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Stiffened plate bending analysis by the boundary element method

G. R. Fernandes, W. S. Venturini

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Abstract In this work, the plate bending formulation of the boundary element method (BEM) based on the Kirchhoff's hypothesis, is extended to the analysis of stiffened elements usually present in building floor structures. Particular integral representations are derived to take directly into account the interactions between the beams forming grid and surface elements. Equilibrium and compatibility conditions are automatically imposed by the integral equations, which treat this composite structure as a single body. Two possible procedures are shown for dealing with plate domain stiffened by beams. In the first, the beam element is considered as a stiffer region requiring therefore the discretization of two internal lines with two unknowns per node. In the second scheme, the number of degrees of freedom along the interface is reduced by two by assuming that the cross-section motion is defined by three independent components only.

Keywords Plate bending, Boundary elements, Building floor structures

1 Introduction

The boundary element method (BEM) is already a well-established numerical technique to deal with an enormous number of complex engineering problems. Among them, analysis of plate bending problems has proved to offer a particularly adequate field of applications for that technique. The BEM is suitable for evaluating internal force concentrations due to loads distributed over small regions, which very often occur in plate bending analysis. Moreover, BEM can deal with deflections, slopes, moments, and shear forces, approximating them by using the same order polynomials. Thus, shear forces are much better evaluated when compared with other numerical methods; they depend only on the adopted boundary value approximation.

The first works discussing the use of direct boundary element formulation, in conjunction with Kirchhoff's theory, are of Bezine [1], Stern [2] and Tottenhan [3]. It is also important to mention some previous studies dealing with plate bending problems in the context of indirect methods [4, 5]. These, as well as several other more recent publications, have pointed out the capability of the method for modelling plates in bending, mentioning further accuracy and reliability.

In order to use BEM to analyse more complex plates, e.g., stiffened plates of building floor structures, one has to extend the BEM formulations to take into consideration arbitrarily displayed beams, general boundary and internal constraints and several kinds of transversal loads acting over the plate surface or part of it. Along these lines, Song [6], Hartmann and Zotemantel [7] have presented interesting approaches, discussing in detail displacement restrictions at internal points and the use of hermitian interpolations. More recently, Oliveira Neto and Paiva [8] have shown a BEM/FEM for analysing building floor structures.

While BEM is strongly recommended for plate bending analysis, in which internal force and displacement fields are always accurately modelled, the natural choice to solve the building floor structures is the BEM/FEM combinations. Boundary elements are recommended to deal with plate elements, which are combined together by enforcing equilibrium and compatibility conditions along the interfaces. The FEM is used to model beam elements. However, for complex floors, characterized by a large number of connected beams and many plate regions of different thickness, the number of degrees of freedom rapidly increases and the solution accuracy diminishes.

To overcome these difficulties, Venturini and Paiva [9] have proposed a scheme to deal with zoned domains without dividing them into sub-regions. In their formulations, only the displacements along the interfaces require approximation. Equilibrium conditions are automatically satisfied, so no approximation of these values is required along the interfaces. In addition, the number of degrees of freedom at interface nodes is divided by two. A given integral representation of displacements can be easily written for the whole body. This technique has been extended to potential and 2D elastic problems [10], preserving the original characteristics: reduction of degrees of freedom and increasing result accuracy. More recently, following the same ideas, Fernandes et al. [11] have presented a plate-bending BEM formulation to deal with a varying thickness problem.

Here, this formulation is modified to consider plate domains stiffened by beams without combining the algebraic equations written separately for each problem. The beams are treated as regions of different thicknesses, which can be very narrow, therefore representing the stiffness variation introduced by this kind of stiffener.

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Their rigidities are automatically taken into consideration by the global integral representations of displacements and rotations, in which the stiffener rigidity influences are given by two-line integrals along the beam sides. To obtain accurately the algebraic equations, quasi-singular integral schemes are required to perform the integral along the beam elements leading to stable numerical solutions. The formulation is further modified by assuming simple displacement approximations in the direction perpendicular to the beam axes. Displacements in this direction are assumed to be linear and, therefore, given in terms of two node values: the deflection and the rotation at the beam skeleton point. These assumptions reduce strongly the number of degrees of freedom of the entire problem.

Finally, some numerical examples are presented to illustrate the accuracy of the results, as well as the problem size reduction in terms of degree of freedom.

