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ANALITIC ELEMENT MODEL FOR GROUNDWATER FLOW IN THE AQUIFERS' RECHARGE ZONE - PART 1: UNCONFINED COASTAL AQUIFERS

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Abstract. *The Analytic Element Method (AEM) is a computational technique which permits to superimpose analytic solutions, generally related to regional-wide scale feature in aquifers, based on linear groundwater flow models. It was originally organized by Strack and Haitjema (1981). Zaardnoordjik and Strack (1993) proposed an extension that represents unsteady line source/sink (e.g. creeks) and finite area source/sink (e.g. irrigation areas). Aquifer recharge zones commonly occur on the narrow strip form, as in the most of the unconfined coastal aquifer cases and in several confined aquifer raising zones. In the first part of this paper the coastal unconfined groundwater flow with accretion is treated. We firstly develop a mathematical model for the unsteady rainfall-recharge effect. Further, the expression is implemented in an analytic element open source program (TIMSL 0.3) and a closed domain is assumed in order to compare the solution f an object problem using AEM and a finite element model. The comparison shows good agreement.*

Keywords: *Environmental flow, Groundwater modeling, Analytic elements*

1. INTRODUCTION

The mention of groundwater flow modeling usually invokes the image of a model grid or an element network. The elegance of classical analytic solutions for groundwater flows has been largely replaced by the versatility of numerical algorithms (e.g. FDM and FEM) and the power of the digital computer. This emphasis on numerical modeling often deprives the hydrogeologists from the insight in basic groundwater hydraulics, which comes from solving elementary flow problems analytically, using tools such as the basic concept of superposition of particular solutions. However, the superposition principle is only valid for linear equations and the groundwater flow equations are not always linear. This looks to be a limitation for analytical solutions of nonlinear problems, which in this case need to be linearized. On the other hand, numerical algorithms can readily be applied to nonlinear differential equations without strong difficulties (although other approximations are made to formulate discretization methods).

A largely used assumption in groundwater flow modeling is the Dupuit-Forchheimer approximation. It allows regional models to be expressed in a two-dimensional form. If the flow occurs in a "conservative field" (see Kellogg, 1953) it may be expressed in terms of its potential. Nonlinear problems like the two-dimensional homogeneous phreatic steady groundwater flow can be expressed in terms of potential discharge in a linear form.

Computer modeling in this area requires the handling of large unbounded domains, simulating the flow in the area of interest, and dealing with oddly shaped internal boundaries. The "numerical method" called Analytic Element Method (AEM) has been developed from an initial idea of distributing some singularities in an attempt to meet the criteria of unbounded domains (Strack and Haitjema, 1981) as in the Figure 1. The method is based on superposition of suitable analytic functions, and has been implemented in several computer programs (e.g. GNU TIMSL), which have been applied successfully to numerous problems of regional flow. The functions implemented in the AEM, which are obtained from mathematical-physics methods (Courant and Hilbert, 1989), are continuous, differentiable and satisfies the original differential equation everywhere. Thus the continuity of flow is guaranteed, not requiring water balance check in the domain.

Since analytical developments in groundwater flow are strongly based on the Dupuit-Forchheimer assumption, local models (three-dimensional or vertical models) are not the focus of AEM because the vertical velocity components are not negligible. In addition, it is quite difficult to define three-dimensional contours analytically, although some vertical models are found in the classic groundwater literature and some analytic elements have been proposed originally by Haitjema (1985). However, recently Luther e Haitjema (2000) presented methods for both, finding approximate analytic solutions for three-dimensional unconfined steady state groundwater flow with accretion near partially penetrating and horizontal wells and for combining those solutions with regional two-dimensional models (Dupuit-Forchheimer models). This proposal was capable, as well, to describe the seepage face at the wells.

