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Constraining EDM and MDM lepton dimension-five interactions in the electroweak sector

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ABSTRACT

We investigate dimension-five Lorentz-violating (LV) nonminimal interactions in the electroweak sector, in connection with the possible generation of electric dipole moment (EDM), weak electric dipole moment (WEDM), magnetic dipole moment (MDM) and weak magnetic dipole moment (WMDM) for leptons. These couplings are composed of the physical fields in the Standard Model and LV tensors of ranks ranging from 1 to 4. The CPT-odd couplings do not generate EDM behavior nor provide the correct MDM signature, while the CPT-even ones are compatible with EDM and MDM behavior, being subject to improved constraining. Tau lepton experimental data is used to constrain the WEDM and WMDM couplings to the level of 10^{-4} $(\mathrm{GeV})^{-1}$, whereas electron MDM and EDM data are employed to improve constraints to the level of 10^{-10} $(\mathrm{GeV})^{-1}$ and 10^{-16} $(\mathrm{GeV})^{-1}$, respectively.

1. Introduction

Electric dipole moment (EDM) physics is a broad field of investigation [1–4] deeply connected with precise experiments and physics beyond the Standard Model (SM) [5]. EDM has as signature the violation of parity (P) and time reversal (T) symmetries, while preserving charge conjugation (C) and the CPT symmetry. In the relativistic context, the electric dipole moment, $\mathbf{d} = g(q/2m)\mathbf{S}$, yields the interaction $d(\mathbf{\Sigma}\cdot\mathbf{E})$, with d, $\mathbf{E}$, $\mathbf{\Sigma}$, being the EDM modulus, the electric field and the Dirac spin operator, respectively. The EDM Lagrangian is represented by the dimension-five term $d(\bar{\psi}i\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\psi)$, where ψ is a Dirac spinor. It is important to mention that the EDM structure is generated by radioactive corrections only at four-loop order [4,6], so that its magnitude (for the electron) is of about $d_e \simeq 10^{-38}\, e\cdot\mathrm{cm}$ in the SM framework. EDM measurements have been progressively improved [7], reaching the level of $10^{-29}\, e\cdot\mathrm{cm}$ for the electron EDM [8], and $10^{-30}\, e\cdot\mathrm{cm}$ for the ^{199}Hg nuclear EDM [9]. Each order of magnitude improvement in the EDM experiments leads to strong phenomenological consequences on a diversity of CP-violating theories. The gap of seven orders of magnitude still remaining between the experimental data and the theoretical evaluations for the electron EDM allows for new CP-violating mechanisms, besides the usual CP violation sources already embedded in the SM. These sources may be relevant for explaining the observed baryon asymmetry of the universe, an issue possibly connected with axions and the strong CP problem [10]. Muon EDM [11] and tau lepton EDM [12–14] have also been under rich investigation in connection with the physics of SM and beyond the SM.

The magnetic dipole moment is $\mu = g\mu_B\mathbf{S}$, where $\mu_B = q/2m$ is the Bohr magneton and $g = 2(1+a)$ is the gyromagnetic factor. Here, $a = \alpha/2\pi \simeq 0.00116$ represents the deviation from the usual Dirac value, $g = 2$. The relativistic magnetic interaction, $\mu(\mathbf{\Sigma}\cdot\mathbf{B})$, has a Lagrangian form, $\mu(\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi)$, which appears in the SM framework at 1-loop order. For the electron MDM, there are very precise theoretical calculations [15] that present astonishing agreement to experimental measurements [16] at the level of 1 part in 10^{12}. The experimental imprecision on the electron MDM is at the level of 2.8 parts in 10^{13} [16], that is, $\Delta a \leq 2.8\times 10^{-13}$, being this value a limit for new theories with repercussions on the eMDM interaction. Concerning the muon MDM, the agreement between theoretical calculations and experimental measures is lower, although still impressive. As observed for any lepton, the SM prediction for the muon factor $a_\mu = (g-2)/2$ is composed of

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three pieces, $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had}$, stemming from the radiative evaluations in the quantum electrodynamics, electroweak and hadronic sectors, yielding the value: $a_\mu^{SM} = 116591830 \times 10^{-11}$. Very precise measures were obtained analyzing the precession of muons in a constant magnetic field in storage rings, $a_\mu^{\exp} = 11659209 \times 10^{-10}$ [17,18]. The theoretical/experimental discrepancy $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} \simeq 260 \times 10^{-11}$ is nearly 10^4 larger than the observed for the electron, being a topic of intensive investigation in contemporary physics. It can be ascribed to supersymmetric particle loops [19] or to new physics, as dark matter scenarios, where a light dark matter photon could couple to known physical particles [20], engendering additional MDM contributions.