2 Basic equations

Let us consider a flat plate of thickness h, referred to a Cartesian system of co-ordinates with axes x_1 and x_2 lying on its middle surface and axis x_3 perpendicular to that plane. The plate domain is denoted by Ω (see Fig. 1), while its boundary is represented by Γ. It is assumed that a distributed load g is acting in the x_3 direction on the plate midplane, with no distributed external moments.

For this plate, the following basic relationships are defined:

- Equilibrium equations in terms of internal forces:

$$m_{ij},_j - q_i = 0 \tag{1}$$

$$q_i,_i + g = 0 \tag{2}$$

where m_{ij} are bending and twisting moments, while q_i represents shear forces, with subscripts taken in the range $i, j = \{1, 2\}$.

- The plate bending differential equation,

$$m_{ij,ij} + g = 0 \tag{3}$$

or

$$w,_{iijj} = \frac{g}{D} \quad (i, j = 1, 2) \tag{4}$$

where $D = Eh^3/(1 - \nu^2)$ is the flexural rigidity and $w,_{iijj} = \nabla^2 w$, is the Laplacian operator.

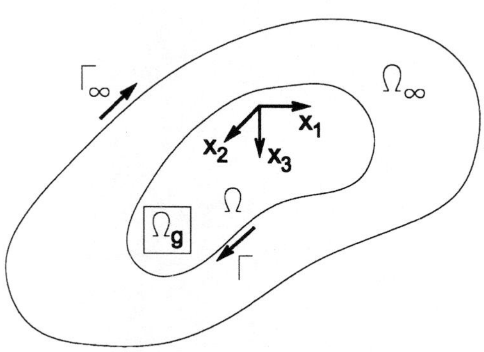

Fig. 1. Plate domain

- The generalized internal force × displacement relations,

$$m_{ij} = -D\big(\nu\delta_{ij} w,_{kk} + (1 - \nu) w,_{ij}\big) \tag{5}$$

$$q_i = -Dw,_{jji} \tag{6}$$

- The effective shear force,

$$V_n = q_n + \partial m_{ns}/\partial s \tag{7}$$

where (n, s) are the local co-ordinate system, with n and s referred to the boundary normal and tangential directions, respectively; no summation is implied.

The problem definition is then completed by assuming the following boundary conditions over Γ: $u_i = \bar{u}_i$ on Γ_1 (generalized displacements, deflections, and rotations) and $p_i = \bar{p}_i$ on Γ_2 (generalized tractions, normal bending moments, and effective shear forces), where $\Gamma_1 \cup \Gamma_2 = \Gamma$.

3 Integral representations

As usual, the integral representations of deflections, slopes, and generalized internal forces for Kirchhoff's plates can be derived from a reciprocity relation written in terms of bending moments and curvatures of two independent mechanical states. The first state is represented by the actual plate bending problem valid over the domain Ω, for which curvatures $w,_{ij}(q)$ and moments $m_{ij}(q)$ at a field point q are defined, as well as the associated boundary values: two generalized displacements, $u_i(Q)$, and two generalized tractions, $p_i(Q)$, referred to a boundary field point Q. The second elastic state is obtained from the fundamental solution $w^*(s, q)$, the deflection at the field point q due to a unit load applied at a source point s. From this state, fundamental values for curvatures, $w^*_{,ij}(s, q)$, and moments, $m^*_{ij}(s, q)$, can be easily derived (1,2,3,11). This elastic state is defined over the infinite space of domain Ω_∞ that contains Ω (see Fig. 1).

In order to obtain all the integral representations, the following reciprocity relation is easily found for the constant rigidity case:

$$\int_\Omega m^*_{ij}(s, q) w,_{ij}(q) \mathrm{d}\Omega(q) = \int_\Omega m_{ij}(q) w^*_{ij}(s, q) \mathrm{d}\Omega(q) \tag{8}$$

The reciprocity relation (8) is valid only for plates exhibiting constant rigidity D, but its extension to a general case is simple [11, 12]. Let us consider a plate element now characterized by exhibiting variable thickness, i.e., $t = t(q)$, which gives corresponding rigidity $D = D(q)$. Following the same steps to derive the reciprocity relations (8), but now assuming $D = D(q)$ variable (continuous or abrupt variation), one can easily obtain,

$$\int_\Omega m_{ij}(q) w,^*_{ij}(s, q) \mathrm{d}\Omega(q) = \int_\Omega \frac{D(q)}{D_o} m^*_{ij}(s, q) w,_{ij}(q) \mathrm{d}\Omega(q) \tag{9}$$

where D_o is a reference value; For simplicity, it will be assumed as the plate rigidity at the collocation point $D(s)$.