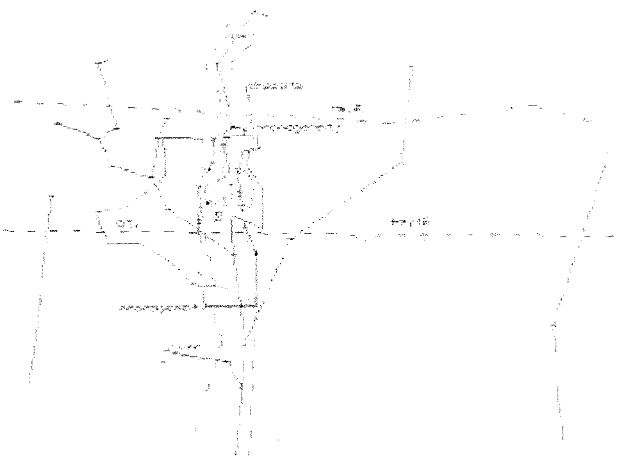

Figure 1 – Element analytic model layout

2. AQUIFER RECHARGE ZONES

According to Toth's solution (Haitjema, 1995 after Thot, 1963) vertical flow indeed plays an important role even in regional flow. There are at least three conditions which do not corroborate with the Dupuit-Forchheimer hypothesis in the aquifer systems (considering a practical water resources viewpoint):

1) The depth and lateral extent of the aquifer are of the same order of magnitude.
2) The water table is controlled by the topography;
3) The hydraulic conductivity is relatively low.

Groundwater recharge areas, by their nature, must exhibit vertical components of flow. Haitjema (1987), comparing results given by Strack (1984) with his three-dimensional approach for circular recharge areas, points that the Dupuit-Forchheimer approximation is acceptable, except if the size of the recharge area is of the same order as or smaller than the aquifer thickness. When considering areal recharge due to rainfall, the recharge area is the entire flow "domain"(!), therefore several orders of magnitude larger than the aquifer thickness (Haitjema, 1995). The abovementioned approximations consider uniform distributions, but it must be emphasized that there are not aquifers with uniform spatial or temporal recharge distributions. As an example of a technique for modeling recharges due to rainfall, Haitjema (1995) shows the use of "source discs" over the whole "domain" (the domain of interest). For the case of inhomogeneous spatial distribution, discs with different strengths should be distributed over.

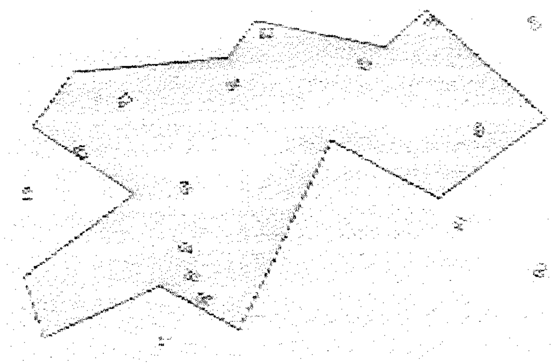

Figure 2 – The area-sink element

In general, these kinds of functions (source/sink disc) are used for any kind of "local" flow accretion (supply) in an aquifer. For example, irrigation areas, circular ponds without direct connection with the aquifer, etc. On the other hand, for a better representation of general shape of reservoirs (without direct connection) and situations of same recharge due to rainfall delineation, "area sink" (source/sink with polygonal shape) is more suitable, as illustrated in Figure 2, though computationally more expensive. A powerful element function for transient "area sink" has been presented in Zaardnoordjik and Strack (1993).

3. THE IMAGE METHOD

One of the most interesting consequences of the superposition principle is the image method. Sometimes it is necessary to consider "pure" reflection along a plane (or a line in two-dimensional problems). This plane (or line) deals as a mirror and elementary functions, such as pumping wells, are arranged "mathematically" in both sides of that mirror to produce a symmetrically response to system of equations used. The "mirror", in the present case, is impervious line. Another kind of contour is the pervious line, which is obtained usin "negative" images (anti-symmetric images).

