

$$\mathbf{C}D \cong \mathbf{C}(D/D') \oplus \Delta_{\mathbf{C}}(D : D')\mathbf{C}E \cong \mathbf{C}(E/E') \oplus \Delta_{\mathbf{C}}(E : E').$$

Since the simple commutative components of $\mathbf{C}D$ and $\mathbf{C}E$ are those of $\mathbf{C}(D/D')$ and $\mathbf{C}(E/E')$ respectively [3, p.36], it follows that $\mathbf{C}(D/D') \cong \mathbf{C}(E/E')$ and thus $|E'| = |D'| = p$. Also $\Delta_{\mathbf{C}}(D : D') \cong \Delta_{\mathbf{C}}(E : E')$.

Since lemma (1.2) shows that all simple components of $\Delta_{\mathbf{C}}(D : D')$ are of the form $M_p(\mathbf{C})$, the same is true for $\Delta_{\mathbf{C}}(E : E')$ and, again because of lemma (1.2), we obtain that $E/Z(E) \cong C_p \times C_p$.

In a similar way, we obtain that $\mathbf{Q}(D/D') \cong \mathbf{Q}(E/E')$ and thus $D/D' \cong E/E'$ [7, theorem III.2.12]. It now follows easily that $E = < u, v, Z(E) >$ where u, v and $Z(E)$ are as described in Theorem (1.2).

In order to simplify notations, from now on, we shall identify E with its image, though an isomorphism $\varphi : \mathbf{Q}E \to \mathbf{Q}D$, in $\mathbf{Q}D$.

(2.3) Lemma. With the notations above, if $\mathbf{Q}D \cong \mathbf{Q}E$ then $\widehat{D'} = \widehat{E'}$.

Proof. Write $\mathbf{Q}D = \mathbf{Q}D.\widehat{D'} \oplus \mathbf{Q}D(1 - \widehat{D'})$, where $\mathbf{Q}D\widehat{D'} \cong \mathbf{Q}(D/D')$ and $\mathbf{Q}D(1 - \widehat{D'}) = \Delta_{\mathbf{Q}}(D : D')$.

Since D/D' is abelian, all simple components of $\mathbf{Q}D.\widehat{D'}$ are commutative. On the other hand, we claim that $\Delta_{\mathbf{Q}}(D : D')$ contains no commutative components. In fact, we have that

$$\mathbf{C}D \cong \mathbf{C} \otimes \mathbf{Q}D \cong \mathbf{C}(D/D') \oplus (\mathbf{C} \otimes \Delta_{\mathbf{Q}}(D : D'))$$

so, commutative components in $\Delta_{\mathbf{Q}}(D : D')$ will imply the existence of commutative components in $\Delta_{\mathbf{C}}(D : D')$ contradicting lemma (1.1).

Thus:

$$\mathbf{Q}D.\widehat{D'} = \mathbf{Q}E.\widehat{E'} \qquad \text{and}$$

$$\mathbf{Q}D(1 - \widehat{D'}) = \mathbf{Q}E(1 - \widehat{E'}).$$

In particular, since $\widehat{D'}$ and $\widehat{E'}$ are the unity elements of $\mathbf{Q}D.\widehat{D'}$ and $\mathbf{Q}E.\widehat{E'}$ respectively, it follows that $\widehat{D'} = \widehat{E'}$. □

Since D and E are groups as described in Theorem (1.2) we can write:

$$Z(D) = C_{p^{n_1}} \times C_{p^{n_2}} \times C_{p^{n_3}}$$

$$Z(E) = C_{p^{m_1}} \times C_{p^{m_2}} \times C_{p^{m_3}}$$

with $n_1, m_1 \geq 1$, $n_i, m_i \geq 0$, $i = 1, 2, 3$.

(2.4) Lemma. With the notations above, if $\mathbb{Q}D \cong \mathbb{Q}E$ then $n_1 = m_1$.