From Eq. (9), one can derive the deflection representation for the general case of varying rigidity plates with or without abrupt rigidity changes. Thus, assuming that the rigidity can vary over the domain, one can integrate Eq. (9) by parts to obtain the deflection integral representation, as follows,

$$
\begin{aligned}
C(S)w(S) &+ \sum_{k=1}^{N_s} \frac{1}{D(S)} \int_{\Gamma_k} \left(D(Q) V_n^*(S,Q) \right. \\
&\left. + 2\frac{\partial D(Q)}{\partial s} M_{ns}^*(S,Q) + \frac{\partial D(Q)}{\partial n} M_n^*(S,Q) \right) w(Q)\mathrm{d}\Gamma(Q) \\
&- \sum_{k=1}^{N_s} \frac{1}{D(S)} \int_{\Gamma_k} D(Q) M_n^*(S,Q) \frac{\partial w}{\partial n}(Q)\mathrm{d}\Gamma(Q) \\
&+ \sum_{C=1}^{N_c} \frac{D_C}{D(S)} R_C^*(S,C) w_C(C) \\
&= -\frac{1}{D(S)} \int_{\Omega} \left(2\frac{\partial D(q)}{\partial x_i} q_i^*(S,q) + \frac{\partial^2 D(q)}{\partial x_i \partial x_j} m_{ij}^*(S,q) \right) w(q)\mathrm{d}\Omega(q) \\
&= \int_{\Gamma} \left(V_n(Q) w^*(S,Q) - M_n(Q) \frac{\partial w^*}{\partial n}(S,Q) \right) \mathrm{d}\Gamma(Q) \\
&+ \sum_{C=1}^{N_c} R_C(C) w_C^*(S,C) + \int_{\Omega_g} (g(q) w^*(S,q))\mathrm{d}\Omega(q)
\end{aligned}
\tag{10}
$$

where the rigidity $D(S)$ at source point S was assumed as D_o, Γ_k is the sub-region boundary which may be defined to make possible abrupt rigidity variation, while N_s is the total number of sub-regions.

It is worth noting that only deflection and rotation values are required at points along the interface. The right-hand side integral over Γ is only performed over the external boundary. Thus, traction values along the interface were eliminated, automatically satisfying equilibrium conditions.

Equation (10) can be particularized for the simple case of zoned domain, in which the rigidities are constant over the sub-regions, therefore the first and second derivatives of the sub-region rigidity in Eq. (10) are assumed null. This particular case has already been discussed in previous publications where alternative strategies have been used to obtain the final integral representation [11, 12].

Thus, for a collocation point S_i belonging to sub-region i and considering only the case of abrupt rigidity changes, Eq. (10) is reduced to:

$$
\begin{aligned}
Cw(S_i) &+ \sum_{k=1}^{N_s} \frac{D_k}{D_i} \int_{\Gamma_{kj}} \left(V_n^* w - M_n^* \frac{\partial w}{\partial n} \right) \mathrm{d}\Gamma + \sum_{C=1}^{N_c} \frac{D_c}{D_i} R_C^* w_C \\
&= \int_{\Gamma} \left(V_n w^* - M_n \frac{\partial w^*}{\partial n} \right) \mathrm{d}\Gamma + \sum_{C=1}^{N_c} R_C w_C^* + \int_{\Omega_g} (g w^*)\mathrm{d}\Omega
\end{aligned}
\tag{11}
$$

where D_i is the rigidity of the sub-region Ω_i taken as the reference value and Γ_{kj} is the interface between the sub-region Ω_k and its adjacent sub-region Ω_j.

Figure 2 shows the case of a zoned domain with a narrow region, which can be degenerated to represent the behaviour of a beam element. For simplicity, single subscripts are adopted to represent the external boundary.