The following figure refers to a two lines bounded domain. The first one represents an impervious wall at the plane $y=0m$ and the other one representing a pervious known head line at the plane $y=2.4x10^4m$.

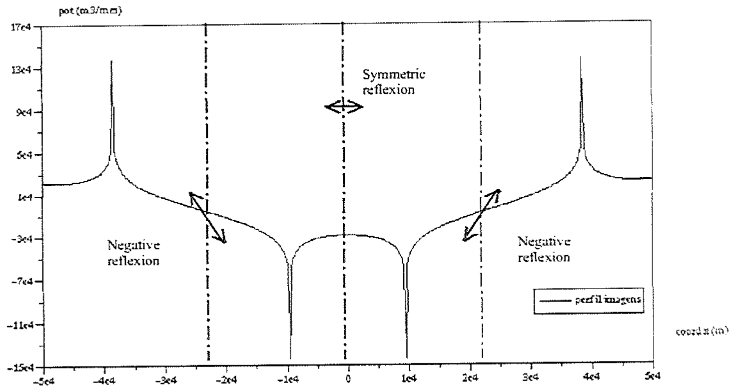

Figure 3 – Image usage for a two-lines bounded domain

4. MATHEMATICAL MODEL FOR UNCONFINED COASTAL AQUIFER

Coastal regions are characterized by sediment deposits. It is possible to represent such narrows sediment areas as a strip with infinite length and convenient mean width. The finite dimension for our proposes is located between two different boundary conditions: the sea (vertical pervious plane) and the base rising (impervious wall). The saline intrusion could be considered, but it s not necessary for the present analysis.

With this geometrical situation only one dimension (ξ) to describe all the flow in the "slab" $0<\xi<L$, where L is the mean distance between the coastal aquifer boundaries.

The present situation is similar to *areal recharge* problems found in the literature. For example, for flow between two infinite rivers (Strack, 1989 and Haitjema, 1995). No-flow boundaries at impervious walls are met taking advantage of the water divisor that appears in the middle of the space between two same head infinite rivers. The equation for steady state potential flow is given by:

$$\Phi(\xi) = -\frac{N}{2}\xi^2 + NL\xi + \Phi(0) \qquad (1)$$

The hydraulic head (h) is then obtained from the potential flow relation

$$\Phi = \frac{Kh^2}{2}. \qquad (2)$$

This simple parabolic shaped equation shows positive values greatest than $\Phi(0)$, but decreasing infinitely beyond the domain of interest ($0<\xi<L$). This indicates that the image tool may be used when considering regional recharge in a groundwater flow model. Figure 4 shows a profile with the superposition of Eq. (1) and the image generated for Figure 3.

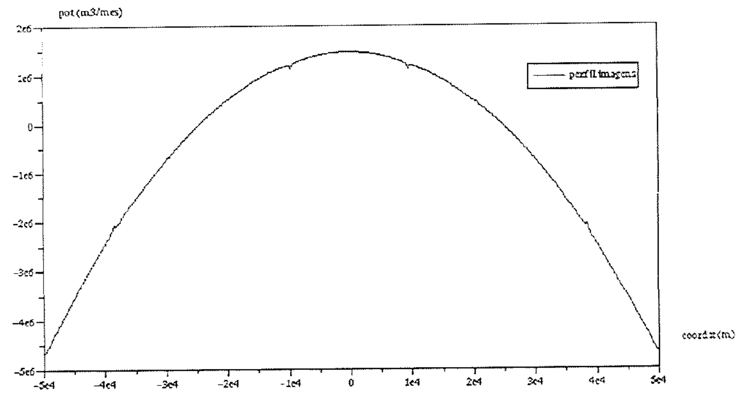

Figure 4 – Superposition of the situation generated for Figure 3 and Eq. 1 with convenient values

We consider now the basic flow in the potential response for a time-dependent recharge uniformly distributed in $0<\xi<L$.

