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Exact transformation of domain integrals into boundary integrals in anisotropic plate bending problems

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Abstract. This paper presents a boundary element formulation without any domain integral for general anisotropic plate bending with loads applied transversely in the plate surface. The domain integral, which comes from loads transversely applied in an area of plate surface, are transformed into boundary integral by exact transformation. Uniformly and linearly distributed loads are considered. A numerical example concerning orthotropic plate bending problems is presented. Results show good agreement when compared with analytical solutions available in literature.

Introduction

In general plate bending boundary element method, domain integrals arise in the formulation due to the distributed load in the domain (see for example Aliabadi [1]). In order to evaluate these integrals, cell integration scheme can give accurate results, as carried out by Shi and Bezine [2] for anisotropic plate bending problems. However, the discretization of the domain into cells reduces the advantage of boundary element method in that only the boundary of the problem needs to be discretized into elements.

In this work, domain integrals which come from distributed loads are transformed into boundary integrals by exact transformation using the radial integration method. This method was initially presented by Venturini [3] in 1988 for isotropic plate bending problems. The most attractive feature of the method is its simplicity since only the radial variable is integrated.

For domain integrals which include unknown variables, the proposed procedure can be performed using radial basis function as in the dual reciprocity method (Gao [4]).

Boundary integral equation for bending problems of anisotropic plates

Using Rayleigh-Green identity, an integral equation for an anisotropic thin plate under transversal load $g(p)$ can be written as (see Shi and Bezine [2]):

$$Kw(Q) + \int_{\Gamma} \left[V_n^* w - m_n^* \frac{\partial w}{\partial n} \right] d\Gamma + \sum_{i=1}^{N_c} R_{c_i}^* w_{c_i} =$$

$$\int_{\Gamma} \left[V_n w^* - m_n \frac{\partial w^*}{\partial n} \right] d\Gamma + \sum_{i=1}^{N_c} R_{c_i} w_{c_i}^* + \int_{\Omega} g w^* d\Omega, \qquad (1)$$

where n is the outward unit normal vector to the boundary Γ, m_n, V_n are respectively the normal bending moment and the Kirchhoff's equivalent shear force on the boundary Γ, R_c is thin plate reactions of corners, w_c is the deflexion of corners, P is the source point, Q is the field point, and the symbol "*" stands for the fundamental solutions.

The constant K is introduced in order to consider that the point Q can be in the domain, on the boundary, or outside the domain. If the point Q is on a smooth boundary, than $K = 1/2$.

Transformation of domain integrals into boundary integrals in anisotropic plate bending problem

Consider a plate under loading g, applied in a Ω_g area. Assuming that loading g has a linear distribution $(Ax + By + C)$ in the area Ω_g, the domain integral can be written as:

$$\int_{\Omega_g} g w^* d\Omega = \int_{\Omega_g} (Ax + By + C) w^* \rho d\rho d\theta \qquad (2)$$

or

$$\int_{\Omega_g} g w^* d\Omega = \int_{\theta} \int_0^r (Ax + By + C) w^* \rho d\rho d\theta, \qquad (3)$$

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where r is the value of ρ in a point of boundary Γ_g.

Defining F^* as the following integral:

$$F^* = \int_0^r (Ax + By + C)w^* \rho d\rho, \tag{4}$$

we can write:

$$\int_{\Omega_g} gw^* d\Omega = \int_\theta F^* d\theta. \tag{5}$$

Considering an infinitesimal angle $d\theta$ (Figure 1), the relation between the arch length $rd\theta$ and the infinitesimal boundary length $d\Gamma$, can be written as:

$$\cos\alpha = \frac{r\frac{d\theta}{2}}{\frac{d\Gamma}{2}}, \tag{6}$$

or

$$d\theta = \frac{\cos\alpha}{r} d\Gamma. \tag{7}$$

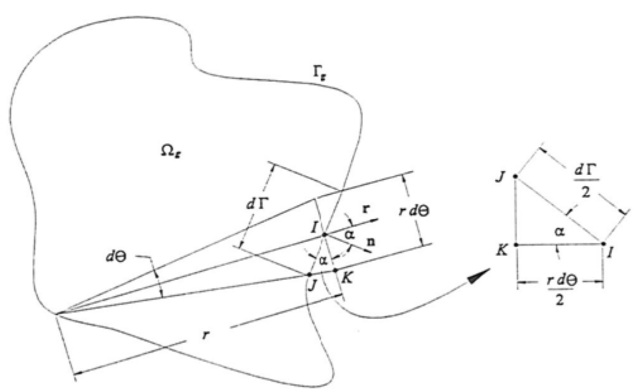

Figure 1: Transformation of domain integral into boundary integral.

