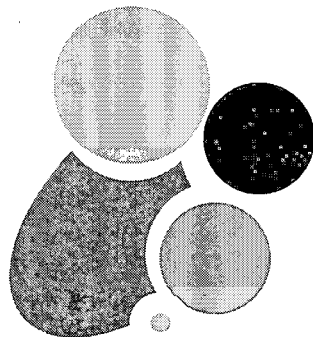


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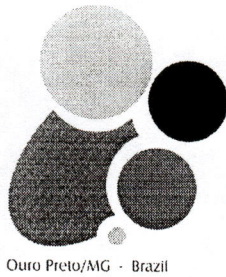


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A NEW CONTACT/ IMPACT IDENTIFICATION ALGORITHM

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Abstract. This work presents a new algorithm to identify the occurrence of impact (or contact) among general structures. Structures are modeled here by finite elements and the resulting model is employed in two-dimensional frame impact problems by the Lagrange Multiplier technique. The proposed algorithm is based on potential theory, largely employed in various fields of physics. Together with the potential theory, integral equations are used, allowing the geometrical evaluation between relative node positions of the projectile structures and domains of the target structures. The present technique is general and can be used to model contact among two or three-dimensional structures and also between structures and rigid obstacles. Numerical results are compared with other references.

Keywords: contact, impact, identification algorithm

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1. INTRODUCTION

The dynamic contact among structures is subject of great interest for various engineering fields, as mechanic, civil and aeronautic. During the last two decades there were a great advance on the numerical modeling of impact, contact or crash problems, mainly using FEM. One of the difficulties that emerge when dealing with impact problem is the friction model to be adopted. Usually, the preferred one is the Coulomb model. The Coulomb model states a relation between normal forces and shear forces at the contact interface. This relation can be linear, non-linear or even discontinuous when the contact situation passes from static to dynamic. The non-linearity associated with more complete models is referred to relative velocity between contact surfaces, i.e., for larger relative velocity smaller relation between normal and shear forces is expected. The contact model used in this paper is based on a geometrical description of Coulomb friction model. Another difficulty that emerges when dealing with numerical modeling of impact is the proper identification, at a specific instant, whether the contact takes place or not. A new algorithm to identify the occurrence of impact (or contact) among general structures is presented. In this work two dimensional straight frame elements have been used in order to demonstrate the technique, but the algorithm is also valid for membrane elements, mixed problems and 3D applications. Rigid bodies are discretized by straight-lines, but any spatial variation could be adopted.

Following a natural presentation order, we start describing the impact identification algorithm in the next section and later the geometrical friction algorithm. At the end of the paper some results are shown in order to validate the proposed technique.

2. PROPOSED IMPACT IDENTIFICATION ALGORITHM

In the specialized literature there are several algorithms that identify the contact by a local procedure, as for example Belytschko & Neal (1991), Wang *et al.* (2001) and Belytschko & Yeh (1993). In this case the contact identification is done for any point of the projectile structure with respect to circular (or other geometry) (2D) or spherical (3D) regions defined in the neighborhood of the target structure nodes, called here influence regions (Zhong & Nilsson, 1996). This kind of prediction can present some problems when the geometry of the studied structures is very complicate. In this situation the referred methodology can present superposition between influence regions dealing to difficulties in the choice of the contact element. The proposed technique does not depend upon local geometry of the colliding bodies, overcoming difficulties of contact identification.

Other sophisticated and important algorithms can be found in literature; for example, references Chaudhary & Bathe (1986), Hughes *et al.* (1976), Bathe & Chaudhary (1985) and Hallquist *et al.* (1985) are related to contact identification and momentum conservation. Recently other important works Wriggers *et al.* (1990) and Mahmoud *et al.* (1998) have been published trying to improve the numerical contact identification and the collision modeling, proving that it is an important subject not totally solved.

The majority of works on collision problems uses FEM as the numerical tool. It is interesting to note that BEM works on the subject use the same contact identification procedures proposed by FEM researchers, Lorenzana & Garrido (1998) and Landenberger & El-Zafrany (1999). This observation is very important because the proposed collision identification procedure is new even for researchers used to potential theory.

The proposed algorithm is based on boundary integral equations written for steady state potential problems, Jaswon (1963) and Symm (1963). The idea is to make an analogy between the geometry of the bodies, involved in the real impact problem, and the boundary of

virtual two-dimensional (or three) bodies with the same geometry. For each virtual body (target) a hypothetical potential integral equation, like the usual ones found in boundary element formulations, is written, having as source points the nodes of the other involved structures (projectiles). If any source point (unit potential source) is inside the virtual domain (representing the target structure) the integral equation must give the number 1 as result, if the source point is outside the virtual domain the result must be zero and if it is on the boundary the result gives a number between 1 and zero depending upon the target structure geometry.

