CILAMCE

Ouro Preto/MG - Brazil

2003

# XXIV IBERIAN LATIN-AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING

# CILAMCE 2003

CILAMCE 2003 was held in the historical and beautiful city of Ouro Preto, Minas Gerais, Brazil. This traditional event started as a conference for Civil Engineers in Rio de Janeiro in 1977 and now involves different engineering applications. The conference aims to create a forum in which engineers, researchers and students can exchange ideas and information about the computational methods, currently available systems and improvements in computer technology to solve various complex practical and theorical engineering problems. The CILAMCE conference has played a major role in the dissemination of the most recent computational applications and computational developments in engineering among professionals, researchers and students of the Iberian Latin American community. CILAMCE 2003, the 24st CILAMCE Conference, is being organized by the Department of Civil Engineering, School of Mines, of the University Federal of Ouro Preto (UFOP).

CONGRESS DATE AND VENUE

The XXIV CILAMCE was held from October 29-31, 2003, at the Parque Metalúrgico - Arts and Convention Center of Federal University of Ouro Preto, in the city of Ouro Preto, State of Minas Gerais, Brazil. Parque Metalúrgico - Arts and Convention Center was created in 2001, after restoration of an old building which belonged to the University and was used mainly for metallurgical operations. Now it places Ouro Preto as a major place for conferences, seminars and business-oriented tourism. For the plenary and keynote lectures a 300-seat auditorium will be used, and for the technical contributions, six 50-seat rooms will be available. The main theatre with 700 seats may be used for the cultural events.

SCIENTIFIC PROGRAM

The scientific program consists of a plenary opening lecture (October 29, 2003), a plenary closing lecture (October 31, 2003), six keynote lectures and contributed papers presented in lecture sessions. Invitations to present contributed papers were made on recommendation by the Scientific Advisory Board, based on their review of submitted abstracts.

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# ANALYTIC ELEMENT MODEL FOR GROUNDWATER FLOW IN THE AQUIFERS' RECHARGE ZONE - PART 2: CONFINED AQUIFERS

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***Abstract.*** *The analytic element method extension presented in the first part of this paper is generalized by taking the effect of the recharges due to rainfall into account in the case of unsteady groundwater flow through confined aquifers recharge zone. The Analytic Element Method (AEM), as described previously, is a computational technique which permits to superimpose analytic solutions of linear equations of regional-wide scale features encountered in aquifers. Aquifer recharge zones look usually like strips, remembering coastal zones, but the way with they release the flow is different from coastal aquifers. We firstly develop a mathematical model for the unsteady rainfall-recharge effect and then, in this study, implement it in an analytic element open source program (TIMSL 0.3) for a comparison between different aquifer regional boundary conditions. The differences observed between the results obtained for entrance confined and unconfined aquifers are significant.*

***Keywords:*** *Environmental flow, Groundwater modeling, Analytic elements*

## 1. INTRODUCTION

Aquifers' recharge zone often presents a strip shape. Aquifers in inland show a wider spectrum of shape occurrence than coastal aquifers, depending on the place and order of the sediment deposition. However, some of important aquifers in Brazil present its recharge zone in a long and narrow strip shape, attaining regional scales. The most popular example in Brazil is the "Guarany Aquifer", considered the biggest one in South America. Its recharge zone involves five Brazilian States, covering a distance of 3.000 Km with a mean width of 29Km (87000Km$^2$). Thus, any regional model considered for this aquifer recharge zone, must take into account the characteristics of the regional boundary conditions.

Once such areas (narrow strip areas) need to be modeled in regional scales, the Analytic Element Method (AEM) seems to be an interesting tool in this sense. In the AEM each feature of the aquifer is modeled individually and the *local* recharge areas are so far considered in the literature.

The main regional recharge sources are generally due to rainfall. Although rainfall recharge happens over all the aquifer top, with all its inhomogeneities, it can be considered by only one "element" in the model instead of several local recharge elements (area-sinks).

References to the narrow strip recharge phenomena are usually described in the literature as occurring over an areal domain of an infinite phreatic aquifer (Polubarinova-Kochina, 1962 and Hantush, 1967). The mentioned authors considered the linearized Boussinesq equation (the Heat Equation) and represented groundwater head responses to unsteady recharges. In the present study a situation of entrance from a phreatic aquifer to a confined aquifer is considered, which represents the zone of recharge due to rainfall of confined aquifers. In such areas, the flow which enters into the confined part differs significantly from the entrance characteristics of flows into phreatic aquifers.