A similar problem is considered by Carslaw and Jaeger (1959) who studied the temperature rising due to heat accretion in a finite rod $-L < \xi < L$. They obtain the solution by a cosine Fourier series. In the present case, considering $0<\xi<L$, the solution is obtained by the following sine Fourier series:

$$\Phi(\xi,\tau)=\frac{4NL^2}{\pi^3}\sum_{1}^{\infty}\frac{1}{n^3}\left\{1-\exp\left[-\tau\left(\frac{\pi na}{L}\right)^2\right]\right\}\mathrm{sen}\left[\xi\left(\frac{\pi na}{L}\right)^2\right] \tag{3}$$

Where, $a^2=\dfrac{HK}{S}$ is the hydraulic diffusivity with H, the mean head in the time (τ) and the domain (ξ), K is the hydraulic conductivity and S, the storage coefficient of the matrix porous. It is interesting to mention that analytical expressions in the AEM are always split in a *discharge parameter* and an *influence function*. In the Eq. 1 and 3, for example, N is the discharge parameter and all other terms compose the influence function. Some of the elements may be applied with unknown discharges (e.g. specified head wells) in the model that need to be solved by a computer program.

5. THE TIMSL PROGRAM

TimSL is an object-oriented analytic element program written in Python language for simulation of groundwater flow. The code is distributed under the GNU Lesser General Public License.

The design of the TimSL code consists of three base "classes": Geometry2d (Interface type), Element, and Aquifer. Geometry2d is the base class for the two-dimensional geometries of the analytic elements. Geometries such as Circle or Line are derived from the Geometry2d class. Element is the base class for the analytic element functions. All analytic elements, such as wells or line-sinks are derived from both Element and a geometry class derived from Geometry2d.

An Universal Modular Language (UML) diagram of the basic design of the TimSL code is shown in Figure 5. All class names start with a capital letter and follow camelback notation (such as HeadWell). Package or module names do not contain capitals, intending to make it easy to distinguish them. Packages are implemented in order to contain related modules (underlined in the Figure 5), which implements their related classes with their required methods and attributes, inherited or not. Here is the main difference and advantage in oriented-object codes, in that, each "constructed" class may inherit any hierarchically above class attributes or methods (see arrows in Figure 5). The last two boxes contain an extension due to the present study. Two modules with the same name "coastalarealrec" have been introduced in the single package and in the transient package.

