

Computational Methods in Engineering' 99



Edited by
Paulo M. Pimenta,
Reyolando M. L. R. F. Brasil,
Edgard S. Almeida Neto

Departamento de Engenharia de Estruturas e Fundações
Escola Politécnica da Universidade de São Paulo

Installation on IBM-PC platforms

Windows 95/98 Installation (with Autorun)

- 1. Put the CD in your CD-ROM drive**
- 2. Wait for the initial screen**

Windows 95/98 Installation (without Autorun)

- 1. Put the CD in your CD-ROM drive**
- 2. Double-click in "My Computer" on your desktop**
- 3. Double-click in your CD-ROM drive**
- 4. Double-click in setup.exe**
- 5. Wait for the initial screen**

Windows 3.1x Installation

- 1. Put the CD in your CD-ROM drive**
- 2. From the Program Manager File menu, chose Run**
- 3. In the Run dialog box, type the following command:**
<Letter of your CD-ROM drive>:\setup.exe
(e.g.: D:\setup.exe)
- 4. Wait for the initial screen**

NT Installation Note:

**In order to install Acrobat Reader (the viewer for the papers)
you must log in with "administrator" privileges.**

F952 i

IMPLICIT AND EXPLICIT ELASTOPLASTIC BEM FORMULATIONS APPLIED TO GRADIENT PLASTICITY

Carlos A. Fudoli*, Wilson S. Venturini**, Ahmed Benallal†

*CENAPAD-SP, State University of Campinas – UNICAMP
Rua Hum 45, 13083-970 Campinas, Brasil
e-mail: fudoli@cenapad.unicamp.br

**São Carlos School of Engineering, University of São Paulo
Av. Carlos Botelho 1465, 13560 São Carlos, Br, e-mail: venturin@sc.usp.br

†Laboratoire de Mécanique et Technologie, E.N.S.-Cachan/C.N.R.S./Paris 6
61 Avenue du Président Wilson, 94235 Cachan cedex,
e-mail: Ahmed.Benallal.ens-cachan.fr

Key words: Boundary Elements, Non-linear Problems, Localization.

Abstract: *In this paper explicit and implicit BEM formulations are extended to incorporate the analysis of bodies where localization phenomena occur. In order to overcome the difficulties associated to local continua, a gradient plasticity model is used. This model involves the second gradient of the cumulated plastic strain in the yield condition. Integral representations for the stress field in plasticity are derived for internal as well as for boundary points. Algebraic systems derived from the integral displacement and plastic multiplier representations are coupled and solved simultaneously. Numerical examples are shown to illustrate the use of BEM in dealing with localization phenomena. Expected mesh dependency for local plasticity is underlined and first results with gradient model are presented and discussed.*

1068 420
270300

SYSNO	1068 420
PROD	003073
ACERVO EESC	

1 INTRODUCTION

Along the last three decades, the Boundary Element Method (BEM) has proved to be appropriate to deal with an enormous number of engineering problems. The technique is nowadays a well-established procedure for the analysis of many practical engineering applications. In particular, the use of BEM to analyse non-linear problems has deserved special attention.

Non-linear phenomena, such as plasticity and visco-plasticity for instance, were treated by BEM in the early eighties^{1,2,3}, after the correct obtention of the free term for the initial strain tensor made by Bui⁴. As the BEM formulations work on the stress space, it is expected that numerical solutions for non-linear analysis are better than other techniques that require differentiation of shape functions to compute the stress field. The boundary element method has already proved to be able to compute high gradients and stress and strain concentrations. Thus, the BEM formulations might be recommended for non-linear analysis that exhibits the mentioned characteristics.

Although proving to give good results, the BEM non-linear approaches, appearing before this decade, were all based on the very simple explicit scheme accomplished by constant matrix procedures. The results obtained by using those simple models seemed to be precise and may suggest that BEM does not require to use better approaches.

Implicit approaches have been proposed more recently. Jim et al.⁵ have used implicit integration for BEM finite deformation plasticity. Telles & Carrer^{6,7} have also proposed an implicit model to solve elasto-plastic problems in the context of dynamic analysis for which they followed mass matrix approach. The CTO (Consistent Tangent Operator) has been introduced recently in the boundary element technique by Bonnet & Mukherjee⁸ using a scheme similar to the one proposed by Simo & Taylor⁹ for finite elements.

One aspect forgotten by the boundary element community up to now is concerned with strain localisation phenomena. This problem is certainly appropriate to be analysed by BEM; it exhibits small areas of interest inside the body, where the dissipation of energy occurs, as well as rather large displacement gradients. Analysis of strain localization has been an important subject in the attempt to improve numerical simulation of structural failures. Material behaviours characterised by constitutive relations that exhibit a softening branch (or a non-associated behaviour) bring great difficulties to classical (local) continuum theories in the description of localisation phenomena¹⁰. The associated boundary value-problem is actually no longer mathematically well posed¹¹ after the onset of localisation, and local continua allow for an infinitely small bandwidth in shear or in front of a crack tip. At the numerical level, these difficulties translate into pathological mesh dependence of solutions^{10,12}. Different approaches have been proposed to overcome these difficulties. One idea is to enrich the continuum with non conventional constitutive relations in such a way that an internal or characteristic length scale is introduced. Cosserat continua¹³, higher gradient theories¹⁵, non-local theories^{16,17,18,19} are among such enrichments.