Proof. Since $|Z(D)| = |Z(E)| = \frac{|D|}{p^2}$ it follows immediately that

$$n_1 + n_2 + n_3 = m_1 + m_2 + m_3.$$

Now:

$$\mathbb{Q}(Z(D))(1 - \widehat{D'}) = \mathbb{Q}(C_{p^{n_1}} \times C_{p^{n_2}} \times C_{p^{n_3}})(1 - \widehat{D'}) =$$

$$= [\mathbb{Q}(C_{p^{n_1}})(C_{p^{n_2}} \times C_{p^{n_3}})](1 - \widehat{D'}).$$

In the notation of lemma (2.1) S_1 is the subgroup of $C_{p^{n_1}}$ of order p, i.e., $S_1 = D'$ and thus, that lemma shows that

$$\mathbb{Q}C_{p^{n_1}}(1 - \widehat{D'}) \cong \mathbb{Q}(\zeta)$$

where ζ is a primitive root of unity of degree $p^{n_1 - 1}(p-1)$.

Hence:

$$\mathbb{Q}(Z(D))(1 - \widehat{D'}) \cong \mathbb{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}).$$

In a similar way we obtain that $\mathbb{Q}(Z(E))(1 - \widehat{E'}) \cong \mathbb{Q}(\zeta_1)(C_{p^{m_2}} \times C_{p^{m_3}})$ where ζ_1 is of degree $p^{m_1 - 1}(p-1)$, thus:

$$\mathbb{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \mathbb{Q}(\zeta_1)(C_{p^{m_2}} \times C_{p^{m_3}})$$

and, since $\mathbf{Q}(\zeta)$ and $\mathbf{Q}(\zeta_1)$ are the components of smaller dimension over $\mathbf{Q}$ in both sides of the expression, we see that $n_1 = m_1$. □

(2.5) Lemma. Let ζ be a root of unity of degree $p^{n-1}(p-1)$ and assume that $\mathbf{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \mathbf{Q}(\zeta)(C_{p^{m_2}} \times C_{p^{m_3}})$. Then either:

(i) $C_{p^{n_2}} \times C_{p^{n_3}} \cong C_{p^{m_2}} \times C_{p^{m_3}}$

or

(ii) $n_1 > \max\{n_2, n_3, m_2, m_3\}$ and $n_2 + n_3 = m_2 + m_3$.

Proof. Assume first that $\mathbf{Q}(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \mathbf{Q}(C_{p^{m_2}} \times C_{p^{m_3}})$ and that $C_{p^{n_2}} \times C_{p^{n_3}} \not\cong C_{p^{m_2}} \times C_{p^{m_3}}$.

Without loss of generality, we may assume that $n_2 = \max\{n_2, n_3, m_2, m_3\}$. Since the groups are not isomorphic we must have that $n_2 > m_2, m_3$.

As in lemma (2.1), $\mathbf{Q}(C_{p^{n_2}} \times C_{p^{n_3}})$ contains a simple component isomorhic to $\mathbf{Q}(\Theta)$ where $\deg(\Theta) = p^{n_2}(p-1)$. Hence, $\mathbf{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \mathbf{Q}(\zeta) \otimes_{\mathbf{Q}} \mathbf{Q}(C_{p^{n_2}} \times C_{p^{n_3}})$ contains a component which is isomorphic to $\mathbf{Q}(\zeta) \otimes_{\mathbf{Q}} \mathbf{Q}(\Theta)$ which, in turn, contains $\mathbf{Q}(\Theta)$.

If $n_1 < n_2$, $\mathbf{Q}(\zeta) \otimes_{\mathbf{Q}} \mathbf{Q}(\Theta) \cong \mathbf{Q}(\Theta)$ and $\mathbf{Q}(C_{p^{m_2}} \times C_{p^{m_3}})$ contains no simple component isomorphic to $\mathbf{Q}(\Theta)$, a contradiction.