In order to write an appropriate number of equations relating boundary and interface values, the usual way is to differentiate equation (11) to obtain other integral representation, finding the free terms that may appear due to differentiation of strong singular kernels [13]. The slope integral representation is the usual relationship frequently adopted to complete the necessary number of relations for the corresponding algebraic system of equations. For an internal point S of sub-region Ω_i the slope equations is given by,

$$
\begin{aligned}
\partial w(s)/\partial m &+ \sum_{k=1}^{N_s} \frac{D_k}{D_i} \int_{\Gamma_{kj}} \left(\frac{\partial V_n^*}{\partial m} w - \frac{\partial M_n^*}{\partial m} \frac{\partial w}{\partial n} \right) \mathrm{d}\Gamma \\
&+ \sum_{C=1}^{N_c} \frac{D_c}{D_i} \frac{\partial R_C^*}{\partial m} w_C = \int_{\Gamma} \left(V_n \frac{\partial w^*}{\partial m} - M_n \frac{\partial^2 w^*}{\partial n \partial m} \right) \mathrm{d}\Gamma \\
&+ \sum_{C=1}^{N_c} R_C \frac{\partial w_C^*}{\partial m} + \int_{\Omega_g} \left(g \frac{\partial w^*}{\partial m} \right) \mathrm{d}\Omega
\end{aligned}
\tag{12}
$$

where m defines the slope direction.

For boundary and interface points the vector m must be properly defined according to the adopted rotation value.

As is well known in the case of constant thickness plate, boundary and interface values have to be approximated before algebraic relations can be obtained from the integral representations (11) and (12). Although using only algebraic deflection relations to find the system of equations has already proved to be an appropriate scheme for analysing plate bending problems [11], in the present formulation it is convenient to also use slope representations for collocations defined along the interfaces. Moreover, at corners and other boundary and interface nodes characterized by the presence of a discontinuity, double nodes are adopted. In this case, an extra equation must be written to make corner reaction and the corresponding deflection independent values. Thus, at those points where discontinuity is assumed, five independent equations are required: three deflection representations and two slope representations. For nodes along the external boundary, two deflection equations are adopted, one for the bound-

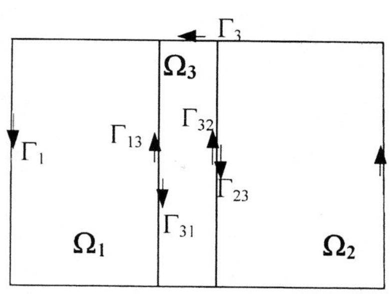

Fig. 2. Zoned domain with a narrow region representing a beam

ary node and the other for an external collocation very close to the boundary. For interface nodes, deflection and slope representations are adopted.

Free term definition is another important aspect requiring discussion. For simplicity, slope equations are only written for smooth boundary and interface nodes, which lead to simple values. However, it is difficult to avoid computing the free terms of the deflection equation for collocations defined at corners. In Fig. 3, the usual cases of external and internal corners for which one needs to derive the free terms are depicted, while the corresponding values are given in Table 1.

In Table 1, D_a is the rigidity of the adjacent sub-region, while β_2 and γ are internal angles of the corresponding sub-regions, values usual in BEM formulations.

To complete the necessary integral relations for the analysis of plate bending problems considering beam inclusion, one can differentiate Eq. (11) twice to obtain the curvature integral representation at internal points, as follows,

$$\begin{aligned}\frac{\partial^2 w(q_m)}{\partial x_i \partial x_j} &+ \sum_{k=1}^{N_s} \frac{D_k}{D_m} \int_{\Gamma_{k\ell}} \left(\frac{\partial^2 V_n^*}{\partial x_i \partial x_j} w - \frac{\partial^2 M_n^*}{\partial x_i \partial x_j} \frac{\partial w}{\partial n} \right) \mathrm{d}\Gamma \\ &+ \sum_{c=1}^{N_c} \frac{D_c}{D_m} \left(\frac{\partial^2 R_c}{\partial x_i \partial x_j} \right) w_c \\ = \int_{\Gamma} &\left(V_n \frac{\partial^2 w^*}{\partial x_i \partial x_j} - M_{nn} \frac{\partial^2}{\partial x_i \partial x_j} \left(\frac{\partial w^*}{\partial n} \right) \right) \mathrm{d}\Gamma \\ &+ \sum_{i=1}^{N_c} R_c \frac{\partial^2 w_c^*}{\partial x_i \partial x_j} + \int_{\Omega_g} \left(g \frac{\partial^2 w^*}{\partial x_i \partial x_j} \right) \mathrm{d}\Omega \end{aligned} \tag{13}$$