In this paper, explicit and implicit BEM formulations will be presented regarding their ability to describe localisation phenomena. In the implicit case, and for J_2 elasto-plastic problems, the consistent tangent operator is constructed. In order to overcome the difficulties

associated to local continua, a gradient plasticity model is adopted. In finite element context, gradient plasticity has received a great amount of attention in the last years, however, there has been none or only limited interest to apply BEM to analyse this kind of problems, without results published so far²⁰.

Examples are solved to illustrate classical pathological mesh dependence associated to local continua. Afterward, numerical examples are shown to illustrate the proposed BEM gradient plasticity formulations.

2 CONSTITUTIVE EQUATIONS FOR GRADIENT PLASTICITY

The small strain gradient plasticity model taken for this work is a simple modification of the flow theory of plasticity^{18,21}. This classical approach is given by the following relations:

- The Cauchy stress tensor increment is given by

$$\dot{\sigma} = C : (\dot{\epsilon} - \dot{\epsilon}^p) \quad (1)$$

where $\dot{\epsilon}$ is the total strain rate, $\dot{\epsilon}^p$ stands for the plastic strain rate and C is the matrix of elastic moduli.

- The yield criterion reads

$$f(\sigma, R(p)) \leq 0 \quad (2)$$

where R is the size of the yield surface and p the cumulated plastic strain defined by:

$$\dot{p} = \sqrt{\frac{2}{3} \dot{\epsilon}^p : \dot{\epsilon}^p} \quad (3)$$

- Plastic flow is given by the normality rule to the plastic potential F , i.e.

$$\dot{\epsilon}^p = \dot{\lambda} \frac{\partial F}{\partial \sigma} \quad (4)$$

where λ is the plastic multiplier;

- The complementary rule

$$\dot{p} = \dot{\lambda} \frac{\partial F}{\partial R} \quad (5)$$

The plastic multiplier in equations (4) and (5) satisfies the Kuhn-Tucker conditions:

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} f = 0 \quad (6a,b,c)$$

When λ is positive, it is obtained by the consistency condition, i.e.

$$\dot{f} = 0 \quad (7)$$

Then, using relations (1)-(5), one obtains:

$$\dot{\lambda} = \frac{\frac{\partial f}{\partial \sigma} : C : \dot{\epsilon}}{h + \frac{\partial f}{\partial \sigma} : C : \frac{\partial F}{\partial \sigma}} \quad (8)$$

where we have defined the plastic modulus h ,

$$h = - \frac{\partial f}{\partial R} \frac{\partial R}{\partial p} \frac{\partial F}{\partial R} \quad (9)$$

Hardening behaviour is defined by a function $R(p)$.

The small strain gradient plasticity is obtained just by modifying equation (2), to make R dependent on p , as well as on its successive gradients. For the sake of simplicity, we consider that R depends just on p and its Laplacian $\nabla^2 p$, therefore the yield criterion becomes:

$$f(\sigma, p, \nabla^2) = 0. \quad (10)$$

Thus an explicit form for λ similar to equation (8) can not be derived. From the consistency condition, one is rather able to derive the following partial differential equation

$$\frac{\partial f}{\partial \sigma} : C : \dot{\epsilon} - H\lambda + \omega \nabla^2 \lambda = 0 \quad (11)$$

where

$$H = h + \frac{\partial f}{\partial \sigma} : C : \frac{\partial F}{\partial \sigma}, \quad \omega = \frac{\partial f}{\partial R} \frac{\partial R}{\partial (\nabla^2 p)} \frac{\partial F}{\partial R} \quad (12a,b)$$

From equation (12) one can realise that the dimension of ω is H times squared length, which gives $\omega = \alpha \ell^2$, being ℓ a characteristic length and α a material parameter.

One important aspect to discuss here is the boundary conditions to be assumed to solve the above partial differential equation. There are clearly two types of boundaries to consider: the region where the plastic zone reaches the actual boundary of the body and the interface between the elastic and plastic regions. For the first case, we assumed that the outward normal flux is zero, while the plastic multiplier boundary values are unknown. In the second case, we assume that the plastic multiplier values are zero, while the normal fluxes are unknown.

Along the actual body boundary and along the internal plastic zone interface one can assume the following boundary conditions, respectively:

$$\frac{\partial \lambda}{\partial n} = 0 \quad \text{and} \quad \lambda = 0 \quad (13)$$

For this work here, we consider a J_2 plasticity model where the yield condition is given by

$$f(\sigma, p, \Delta p) = J_2(\sigma) - Hp + \omega \nabla^2 p \quad (14)$$

with the hardening parameter H and ω constants.

3 INTEGRAL REPRESENTATION FOR GRADIENT PLASTICITY

3.1 Integral representation of displacements and stresses

Let us first consider an elastic body associated with many possible elastic states satisfying the Navier's equations, i.e.:

$$(-L_{ij}u_j) = -C_{ijkl}u_{k,lj} = -Gu_{i,ii} - \frac{G}{1-2\nu}u_{i,ii} = b_i \quad (15)$$

where u_k represents the components of the displacement field; G is the shear modulus and ν is Poisson's ratio.