MDM and EDM physics may also be related to Lorentz-violating theories, investigated in the broader framework of the Standard Model extension (SME), developed by Colladay and Kostelecky [22]. The SME incorporates dimension-four and dimension-three LV terms in all sectors of the Standard Model, including fermions [23–25], photons [26–29], photon-fermion interactions [30,31], and electroweak (EW) processes [32–34]. Beyond the minimal SME, there are nonminimal extensions encompassing couplings with higher-order derivatives [35] and higher-dimensional operators [36–39].

Lorentz violation can work as a source of CP violation and generate EDM via radiative corrections [40], or even at tree level via dimension-five nonminimal (NM) couplings [41,42]. Dimension-five nonminimal couplings have been proposed as non usual QED interactions between fermions and photons, yielding EDM Lagrangians pieces as $\lambda\bar{\psi}(K_F)_{\mu\nu\alpha\beta}\Gamma^{\mu\nu}F^{\alpha\beta}\psi$, $\lambda_1\bar{\psi}T_{\mu\alpha}F^{\alpha}{}_{\nu}\Gamma^{\mu\nu}\psi$, where $(K_F)_{\mu\nu\alpha\beta}$ and $T_{\mu\alpha}$ are CPT-even LV tensors, with $\Gamma^{\mu\nu} = \sigma^{\mu\nu}$ or $\sigma^{\mu\nu}\gamma_5$ [42]. Electron EDM experimental data has yielded upper bounds as tight as $10^{-25}\,(\text{eV})^{-1}$ on the magnitude of these couplings. Considering the Schiff screening theorem [43], anisotropic electrostatic interactions were taken into account in order to engender LV corrections on the nuclear EDM and Schiff moment [44]. Recently, general dimension-six nonminimal fermion-fermion couplings were proposed [45] and constrained at the level of 10^{-15} $(\text{GeV})^{-2}$ by EDM data [46], considering these couplings as electron-nucleon P-odd and T-odd atomic interactions. LV contributions to MDM physics were also examined [47,48].

If the Standard Model is addressed as a low-energy effective theory, it is worth considering higher dimensional terms in the Lagrangian. Extensions of the electroweak model containing higher dimension terms (mainly dimension-six) have been analyzed as effective theories since the 1980s [49]. Lists of dimension-six EW and strong couplings have been presented and updated so as to involve top quark physics and interactions with the Higgs [50,51]. CP-violating couplings in the Higgs sector, which comprises CP-violating interactions with quarks and the tau lepton, are also represented by dimension-six operators. Such couplings can generate EDM, providing an effective route of constraining it [52]. Some of the best bounds on the anomalous CP-violating Higgs interactions come from EDM measurements. A plethora of dimension-six terms yielding electroweak baryogenesis and CP violation has been considered in connection with the baryon asymmetry of the universe [53]. The role of EDM physics in electroweak interactions and electroweak baryogenesis has been a topical issue in the latest years [54,55].

Dimension-five nonminimal couplings in the Glashow-Salam-Weinberg (GSW) electroweak model have also been proposed in connection with CPT and Lorentz symmetry violation [56,57]. Such couplings have been constrained by weak decay data at the level of $10^{-5}\,(\text{GeV})^{-1}$. The repercussions of such nonminimal couplings on MDM and EDM physics have not been examined yet, and can be used to improve the constraining on these couplings. In the electroweak sector, the weak magnetic moment (WMDM) and weak electric dipole interaction (WEDM) involve interaction with the Z boson field, being given by the effective Lagrangian [58,59]:

$$\mathcal{L}_{EW} = \frac{1}{\sin 2\theta}\bar{\psi}\left[\alpha_w \frac{e}{2m_l}\sigma^{\mu\nu}Z_{\mu\nu} + i d_w \sigma^{\mu\nu}\gamma_5 Z_{\mu\nu}\right]\psi, \tag{1}$$

where α_w and d_w represent the WMDM and WEDM magnitudes, θ is the Weinberg angle and $Z_{\mu\nu}$ is the $U(1)$ boson field strength. High energy experiments for constraining EDM and WEDM are based on the search for CP-violating electroweak sources, whose magnitude scales as the mass of the lepton (being more significant for the tau lepton). Tau lepton EDM, WEDM and WMDM data is obtained from electron-positron scatterings, $e^- + e^+ \to \tau^+ + \tau^-$, $e^- + e^+ \to \tau^+ + \tau^- + e^- + e^+$ [13,14], [58,59]. Such experiments yield $\alpha_\tau^w < 1 \times 10^{-3}$ and $d_\tau^w < 10^{-17}\,e \cdot \text{cm}$.

In this work, we analyze a few dimension-five LV couplings in the GSW electroweak model concerning the possibility of generating EDM, WEDM and MDM, WMDM for leptons. While we propose CPT-odd and CPT-even couplings, only the latter ones generate EDM- or MDM-compatible terms. Using tau WEDM and WMDM experimental data, some couplings are constrained to the level of 10^{-4} $(\text{GeV})^{-1}$, while the electron EDM and MDM yield upper bounds to the level of 10^{-16} $(\text{GeV})^{-1}$ and 10^{-10} $(\text{GeV})^{-1}$, respectively.

