Modeling and Analysis of Inductive "Kickback" in Low Voltage Circuits

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Abstract*—*Contribution: **A didactic methodology, based on analytical expressions and experimental validation, to describe the process of abrupt current interruption in a series *RL* circuit, that considers real passive components and uses a toggle switch as disconnecting device.**

Background: **In undergraduate courses, circuits adopted in transient analysis usually assume electrical switches in their ideal form, disregarding electrical impedance and its variation over time. Texts and instructors rarely point out these simplifications, so students can become confused. There is thus a need for a suitable switch model that yields a behavior consistent with real devices, while permitting a mathematical formulation of the inductor current.**

Intended Outcomes: **That students should understand electrical component modeling, and how models can represent the physical phenomena that occur in electrical circuits.**

Application Design: **An experiment, designed to evaluate students' perceptions of switch modeling, modeled a switch by using three different equivalent circuits of increasing complexity, to allow a more realistic simulation. Electrical switch modeling is described, with the models being based on measurements performed on a real circuit.**

Findings: **Student performance was evaluated qualitatively through test scores, and quantitatively through a post-class questionnaire. The results indicate that 96.15% of the students were able to propose an accurate model of the electrical switch.**

***Index Terms*—Active learning, electrical engineering, modeling, power system transients, simulation, SPICE, switches, student experiments.**

I. INTRODUCTION

ELECTRICAL circuit theory is a fundamental course in electrical engineering undergraduate courses [1]–[6]. Many students consider transient response analysis more difficult than the steady-state analysis and the solutions of differential equations yield complex expressions that do not always match the waveforms measured in real circuits when the models are chosen based on ideal components.

Despite educators' best efforts [2], [5], misconceptions can be created when rudimentary models are employed to avoid detailed characterizations and simplify analytical expressions. Because texts and instructors often do not point out these simplifications, students can become confused. This problem is particularly evident in the case of a current interruption in an inductive circuit, when energy is released over a short time creating a large voltage known as "inductive kickback".

The interruption means the conducting circuit is forced to a steady-state condition of zero current. This sounds simple, but often the actual details of the physics of the process are obscured by the seeming triviality of the switching action. Circuits used in transient analysis usually assume electrical switches in their ideal form, disregarding electrical impedance and its variation over time. With that approach, the inductor current would change instantaneously, and its rate of change and the induced voltage would be infinite—which is not possible because the inductor current cannot change as a step function, but must vary continuously.

The challenge is to find a suitable switch model that yields a behavior consistent with real devices, while permitting a mathematical formulation of the inductor current [7].

A didactic methodology, based on analytical expressions and experimental validation, is proposed to describe the process of abrupt current interruption in a low voltage-fed series *RL* circuit with real passive components and a toggle switch as the disconnecting device. Three different switch models of increasing complexity are proposed to match computer simulations with experimental measurements. The network is modeled using linear elements.

This paper is organized as follows: Section II presents the various approaches to modeling the toggle switch; Section III gives the equations of a series *RL* circuit with the electrical switch modeled as a capacitor; Section IV briefly describes the instrumentation required to characterize the switch, and the models used in the simulations; Section V presents the results of the characterization, simulation and test of the *RL* circuit triggered by the modeled switch; Section VI discusses application of this work in the classroom and the questionnaire answered by the 26 student participants in the practical experiment; finally, Section VII draws conclusions.

II. SWITCH MODELING

Any inductive circuit with current flowing through it stores energy that must be discharged when the current is removed. It is the release of this stored energy that can damage the element switching the current on and off.

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Physical phenomena occurring upon current interruption are very complex. The current is not always interrupted when the switch contacts start moving away from each other, but continues to flow through an electric arc that ignites in the gap between the opening contacts. In a circuit breaker, the electric arc is forced to leave the breaker contacts as soon as possible to minimize damage. To complete the circuit break, the arc will be rapidly relocated to an extinguishing chamber, to eventually be extinguished. Arc models for circuit breaker design and insulation that take this complexity into account, such as the Mayr/Cassie mathematical models developed in the middle of the last century [8]–[11], are thus challenging to use. In the *RL* circuit analyzed here, the current to be interrupted is too low to form an arc [12]–[14], so switch models based on a single circuit element can be used to obtain a consistent analytical formulation for turning off the switch; three approaches to such a model are introduced here.