There are three types of virtual domains to be considered, the finite, the infinite and the finite with cavities. The difference between the first two is given by the sense of boundary integration. If the integration is done in clockwise sense the body is infinite otherwise it is finite. If a simple crash between two ordinary bodies is to be considered both virtual domains must be considered finite. But if one desires to study the behavior of a structure confined inside a closed region, as for example, a snooker ball colliding with the table limits, the ball must be considered as a finite virtual region and the table limits as an infinite region, see figure 1a and 1b. Obviously this virtual infinite region is modeling a frame or a cavity inside an infinite medium. The third type is a mixed problem and is easily treated by doing a clockwise integration for internal boundaries and anti-clockwise for external boundaries, see figure 1c.

In figures 1a, 1b, 1c letters η e Γ represent, respectively, normal and tangential directions regarding the virtual boundary. The virtual boundary, straight elements have been adopted (like boundary elements). In this work the virtual boundary elements geometry coincides with the adopted finite elements at the boundary of the structure. In the case of not connected structures additional "virtual boundary elements" should be provided in order to close de virtual domain. It is worth noting that if a structure is only a projectile (users decision) it does not need associated virtual boundaries. For multi-body crash one needs to give information about the structure status. For example when one has three structures named A, B and C he may decide that A is target for B and C, B is target for C (but is projectile for A) and C is only projectile. In this way the nodes of B and C will be virtual sources for A and the nodes of C will be virtual sources for B.

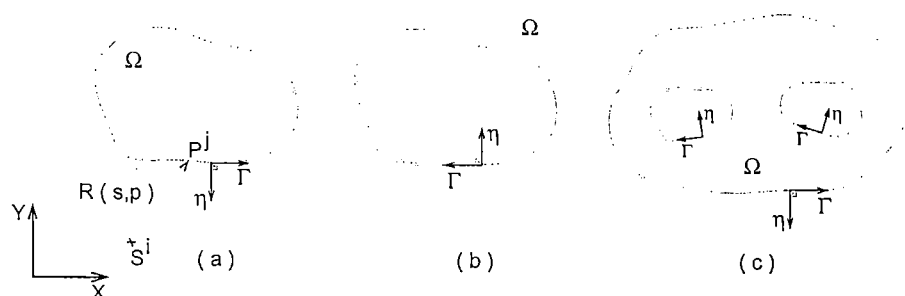


Figure 1 - Kinds of virtual bodies and sense of integration

Self-contact can be handled by assuming dummy virtual domains as exposed in a recent work accepted to be published (Greco, Coda and Venturini, 2003).

As mentioned before it is considered that each node of a projectile structure is a source point (S), as in boundary element procedures, for a target structure. So one can easily write the following potential boundary integral equation (Kellogg, 1929) for each node (S) of the projectile structure and for each boundary (Γ) of the target structure.

$$c(s)p(s) + \int_{\Gamma} q^* p d\Gamma = \int_{\Gamma} p^* \frac{\partial p}{\partial \eta} d\Gamma \quad (1)$$

Where (p) is the potential, (q) is the flux, defined in Eq. (2), and the superscript star stands for fundamental values, given in Eq. (3) and (4).

$$q = \frac{\partial p}{\partial \eta} \quad (2)$$

$$p^* = \frac{-1}{2\pi} \ln R \quad (3)$$

$$q^* = \frac{-1}{2\pi R} \frac{\partial R}{\partial \eta} \quad (4)$$

The constant $(c(s))$ of Eq. (1) depends upon the position of source point (S) , remembering that S is a node of projectile structure the following result is generated.

$$\begin{cases} c(S) = 0 & \text{for } S \text{ external to the target domain} \\ c(S) = \frac{\alpha}{2\pi} & \text{for } S \text{ on the boundary } (\alpha \text{ is the angle at non-smooth surface}) \\ c(S) = \frac{\pi}{2\pi} = 0.5 & \text{on smooth boundary} \\ c(S) = 1 & \text{for } S \text{ internal to the target domain} \end{cases} \quad (5)$$

This is the simple proposed test that should be done in order to identify if any point of the target structure is colliding or not at any instant.