For a domain Ω with boundary Γ , standard integral representations are derived by applying Green's second identity (Betti's principle). Particularly, displacement and stress integral representations are easily derived and may be found in Brebbia³:

$$c_{ik}u_k = -\int_{\Gamma} p_{ik}^* u_k d\Gamma + \int_{\Gamma} u_{ik}^* p_k d\Gamma + \int_{\Omega} u_{ik}^* b_k d\Omega \quad (16)$$

$$\beta \sigma_{ij} = -\int_{\Gamma} S_{ijk} u_k d\Gamma + \int_{\Gamma} D_{ijk} p_k d\Gamma + \int_{\Omega} D_{ijk} b_k d\Omega \quad (17)$$

p_k and b_k are traction and body force components respectively; the symbol "*" is related to the fundamental solution corresponding to a Dirac delta load applied in the collocation point (the second considered elastic state); the free terms c_{ik} and β are dependent upon the boundary geometry; D_{ijk} and S_{ijk} are kernels derived from equation (16).

For non-linear problems, Betti's principle can not be directly applied. Moreover, in plasticity, the state variables are history dependent. In this case, splitting the total strain into its elastic and plastic components, the Navier operator applies only to the elastic part, so that equation (15) becomes:

$$-L_{ij}\dot{u}_j = \dot{b}_i - C_{ijkl}\dot{\varepsilon}_{kl,j}^p \quad (18)$$

where ε_{kl}^p is the plastic strain tensor.

In equation (18), the plastic strains act therefore as fictitious body forces. This is referred as the initial stress method and usually equation (18) is written as

$$-L_{ij}\dot{u}_j = \dot{b}_i - \dot{\sigma}_{ij,j}^p \quad (19)$$

Thus, Green's second identity is now applied, leading to the Somigliana's identity for plasticity based on the initial stress approach, as follows:

$$c_{ik}\dot{u}_k = \int_{\Gamma} u_{ik}^* \dot{p}_k d\Gamma - \int_{\Gamma} p_{ik}^* \dot{u}_k d\Gamma + \int_{\Omega} u_{ik}^* \dot{b}_k d\Omega + \int_{\Omega} \varepsilon_{ijk}^* \dot{\sigma}_{jk}^p d\Omega \quad (20)$$

As for elasticity, the integral representation of stress rates can be obtained by differentiating Green's second identity with respect to space co-ordinates and applying Hooke's law. Thus, one obtains,

$$\dot{\sigma}_{ij} = \int_{\Gamma} D_{ijk} \dot{p}_k d\Gamma - \int_{\Gamma} S_{ijk} \dot{u}_k d\Gamma + \int_{\Omega} D_{ijk} \dot{b} d\Omega + \int_{\Omega} E_{ijmk} \dot{\sigma}_{mk}^p d\Omega + g_{ij}(\dot{\sigma}_{mk}^p) \quad (21)$$

where the kernel E_{ijmk} comes from the differentiation of the plastic integral and $g_{ij}(\dot{\sigma}_{mk}^p)$ is a free-term that appears due to the strong singularity of this kernel.

Equation (21) was derived only for internal points. For boundary nodes, one must find the limit when q , internal collocation point, goes to Q , on the boundary. Several schemes have already been discussed to derive the stress representation for boundary points. The simplest scheme very often adopted consists of writing only the algebraic representation using traction components (Cauchy's formula) and numerical differences of displacements. In order to obtain more accurate results we are adopting a scheme to compute boundary stresses by using a proper integral representation.

Although several alternatives could be followed, depending on the way the hypersingular terms are treated (reduced kernels, addition of an extra boundary value, etc.²²), we decided to work on the hypersingular term to transform it into a regularised one. For this scheme, the continuity of the displacement derivatives at collocation points taken along the boundary must be assumed (Guiggiani²³). Using this alternative, we have been forced to define stresses at points inside the element. We can therefore apply the algorithm described by Guiggiani to integrate the hypersingular kernel over the intrinsic space by using an expansion in terms of a Laurent series at the source points.

It is important to mention that to complete the integral representation a new free term corresponding to boundary points has been derived as well. The free term for smooth boundary nodes is given in 2D situations by:

$$g_{ij}(\dot{\sigma}_{mk}^p) = -\frac{1}{8} (2(1+\nu) \dot{\sigma}_{ij}^p + (1-3\nu) \dot{\sigma}_{ll}^p \delta_{ij}) \quad (22)$$

3.2 Integral representation of the plastic multiplier

In order to solve a gradient plasticity problem one has to take into consideration equation (11) that governs the plastic multiplier. In this case, the scalar value λ is not dependent only upon the local state of stress, as assumed by the standard procedure, equation (8), but it represents the solution of a scalar partial differential equation (11).

Transforming that differential equation (11) into an integral representation is not difficult. One must only follow the usual steps given in well-known references³. For this work, we are going to use the collocation formulation. Even choosing this simple formulation, we could have several representations by adopting convenient different fundamental solutions. For sake of simplicity, we limit the analysis here to two-dimensional situations. Extension to three-dimensional setting poses no difficulty.