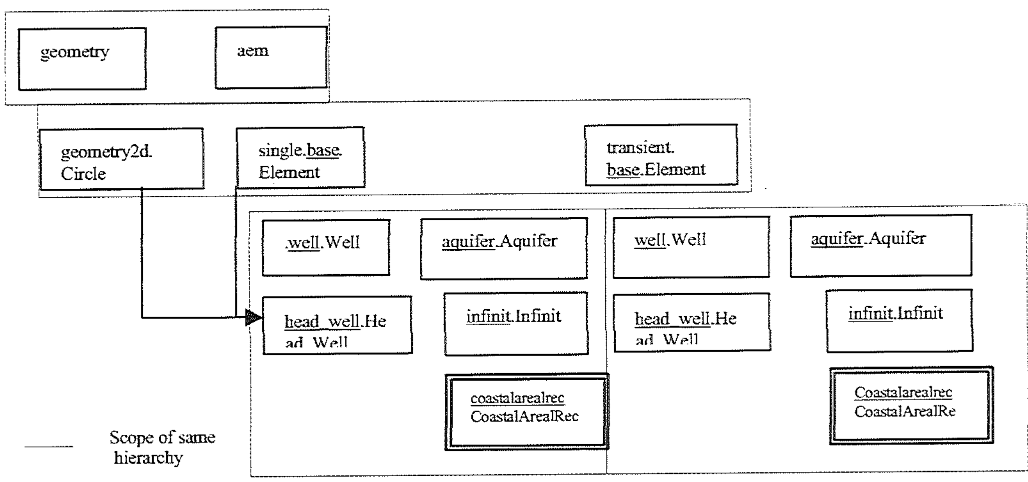

Figure 5 – Illustrative TimSL's UML Diagram

The first element given to the model is often an aquifer type. The other elements assign that aquifer as a "parent" in order to be included in the parent elementList. Because all elements are in the Aquifer elementList attribute, the solve methods can find unknown parameters for all elements in the aquifer. As long as any additional element calls the addElement method upon their creation, it is stored in the elementList and the potential due to all elements (superposition of influences) can be calculated.

7. OBJECT PROBLEM

In Brazil, the northeastern coastline concentrates most of the local population and has important sedimentary deposits of soil. This deposits constitutes the "Barreiras" aquifer system, the main system in the Brazilian coastal aquifers, with about 33000 Km^2 distributed since the State of Maranhão until the State of Rio de Janeiro.

Sedimentary deposits rest on impervious bases that commonly are not completely covered by them. It characterizes a restricted (in one direction), though still infinite domain (semi-infinite domain). Although AEM is recommended for unbounded domain, a fictitious closed domain is considered here in order to compare the results obtained by the new AEM extension and with an equivalent finite element model (FEM). The problem is stated as follows (based on a situation observed at the Brazilian northeast coast):

"A well produces $72x10^3$ m^3/month and is located at a pond neighborhood like disposed in the Figure 6. The pond water budget is given in $230x10^3m^3$/month negatively. This area receives constantly 5 mm/month uniformly distributed over a confined aquifer recharge zone. In order to do comparison with coastal aquifers, take h_0=0.0m at the exit boundary, which is 24Km distant from impervious wall and the impervious base at quote 20m below. What should be the steady water level in the pond?"

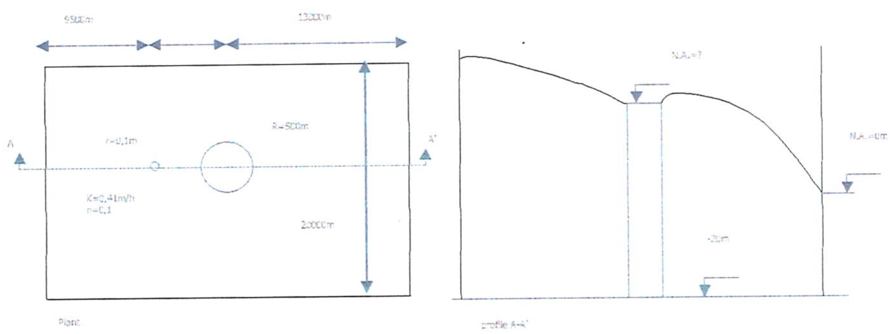

Figure 6 – Didactic scheme for the problem considered in this study

Figure 7 and 8 show the comparison between the AEM and FEM formulation for steady flow in coastal domains. For the AEM the pond water level was met at the quote 58.55m. The results show qualitative good agreement between the contour lines obtained for each situation, in that contour line shapes follows the same trends.

For the FEM, the pond water level was met at the quote 57.72m, that is 0.83m lower than the AEM has predicted. Although this difference must be explained, it is very positive that contour lines and general trends are similar for both situations. Some numerical characteristics of the FEM may produce disturbances such as numerical diffusion. The element network, in the present case, was composed by 240x200 elements (100x100 square me). The mentioned disturbance may be some of the source of the observed difference. It is still important to note that the FEM simulation is done with the original nonlinear Boussinesq equation while the AEM consider the linearized problem.