At this point our impact identification algorithm is summarized in calculating $c(s)$ that is easily done, as follows:

One has to substitute the exact boundary by the approximated one, in this case straight lines and choose an approximate pattern for potential and flux, in the present work it is considered constant along the virtual boundary elements. So, Eq. (1) is rewritten as follows:

$$c(s)p(s) + \sum_{j=1}^N \left(\int_{\Gamma} q^* d\Gamma \right) p_j = \sum_{j=1}^N \left(\int_{\Gamma} p^* d\Gamma \right) q_j \quad (6)$$

Where (j) are the virtual boundary elements. The integral inside parenthesis can be done analytically (for this case) and is represented in an algebraic form as:

$$c(s)p(s) + \sum_{j=1}^N \hat{H}_j p_j = \sum_{j=1}^N G_j q_j \quad (7)$$

For a particular problem the term in the right side of Eq. (7) (flux) can be considered zero and all potentials set equal to the unity. From these considerations follows a simple equation that identifies the relative position of a source point (projectile) regarding the virtual body (target), that is:

$$c(s) = - \sum_{j=1}^N \hat{H}_j \quad (8)$$

To discover, for any instant, if the projectile node is inside or outside the target structure one should perform Eq. (8) and see if the result is 1 (inside), 0 (outside) and other number (over the boundary). The last case (over the boundary) is a particular case that does not deserves return action. The case for which the projectile node is inside the target deserves a return action to the nearest target element. This element is indicated by potential theory results, i.e., it is the one that presents the nearest to 1/individual integral value.

The values of $\hat{H}_j$ are obtained analytically due to the simplicity of straight elements and two-dimensional applications, for three-dimensional problems or curved elements numerical evaluation can be adopted, as usually done in Boundary Element Techniques (Banerjee, 1994). The analytical integration process follows the general scheme depicted in figure 2. In this figure global and local co-ordinates reference axis (XY) and ($\bar{X}\bar{Y}$), respectively, are depicted.

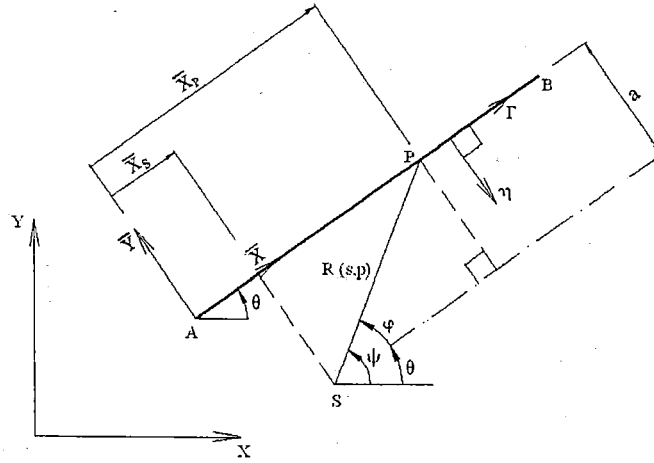


Figure 2 - Element integration scheme

For each boundary element the $\hat{H}_j$ integral is written as:

$$\hat{H}_j = \int_{\Gamma} \frac{-1}{2\pi R} \frac{\partial R}{\partial \eta} d\Gamma = \int_{\Gamma} \frac{-1}{2\pi R} \left(\frac{\partial R}{\partial X} \eta_X + \frac{\partial R}{\partial Y} \eta_Y \right) d\Gamma \quad (9)$$

Looking at figure 3 one writes Eq. (9) in a more convenient form as:

$$\hat{H}_j = \int_{\Gamma} \frac{-1}{2\pi R} \left(\frac{\partial R}{\partial X} \eta_X + \frac{\partial R}{\partial Y} \eta_Y \right) d\Gamma = \int_{\Gamma} \frac{-1}{2\pi R} (\cos \psi \sin \theta - \sin \psi \cos \theta) d\Gamma \quad (10)$$

By the local system of co-ordinates one knows that a is negative, and then:

$$R = \frac{-a}{\sin \varphi} \quad (11)$$

The local co-ordinate of a field point (P) is written as a function of φ as follows:

$$\bar{X}_P = \bar{X}_S - a \frac{\cos \varphi}{\sin \varphi} \quad (12)$$

The differential relation between the infinitesimal integrands can be obtained by simple differentiation:

$$d\bar{X}_P = d\Gamma \quad (13)$$

And:

$$\frac{d\bar{X}_P}{d\varphi} = \frac{a}{\sin^2 \varphi} \quad (14)$$

Putting together (13) and (14), results:

$$d\Gamma = \frac{a}{\sin^2 \varphi} d\varphi \quad (15)$$

Replacing Eq. (11) and (15) into (10) the integral referred to the angle φ becomes:

$$\hat{H}_j = \int_{\Gamma} \frac{l}{2\pi \sin \varphi} (-\sin(\psi - \theta)) d\varphi = \int_{\Gamma} \frac{l}{2\pi \sin \varphi} (-\sin \varphi) d\varphi = \int_{\varphi_A}^{\varphi_B} \frac{-l}{2\pi} d\varphi = \frac{\varphi_A - \varphi_B}{2\pi} \quad (16)$$

In Eq. (16), φ_A is the angle φ measured regarding the initial node of the boundary element (node A) and φ_B is the angle measured at the final node (node B). Equation (16) is very simple, but an extra care should be taken when the interval $-\pi \leq \varphi_A - \varphi_B \leq \pi$ is not respected. In this case the following rule should be imposed.