## 2. MATHEMATICAL MODEL FOR CONFINED AQUIFER RECHARGE

Recharge due to rainfall happens over regions where groundwater flow takes place under atmospheric pressure. On reaching the saturated zone, the water is driven to the aquifer's confined region and the flow becomes under pressure. To characterize the relationship between confined and unconfined flow, we shall seek the *potential flow* formula as follows:

$$\Phi' = \frac{Kh'^2}{2} \tag{1}$$

$$\Phi'' = Kbh'' \tag{2}$$

Where $h'$ and $h''$ are the hydraulic head in the unconfined and confined regions, respectively.

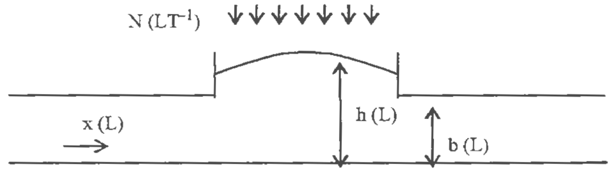


Figure 1 – Hydraulic scheme of confined aquifer recharge

In the picture, we denote $x$ the positions taken in the entire infinite domain and the positive flow direction (L); $h$, the hydraulic head (L); $b$, the confined aquifer thickness (L) and $N$, the flow accretion ($LT^{-1}$). The only source in the problem is the accretion ($N$) in the aquifer recharge zone. The discharge $Q$ ($L^2T^{-1}$), Eq. 3 and 4, at the left side equals the discharge at the right side. In order to express the flow for these conditions, we verify *potential flow* continuity, $\Phi' = \Phi''$ around the boundaries, by expressing the discharge continuity, $Q' = Q''$ around the boundaries as well. Thus, in terms of head using Darcy's law with $K$ ($LT^{-1}$) the hydraulic conductivity:

$$Q' = -Kh'\frac{dh'}{dx} \tag{3}$$

$$Q'' = -Kb\frac{dh''}{dx} \tag{4}$$

Focusing only the domain of interest (the recharge zone) and we denote positions within the recharge zone as $\xi$. Thus the solution of head distribution for steady flow within $\xi_a < \xi < \xi_b$ may be written as follows:

$$\Phi(\xi) = -\frac{N}{2}(\xi^2 - \xi_0^2) + \frac{N}{2}(\xi_a + \xi_b)(\xi - \xi_0) + \Phi_0 \tag{5}$$

$\xi_a$ and $\xi_b$ are the position of the recharge area boundaries; $\xi_0$ is the position in the domain where the potential $\Phi_0$ may be previously determined.

Boussinesq equation (Eq. 6) expresses the unsteady situation in which $N$ is time-dependent in a finite scope of position $x$. The linearized Boussinesq equation (Eq. 7) is solved in Polubarinova-Kochina (1962) for a spreading strip.

$$\tfrac{1}{2}K\frac{\partial^2 h^2}{\partial \xi^2} + N = n\frac{\partial h}{\partial \tau} \tag{6}$$

$$a^2\frac{\partial^2 h}{\partial \xi^2} + \frac{N}{n} = \frac{\partial h}{\partial \tau} \tag{7}$$

Where $a^2 = \frac{HK}{n}$ with $H$, the mean head in the time ($\tau$) and the domain ($\xi$), $K$ is the hydraulic conductivity and $n$, the storage coefficient of the matrix soil for phreatic flow, it is the porosity.

The differential equation for the potential $\Phi = \frac{Kh^2}{2}$ is obtained in an identical form as the linearized Boussinesq resulting:

$$a^2 \frac{\partial^2 \Phi}{\partial \xi^2} + \frac{NHK}{n} = \frac{\partial \Phi}{\partial \tau} \tag{8}$$

Though the differential equation was written by Polubarinova-Kochina (1962) for hydraulic heads, the solution may also be translated in terms of potential, yielding:

$$\Phi(\xi,\tau) = \frac{Na^2\tau}{\sqrt{\pi}}\left\{ erf\left(\frac{L-\xi}{2a\tau}\right) - \frac{(L-\xi)^2}{2L^2} erfc\left(\frac{L-\xi}{2a\tau}\right) + \frac{(L-\tau)}{a\sqrt{\pi\tau}}\exp\left[-\frac{(L-\xi)^2}{4a^2\tau}\right] + \right.$$
$$\left. + erf\left(\frac{L+\xi}{2a\tau}\right) - \frac{(L+\xi)^2}{2L^2} erfc\left(\frac{L+\xi}{2a\tau}\right) + \frac{(L+\tau)}{a\sqrt{\pi\tau}}\exp\left[-\frac{(L-\xi)^2}{4a^2\tau}\right]\right\} \tag{9}$$

Where, $a^2 = \frac{HK}{n}$ is called the hydraulic diffusivity.