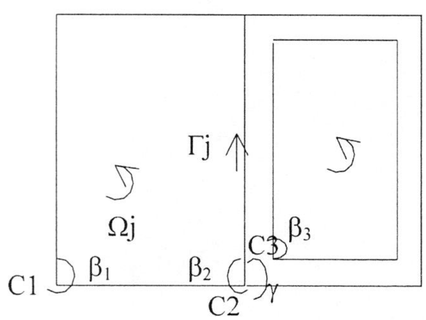

Fig. 3. Internal and external corner type cases

Table 1. Free terms $K(Q)$ for usual corners in zoned domains

$K(Q)$	Type of corner
0.5	Smooth external boundary collocation node Q
$\frac{1}{2}\left(1 + \frac{D_a}{D_j}\right)$	Smooth interface collocation node Q; coefficient referred to region Ω_j
$1 + \frac{\beta_3}{2\pi}\left(\frac{D_a}{D_j} - 1\right)$	Type C_3 corner collocation node Q
$\frac{\beta_2}{2\pi} + \frac{D_a}{D_j}\frac{\gamma}{2\pi}$	Type C_2 corner collocation node Q; β_2 is referred to the plate with rigidity D_j; γ is referred to the plate with rigidity D_a
$\frac{\beta}{2\pi}$	Corner collocation along external boundary; node type C_1

Bending and twisting moment integral representations are obtained from Eq. (13) by simply applying the definition given in Eq. (5). To complete the internal force values at internal points, one can differentiate Eq. (13) once more and apply the definition of shear forces of Eq. (6) to find its integral representation.

The derivatives appearing in Eq. (13) and in the shear representation must be performed carefully; free terms always arise when performing derivatives of strong and hypersingular integrals. Relevant information about the correct procedure for extending Eq. (13) can be found elsewhere [13].

4
Algebraic equations

As usual for any BEM formulation, the integral representations (11), (12), (13) and the ones written to compute internal forces can be transformed into algebraic expressions after discretizing the boundary into elements. In the present case, plate boundary and interface have been discretized into geometrically linear elements over which the boundary values have been approximated by quadratic shape functions. The approximations of the boundary and interface values enable us to write algebraic representations of deflections, rotations, moments, and shear forces for any collocation point taken inside the domain, along the boundary, along the interfaces, and outside the plate. After selecting an appropriate number of algebraic equations, one can assemble a convenient set of equations to solve the problem in terms of boundary and interface values. This set of algebraic equations has been defined by writing two relations per node plus an extra relation at each internal or external corner. Only the deflection representations are defined for boundary nodes while, for interface boundary nodes, slope representations are also adopted. The collocations related with boundary nodes are taken either along the boundary or outside the body, while for interface nodes the collocations are always defined along the interface line.

The outside collocations have been placed very close to the boundary. To guarantee that the algebraic representations are accurate, a sub-element scheme has been used to compute the boundary and interface element integrals. In this case, sub-element lengths must be smaller than half of the minimum distance of the collocation to any sub-element point. This scheme has proved to be very precise and guarantees the quality of the final results.

After performing the element integrals, the algebraic set of equations reads:

$$\mathbf{HU} = \mathbf{GP} + \mathbf{T} \tag{14}$$

where $\mathbf{U}$ contains deflection and rotation boundary and interface nodal values, while $\mathbf{P}$ contains boundary node tractions only; $\mathbf{T}$ is the independent vector due to the applied loads.

This scheme is simple and can be applied to analyse plates stiffened by beam elements accurately. One needs only to model the beams as narrow plate regions, which leads to a system of equations with a reduced number of unknowns. There are two important aspects that deserve special attention when using this scheme. Quasi-singular

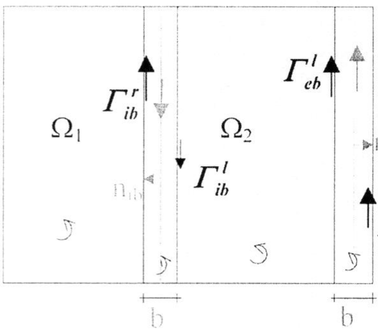

Fig. 4. Plate domain stiffened by beams represented by narrow regions

integrals are necessarily present due to the narrowness of some regions. They must be performed accurately by using either analytical expressions or numerical schemes with an appropriate sub-elementation procedure. For practical building floor applications, performing numerically the integrals has proved to be accurate enough.