The fundamental solution to be adopted to derive an integral representation is given by the following equation:

$$-\nabla^2 \lambda^*(q, p) + \frac{H}{\omega} \lambda^*(q, p) = \delta(q, p) \quad (23)$$

for the infinite 2D domain with an unit load applied at a single point q . Thus, the fundamental solution for this case is given by

$$\lambda^* = \frac{I}{2\pi} K_0\left(\sqrt{\frac{\omega}{H}} r\right) \quad (24)$$

where K_0 is the modified zero-order Bessel function of the second kind and r is the distance between load and field points (q, p) .

From equation (11) one is able to derive an integral representation for the plastic multiplier λ as

$$c\lambda = \int_{\Gamma} \lambda^* \frac{\partial \lambda}{\partial n} d\Gamma - \int_{\Gamma} \lambda \frac{\partial \lambda^*}{\partial n} d\Gamma + \frac{I}{\omega} \int_{\Omega} \lambda^* \frac{\partial f}{\partial \sigma} : E : \dot{\epsilon} d\Omega \quad (25)$$

where c is a free term similar the one used in equation (16).

Equation (25) governs the plastic multiplier field in the plastic zone, i.e., over the region where the plastic phenomenon takes place. Thus, the adopted boundary, Γ_p , in equations (25) represents the plastic zone boundary, which can move during the loading process. This integral equation is clearly the appropriate representation of a moving boundary problem, in which the final position of Γ_p is also a problem unknown.

4 NUMERICAL FORMULATION

4.1 Explicit elastoplastic BEM

As it is well known, equations (18) and (19) of the precedent section can be transformed into algebraic representations by approximating u_k and p_k along the boundary duly divided into elements, as well as b_k and σ_{mk}^p over the domain now divided into cells. One can write as many algebraic equations as needed. Similarly, we can write an appropriate number of algebraic stress equations, the ones where the stress values are required to solve the problem. Moreover, without loss of generality, we continue this description removing the rate symbol; It is important to note that the final algebraic representations to be achieved can be applied to corresponding rate or incremental problems.

Thus, using shape functions to approximate all variables, equations (20) and (21) become³

$$\mathbf{H}\mathbf{U} = \mathbf{G}\mathbf{P} + \mathbf{T}\mathbf{B} + \mathbf{E}\sigma^p \quad (26)$$

$$\sigma = -\mathbf{H}'\mathbf{U} + \mathbf{G}'\mathbf{P} + \mathbf{T}'\mathbf{B} + \mathbf{E}'\sigma^p \quad (27)$$

where $\mathbf{U}$ and $\mathbf{P}$ are vectors containing the nodal values for displacements and tractions, respectively; $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^p$ are the stress and the initial stress vectors; $\mathbf{H}$, $\mathbf{H}'$, $\mathbf{G}$, $\mathbf{G}'$, $\mathbf{T}$, $\mathbf{T}'$, $\mathbf{E}$ and $\mathbf{E}'$ are the influence matrices arising from the numerical integration over elements and cells.

Applying the boundary conditions, equations (26) and (27) become

$$\mathbf{AX} = \mathbf{F} + \mathbf{E}\boldsymbol{\sigma}^p \quad (28)$$

$$\boldsymbol{\sigma} = -\mathbf{A}'\mathbf{X} + \mathbf{F}' + \mathbf{T}'\mathbf{B} + \mathbf{E}'\boldsymbol{\sigma}^p \quad (29)$$

Where $\mathbf{X}$ is the vector of boundary unknowns; $\mathbf{A}$ and $\mathbf{A}'$ contain the coefficients of the unknown boundary values and $\mathbf{F}$ and $\mathbf{F}'$ are independent vectors computed from the prescribed boundary conditions and body forces.

Equations (28) and (29) can be reduced to:

$$\mathbf{X} = \mathbf{M} + \mathbf{R}\boldsymbol{\sigma}^p, \quad \boldsymbol{\sigma} = \mathbf{N} + \mathbf{S}\boldsymbol{\sigma}^p \quad (30)$$

where $\mathbf{M}$ and $\mathbf{N}$ are the elastic solutions (displacements and stresses); $\mathbf{R}$ and $\mathbf{S}$ represent the influences of the initial stresses.

For elastoplastic solutions, one must realise that the plastic stress increments are computed following the proper elastoplastic relations given in section 2.

4.2 Implicit BEM formulation

Telles and Carrer^{6,7} were among the first to propose an implicit model to solve elastoplastic problems. They proposed an algorithm based on a continuous tangent operator. After that Bonnet and Mukherjee⁸ have used, for the first time, the concept of consistent tangent operator together with an initial strain approach.

The formulation implemented in this work is similar to the one proposed by Bonnet, but conveniently modified to work with initial stress fields.

Initially, we start by implementing a scheme to obtain implicitly the return to the yield surface. The scheme is developed here for the yield function given by equation (14). For this situation, the procedure is easily derived. The prediction is defined iteratively, assuming first the load increment entirely elastic (elastic trial), and then correcting if necessary (to return to the yield surface by the radial algorithm).