- If $\Delta\varphi = \varphi_A - \varphi_B \geq \pi$, Then: $\Delta\varphi = \Delta\varphi - 2\pi$.
- If $\Delta\varphi \leq -\pi$, Then: $\Delta\varphi = \Delta\varphi + 2\pi$.

Therefore, Eq. (8) can be simplified to:

$$c(s) = -\sum_{j=1}^N \hat{H}_j = \frac{-l}{2\pi} \sum_{j=1}^N \Delta\varphi_j \quad (17)$$

Equation (17) finishes the identification procedure together with algorithm (5).

3. GEOMETRICAL FRICTION MODEL AND RETURN ALGORITHM

Before describing the proposed friction algorithm it is worth describing the dynamic integration procedure used to achieve stable solutions for both proposed and compared formulations.

In figure 3, a collision between two bodies is depicted. Γ_1 is the surface of prescribed forces, Γ_2 is the surface of prescribed displacement and Γ_C is the contact region.

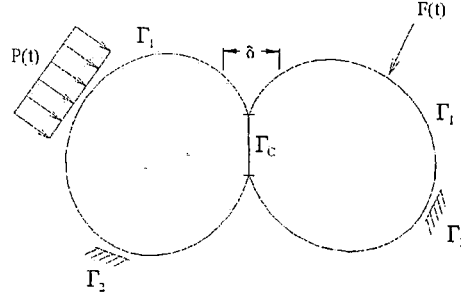


Figure 3 - Impact between two elastic bodies

The nil penetration condition is a necessary constraint for the collision analysis, when penetration occurs the following equation holds:

$$R_C X_{S+l} = X_T \quad \delta = 0 \quad (18)$$

Where δ is the initial distance between two considered contact points, X_T is the target position, X_{S+l} is the position vector and R_C is the constraint direction of the contact. The penetration condition is determined as described before.

The dynamic equilibrium equation can be written as:

$$\left. \frac{\partial U_T}{\partial X} \right|_{S+l} + M\ddot{X}_{S+l} + C\dot{X}_{S+l} = F_{S+l}^{EXT} + R_C \lambda_{S+l} \quad X_{S+l} \in \Gamma_C \quad (19)$$

Where C is the damping matrix (Rayleigh) as given by Clough & Penzien (1993) and λ_{S+l} is the Lagrange multiplier. The Lagrange multiplier can be interpreted as an external force that guaranties the nil penetration condition.

More details about Eq. (19) can be found in 'An easy large deflection formulation for frames by the Finite Element Method' and 'A novel dynamic fem formulation for 2D frame analysis with large deflections and rotations' (Greco & Coda, 2003).

A classical Lagrange multiplier formulation Chaudhary & Bathe (1986) makes use of FEM together with the Newmark time integration algorithm with constants $\gamma=0.5$ and $\beta=0.5$. The positions, velocities and accelerations are related each other by the classical Newmark equations.

$$X_{S+l} = X_S + \Delta t \dot{X}_S + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \ddot{X}_S + \beta \ddot{X}_{S+l} \right] \quad (20)$$

$$\dot{X}_{S+l} = \dot{X}_S + \Delta t (1 - \gamma) \ddot{X}_S + \gamma \Delta t \ddot{X}_{S+l} \quad (21)$$

For usual structures these coefficients are efficient, but in the studies of Carpenter *et al.* (1991) and Taylor & Papadopoulos (1993), these adopted parameters are criticised. These works present some alternatives in order to improve the time integration in collision analysis.

The time scheme integration used in this paper is based on Hu (1997); modified parameters for the Newmark time integration is presented, including damping and plasticity, not previously considered. The proposed modification is based on the observation that the return to nil penetration, from a penetration numerical trial position, should not be applied instantaneously. If it is done instantaneously the deformed shape concentrates in the first element (in the contact) and is not appropriately spread into the body, dealing to numerical oscillations. Following this reasoning it is desirable to spread the influence of the Lagrange multiplier (λ_{s+1}) to through the next time step. The easier way to spread this influence is to use the Newmark coefficients as $\gamma=1.5$ and $\beta=1.0$.