The first approach, the *ideal switch model*, found in most introductory circuit courses, has the features: *i*) the switching time of state transitions between "ON" and "OFF" should be zero, *ii*) in the "ON" state, the voltage drop across the device should be zero, and the current handling capability should be infinite, and *iii*) in the "OFF" state the current through the device should be zero, and the voltage withstand capability should be infinite. This model seems practical but is unreal. It has the advantage of reducing modeling complexity but introduces discontinuities in the network, and the resulting response may contain impulses.

The second approach, the *fixed capacitance switch model*, considers the electrical switch as a capacitor with constant capacitance value. This condition takes into account the non-ideal electrical switch features, such as the value of the capacitance between the plates of the switch. The insertion of the capacitor transforms the *RL* circuit into a series *RLC* circuit, described by a second-order differential equation, thus named a second-order circuit. The advantage of this model is that it can represent the oscillatory behavior that occurs during the switching. A drawback is that it does not include the switch opening and closing times.

The third and more sophisticated approach to electrical switch modeling, the *time-varying capacitance model*, includes both impedance values and switching time. In this condition, the electrical switch is replaced by a time-varying capacitance. This model allows a proper representation of the circuit's dynamic behavior during the switching action.

III. Theoretical Analysis

A. The Ideal Switch and "Inductive Kickback"

For a series *RL* circuit connected to a DC voltage source E by an ideal switch, Fig. 1, in steady state condition, when the switch is in the closed position, the inductor current i is constant and, according to Faraday's law (1), the voltage v_L across the inductor is zero.

$$v_L = \frac{d\psi}{dt} = L\frac{di}{dt}, \tag{1}$$

where ψ is the inductor's linked magnetic flux.

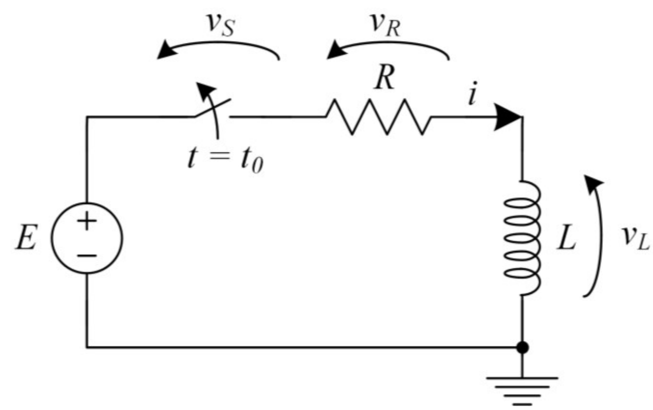

Fig. 1. Series *RL* circuit connected with a DC voltage source by an ideal switch.

When opening the switch, the inductor current will be abruptly cut off, and the inductor voltage v_L will rise steeply, trying to maintain the initial current. This very rapid change in voltage across the inductor when current flow is interrupted is called the "inductive kickback".

The current discontinuity can be represented as a step with amplitude equal to $-E/R$. Therefore,

$$v_L(t) = L\frac{d\left(-\frac{E}{R}H(t)\right)}{dt}, \tag{2}$$

$$v_L(t) = -\frac{L}{R}E\delta(t), \tag{3}$$

where, $H(t)$ and $\delta(t)$ represent the Heaviside step function and the Dirac impulse, respectively.

The ideal switch implies an instantaneous current transition and the voltage amplitude at instant $t = 0$ becomes ideally infinite. In practice, with the switch in open state the current would try to flow through an electric arc eventually established between the switch contacts. However, real devices such as mechanical switches need a toggling time from one state to another, and even semiconductor switches do not act instantaneously. In the absence of an electric arc, a more realistic approach is to model the switch during opening action by a capacitance between contacts.

B. Fixed Capacitance Switch Model and Circuit Analysis

In DC-operated circuits the impedance of a mechanical switch in the ON and OFF states can be represented by a small (almost zero) resistance when switch contacts are touching, and by a very high (almost infinite) resistance when switch contacts are separate. When the two air-separated contacts are at different electric potentials in the OFF state, there is an associated electrostatic field. In this condition, the contacts form a capacitor, whose capacitance C is a function of the contact area and contact separation distance. Fig. 2 shows the equivalent circuit with the *fixed capacitance switch model*.