For gradient plasticity we have to consider the following partial differential equation to govern the plastic multiplier:

For gradient plasticity we have to consider the following the partial differential equation to govern the plastic multiplier:

$$\left(\|\mathbf{S}_{n+1}^t\| - \sigma_0 - H\lambda_n + \omega (\nabla^2 \lambda)_n \right) - (3\mu - H)\Delta \lambda_n + \omega \Delta (\nabla^2 \lambda)_n = 0 \quad (31)$$

Equation (25) can also be transformed into an algebraic representation following the standard steps of the boundary methods as made to obtain equation (26).

For the implicit scheme, the plastic multiplier given by the return algorithm is used to achieve the local consistent tangent operator C^{ep} to be used to correct the global matrix:

$$C^{ep} = \frac{\partial \sigma^{arr}}{\partial \Delta \epsilon} \quad (32)$$

where σ^{arr} comes from the return algorithm.

Note that for the gradient plasticity case, C^{ep} can not be differentiated analytically. Its derivatives can be only performed numerically by using equation (26) together with equation (1).

To obtain the implicit algorithm using the CTO procedure we can start by writing equation (30b) in its incremental form. Then, the plastic stress tensor σ^p is now added to the both sides of equation (30b), using properly relation (1) to give:

$$\{Y(\Delta \epsilon_n)\} = -[C]\{\Delta \epsilon_n\} + \{N_n\} + [\bar{S}]\{[C][\Delta \epsilon_n] - [\Delta \sigma_n]\} = 0 \quad (33)$$

where $[C]$ is the usual Hookean elastic tensor, $[\bar{S}] = [S] + [I]$ with $[I]$ being the identity matrix, Δ indicates increments and the subscript n gives the increment number.

The stress and strain tensor increments, $\Delta \epsilon_n$ and $\Delta \sigma_n$, that cumulate into the stress and strain values at interaction n , lead to their up-dated values, as follows.

$$\epsilon_{n+1} = \epsilon_n + \Delta \epsilon_n, \quad \sigma_{n+1}^{arr} = \sigma_n + \Delta \sigma_n \quad (34a,b)$$

where $\sigma_{n+1}^{arr}(\Delta \epsilon_{n+1})$ is achieved by the return algorithm and σ_n is given by the BEM algebraic relation.

Rearranging equation (33) to take into account relations (34), the following expression is found:

$$\{Y(\Delta \epsilon_n)\} = -[C]\{\Delta \epsilon_n\} + \{N_n\} + [\bar{S}]\{[C][\Delta \epsilon_n] - \{\sigma_{n+1}^{arr}\} + [\sigma_n]\} = 0 \quad (35)$$

Equation (38) can be solved using the Newton Raphson scheme. For that one needs to define:

$$\Delta \epsilon_n^{i+1} = \Delta \epsilon_n^i + \delta \Delta \epsilon_n^i \quad (36)$$

where the superscript i gives the iteration.

Considering only the first variation of $\{Y(\Delta \epsilon_n)\}$, equation (35) becomes:

$$\{Y(\Delta \epsilon_n^i)\} = \{[C] - [\bar{S}]\{[C] - [\partial \sigma_n^{arr} / \partial \Delta \epsilon_n^i]\}\} \{\delta \Delta \epsilon_n^i\} \quad (37)$$

where the term between parenthesis in the right hand side is the consistent tangent operator for BEM elastoplastic formulation, i.e.,

$$[C^{ep}] = \{[C] - [\bar{S}]\{[C] - [\partial \sigma_n^{arr} / \partial \Delta \epsilon_n^i]\}\} \quad (38)$$

Note that, for the initial stress algorithm, the elastic tensor must multiply the results of equation (37).

It is also important to notice that the system of equations (38) can be solved just for the nodal points with a non-zero plastic component of the current strain increment. This is because $C^{ep}=C$ for elastic points, allowing a block decomposition. The correction for elastic points can be calculated once the plastic values are known, saving computation time.

5 NUMERICAL ANALYSIS

In this section, a classical example in localisation studies is analysed in the framework of BEM. A rectangular block subjected to compression loading as shown in figure 1 is considered. The block is analysed in plane conditions with a zero poisson's ratio. In order to trigger localisation, an internal small region near the center of the block is weakened by reducing its yield stress. The yield stress of the material is equal to 2.0Mpa everywhere except in the weaker internal square. The reduction is 0.2Mpa at the central nodes and varies linearly from the center to the square region boundary. Discretizations exhibiting from 128 to 2048 cells have been used, even though not all of them are presented here. The Young modulus assumed was $E= 2,000.00\text{N/mm}^2$, while the softening modulus was taken as $h= -0.0125 E$. The block is loaded by applying a uniform displacement $\delta=-0.24\text{mm}$ as shown in Figure 1. Several increment sizes have also been tested to analyze how the developed procedure behaves.