Replacing Eq. (20) and (21) into Eq. (18) and (19) is possible to obtain expressions written in function of acceleration, Eq. (22) and (23).

$$\beta\Delta t^2 R_C \ddot{X}_{s+l} = X_T - R_C b_0 \quad (22)$$

$$(M + \gamma\Delta t C) \ddot{X}_{s+l} = F_{s+l}^{EXT} + R_C \lambda_{s+l} - \left. \frac{\partial U_T}{\partial X} \right|_{s+l} - C b_l \quad (23)$$

$$b_0 = X_s + \Delta t \dot{X}_s + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{X}_s \quad (24)$$

$$b_l = \dot{X}_s - \Delta t(1 - \gamma) \ddot{X}_s \quad (25)$$

In classical formulations the use of small time steps induces oscillations in the numerical response and large time steps result in non-accurate values. In the proposed formulation it is desirable to assume small time steps (Δt) in order to reduce the undesirable numerical damping present when these Newmark constants are adopted, Hu (1997).

Equations (22) and (23) can be rewritten in a matrix form as:

$$\begin{bmatrix} (M + \gamma\Delta t C) & -R_C \\ \beta\Delta t^2 R_C & 0 \end{bmatrix} \begin{Bmatrix} \ddot{X}_{s+l} \\ \lambda_{s+l} \end{Bmatrix} - \begin{Bmatrix} F_{s+l}^{EXT} - \left. \frac{\partial U_T}{\partial X} \right|_{s+l} - C b_l \\ X_T - R_C b_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \psi \quad (26)$$

Considering the non-linear characteristic of Eq. (26), one may use Taylor expansion of first order achieving a trial and corrector algorithm, as follows:

$$\psi^{i+l} = \psi^i + \frac{\partial \psi^i}{\partial X_{s+l}} \Delta X = 0 \Rightarrow \frac{\partial \psi^i}{\partial X_{s+l}} \Delta X = -\psi^i \quad (27)$$

$$\frac{\partial \psi^i}{\partial X_{s+l}} = \frac{\partial \psi^i}{\partial \ddot{X}_{s+l}} \frac{\partial \ddot{X}_{s+l}}{\partial X_{s+l}} = \begin{bmatrix} (M + \gamma\Delta t C) & 0 \\ \beta\Delta t^2 R_C & 0 \end{bmatrix} \frac{1}{\beta\Delta t^2} + \begin{Bmatrix} \left. \frac{\partial^2 U_T}{\partial X^2} \right|_{s+l} \\ 0 \end{Bmatrix} \quad (28)$$

One finds incremental equation for displacements considering Eq. (27) and (28) together:

$$\left[\begin{pmatrix} \frac{M}{\beta \Delta t^2} + \frac{\gamma C}{\beta \Delta t} + \frac{\partial^2 U_T}{\partial X^2} \Big|_{s+l} \\ R_C \end{pmatrix} - R_C \right] \begin{Bmatrix} \Delta X \\ \lambda_{s+l} \end{Bmatrix} = \begin{Bmatrix} F_{s+l}^{EXT} - \left(\frac{\partial U_T}{\partial X} \Big|_{s+l} + M\ddot{X}_{s+l} + C\dot{X}_{s+l} \right) \\ X_T - R_C X_{s+l} \end{Bmatrix} \quad (29)$$

During the iterations:

$$X_{s+l} = X_{s+l} + \Delta X \quad (30)$$

$$\dot{X}_{s+l} = \dot{X}_{s+l} + \Delta \dot{X} = \dot{X}_{s+l} + \frac{\partial \dot{X}_{s+l}}{\partial X_{s+l}} \Delta X = \dot{X}_{s+l} + \frac{\gamma}{\beta \Delta t} \Delta X \quad (31)$$

$$\ddot{X}_{s+l} = \ddot{X}_{s+l} + \Delta \ddot{X} = \ddot{X}_{s+l} + \frac{\partial \ddot{X}_{s+l}}{\partial X_{s+l}} \Delta X = \ddot{X}_{s+l} + \frac{I}{\beta \Delta t^2} \Delta X \quad (32)$$

The contact force modulus, in the exact representation, is the Lagrange multiplier but in the approximation level, mainly in the linear representation, it loses this precision, Chaudhary & Bathe (1986). Following this reasoning the Lagrange multiplier is used to constrain the movement while the contact force is calculated directly from the equilibrium equation for the impact time step ($s+l$), Eq. (19).