If $n_1 = n_2$ then $\mathbf{Q}(\zeta)$ is a splitting field for $C_{p^{m_2}} \times C_{p^{m_3}}$ but not for $C_{p^{n_2}} \times C_{p^{n_3}}$ so the corresponding group algebras cannot be isomorphic. □

Conversely, if (i) holds the statement is obvious so assume that (ii) holds. As before, set $n_2 = \max\{n_2, n_3, m_2, m_3\}$. If $n_1 > n_2$ then $p^{n_1}(p-1) \geq p^{n_2}$

and $\mathbf{Q}(\zeta)$ is a splitting field for both groups so:

$$\mathbf{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \underbrace{\mathbf{Q}(\zeta) \oplus \ldots \oplus \mathbf{Q}(\zeta)}_{(n_2+n_3)\ times} \cong \mathbf{Q}(\zeta)(C_{p^{m_2}} \times C_{p^{m_3}})$$

□

We are now ready to prove the main result of this section.

(2.6) Theorem. Let D be a group as described in Theorem (1.2) and let E be another group. If $\mathbf{Q}D \cong \mathbf{Q}E$ then

(i) $D/D' \cong E/E'$

and either:

(ii) $Z(D) \cong Z(E)$

or

(ii') with the notations above $n_1 = m_1 > \max\{n_2, n_3, m_2, m_3\}$ and $n_2 + n_3 = m_2 + m_3$.

Conversely, if E is also as in Theorem (1.2) and the conditions above hold, we have that:

(1) If $p \neq 2$ then $\mathbf{Q}D \cong \mathbf{Q}E$.

(2) If $p = 2$ and $n_1 > 1$ then $\mathbf{Q}D \cong \mathbf{Q}E$.

vspace0.5cm **Proof.** The necessity of the conditions follows directly from the previous lemmas.

To prove the converse, we recall from Lemma (1.2) that every irreducible representation of D and E has degree equal to either 1 or p. Also, as shown in Lemma (2.3), $\mathbf{Q}E(1 - \hat{E'})$ and $\mathbf{Q}D(1 - \widehat{D'})$ contain no commutative simple components, hence:

$$\mathbb{Q}D.\widehat{D'} \cong F_1 \oplus \ldots \oplus F_r$$

$$\mathbb{Q}E.\widehat{E'} \cong K_1 \oplus \ldots \oplus K_s$$

$$\mathbb{Q}D(1-\widehat{D'}) \cong A_1 \oplus \ldots \oplus A_u$$

$$\mathbb{Q}E(1-\widehat{E'}) \cong B_1 \oplus \ldots \oplus B_v$$

where F_i, K_j are fields and A_i, B_j are simple algebras of dimension p^2 over their centers.

Since $\mathbb{Q}D.\widehat{D'} \cong \mathbb{Q}(D/D')$ and $\mathbb{Q}E.\widehat{E'} \cong \mathbb{Q}(E/E')$, condition (i) implies that $r = s$ and, in a convenient reordering $F_i \cong K_i$, $1 \leq i \leq r$.

Conditions (ii) or (iii) imply that $\mathbb{Q}(\zeta)(C_{p^{n_2}} \times C_{p^{n_3}}) \cong \mathbb{Q}(\zeta)(C_{p^{m_2}} \times C_{p^{m_3}})$ and, as in Lemma (2.3), we see that:

$$\mathbb{Q}(Z(D))(1-\widehat{D'}) \cong \mathbb{Q}(Z(E))(1-\widehat{E'})$$

so

$$Z(\mathbb{Q}D(1-\widehat{D'})) \cong Z(\mathbb{Q}E(1-\widehat{E'}).$$

Hence, $u = v$ and, in a convenient reordering, $Z(A_i) \cong Z(B)$, $1 \leq i \leq u$. If $p \neq 2$ this implies also that $A_i \cong B_i \cong M_p(Z(A_i))$, $1 \leq i \leq u$ (see [2, p.5]) and thus $\mathbb{Q}D(1-\widehat{D'}) \cong \mathbb{Q}D(1-\widehat{E'})$; consequently, we also have that $\mathbb{Q}D \cong \mathbb{Q}E$.

If $p = 2$ the simple components are algebras of dimension 4 over their centers and can thus be isomorphic eiter to a full matrix algebra or a quaternion algebra.