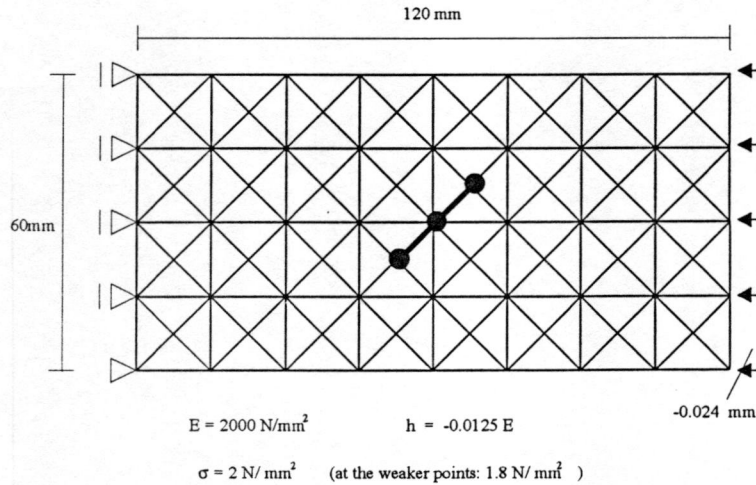


Figure 1. Rectangle: Size and discretizations.

Figure 2 gives the displacement x reaction curves captured for the two finest discretizations, 512 and 2048 cells, respectively. The total reactions along the clamped end were computed by integrating the obtained tractions. The mesh dependence is clearly demonstrated in this figure for the local plasticity model. Using other courser meshes different curves, for the softening branch, have been captured. The results obtained by following the consistent tangent operator is far better when compared with the ones computed by the

classical constant matrix BEM. Using the standard scheme the mesh dependence can also be detected, but the results are more unstable in comparison with the previous scheme.

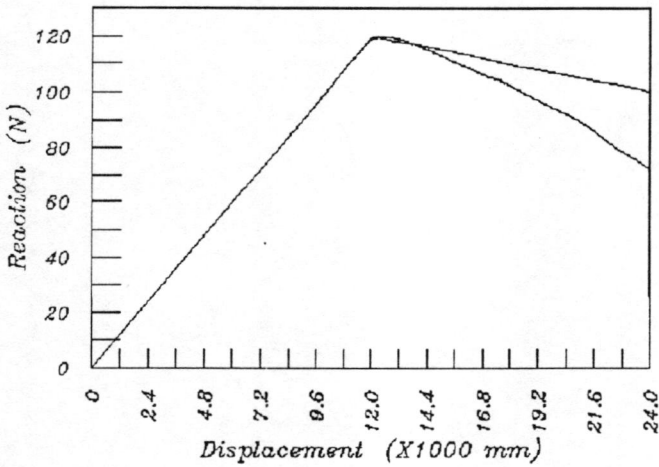


Figure 2. Reaction versus displacement curve for the 512 and 2048 cell cases.

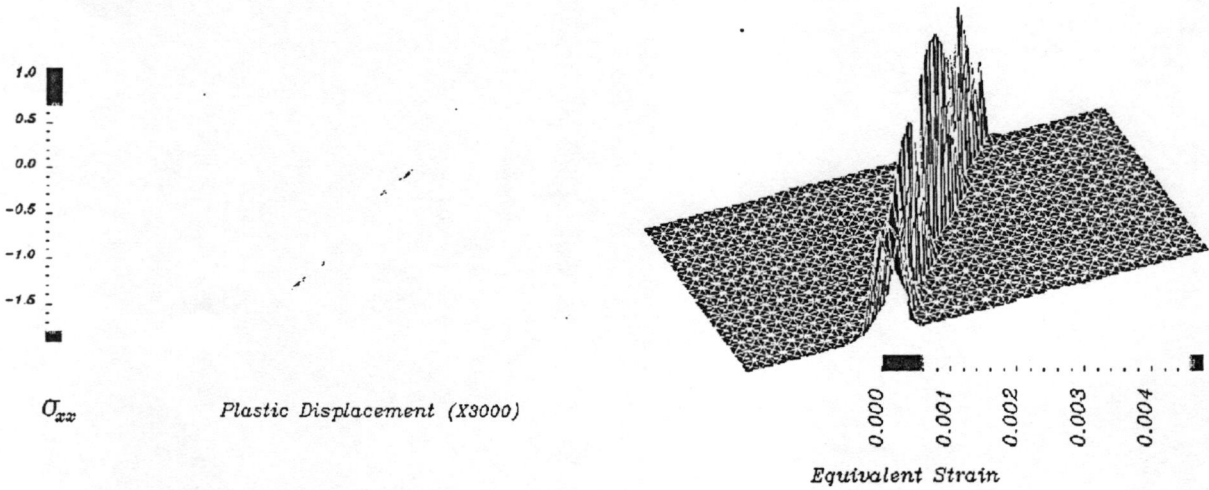


Figure 3. a) Plastic displacements and normal stress component σ_{xx} distribution.
b) 3D visualization for the final equivalent plastic strain field.

Another important aspect of this BEM analysis is concerned with the final plastic strain configuration achieved. Again, the solution obtained by using the CTO scheme is the expected one, with a clear localized inclined narrow zone. This narrow zone is always precisely defined over a row of cells. Thus, the width of this zone is exactly the mesh size.

This solution is given in Figure 3b, where the concentration of the plastic strain over the narrow band across the body is clearly defined. No plastic strain develops outside that zone. On the other hand, using the classical BEM approach with constant matrix the localized zone can be achieved, but they are not well defined (for some discretization more than one localized zones have been detected). We have obtained, for all discretizations tested, plastic strain developed over a much wider zone, when the explicit scheme was used together with constant matrix.