Using the properties of internal product of unity vectors $\mathbf{n}$ and $\mathbf{r}$, indicated in Figure 1, we can write:

$$d\theta = \frac{\mathbf{nr}}{r} d\Gamma. \tag{8}$$

Finally, substituting equation (8) into equation (5), the domain integral of equation (1) can be written as boundary integral given by:

$$\int_{\Omega_g} g w^* d\Omega = \int_{\Gamma_g} \frac{F^*}{r} \mathbf{nr} d\Gamma. \tag{9}$$

Provided that

$$x = \rho \cos\theta \tag{10}$$

and

$$y = \rho \sin\theta, \tag{11}$$

the integral F^* can be written as

$$F^* = \int_0^r \frac{1}{8\pi} (A\rho\cos\theta + B\rho\sin\theta + C) \left[C_1 R_1 + C_2 R_2 + C_3 (S_1 - S_2)\right] \rho d\rho, \tag{12}$$

where C_1, C_2, and C_3 are the same functions of equation (12) Equation (12) can be rewritten as:

$$\begin{aligned} F^* &= \frac{1}{8\pi} \left\{ (A\cos\theta + B\sin\theta) \int_0^r \rho^2 \left[C_1 R_1 + C_2 R_2 + C_3 (S_1 - S_2)\right] d\rho + \right. \\ & \left. C \int_0^r \rho \left[C_1 R_1 + C_2 R_2 + C_3 (S_1 - S_2)\right] d\rho \right\}. \end{aligned} \tag{13}$$

Numerical results

Consider a square plate of side length $a = 1$ m and thickness $h = 0.01$ m. The material is orthotropic and its material properties are: $E_x = 2.068\ 10^{11}$ Pa, $E_y = E_x/15$, $\nu_x = 0.3$, $G_{xy} = 6.055\ 10^8$ Pa. The square plate is considered simply supported on its four edges under uniformly distributed load $q = 1$ Pa applied along its domain (Figure 2). For this case the results obtained by BEM will be compared with the solution obtained by

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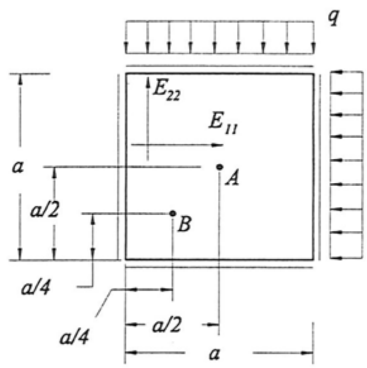

Figure 2: Square plate with simply-supported edges under uniformly distributed load.

Timoshenko and Woinowski-Krieger [5] which solve this problem using a series solution.

In order to assess convergency, the problem is solved using different meshes and the results for deflexions at point A and at point B are compared with series solutions using $N = 19$ and $M = 19$. This series solution for point A is $w_{se.} = 8.1258\ 10^{-7}$ m and for point B is $w_{se.} = 4.5211\ 10^{-7}$ m. Table shows deflexions computed by the present BEM technique using different meshes and their respective errors compared to Timoshenko and Woinowski-Krieger [5] series solutions.

Table 1: Accuracy of deflexions obtained by BEM for different number of elements (NE) of the orthotropic square plate with simply supported edges under uniformly distributed loads.

NE	Deflexions and errors			
	w at A [m]	Er. at A [%]	w at B [m]	Er. at B [%]
8	$9.22\ 10^{-7}$	13.45	$5.40\ 10^{-7}$	19.38
16	$8.04\ 10^{-7}$	1.03	$4.58\ 10^{-7}$	1.35
24	$8.04\ 10^{-7}$	1.01	$4.46\ 10^{-7}$	1.25
32	$8.06\ 10^{-7}$	0.77	$4.47\ 10^{-7}$	1.09
40	$8.08\ 10^{-7}$	0.59	$4.52\ 10^{-7}$	0.88

As it can be seen in Table , results are very poor when 8 elements (2 elements per side) are used. However, they converge fastly to the series solutions if the number of the element is increased. When 40 boundary elements are used, deflexions in both points present errors below 1 % if compared with series solutions.

Conclusions

In this work, the use of radial transformation in boundary element formulation for the analysis of anisotropic plate bending problems was presented. Domain integrals which come from linearly and uniformly distributed loads were transformed into boundary integrals by exact radial transformation. A numerical example was shown for orthotropic materials. The numerical results obtained with the present BEM technique were compared with results obtained analytically and show good agreement.

Aknowlegment

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