The coil inductor to be used in the experimental setup is formed by multiple layers of insulated copper wires wound close together. The voltage differences between turns and between winging layers create distributed parasitic capacitances that can be modeled by a lumped capacitance connected

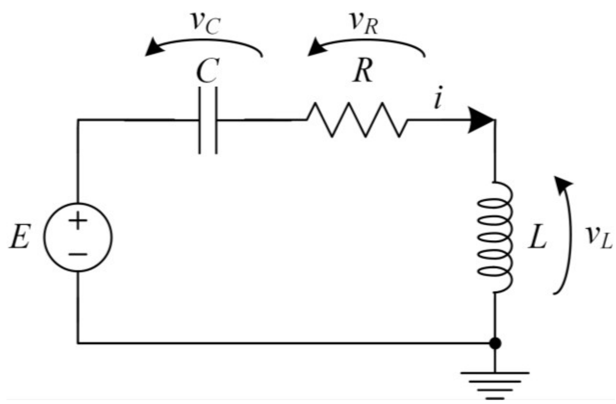

Fig. 2. Series *RL* circuit connected with a DC voltage source by a switch modeled as a capacitance *C*.

between the winding terminals. However, its value is usually very small and becomes relevant only for high frequencies (typically in the order of MHz), so will be disregarded in this case. The circuit resistance R will be the sum of the resistances of the inductor's winding and the physical resistor present in the circuit.

The *RLC* circuit shown in Fig. 2 is governed by the second order linear differential equation given by:

$$L\frac{di(t)}{dt} + R \cdot i(t) + \frac{1}{C}\int_0^t i(\tau) \cdot d\tau + v_C(0) = E(t). \tag{4}$$

Taking derivatives throughout yields:

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dE(t)}{dt}. \tag{5}$$

Defining the parameters: $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$, and assuming $i_g(t) = \frac{1}{L}\frac{dE(t)}{dt}$, (5) can be rewritten as:

$$\frac{d^2 i(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = i_g(t). \tag{6}$$

The circuit natural frequencies can be obtained applying the Laplace transform and are the roots of the characteristic equation:

$$s^2 + 2\alpha \cdot s + \omega_0^2 = 0. \tag{7}$$

Assuming the circuit operates in underdamped conditions, the discriminant of (7) becomes negative and therefore:

$$\Delta \ = 4\alpha^2 - 4\omega_0^2 < 0 \Leftrightarrow L > \frac{C \cdot R^2}{4}. \tag{8}$$

Defining $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, the roots of the characteristic equation are given by:

$$s_{1,2} = -\alpha \pm j\omega_d. \tag{9}$$

The solution of (5) is:

$$i(t) = I_1 e^{(s_1 t)} + I_2 e^{(s_2 t)} + i_p(t), \tag{10}$$

where, $i_p(t)$ is a particular solution, based on the source excitation, and I_1 and I_2 are constant values related to the initial conditions of the problem.

The constants I_1 and I_2 are obtained using the initial conditions $i(0)$ and $\frac{di}{dt}(0)$ resulting in:

$$i(0) - i_p(0) = I_1 + I_2, \tag{11}$$

$$\frac{di(0)}{dt} - \frac{di_p(0)}{dt} = s_1 I_1 + s_2 I_2. \tag{12}$$

Defining $a = i(0) - i_p(0)$ and $b = \frac{di(0)}{dt} - \frac{di_p(0)}{dt}$, (10) can be expressed as:

$$i(t) = \frac{s_2 a - b}{s_2 - s_1}e^{(s_1 t)} + \frac{-s_1 a + b}{s_2 - s_1}e^{(s_2 t)} + i_p(t) \tag{13}$$

Substituting (9) in (11), after some algebraic manipulation (13) can be written as:

$$i(t) = Ie^{-\alpha t}\cos(\omega_d t + \phi) + i_p(t), \tag{14}$$

where:

$$I = \sqrt{a^2 + \left(\frac{\alpha \cdot a + b}{\omega_d}\right)^2}, \tag{15}$$

$$\phi = \operatorname{atan2}\left(-\frac{\alpha \cdot a + b}{\omega_d}, a\right). \tag{16}$$

The function atan2(y, x) returns the argument of the complex number $x + jy$, and it is used instead of the usual arctangent function, atan(y/x), that always returns an angle between $-\pi/2$ and $\pi/2$.