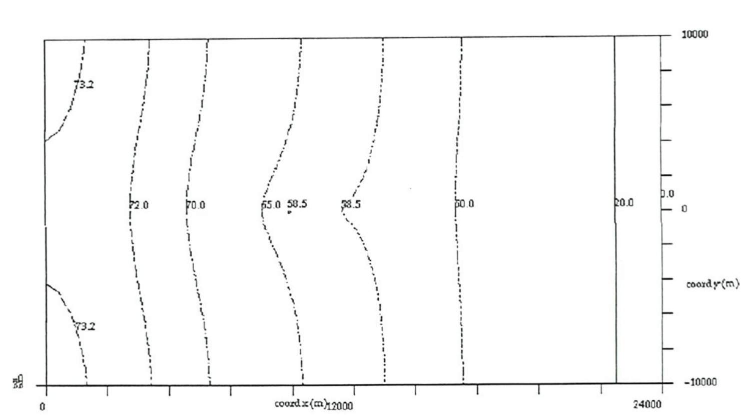

Figure 7 – AEM results for the example in a closed domain

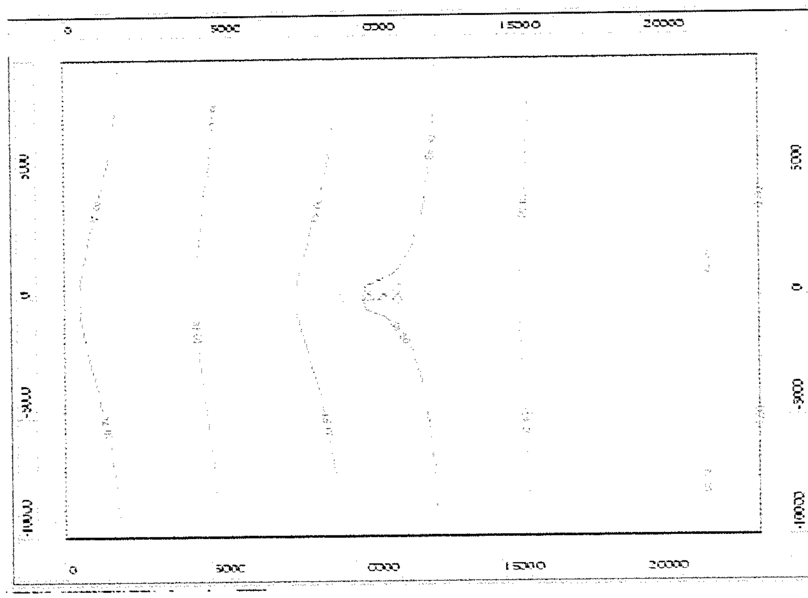

Figure 8 – Finite Element Model for the example

A further unsteady situation is discussed. The fictitious problem is posed again with the same hydraulic parameters used for the steady flow case. Once more, considering situations related to the Brazilian northeast coast the following sequence of "step" is modeled:

"Supposing the same hydraulic parameters, let the recharge due to rainfall be null during 42 months due a dry period. As a consequence, 24 months after the beginning of the dry period, the local water resource agency decides to reduce the well pumping to 50% of the mean yielding (given the in the steady state situation). In addition, because of this dry weather, the water budget is still decreased to minus 300×10^3 m^3/month. When the dry season finishes, the recharges reaches 10 mm/month and the water budget is elevated to 150×10^3 m^3/month (still negative). Thus the water resources agency allows, 6 months after the end of the dry period, the yield backs to the mean values. Table 1 shows the data summary for this problem. How does the water level in the pond vary with time? Furthermore, what is the pond water level 30 months after the end of the dry?"

Table 1 – Data Summary

Time	Recharge (mm/month)	Lake's Water Budget (m^3/month)	Well's Discharge (m^3/mês)
Initial Condition	5	230000	72000
1^{st} month (dry)	0	300000	72000

25^{th} month (dry)	0	300000	36000
43^{rd} month (wet)	10	150000	36000
49^{th} month (wet)	10	150000	72000

Results point to differences between the considered methods. Results from AEM show that, at the end of the analysis, the water level met the quote 58.99m, which differs 1.06m from the FEM prediction. The greatest difference observed between both methods during this analysis was 1.98m.