Figure 3a illustrates as well the capability of the proposed BEM scheme. One can see that stress field is disturbed only over the narrow dissipation zone. In addition, this figure shows that the body is split into two elastic parts with no displacement due to plastic strain evolution inside the narrow band.

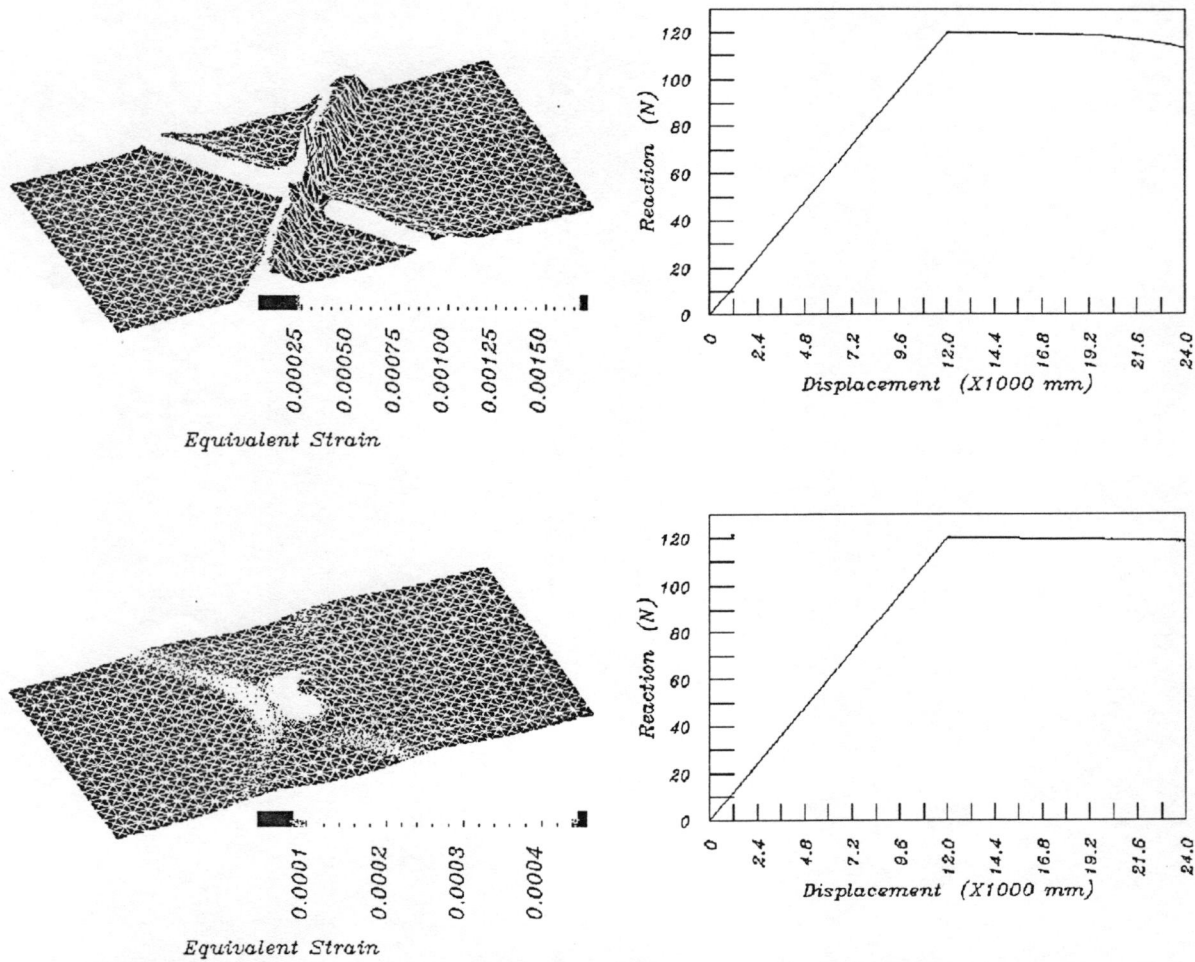


Figure 4. Comparison between the local and gradient plasticity models.
a) Local plasticity (above). b) Gradient plasticity (below).

Figure 4 illustrates the effect of the gradient plasticity on the solution. The local scheme (fig. 4a) is not capable to capture precisely the critical solution. The concentration of plastic strain clearly occur and consequently the mesh dependency. Using the gradient plasticity model (fig. 4b) gives a much larger dissipation band with less concentrated plastic strains.

6 CONCLUSIONS

Application of BEM to analyse localization phenomena has been considered in this paper. In order to overcome some difficulties observed with local continua, a gradient plasticity model was used. An integral representation for boundary stress evaluation was used to avoid inaccurate calculation of the stress field. Implicit and explicit elastoplastic BEM formulations have been discussed for both classical and gradient plasticity. The implicit schemes were proposed together with the definition of a consistent tangent operator. The integral representation of the plastic multiplier field has been discussed for gradient plasticity with explicit and implicit schemes. Numerical strategies for the solution of the BEM algebraic equations of the plastic multiplier were also discussed.