Consider the switch closed for a long time with the series RL circuit in steady state. When the switch is opened at $t = 0$, the initial conditions for the open switch *RLC* model are given by: $i(0_+) = \frac{E}{R}$; $\frac{di(0_+)}{dt} = \frac{E - Ri(0_+) - v_C(0_+)}{L} = 0$; $i_p(0_+) = 0$; and $\frac{di_p}{dt}(0_+) = 0$.

Applying the previous general expressions to the model, and assuming $E > 0$, it is possible to use atan instead of atan2.

Substituting the circuit initial conditions in (11) and (12), the current $i(t)$ and voltage drops on the resistor $v_R(t)$ and on the inductor $v_L(t)$ are given by (17), (18) and (19), as shown in the Appendix:

$$i(t) = \frac{E}{R}\sqrt{\frac{1}{1 - \frac{R^2 C}{4L}}}e^{-\frac{R}{2L}t}\left[\cos\left(\begin{array}{c}\frac{1}{2L}\sqrt{\frac{4L - R^2 C}{C}}t \\ + \operatorname{atan}\left(-R\sqrt{\frac{C}{4L - R^2 C}}\right)\end{array}\right)\right] \tag{17}$$

$$v_R(t) = E\sqrt{\frac{1}{1 - \frac{R^2 C}{4L}}}e^{-\frac{R}{2L}t}\left[\cos\left(\begin{array}{c}\frac{1}{2L}\sqrt{\frac{4L - R^2 C}{C}}t \\ + \operatorname{atan}\left(-R\sqrt{\frac{C}{4L - R^2 C}}\right)\end{array}\right)\right] \tag{18}$$

$$v_L(t) = -\frac{E}{R}\sqrt{\frac{1}{1 - \frac{R^2 C}{4L}}}e^{-\frac{R}{2L}t}\sqrt{\frac{L}{C}}\sin\left(\frac{1}{2L}\sqrt{\frac{4L - R^2 C}{C}}t\right) \tag{19}$$

The voltage on the real inductor is the sum of the ideal inductor voltage (19) and the fraction of the total resistor voltage (18) that corresponds to the voltage drop on the winding resistance.

C. Time-Varying Capacitance Switch Model

During the opening of the switch, as its contacts progressively move away, the capacitance value varies. A *time-varying capacitance switch model* can be simulated numerically in PSPICE software.

IV. Materials and Methods

A. Instrumentation for the Circuit Characterization

A Keysight E4980AL *LCR* meter was used to measure the *RL* circuit parameters and switch OFF mode capacitance with a precision of 0.1%. The OFF mode switch capacitance was measured at 200 kHz using the parallel-equivalent circuit model (C_p-R_p). This frequency is close to the natural ringing frequency of the circuit measured during switch opening in the experimental setup.

B. Experimental Procedure

A practical experiment was carried out by 26 students enrolled in the Electrical Circuits course at the Polytechnic School of the University of São Paulo, Brazil, in 2018. They were first asked to obtain the parameters of an assembled *RL* circuit connected to a constant DC voltage source by a mechanical switch. Next they had to propose a first-order model of the circuit, obtain analytical expressions and perform simulations to obtain the inductor voltage after current interruption, adopting the ideal switch model. They then measured this voltage on the real circuit and proposed alternative switch models that better represented the transient circuit response. After comparing the ringing period of the measured inductor voltage with the analytical expression (19), the students had to propose a fixed capacitance switch model and repeat the simulations. To refine the results, they were told to use a first-order linear variable capacitance model.

V. Results and Discussions

A. Initial Tests on the Electrical Switch

The students' first test is to check the reproducibility of the switch impedance measurements, after successive turn on and off operations. The switch in the OFF position shows a capacitance of 1.0 pF and a parallel resistance of 14.21 MΩ. In the ON position the switch electrical resistance drops to 0.23 Ω, and a 95.17 nH inductance was measured with the *LCR* meter in the series-equivalent circuit mode (L_s-R_s). This procedure was repeated three times, yielding the same measurements each time, indicating that operating the switch does not affect the electrical parameters.