When the beam is too narrow, the algebraic system of equations becomes unstable: displacement equations of any two close collocation points are so similar that they lead to inaccurate numerical solutions. In this case, to improve the quality of the results one has to eliminate all strongly dependent degrees of freedom, eliminating as well the corresponding algebraic relations. The simplest way to reduce the number of unknowns is given by assuming rigid rotation of the beam cross-section, making deflections and rotations of two opposite points no longer independent values. Instead, they are written in terms of the values of nodes defined along the beam skeleton lines. Rotation will be assumed constant over the section, while deflections are given by the skeleton values corrected according to the distance from this line. Figure 4 defines narrow regions as representing beams. The cross-section rigid rotation is illustrated in Fig. 5, from which one may write a simple relation between rotation, deflection values taken on the right (Γ^r_{ib}), left (Γ^l_{ib}) interfaces, and along the skeleton.

Thus, according to these kinematic assumptions, deflections and rotations along both sides (Γ^r_{ib} and Γ^l_{ib}) of the narrow region of an internal beam read:

$$\alpha_l = -\alpha_r = \alpha_c \tag{15a}$$

$$w_r = w_c + \alpha_c b/2, \quad w_\ell = w_c - \alpha_c b/2 \tag{15b}$$

where w_r, w_ℓ and w_c are deflections on the right, left, and along the skeleton, respectively, while α_r, α_ℓ and α_c are the rotations.

In the case of an external beam, the deflections along the sides (Γ^r_{eb} and Γ^l_{eb}) are also given by the expression (15b); however, for the rotations one has:

$$\alpha_l = \alpha_r = \alpha_c \tag{15c}$$

With this approximation only two algebraic relations are required at each beam node. Thus, the collocation point is now placed along the beam skeleton, while w_c and α_c are the assumed unknown values. The intersection regions between two or more beams were also assumed rigid and, consequently, only three displacements are left to describe

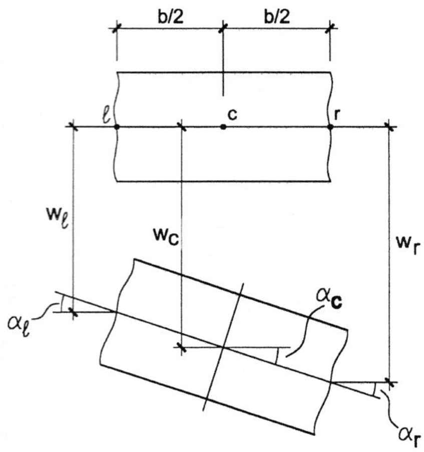

Fig. 5. Assumed deflections and rotations over the beam cross-section

the whole displacement filed over these regions. Several other alternatives have been tested, for which more degrees of freedom to model the intersection zones were required. It is important to observe that the skeleton line nodes and the geometry of the beam cross-sections are the only data required to define the narrow region, i.e., the beam. The two close actual interfaces can then be properly generated to perform the integrals. The final system of equations is now given in terms of four boundary values ($w, \partial w/\partial n$, M_n and V_n) and two interfaces values ($w, \partial w/\partial n$).

No limit to reduce the integrals to one single line has been set. The only approximation assumed is the rigid rotation given by Eq. (15). This scheme leads to very small system of algebraic equations without introducing too many variable approximations.

5 Examples

The stiffened plate analysed here and the two discretizations following the schemes described in the previous sections are defined in Fig. 6. One central beam plus two others placed along the boundary are adopted to stiffen this rectangular plate. The two stiffened sides are free, while the two others are simply supported. The load is given by the distributed moment $M_n = 10$ kNm/m applied along the simply supported sides. Plate and beam materials are assumed elastic with Young modulus $E = 3\,000\,000$ kN/cm^2 and Poisson ration $\nu = 0.16$. The plate thickness is $h_p = 8$ cm, while the beam heights are $h_b = 20$ cm. The discretizations are specified by: (a) using the scheme where interface collocations are considered, leading to 38 elements and 92 nodes (see Fig. 6b) and; (b) defining collocations only along the beam skeleton lines, leading to 22 elements and 36 nodes (see Fig. 6a). Figures 7 and 8 give the displacements and the moments in x and y directions, respectively. The values were obtained along the central beam (V_2) by using the two discussed discretization schemes. As one can see, the results given in terms of deflections obtained by using both schemes compare very well (Fig. 7). Comparing the moment distribution along the central beam obtained for both cases,