$$F_{s+l}^{CON} = F_{s+l}^{EXT} - \left(\frac{\partial U_T}{\partial X} \Big|_{s+l} + M\ddot{X}_{s+l} + C\dot{X}_{s+l} \right) \quad X_{s+l} \in \Gamma_C \quad (33)$$

It is usual in friction contact procedures to describe friction from forces definition, see for example Heege *et al.* (1995) and Curnie (1994). In these approaches the tangential forces are written as function of the normal forces that are developed during the contact evolution, Simo & Laursen (1992) and Curnier & Alart (1988). The dependence can be more or less complicated, and the simplest one is the Coulomb model, where this dependence is linear. During a dynamic analysis, this approach, despite not microscopic, needs information about the behavior of friction forces for several different situations that can be very complicated, for example, with temperature changes, local yielding etc. Another difficulty that arises when dealing with friction impact is the worse behavior of the existent technique regarding stability when compared with the frictionless counterpart. The stability of the referred techniques, as well as the one proposed here, is much improved following the time integration proposed in Hu (1997) as described before.

Following the above comments, this work presents a macroscopic approach for friction, i.e., a methodology that considers the global behavior of experimental results in order to model similar situations, together with Lagrange multiplier and Hu's time integrator. For this procedure only one friction parameter should be considered to model impact among bodies. The previous features associated with the proposed impact prevision algorithm results a stable and reliable friction algorithm for general contact impact problems.

In figure 4 are presented the main angles used in the friction model. Anti-clockwise sense is used to measure the angles.

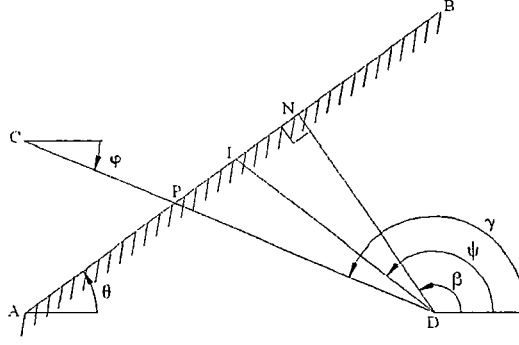


Figure 4 - Return angles used in bi-dimensional contact

The angle φ is a structural node path CD ; β indicates the normal direction between target surface AB and point D .

$$\beta = 90^\circ + \theta \quad (34)$$

The angle γ indicates the direction DC .

$$\gamma = 180^\circ + \varphi \quad (35)$$

The proposed friction model is based on a return coefficient (R) that varies between 0 and 1, positioning the return point I into the segment PN . Therefore, the return angle ψ depends on the return coefficient.

$$\psi = 90^\circ + \theta + R(90^\circ - \theta + \varphi) \quad (36)$$

Angle ψ can vary from the normal to the target surface return direction representing the frictionless situation ($R=0$) until the line CD direction, representing the adherence condition ($R=1$). The maximum adherence condition may be violated by Coulomb model if not foreseen.

4. NUMERICAL EXAMPLES

Two applications involving impact between structures and rigid obstacles are presented in this work. Structures are represented by frame elements. The rigid obstacles are discretized in the 'virtual space' resulting into virtual rigid bodies where structures cannot enter.

4.1 Impact between three rings

This example is used to demonstrate that the proposed technique can also be applied to solve multi body collision problems. It shows the collision among three bodies (rings) modeled by frame elements. In order to simplify the description, the left structure is called structure A , the middle one is called B and the right one C . Structure A is initially moving with a constant velocity from left to right, structures B and C are initially at rest. Each structure is

discretized by 32 straight frame finite elements. The initial distance between structure *A* and *B* is $\delta=0.01m$. Structures *B* and *C* are initially in contact. Physical, geometric and cinematic properties of the analysis are given in figure 5. The adopted time step is $\Delta t=0.001s$. No friction is considered. To implement the collision detection formulation the following information is given. Structure *B* is target of *A* but projectile for *C*. Structure *C* is target of *A* and *B*. The co-ordinate *X* is defined to state the initial point of impact.