B. Switch Modeling and Simulation

The students then had to create a first-order *ideal switch model* of the *RL* circuit, calculate the inductor voltage v_L transient response at switch opening, and simulate the circuit in PSPICE software.

The physical *RL* circuit consists of a 1 kΩ resistor, *R*, in series with a coil inductor *L*, modeled as a 13.3 mH inductance in series with a 600 Ω resistance measured at 200 kHz. The coil DC resistance is 1.6 Ω DC, but its value increases with frequency due to skin effect. The electrical switch mounted on the wooden baseboard connects the *RL* circuit to a 13 V lead-acid battery, Fig. 3. The calculated and simulated values are compared with measurements on the physical circuit.

The inductor and switch voltages are measured using a DSO-X2002A Keysight oscilloscope, set up for single shot capture, with the trigger in the rising edge mode, and with the time base adjusted to display the whole circuit transient.

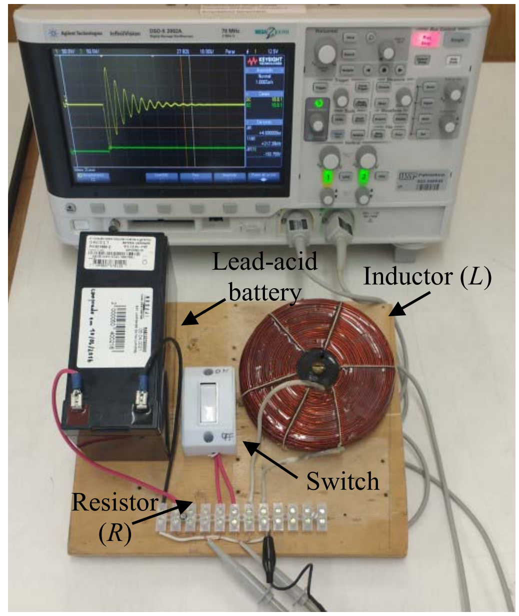

Fig. 3. Measurement instrumentation and 1st order *RL* circuit electrical circuit.

The next step is to use the *fixed capacitance switch model* to determine the switch capacitance and its turn-off dynamic response. The fixed capacitance value for the switch is obtained by equating the first cycle period of the measured switch voltage damped oscillation with the argument of the sine function described in (19).

Note that the first cycles have lower frequencies than the last ones, because as the switch opens the capacitance decreases. This switch characterization method only uses the oscilloscope, without the need of the *LCR* meter. Applying this procedure, a capacitance of $C = 42.95$ pF was obtained for the fixed capacitance switch model.

Knowing the fixed capacitance value and the period of the damped oscillation final cycles, the capacitance decrease can be approximated by a first-order polynomial (20) that represents the capacitance of the variable capacitance model in the interval of $t = 0$ to 0.060 ms:

$$C = 42.95 - 128.67 \times 10^3 \cdot t \ \left[pF\right] \tag{20}$$

Students repeated the inductor voltage calculations and simulations using both *fixed* and *time-varying capacitance switch* models. Finally, they compared the measured inductor voltage with the simulated results obtained with the ideal, the fixed capacitance and the variable capacitance switch models, Figs. 4-6.

Fig. 4 shows the comparison between the measured inductor voltage and the ideal response described in (3). The students noticed that the theoretical solution does not correspond to the measured result, with this disagreement raising much curiosity. Moreover, students' observation of the measured inductor voltage confirms that the circuit seems to satisfy a second-order differential equation.

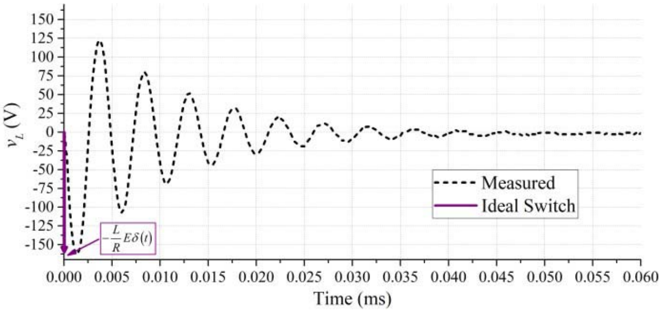

Fig. 4. Comparison between the measured and ideal inductor voltages (v_L).

