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Fractal Measures of Complex Networks Applied to Texture Analysis

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Abstract. This work proposes a novel technique for the extraction of descriptors of a texture image. The method consists in modeling the image through a complex network, based on the distribution of gray-level intensities. Therefore, we process the adjacency matrix as a geometrical object and compute its fractal dimension. Thus we apply a multiscale transform over the power-law fractal curve, obtaining the descriptors of the image. The method is tested in a classification task with a comparison to other classical texture descriptors.

1. Introduction

We observe that complex network modeling [1] has called a significative attention of computer vision community. Complex networks turn possible the analysis of an object represented in an image as being a system from which we can extract measures capable of characterizing precisely an intrinsic structure in the original image. In this way, this modeling can extract features representative of aspects like spatial and pixel intensity distribution, texture patterns and irregularities, etc. Finally, these features allow an accurate description and discrimination of textures, as demonstrated in works like [2, 3, 4, 5].

An important challenge in complex network modeling is what kind of measure could be extracted from the network. Although works like [4, 5] use simple metrics, it is noticeable that more complicated problems require the employment of more elaborate measures [6]. In this context, we propose the use of fractal dimension as a valuable measure of a complex network. This use is explained by the fact that the fractal dimension estimates the complexity of the network. In the texture modeling, this metric captures the propagation of details under different scales in the network.

The proposed method consists in modeling the texture image through a complex network, representing each pixel intensity value as a vertex and connecting two vertices if there are pixels with the corresponding intensities in an 8-neighborhood. Therefore, we analyze the adjascency metrix as an object of interest in a binary image and estimate the fractal dimension of this object. In the following, we apply a multiscale transform over the dimension obtaining the texture descriptors.

We verified the efficiency of the proposed methodology in the classification of a well-known texture dataset, that is, Brodatz data [7]. The results are compared to other classical texture descriptors [8], to know, Gabor wavelets, Fourier descriptors, Laws and a multifractal approach.

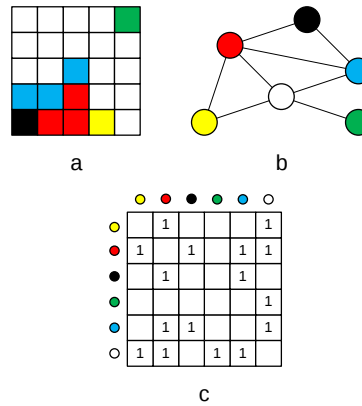


Figure 1. Simple example of color/vertex modeling. (a) 5x5 image; (b) Resultant CAG for 8-neighborhood rule; (c) Adjacency matrix

2. Complex Networks

Given an image I , consisting of a set of pixels $I = \{p_1, p_2, \dots, p_N\}$, before using measures of networks, it is necessary the definition of a way to turn it into a complex network. This requires the definition of the set of vertices V and edges E and their properties.

A simple way to accomplish this transformation is when each node v is a color component of the image in a given color space. An edge is then added between two vertices if the colors represented by them are adjacent in an 8-neighborhood rule in the image (self connections are ignored). Figure 1 exemplifies this transformation. In the Figure 1a, we suppose that each colored square corresponds to a pixel with a specific RGB color. In 1b, we show the corresponding network and in 1c, the adjacency matrix is depicted, where each value 1 corresponds to a connection.

This is not a novel form of modeling. The work [9] shows the first use of this type of transformation calling it as *Color Adjacency Graph* (CAG). Nevertheless, in that work, the authors use this type of modeling in an application to scene objects locating and recognition, while in our work the use is for texture analysis.

In this representation, the number of nodes is equal to the number of colors, thus we need make an image quantization to reduce the number of nodes obtained and reduce the computational cost. Furthermore, as we are dealing with gray-level images, we consider each color as being a gray-level intensity. In this study, we approximate the image gray-levels by using minimum variance quantization. The quantized image has at most 64 levels.