Since $E' = D' = \{1, e\}$ we have that $1-\widehat{D'} = \frac{1-e}{2}$ is the identity element of $\mathbb{Q}D(1-\widehat{D'})$. If $n_1 > 1$ there exists an element $\alpha \in C_p^{n_1}$ such that $\alpha^2 = \ell$. Hence, $\alpha(\frac{1-e}{2}) \in Z(\mathbb{Q}D(1-\widehat{D'}))$ is such that $[\alpha\frac{(1-e)}{2}]^2 = -(1-\widehat{D'})$; this means that the centers of all components are fields containing a square root of -1. Thus, the algebras under consideration cannot be quaternion algebras. Consequently, $\mathbb{Q}D(1-\widehat{D'}) \cong \mathbb{Q}E(1-\widehat{E'})$ and we conclude again that $\mathbb{Q}D \cong \mathbb{Q}E$.□

Notice that in the case where $p = 2$, if $n = 1$ the conditions of the theorem above are not sufficient. In fact, if D_4 and K_8 denote the dihedral and quaternion groups of order 8 respectively, we see that $|D_4'| = |K_8'| = 4$, $Z(D_4) \cong Z(K_8) \cong C_2$ and $D_4/D_4' \cong K_8/K_8' \cong C_2 \times C_2$ but $\mathbb{Q}D_4 \not\cong \mathbb{Q}K_8$.

To study this situation we need a few remarks.

Let E be a group such that $\mathbb{Q}D \cong \mathbb{Q}E$. As in Theorem (2.6), we have that $D/D' \cong E/E'$ and thus $|E'| = |D'| = 2$ so, Lemma (1.2) shows that $E/Z(E) \cong C_2 \times C_2$. Now,

$$Z(\Delta(D:D')) = Z(\mathbb{Q}D(1-\widehat{D'})) = \mathbb{Q}(Z(D)(1-\widehat{D'})$$

and

$$Z(\Delta E:E')) = Z(\mathbb{Q}E(1-\widehat{E'})) = \mathbb{Q}(Z(E))(1-\widehat{E'}).$$

Since $\mathbb{Q}(Z(D)(1-D')) \cong \mathbb{Q}(Z(E)(1-\widehat{E'})$, Lemma (2.4) shows that $Z(D) \cong Z(E)$.

In what follows we shall need information about group algebras of indecomposable groups of order 16 such that, factored by their centers, are isomorphic to $C_2 \times C_2$. There exist two such groups, those of type 16/9 and 16/10 in the table of groups of orders 16 in [8], which we shall denote simply as A and B respectively.

(2.7) Lemma. With the notation above, we have that:

$$QA = 2(Q \oplus Q \oplus Q(i))$$

$$\oplus M_2(Q) \oplus M_2(Q)$$

$$QB = 2(Q \oplus Q \oplus Q(i)) \oplus M_2(Q) \oplus H$$

where H denotes the quaternion algebra over the rational numbers.

Proof. As before:

$$QA \cong Q(\Delta/A') \oplus \Delta(A : A')$$

$$QB \cong Q(B/B') \oplus \Delta(B : B').$$

Since $A/A' \cong B/B' \cong C_2 \times C_4$ it follows easily that

$$Q(A/A') \cong Q(B/B') \cong 2(Q \oplus Q \oplus Q(i)).$$

Set $Z(A) =< e > \times < r >$ where $A' =< e >$ and $o(e) = o(r) = 2$. Then $A/ < r > \cong A/ < er > \cong D_4$. Hence, we have isomorphisms:

$$QA(\frac{1+r}{2}) \overset{\varphi}{\cong} QD_4$$

$$QA(\frac{1+er}{2}) \overset{\psi}{\cong} QD_4.$$

So:

$$QA(\frac{1+r}{2})(\frac{1-e}{2}) \cong QD_4(\frac{1-\varphi(e)}{2}) \cong M_2(Q)$$

$$QA(\frac{1+er}{2})(\frac{1-e}{2}) \cong QD_4(\frac{1-\psi(e)}{2}) \cong M_2(Q).$$

Consequently, QA contains two different simple components which are both isomorphic to $M_2(Q)$ then, QA is as stated.