Numerical examples, for explicit and implicit BEM formulations and local elastoplasticity, have been performed to verify their capability of achieving accurate stress field for problems exhibiting strain concentrations and employing coarse and fine discretizations. It has been found that only implicit BEM is able to capture precisely plastic strain concentrations. This scheme was also able to exhibit clearly the classical problem of mesh dependence. On the other hand the standard procedure, based on explicit scheme and constant matrix, does not present the same accuracy; therefore it is not recommended to investigate elastoplastic problems exhibiting plastic strain concentrations.

Although explicit scheme is not very accurate, its was used, together with a simple strategy to consider gradient plasticity. The adopted scheme, although simple, was used to illustrate how gradient plasticity can affect the solution of numerical examples.

Implementations of all the schemes and strategies discussed are currently in progress and possible alternatives are under investigation or evaluation.

REFERENCES

- [1] Telles, J.C.F., 1983, *The boundary element method applied to inelastic problems*. Springer-Verlag.
- [2] Venturini, W.S., 1983, *Boundary element method in geomechanics*. Springer-Verlag.
- [3] Brebbia, C.A., Telles, J.C.F and Wrobel, L.C., 1984, *Boundary element techniques. Theory and applications in engineering*. Springer-Verlag.
- [4] Bui, H.D., 1978, Some remarks about the formulation of three-dimensional thermoelastic problems by integral equations. *Int. J. Solids and Structures*, 14, 935-939.
- [5] Jim H., Runesson, K. & Matiasson, K., 1989, Boundary element formulation in finite deformation plasticity using implicit integration. *Comp. & Structures*, 31, 25-34.
- [6] Carrer, J.A.M. & Telles, J.C.F., 1991, Transient dynamic elastoplastic analysis by the Boundary element method. In: *Boundary Element Technology VI, Proceedings*, ed. C.A. Brebbia. CMP, UK, and Elsevier, p.265-277.

- [7] Telles, J.C.F. & Carrer, J.A.M., 1994, Static and dynamic non-linear stress analysis by the boundary element method with implicit techniques. *Engineering Analysis with Boundary Elements*, 14, p.65-74.
- [8] Bonnet, M. & Mukherjee, S., 1996, Implicit BEM formulation for usual and sensitivity problems in elasto-plasticity using the consistent tangent operator concept. *Int. J. Solids and Structures*, 33, 4461-4480.
- [9] Simo, J.C. & Taylor, R.L., 1985, Consistent tangent operators for rate-independent elastoplasticity. *Comp. Meth. Appl. Mech. Engng*, 48, 101-118.
- [10] Bazant, Z.P., Belytschko, T.B. & Chang, T.P., 1984, Continuum theory for strain softening. *J. Engng Mech. Div., ASCE*, 110, 1666-1692.
- [11] Benallal, A., Billardon, R. & Geymonat, R., 1988, Some mathematical aspects of the damage softening problem. In: *Cracking and Damage*, J. Mazars and Z. P. Bazant, ed., Vol.1, 247-258.
- [12] Borst, R. de, 1988, Bifurcations in finite element models with a non-associated flow law. *Int. J. Numer. Anal. Meth. Geomech.*, 12, 99-116.
- [13] Eringen, A. C. and Edelen, D. G., 1972, On nonlocal elasticity. *Int. J. Engng Sci.* 10, pp. 233-248.
- [14] Fleck, N. A. and Hutchinson, J. W., 1993, A phenomenological theory of strain gradient plasticity. *J. Mech. Phys. Solids* 41, pp. 1825-1857.
- [15] Triantafyllidis, N. and Aifantis, E., 1986, A gradient approach to localization of deformation. I-Hyperelastic materials. *J. Elasticity* 16, pp. 225-237.
- [16] Pijaudier-Cabot, G. and Bazant, A. P., 1987, Nonlocal damage theory. *J. Engng Mech. ASCE*, 113, pp. 1512-1533.
- [17] Lasry, D. and Belytschko, T., 1988, Localization limiters in transient problems. *Int. J. Solids Struct.* 24, pp. 581-597.
- [18] Mulhlaus, H. B. and Aifantis, E., 1991, A variational principle for gradient plasticity. *Int. J. Solids Struct.* 28, pp. 845-857.
- [19] de Borst, R. and Mulhlaus, H. B., 1992, Gradient-dependent plasticity: formulation and algorithmic aspects. *Int. J. Num. Meth. Engng.* 35, pp. 521-539.
- [20] Maier, G., Miccoli, S., Novati, G. & Perego, U., 1995, Symmetric Galerkin boundary element method in plasticity and gradient plasticity. *Comp. Mech.* 17, 115-129.
- [21] Benallal, A. & Tvergaard, V., 1995, Nonlocal continuum effects on bifurcation in the plane strain tension-compression test. *J. Mech. Phys. Solids*, 43, 741-770.
- [22] Cruse, T.A. & Richardson, J.D. 1996, Non-singular Somigliana stress identities in elasticity. *Int. J. Num. Meth. Engng.*, 39, 3273-3304.
- [23] Guiggiani, M., 1994, Hypersingular formulation for boundary stress evaluation. *Engng. Analysis Boundary Element*, 13, 169-179.