The resulting network with n vertices can be easily represented by an adjacency matrix A of $n \times n$ (Figure 1c):

$$a_{ij} = \begin{cases} 1, & \text{if } \{i, j\} \in E, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where i and j are two vertices.

3. Fractal Descriptors

Fractal descriptors [10, 11, 12] method is a technique to extract features of an image based on the power-law relation of fractal dimension estimation methods.

Generally, the fractal dimension of an object in a digital image is estimated by the following generic expression:

$$D = \lim_{\epsilon \rightarrow 0} \frac{\log(\mathfrak{M})}{\log(\epsilon)}, \quad (2)$$

being $\mathfrak{M}$ a measure depending on the dimension method and ϵ the scale at which this measure is taken. In practice, this limit is estimated by plotting the values of $\mathfrak{M}$ and ϵ in a log-log scale, as described in the following.

The fractal descriptors are provided from the power-law function u :

$$u : \log(\epsilon) \rightarrow \log(\mathfrak{M}). \quad (3)$$

In the proposed work, we extracted the descriptors from the Bouligand-Minkowski dimension method [10]. In this approach, the object of interest in the image is dilated by a variable radius r . For each r , we compute the influence area A , corresponding to the area of the dilated object. Thus the value of $A(r)$ is taken as the measure $\mathfrak{M}$ in the Equation 3 and the scale ϵ is represented by r .

In order to extract the descriptors, the function u can be used directly or submitted to some sort of transform to emphasize some aspect relevant to the specific application.

4. Proposed Method

Here, we employed the fractal descriptors technique over the adjacency matrix of the complex network used to model each texture image.

Initially, we model the original gray-level texture image into a complex network using the approach described in the Section 2.

In the following, we extract fractal descriptors $u(t)$ from the modelled network. Here, we used the function u , followed by a multiscale transform. Essentially, the multiscale transform of a signal $u(t)$ is an operation which maps into a bidimensional function $U(b, a)$, where b is related to t and a corresponds to the scale of analysis. This operation turns possible a local analysis of the fractal power-law, providing a more complete description of the fractal behavior along the scales in the network and, as a consequence, allows a richer modeling of the original texture.

We employed a specific type of multiscale, named time-scale approach. This is based on the wavelets transform. The result of the operation $U(b, a)$ is given through:

$$U(b, a) = \frac{1}{\sqrt{(a)}} \int_{\mathbb{R}} \psi^*\left(\frac{t-b}{a}\right) u(t) dt, \quad (4)$$

where ψ is the wavelet basis function.

Finally, we apply a threshold over the multiscale response, obtaining the final descriptors used in the discrimination of the textures. The threshold point is obtained empirically, from the training data set, as being that which provided the most precise result.

5. Experiments

The proposed technique was tested in the classification of the well-known Brodatz data set [7]. This is composed by 1110 images divided into 111 classes with 10 images in each one. The classification itself was performed by the Linear Discriminant Analysis (LDA) technique and the result was compared to other classical texture descriptors methods [8], that is, Gabor, Fourier, Laws and a multifractal approach.

The classification is carried out in a hold-out scheme, with one half of the data to train and the another one to test. The results are compared in terms of the correctness rate.

6. Results

The Table 1 shows the correctness rates of each technique in a comparison to the proposed method. We see that the novel descriptors have overcome the classical methods in the classification results.

Method	Correctness Rate (%)	Number of descriptors
Gabor	90.0901	20
Fourier	74.5946	100
Laws	87.0270	15
Multifractal	37.4775	101
Proposed method	<u>92.0721</u>	39

Table 1. Correctness rate for Brodatz dataset.

Such good performance was expected, given the fact that we are summing up the richness of two efficient approaches, that is, fractal descriptors and complex networks modelling. The method combines the distribution of pixel intensities along the image with the complexity of the spatial distribution of patterns and irregularities in the texture. These are fundamental features for the description and discrimination of texture images, allowing a precise and robust result for the proposed task.