In a similar way, if we write $Z(B) =< f > \times < s >$ where $B' =< f >$ and $o(f) = o(s) = 2$ we see that $B/ < s > \cong K_8$ and $B/ < fs > \cong D_4$ so the result follows.□

Back to our original question, to fully describe the situation where $p = z$, $n = 1$, we shall discuss separately three cases, according to the rank of D, so set $Z(D) =< e > \times < r > \times < s >$ where $o(e) = 2$, $o(r) = 2^m$, $o(s) = 2^n$.

Case 1. Assume $m = n = 0$.

Since this means that $|D| = 8$ it is clear that $QD \approx QE$ if and only if $D \approx E$.

Case 2. Assume $m \geq 1$, $n = 0$.

The proofs of Lemma (2.4) and Lemma (2.1) show that

$$Z(\Delta(D : D')) \cong Q(C_{2^m}) \cong \oplus_{i=0}^{m} Q(\zeta_i).$$

Now, $\Delta(D : D')$ is a direct sum of simple algebra of dimension 4 over their respective centers. Since $m > 1$ implies that $Q(\zeta_i)$ contains a square root of -1, all the corresponding algebras must be isomorphic to $M_2(Q(\zeta_i))$. For $m = 0$ or $m = 1$ the corresponding algebras can be isomorphic to either $M_2(Q)$ or H. Hence, the isomorphic class of QD will be determined by these two simple algebras.

Now, denote $< r^2 >= H$. Then $\mathbb{Q}D\widehat{H} \cong \mathbb{Q}(D/H)$ and it is easily seen that D/H is indecomposable, of order 16 and that factored by its center is isomorphic to $C_2 \times C_2$. Hence D/H is isomorphic to either A or B and Lemma (2.7) implies that this fact will determine the isomorphism class of $\mathbb{Q}D$.

Case 3. Assume $m \geq 1$, $n \geq 1$.

Once again we can write $\mathbb{Q}D \cong \mathbb{Q}(D/D') \oplus \Delta(D : D')$ and we know that $Z(\Delta(D : D')) \cong \mathbb{Q}(C_{2^m} \times C_{2^n})$ so, it will contain four simple components isomorphic to $\mathbb{Q}$ and others, isomorphic to fields of the form $\mathbb{Q}(\zeta)$. As in the previous case, we need only to determine the simple components whose centers are isomorphic to $\mathbb{Q}$.

Write $H =< r^2 > \times < s^2 >$ and $\overline{D} = D/H$. Then, $\overline{D}$ is a group of order 32 fullfilling the same properties as D. Again, [8] shows that there exists only one such group, that of type 32/18 in their notation, which we shall call G.

Since $\mathbb{Q}D \approx \mathbb{Q}\overline{D} \oplus \Delta(D : H)$, $\mathbb{Q}\overline{D} \cong \mathbb{Q}G$ and

$$Z(\Delta(G : G')) \cong \mathbb{Q}(C_2 \times C_2) \cong \mathbb{Q} \oplus \mathbb{Q} \oplus \mathbb{Q} \oplus \mathbb{Q}$$

we see that the four simple components under consideration, are determined by G. Hence, there is only one possible isomorphism class for $\mathbb{Q}D$.

§3 - Loop Algebras.

We shall now extend the results of the previous section to loop algebras.

From now on L will denote an R.A. loop and $D \subset L$ a group such that $[L : D] = 2$, $D' = \{1, e\} = L'$ and $Z(D) = Z(L)$. Furthermore, we shall assume that it can be written as the group D in Theorem (2.1).

Writing:

$$\mathbf{Q}L = \mathbf{Q}L(\frac{1+e}{2}) \oplus \mathbf{Q}L(\frac{1-e}{2})$$

we obtain, as in §2 that:

$$\mathbf{Q}L \cong \mathbf{Q}(L/L') \oplus \Delta_{\mathbf{Q}}(L : L')$$

where all simple commutative components of $\mathbf{Q}L$ are present in the decomposition of $\mathbf{Q}(L/L')$. Hence

$$Z(\mathbf{Q}L) \cong \mathbf{Q}(L/L') \oplus Z(\Delta_{\mathbf{Q}}(L : L')).$$

(3.1) Lemma. $Z(\Delta_{\mathbf{Q}}(L : L')) = \mathbf{Q}(Z(D))(\frac{1-e}{2})$.