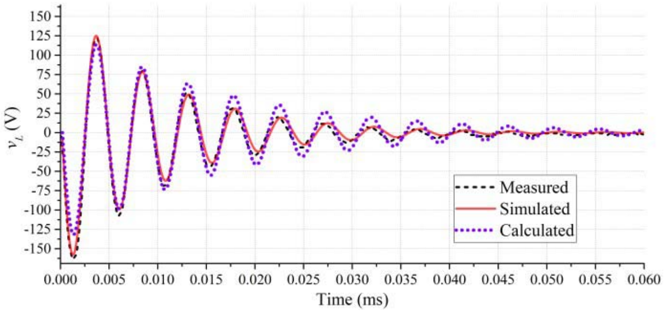

Fig. 5. Measured and calculated/simulated inductor voltages (v_L) using the *fixed capacitance switch model*.

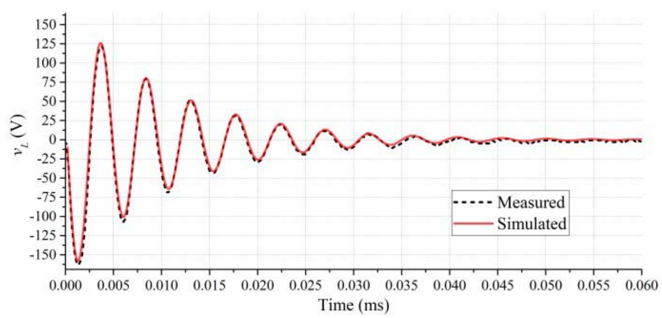

Fig. 6. Measured and simulated inductor voltages (v_L) using the *time-varying capacitance switch model*.

Fig. 5 shows the measured, calculated and simulated inductor voltages using the electrical switch fixed capacitance model approximation. The inductor initial current adopted in the simulation was 8.12 mA, which corresponds to 13 V divided by the total circuit resistance (1600 Ω) measured at 200 kHz. Although both curves match well, a tiny frequency difference can be detected after the third cycle.

Fig. 6 shows the measured and simulated inductor voltages under the electrical switch variable capacitance approximation. PSPICE and LTSPICE software accept time-varying capacitance values. Note that both curves match perfectly, indicating that the variable capacitance model is the best approach to describe the behavior of the real switch.

VI. Classroom Applications

A. Practical Application: Modeling an Electrical Switch

A test evaluated students' perception of switch modeling, by asking four questions (Q1, Q2, Q3, and Q4). Two (Q1 and Q2) were posed before the experiment.

- Q1: What is the order of the electrical circuit presented in Fig. 1?
- Q2: For the circuit of Fig. 1, what is the expected inductor voltage waveform v_L when the switch opens at time $t = t_0$?
- Q3 and Q4 were posed after measuring the inductor voltage:
- Q3: Based on the measurements performed, what is the order of this circuit?
- Q4: From the previous results, propose an equivalent circuit (model) for the electrical switch that explains the behavior of the real circuit.

Of the 26 student participants, 24 answered Q1 correctly, but only 18 were able to sketch the waveform of v_L correctly for Q2. Most of the errors were in voltage polarity and waveform shape. Q3 and Q4 were answered correctly by 25 and 24 students, respectively.

The impact of the switch modeling activity was evaluated through student feedback elicited in a post-class questionnaire; this consisted of five statements rated on a five-level Likert scale (0–5), where 1 corresponds to completely disagree (very poor), 2 corresponds to disagree (poor), 3 corresponds to indifferent (fair), 4 corresponds to agree (good), and 5 corresponds to completely agree (excellent).

The statements (S1, S2, S3, S4, and S5) were:

- S1: The experiment helped me to understand the effects of electrical and electronic components in practice.
- S2: The use of the SPICE simulator helped me to understand the importance of computational tools in electrical circuit modeling.
- S3: The concepts learned in the electrical circuits discipline are adequate to model the circuits proposed in the experiment.
- S4: The experiment was a good bridge between the theoretical discipline of electrical circuits and the practical disciplines in the laboratory.
- S5: The experiment increased my motivation to study electrical circuits.