We also notice that the proposed technique uses a reasonable number of descriptors. Although it provides more values than Gabor method, in this case, the difference is not significant given the size of the dataset. Therefore, the proposed method presents a low computational classification cost and avoids problems like the dimensionality curse or classification bias.

In the Figure 2, we show the respective confusion matrices of each compared method, illustrating the performance per each class in the analyzed data set.

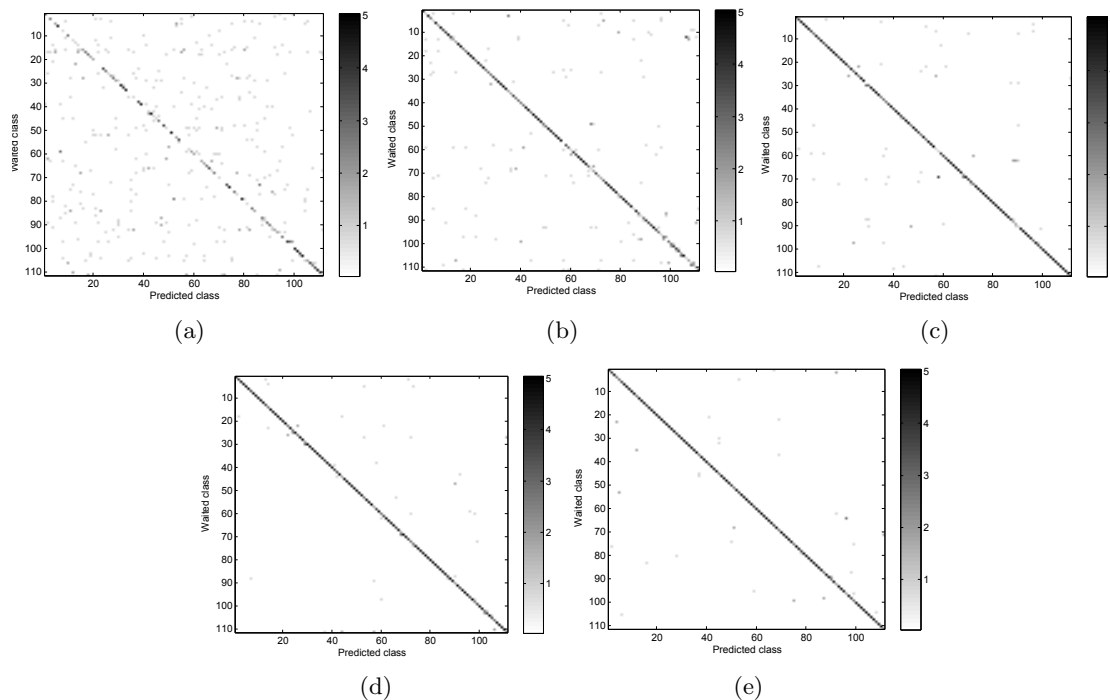


Figure 2. Confusion matrices of the compared methods. a) Multifractal. b) Fourier. c) Laws. d) Gabor. e) Proposed.

The matrices corroborate the efficiency of the proposed descriptors, as this technique showed few gray points outside the diagonal and a more continuous diagonal, showing that the proposed descriptors achieved a great performance in all the tested classes.

7. Conclusions

This work proposed a novel methodology for the extraction of descriptors from a texture gray-scale image. The method is based on the modeling of the image by a complex network. The network is composed in a way that each vertex corresponds to a gray-level intensity and the edges are connected if two pixels with the same intensity are 8-connected. In the following, we obtain the descriptors by applying a method which combines the fractal dimension power-law with a multiscale transform over the adjacency matrix.

The results of a comparison to other classical texture descriptors illustrated the power of the proposed method. The novel technique achieved the best success rate in the classification of a well-known texture data set.

8. Acknowledgements

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