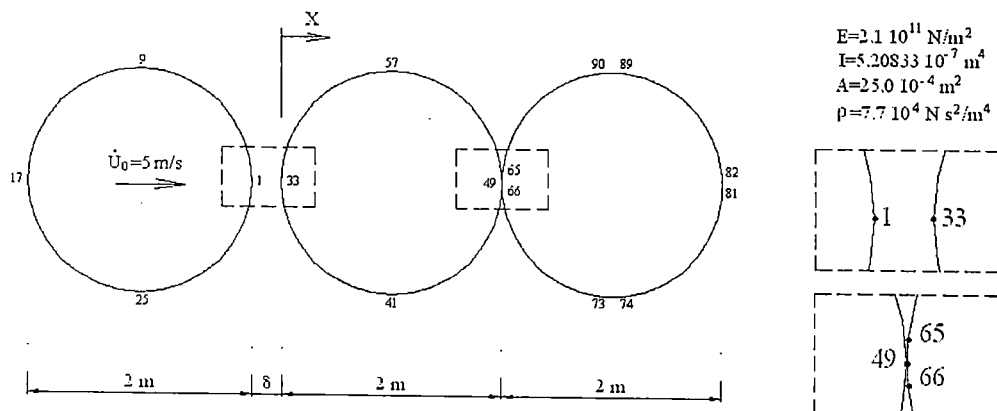


Figure 5 - Impact between three rings

In figure 6(a) there are presented horizontal displacements of nodes 1, 33, 49 and 65. In figure 6(b) the contact forces for nodes 1 and 49 are also shown. The impact between structures *A* and *B* finish at time $t \approx 0.15s$. The impact between structures *B* and *C* has a shorter duration and stops at $t \approx 0.14s$. It presents a temporary separation duration extended from $t \approx 0.017s$ to $t \approx 0.042s$.

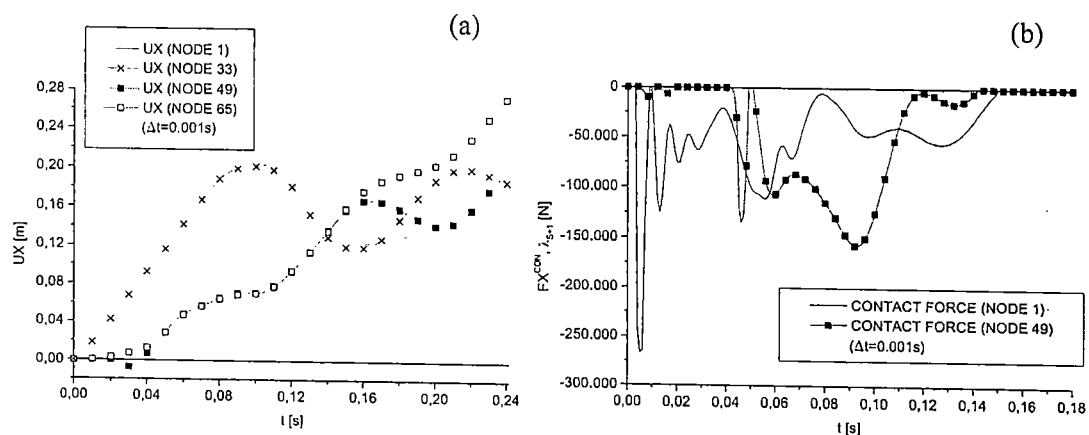


Figure 6 - Displacements (a) and contact forces (b) in direction *X* for the impacting nodes

4.2 Impact of an elastic ring on a rigid obstacle

The example shows the impact between a ring structure and a rigid ground (modeled by a virtual body), and can be found in Wriggers *et al.* (1990). Initially, the structure moves in an inclined path with constant velocity. It has been used 16 straight finite elements to model the ring. The initial position and physical, geometric and cinematic properties of the analysis are depicted in figure 7. The adopted time step is $\Delta t = 0.05$. The problem is analyzed considering two situations: frictionless and frictional case. Only four virtual boundary elements are used to model the obstacle.

In figure 8 are presented the reference responses for the frictionless case (on the left) and for the same problem with friction (on the right). In figure 9 are presented the responses obtained from the proposed formulation for the same two cases. In both cases, results are in good agreement. For frictionless case the achieved reflection angle is 49.83° and for frictional case the reflection angle is 34.86° . The reference presents reflection angles equal 50° and 35° , as presented in figure 8. In the proposed return algorithm, for the frictional case, an equivalent return coefficient ($R = 0.10$) is used to simulate the friction conditions presented in the reference.