## 3. THE TIMSL PROGRAM

An Universal Modular Language (UML) diagram of the basic design of TIMSL is shown in Figure 2. All class names start with a capital letter and follow camelback notation (such as HeadWell). Package or module names do not contain capitals, intending to make it easy to distinguish them. Packages are implemented in order to contain related modules (underlined in Figure 2), which implements their related classes with their required methods and attributes. The TimSL code is extended here with two modules with the same name "contarealrec" in the single package and the transient package.

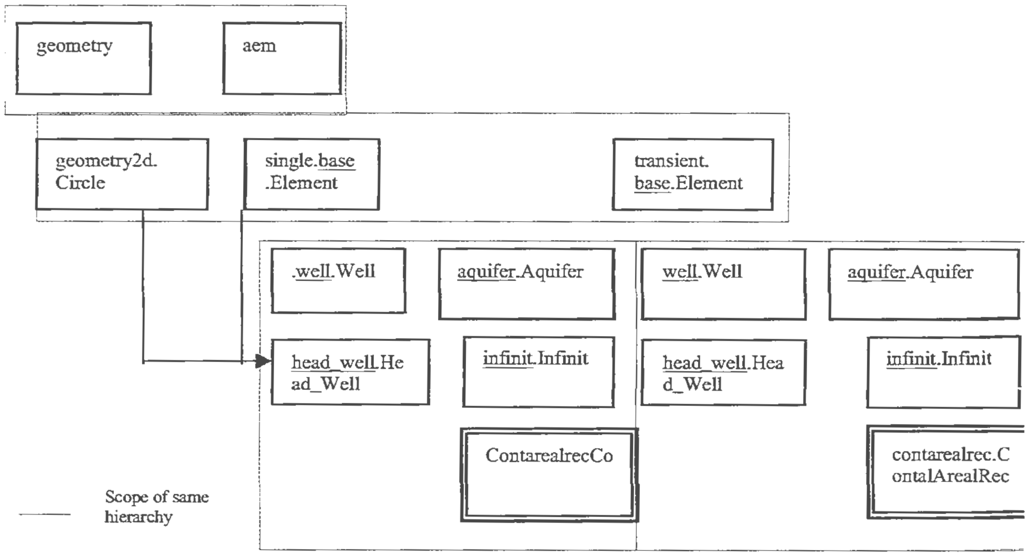


Figure 02 – Extention to TimSL - UML Diagram

## 4. OBJECT PROBLEM

In order to get more insight on practical situations, let consider the same situation given in part I of this paper, as stated bellow:

"A well produces 72x10$^3$ m$^3$/month and is located at some pond neighborhood like disposed in the Figure 3. The pond water budget is given in 230x10$^3$m$^3$/month negatively. This area receives constantly 5 mm/month uniformly distributed over a confined aquifer recharge zone. In order to do comparison with coastal aquifers, take $h_0$=0.0m at the exit boundary, which is 24Km distant from impervious wall and the impervious base at quote 20m below. What should be the steady water level in the pond?"

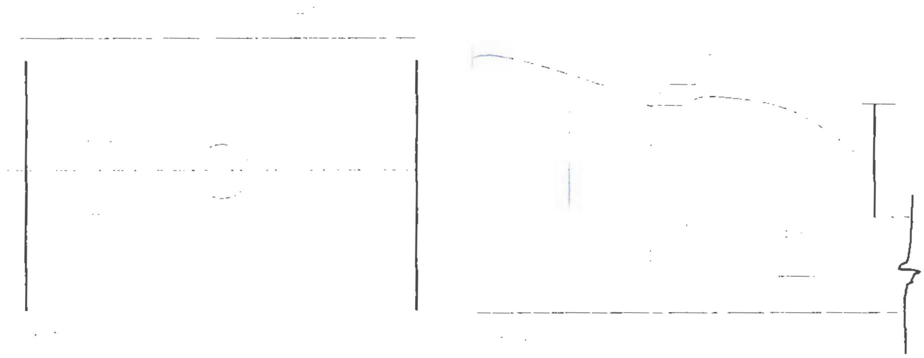

Figure 3 – Didactic Scheme

This region is treated here as a recharge area. To verify the usefulness of the AEM for different boundary conditions and its sensitivity to slight variations related to these boundaries, the entrance region to unconfined and confined aquifers is considered here.