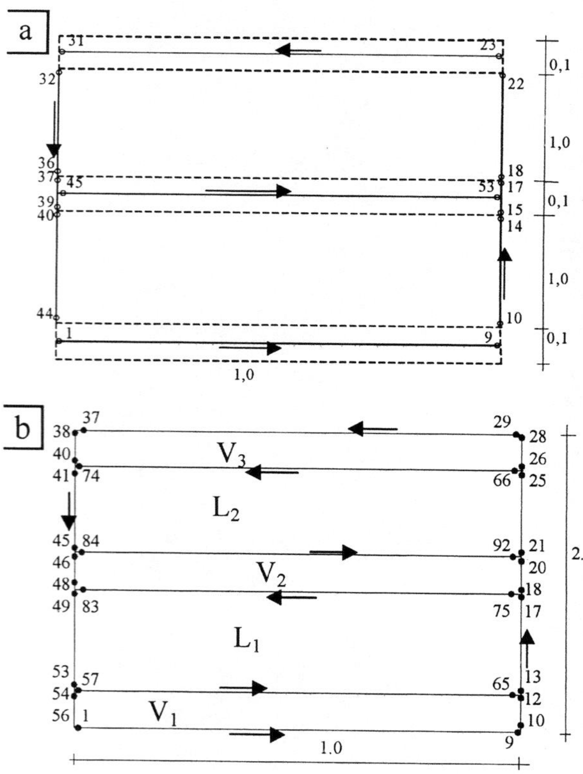

Fig. 6. Stiffened plate

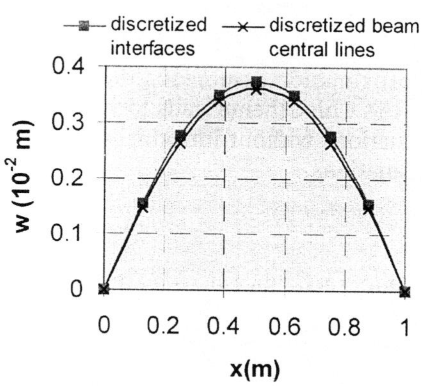

Fig. 7. Deflections along the central beam (V_2)

very small differences are observed (Fig. 8). Thus, the rigid rotation hypothesis assumed for the second scheme introduces negligible differences in the actual beam stiffness as well as in the whole structure. Assuming that the results are enough adequate for analysing stiffened building floors for instance, the second scheme is recommended since it requires a reduced number of algebraic equations, consequently reducing the computations.

To run the second example, we have borrowed several data from the previously analysed problem: the applied boundary moment M_n, the Young modulus E, the Poisson's ratio ν, the plate thickness h_p, and the beam height h_b. Here, the plate is stiffened by beams placed along all sides and by two internal ones, as seen in Fig. 9. Boundary values given in terms of moments are prescribed along the simply supported sides connected to beams V_1 and V_3; the

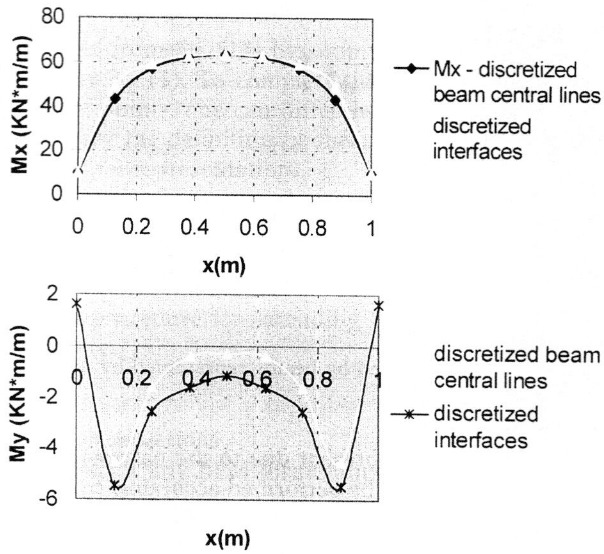

Fig. 8. Moments M_x and M_y along the central beam (V_2)

other two sides are free. The problem has also been solved by the two proposed schemes. Using the first scheme, characterized by defining collocations along the interfaces, 275 nodes were defined along 116 boundary and internal elements. When the second scheme is adopted, the total number of nodes is reduced to 123 with 53 elements (see Fig. 9). The results of both analyses in terms of deflections are displayed in Fig. 10, while Fig. 11 gives the corresponding distribution of moments, M_x and M_y, computed along the central beam (V_5). Again, the computed deflections using both schemes compare very well. The numerical results in terms of moment components appear to be quite reasonable for this case. The only difficult appears to be the accurate computation of the moment values at the beam intersection. The differences observed at this point are introduced by the assumed rigid rotations.