In Figure 9 the pond water level evolution is shown. The difference at the beginning of the analysis refers to the steady state situations, but it may be still observed accretion in the distance between the curves.

This differences may once more be related to the numerical disturbances and to the basic difference between AEM and FEM, which in the use of linearized equations for the first method and nonlinear equations for the second.

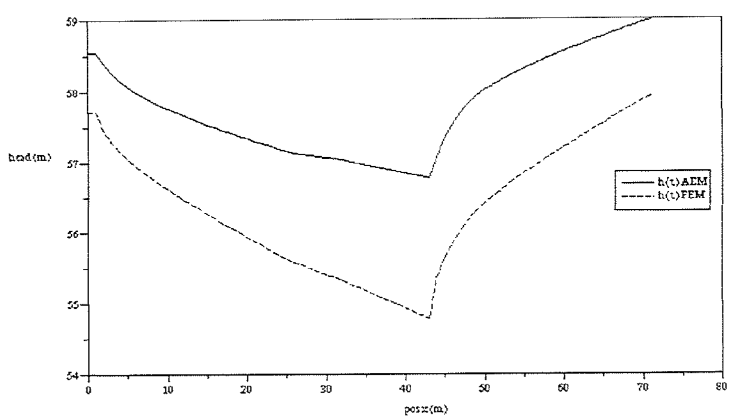

Figure 9 – Punctual head evolution at the pond edge for the example

Finally, considering a situation closer to the real coastal areas, the contour boundaries of the domain are relaxed in order to analyze the same example for semi-infinite areas (in the sense here defined).

The fact of extinguish the "lateral" contour boundary produces, of course, contour head lines changes. Figure 10 shows a part of the area of interest in order to view some of these changes. The contours generated by images are the strait line $h=0m$ and the symmetry of the line $h=78m$ related to the impervious wall on the plane $x=0m$. As an effect of the boundaries relaxation, contour lines are not perpendicular to the planes y=10000m and y= -10000m any more.

An important difference is observed for the pond water level. With the semi-infinite case, the level rises to the quote value of 61.88m, which is 3.33m above the first case (closed domain).

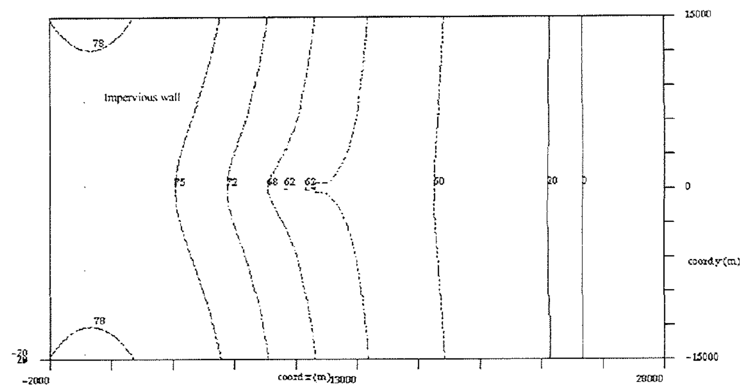

Figure 10 – Contour lines for steady state flow in a coastal aquifer

8. CONCLUDING REMARKS

It may be concluded that:

1) The Analytic Element Method (AEM) is adequate for the study of coastal aquifers in a regional scale;
2) The use of semi-infinite domains or closed domains may produce significant changes for aquifers in regional scales. The semi-infinite domain is more adequate for long narrow sediment deposits;
3) It was observed a difference of 3.33m for the pond water level in the example that was run in this study, comparing semi-infinite and closed domains;
4) The comparison between AEM and finite element (FEM) for the same study cases showed that differences related to numerical disturbances and/or effects of linearization are present. The differences are more pronounced for unsteady situations.

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