Figure 4 shows the solution of the stated problem for steady state condition and fixed $\Phi_0$

of Eq. 5. This is the entrance of a confined aquifer. The asked pond level has the quote of 61.88m, obtained for a semi-infinite domain.

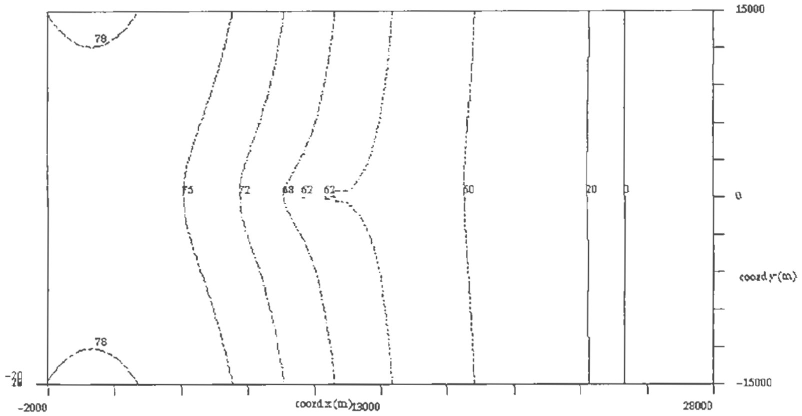


Figure 4 – Contour lines for steady state flow in a coastal aquifer

Figure 5, on the other hand, shows the exit boundary condition treated in the part I of the paper, represented by a constant hydraulic head. In fact the contour lines for the water table coincide exhibiting the expected situation for steady states, because it depends on only one reference head (confined or unconfined), and, of course, the hydraulic parameters.

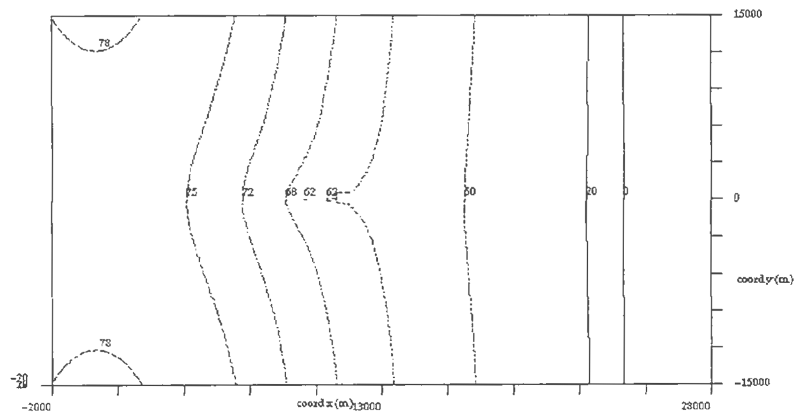


Figure 5 – Contour lines for steady state flow in a continental aquifer

However, the situation changes when the unsteady situation is considered. The object problem is posed again with the same hydraulic parameters that before with the confined aquifer entrance at the boundary. We have stated the problem for:

"Supposing the same hydraulic parameters yet, let the recharge due to rainfall be null during 42 months due a dry period. As a consequence, 24 months after, the local water resource agency decides to reduce the well pumping to 50% of the mean yielding (given in the steady state situation). In addition, because of this dry weather, the water budget is still decreased to minus $3x10^5$ $m^3$/month. When the dry season finishes the recharges reaches 10 mm/month and the water budget is elevated to $1.5 \times 10^5$ $m^3$/month (still negative). Thus the water resources agency allows, 6 months after the end of the dry period, the yield to back to the mean values. How does the water level in the pond varies with time? Furthermore, what is the pond water level 30 months after the end of the dry? Table 1 shows the data summary for this problem"