6
Conclusions

The BEM formulation for analysing zoned plate-bending problem has been extended to deal with plate stiffened by beams. Beam rigidity is taken into account by assuming narrow sub-regions, without dividing the stiffened plate into beam and plate elements. Equilibrium and compatibility conditions are automatically guaranteed by the global integral equations. Two schemes regarding the displacement approximation assumed along the beams were proposed. The formulation avoids unnecessary approximations usually present when treating this problem with the standard sub-region technique, which reduces the number of unknowns and increases the accuracy of the results. The rigid cross-section assumption made for the second scheme reduces even further the number of unknowns and preserves the quality of the results when analysing stiffened plates often adopted for building floor structures showing their adequacy for dealing with practical problems, thus encouraging the BEM researchers to go ahead with this kind of formulation.

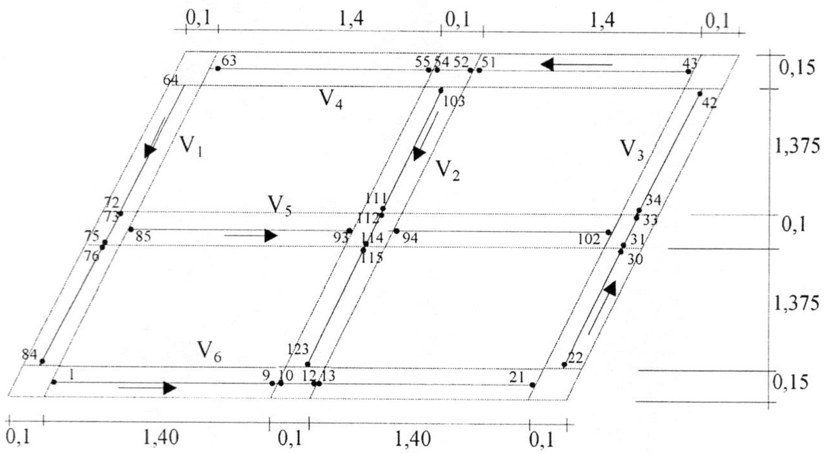

Fig. 9. Stiffened plate

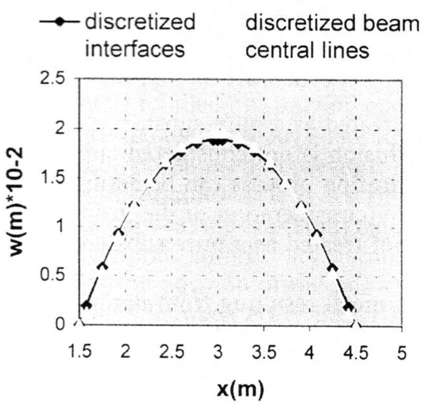

Fig. 10. Deflections along the central beam (V_5)

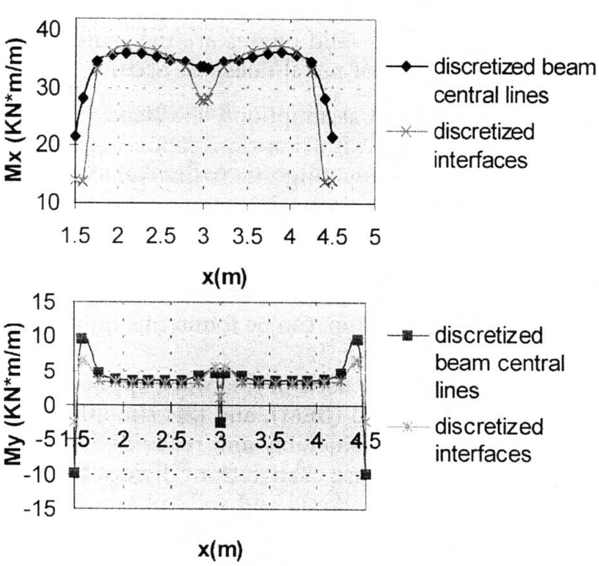

Fig. 11. Moments M_x and M_y along the central beam (V_5)

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