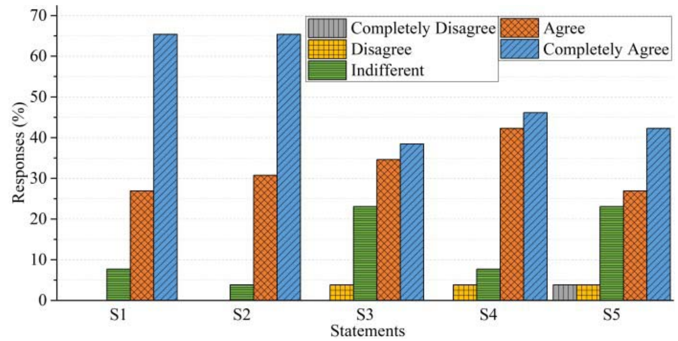

Fig. 7. Student questionnaire responses.

The responses, Fig. 7, indicate that students were satisfied with the hands-on experiment. Excellent scores (> 65% of "completely agree") were obtained for statements S1 and S2, which demonstrates the potential of modeling applications associated with the use of the SPICE circuit simulation

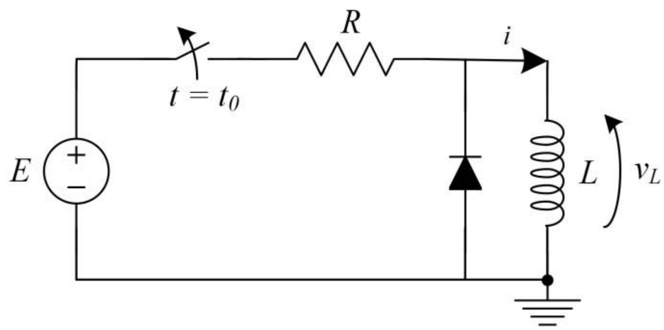

Fig. 8. Diode surge protection against the inductive kickback.

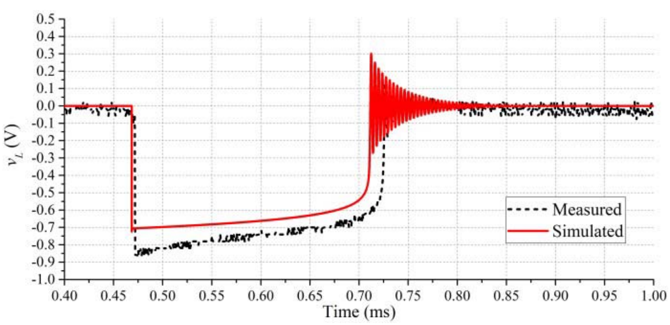

Fig. 9. Inductor voltage when switch turns off with diode in anti-parallel to the inductor. The simulated results use the *fixed capacitance switch model*.

software. Although many students correctly modeled the electrical switch as a capacitor, the high level of "indifferent" answers to S3 reveals a difficulty in applying theoretical concepts to real problems.

Statement S4 confirms the effectiveness of the experiment during the Electrical Circuits course. Finally, given the difficulty of motivating students in electrical circuits courses, the answers to S5 suggest that activities such as those presented here may make classes more interactive and encourage students to participate.

B. Practical Application: Inductive Kickback Protection With a Reverse Diode

A simple and common way to protect the circuit against the effect of inductive kickback is to connect a diode—a semiconductor device that allows current to flow easily in one direction but severely restricts it in the other—in anti-parallel to the inductor [15]–[19]. In normal operation, the current flows solely through the inductor branch, Fig. 8. During switch opening, the inductor current has an alternative path through the diode, so does not vary abruptly. This prevents arcing in the switch and, consequently, premature wear of the switch.

Students checked the circuit protection by connecting a 1N4148 diode in anti-parallel to the inductor. The measured and simulated results, Fig. 9, show that the 125 V surge voltage was replaced by a slight negative voltage (of approximately −0.8 V). The low amplitude oscillatory transient voltage occurs because of the diode capacitance. The diode coped with the initial current and ensured the switch integrity.

VII. Conclusion

A classroom experiment on an electrical circuit confronted the students with the "inductive kickback" problem caused by an abrupt current interruption in highly inductive circuits. The activities were led by an instructor who acted as facilitator, asking questions and drawing attention to interesting results.

Students found that when they used the first-order model to represent the *RL* circuit with the electrical switch, they could not explain the waveform measured. Modeling the switch as a capacitor allowed them to fit the analytical results and the simulations. With a more sophisticated model, consisting of a time-variant capacitor, simulations and measurements matched perfectly.