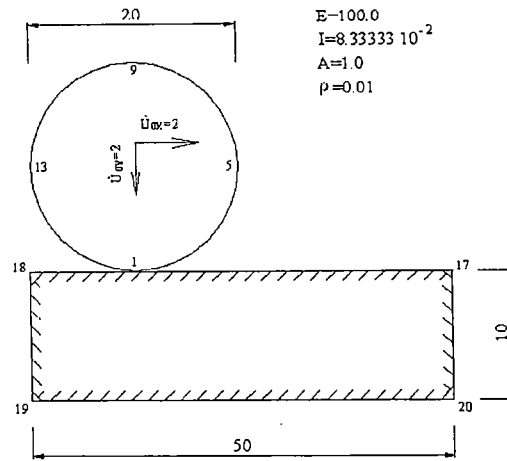


Figure 7 - Impact of a ring on a rigid obstacle modeled by a closed domain

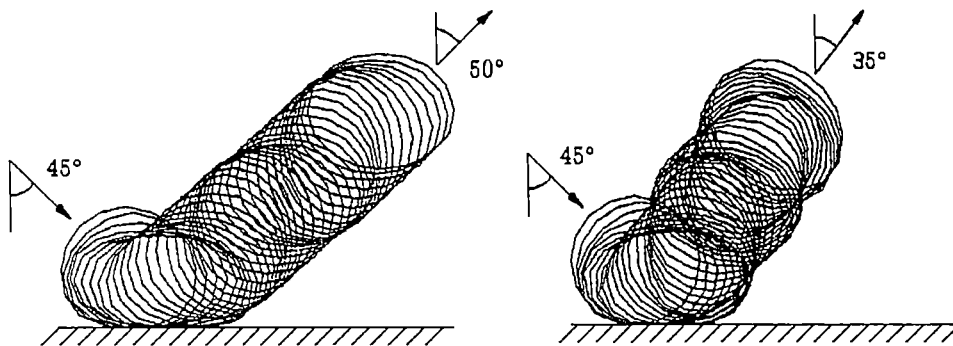


Figure 8 - Reference responses for frictionless and frictional cases

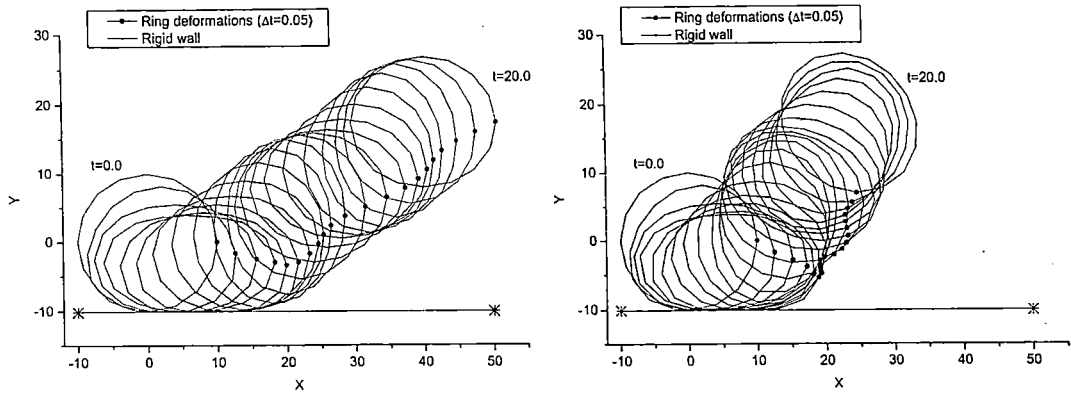


Figure 9 - Obtained responses for frictionless and frictional cases

In figure 10 is shown the ring structure impact in a bitten rigid wall (frictional case with $R=0.10$). It is interesting to note the complexity of the ring movement for the bitten wall case.

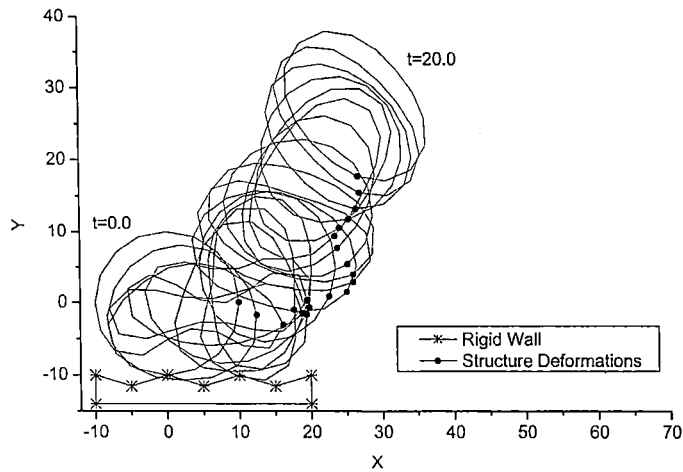


Figure 10 - Bitten wall frictional case.

5. CONCLUSIONS

It has been proposed a simple and original penetration prevision algorithm for general impact problems. Examples demonstrate the applicability of the technique. The algorithm is precise, general and is of straightforward implementation. The necessary integrals can be done analytically for the two-dimensional case with straight elements. For three-dimensional problems or curved elements standard numerical integration procedures can be adopted.

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