Proof. Since $Z(\mathbf{Q}D) \cong \mathbf{Q}(D/D') \oplus \mathbf{Q}(Z(D))(\frac{1-e}{2})$ it follows readily that $\mathbf{Q}(Z(D))(\frac{1-e}{2}) \subset Z(\Delta_{\mathbf{Q}}(L : L'))$.

Computing dimensions, we obtain:

$$[Z(\mathbf{Q}L) : \mathbf{Q}] = |Z(L)| + \frac{|L| - |Z(L)|}{2} = |Z(D)| + \frac{2|D| - |Z(D)|}{2} = \frac{9}{8}|D|.$$

Hence:

$$[Z(\Delta_{\mathbf{Q}}(L : L')) : \mathbf{Q}] = [Z(\mathbf{Q}L) : \mathbf{Q}] - [\mathbf{Q}(L/L') : \mathbf{Q}] = \frac{1}{8}|D|.$$

Since we also see that $[\mathbf{Q}(Z(D))(\frac{1-e}{2}) : \mathbf{Q}] = \frac{1}{8}D$, equality follows.□

(3.2) Theorem. Let L be an R.A. loop and $D \subset L$ a group, as above. Furthermore, assume that there exists an element $\alpha \in Z(D)$ such that $\alpha^2 = e$. Let M be another loop. Then $\mathbf{Q}L \cong \mathbf{Q}M$ if and only if $L/L' \cong M/M'$ and $Z(\mathbf{Q}L) \cong Z(\mathbf{Q}M)$.

Proof. Assume first that $\mathbf{Q}L \cong \mathbf{Q}M$ since $\mathbf{Q}(L/L')$ and $\mathbf{Q}(M/M')$ are the sums of the corresponding commutative components, it follows that $\mathbf{Q}(L/L') \cong \mathbf{Q}(M/M')$, thus $L/L' \cong M/M'$. Also, since

$$\mathbf{Q}(L/L') \oplus \mathbf{Q}(Z(D))(\frac{1-e}{2}) \cong \mathbf{Q}(M/M') \oplus \mathbf{Q}(Z(E))(\frac{1-e}{2})$$

where $E \subset M$ denotes a convenient subgroup of M, we obtain:

$$\mathbf{Q}(Z(D))(\frac{1-e}{2}) \cong \mathbf{Q}(Z(E))(\frac{1-e}{2})$$

using Theorem (2.6) we obtain $Z(L) \cong Z(M)$.

Conversely, assume that $L/L' \cong M/M'$ and $Z(L) \cong Z(M)$. Then

$$\mathbf{Q}(Z(D))(\frac{1-e}{2}) \cong \mathbf{Q}(Z(L))(\frac{1-e}{2})$$

$$\mathbf{Q}(L/L') \cong \mathbf{Q}(D/D')$$

so

$$Z(\mathbf{Q}L) \cong Z(\mathbf{Q}M).$$

Since $\mathbf{Q}L$ and $\mathbf{Q}M$ are semisimple alternative algebras [5, Corollary 8] we can write:

$$\mathbf{Q}L \cong \mathbf{Q}(L/L') \oplus A_1 \oplus \ldots \oplus A_n$$

$$\mathbf{Q}M \cong \mathbf{Q}(M/M') \oplus B_1 \oplus \ldots \oplus B_m.$$

Comparing the respective centers, we obtain $n = m$ and $Z(A_i) = Z(B_i)$, $1 \leq i \leq n$. Since $\alpha^2 = e$ we see, as in Theorem (2.6) that the algebras A_i, B_i must be split Cayley algebras and these are unique over each center [6, Lemma (3.16)], hence $A_i \cong B_i$, $1 \leq i \leq n$ and thus $\mathbf{Q}L \cong \mathbf{Q}M$.

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