Student performance was evaluated qualitatively based on test scores, and quantitatively by their feedback on a post-class questionnaire. The students considered the experiment interesting and suggested it be included in the course. The analysis showed that the experiment helped students to improve their laboratory and simulation skills and to learn a new approach to solving electrical circuit problems.

Appendix

A. Inductor Voltage

The ideal inductor's voltage initial conditions, $v_L(0_+)$, $\frac{dv_L(0_+)}{dt}$, $v_{Lp}(0_+)$, and $\frac{dv_{Lp}}{dt}(0_+)$ can be obtained from expression (19), using the same methodology as used in $i(t)$.

An alternative formulation is differentiating expression (14):

$$\frac{di}{dt}(t) = Ie^{-\alpha t}[-\alpha\cos(\omega_d t+\phi) - \omega_d\sin(\omega_d t+\phi)] + \frac{di_p}{dt}(t)$$

Adding sine and cosine terms:

$$\frac{di}{dt}(t) = I\sqrt{\omega_d^2+\alpha^2}e^{-\alpha t}\cos(\omega_d t+\phi+\operatorname{atan2}(\omega_d,-\alpha)) + \frac{di_p}{dt}(t)$$

Assuming $t > 0$: $i_p(t) = 0$, $a = i(0_+)$ and $b = 0$. Using (15):

$$\frac{di}{dt}(t) = i(0_+)\sqrt{\frac{\omega_d^2+\alpha^2}{\omega_d^2}}\sqrt{\omega_d^2+\alpha^2}e^{-\alpha t} \times \cos\begin{pmatrix}\omega_d t \\ +\operatorname{atan2}\left(-\frac{\alpha\cdot a}{\omega_d}, a\right) \\ +\operatorname{atan2}(\omega_d,-\alpha)\end{pmatrix}$$

Making use of the relations: $\omega_d = \sqrt{\omega_0^2-\alpha^2}$,

$$a = \frac{E}{R} > 0 \Rightarrow \operatorname{atan2}\left(-\frac{\alpha\cdot a}{\omega_d}, a\right) = \operatorname{atan}\left(-\frac{\alpha}{\omega_d}\right),$$
$$\alpha > 0 \Rightarrow \operatorname{atan2}(\omega_d,-\alpha) = \operatorname{atan}\left(-\frac{\omega_d}{\alpha}\right) + \pi,$$

and $\operatorname{atan}(x) + \operatorname{atan}(\frac{1}{x}) = \frac{\pi}{2} * \operatorname{sgn}(x)$, results the expression:

$$\frac{di}{dt}(t) = -i(0_+)\frac{\omega_0^2}{\omega_d}e^{-\alpha t}\sin(\omega_d t)$$

From the relations: $\cos(\phi) = \frac{a}{\sqrt{a^2+\frac{(\alpha a)^2}{\omega_d^2}}} = \frac{\omega_d}{\omega_0}$ and $i(0_+) = I\cos(\phi)$, the equivalent expression to (19) is obtained: $v_L(t) = L\frac{di}{dt}(t) = -LI\omega_0 e^{-\alpha t}\sin(\omega_d t) = -I\sqrt{\frac{L}{C}}e^{-\alpha t}\sin(\omega_d t)$.

B. Convergence to the Ideal Solution

When the capacitor value becomes close to zero, the response approximates (3):

$$\int_{0_+}^{\infty} e^{-\alpha t}\cos(\omega_d t + \phi)dt = -\frac{\omega_d \sin(\phi) - \alpha\cos(\phi)}{\alpha^2 + \omega_d^2}$$

$$\int_{0_+}^{\infty} e^{-\alpha t}\sin(\omega_d t)dt = \frac{\omega_d}{\alpha^2 + \omega_d^2} = \frac{\omega_d}{\omega_0^2}$$

$$\int_{0_+}^{\infty} v_L(t)dt = -LI\frac{\omega_d}{\omega_0} = -LI\cos(\phi) = -Li(0_+) = -\frac{LE}{R}$$

Another way to confirm this fact is:

$$\int_{0_+}^{\infty} v_L(t)dt = \int_{0_+}^{\infty} L\frac{di}{dt}dt = L\big[i(\infty) - i(0_+)\big] = -\frac{LE}{R}.$$

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