Table 1 – Summarized data

| Time | Recharge (mm/month) | Lake's Water Budget ($m^3$/month) | Well's Discharge ($m^3$/mês) |
|---|---|---|---|
| Initial Condition | 5 | 230000 | 72000 |
| 1st month (dry) | 0 | 300000 | 72000 |
| 25th month (dry) | 0 | 300000 | 36000 |
| 43rd month (wet) | 10 | 150000 | 36000 |
| 49th month (wet) | 10 | 150000 | 72000 |

The results point to differences between the considered domains. For the 72th month, the water level met in the pond for confined aquifer recharge zones is at quote 62.25m, which differs -0.05m from the coastal boundary condition case. The greatest difference during this analysis is 0.14m. In Figure 6 the water level is presented as a comparison between results for both boundary conditions. The results show that the AEM is sensitive to slight variations at the boundary conditions. Additionally it is observed that inland recharge areas, represented by confined aquifer entrances may present dynamic behaviors of the water table which differs from coastal situations, a result which is not observed for steady state cases.

It is shown two profiles (Figure 7), for unconfined aquifers entrance zones (coast) and for confined aquifers entrance zone (inland), using the data of the object problem. Both profiles show the differences between the water table at the end of the dry period and its beginning (or the steady state situation).

A largest difference at the flow exit boundary itself is exhibited. For the object, we find a difference of 2.53 m. An increasing difference is observed with the distance from the impervious wall.

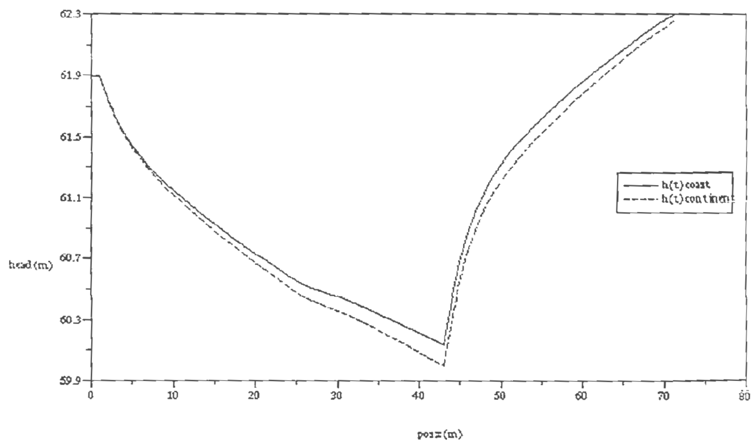


Figure 6 – Comparison of the head evolution in both coastal and continental aquifers

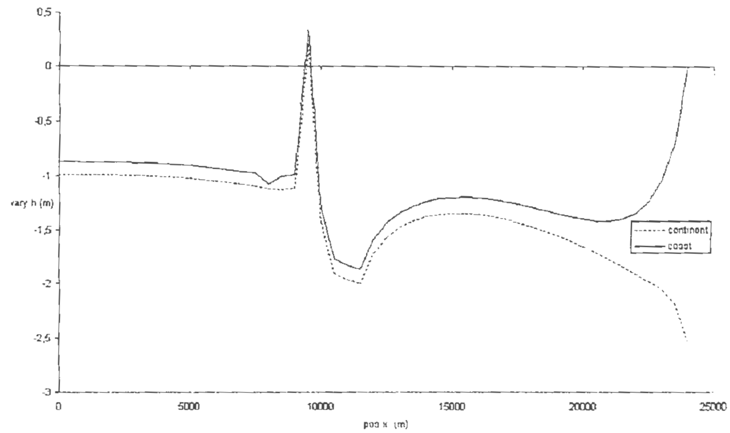


Figure 7 – Comparison of head profile on the *x*-axis for both coastal and continental aquifers

## 5. CONCLUDING REMARKS

Water resource designs in aquifer recharge zones may fall into errors if the boundary conditions do not represent the regional aquifer condition. It is shown that different boundary regional conditions in phreatic aquifers, which conduce to similar behaviors for steady-states situations, may produce different results for unsteady states. It is still important to mention that inland recharge zones represent a challenge for large confined aquifers. In Brazil the recharge zone of the "Guarany Aquifer" overtakes political boundaries even internationally. The effective water entrance from recharge zones to the aquifer confined parts passes necessarily through modeling characteristics of such outlet zones.

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