2. The Glashow-Salam-Weinberg electroweak model

In the GSW model, the left-handed leptons are disposed in isodublets ($T = \frac{1}{2}, T_3 = \pm\frac{1}{2}$), while the right-handed leptons are represented by isosinglets ($T = 0$) under the $SU(2)$ group,

$$L_l = \begin{bmatrix} \psi_{\nu_l} \\ \psi_l \end{bmatrix}_L = \frac{1-\gamma_5}{2}\begin{bmatrix} \psi_{\nu_l} \\ \psi_l \end{bmatrix}, \tag{2}$$

$$R_l = (\psi_l)_R = \left(\frac{1+\gamma_5}{2}\right)\psi_l, \tag{3}$$

with the generators, $\mathbf{T} = (T_1, T_2, T_3)$, fulfilling the relation, $[T_i, T_j] = i\varepsilon_{ijk}T_k$. The GSW Lagrangian is

$$\mathcal{L} = \bar{L}_l\gamma^\mu iD_\mu L_l + \bar{R}_l\gamma^\mu iD_\mu R_l - \frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{4}$$

where the field strengths for the $U(1)$ and $SU(2)$ gauge fields, B_μ and $\mathbf{W}_\mu$, are $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, and

$$\mathbf{W}_{\mu\nu} = \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu + g\left(\mathbf{W}_\mu \times \mathbf{W}_\nu\right). \tag{5}$$

Knowing that the $U(1)$ field is a combination of the electromagnetic and the boson Z field, $B_\mu = \cos\theta A_\mu - \sin\theta Z_\mu$, one has

$$B_{\mu\nu} = (\cos\theta)\,F_{\mu\nu} - (\sin\theta)\,Z_{\mu\nu}, \tag{6}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. The usual covariant derivative is

$$D_\mu = \partial_\mu - ig\,(\mathbf{T}\cdot\mathbf{W})_\mu - i\frac{g'}{2}YB_\mu, \tag{7}$$

where Y is the $U(1)$ generator. We have $Y_L = -1$ for (e_L, μ_L, τ_L) and $Y_R = -2$ for (e_R, μ_R, τ_R). Replacing the covariant derivative in the Lagrangian (4), we obtain

$$\mathcal{L} = i\bar{L}_l\gamma^\mu\partial_\mu L_l + i\bar{R}_l\gamma^\mu\partial_\mu R_l + L_{int}^{(l)}, \tag{8}$$

with the interaction piece being

$$\mathcal{L}_{int}^{(l)} = g\left(\bar{L}_l\gamma^\mu\mathbf{T}L_l\right)\cdot\mathbf{W}_\mu - \left[\frac{g'}{2}\left(\bar{L}_l\gamma^\mu L_l\right) + g'\left(\bar{R}_l\gamma^\mu R_l\right)\right]B_\mu. \tag{9}$$

Table 1
Classification under C, P, and T for the CPT-odd nonminimal couplings of Lagrangian (12).

Coupling	$g'_1 C^0$	$g'_1 C^i$	$g'_1 C^0_A$	$g'_1 C^i_A$
P	+	−	−	+
C	−	+	−	−
T	+	+	−	+

3. CPT-odd dimension-five NM LV electroweak coupling

We investigate a few CPT-odd nonminimal couplings. They do not generate EDM nor possess the correct MDM signature under C, P, and T operators. This can be argued by analyzing rank-1 or rank-3 nonminimal couplings, which are the simplest ones to be proposed.

3.1. Rank-1 CPT-odd NM couplings

Rank-1 CPT-odd and dimension-five nonminimal couplings in the EW sector were proposed in Ref. [56], as

$$\mathcal{L}_{int} = g'_1\left(\bar{L}_l \gamma^\mu B_{\mu\nu} C^\nu L_l\right) + 2g'_1\left(\bar{R}_l \gamma^\mu B_{\mu\nu} C^\nu R_l\right), \tag{10}$$

where C^ν is a fixed LV background, and $Y=-1$ and $Y=-2$ for left-handed and right-handed fermions, respectively. Using Eq. (2) and (3), we obtain the Lagrangian:

$$\mathcal{L}_{int} = \frac{1}{2} g'_1 \bar{\psi}_{\nu_l} \gamma^\mu (1-\gamma_5)\psi_{\nu_l} B_{\mu\nu} C^\nu + \frac{1}{2} g'_1 \bar{\psi}_l \gamma^\mu (3+\gamma_5)\psi_l B_{\mu\nu} C^\nu. \tag{11}$$

Here, it is important to note that the γ_5 operator changes the behavior of the coupling in relation to the C, P and T operators. Thus, it is suitable to rewrite the Lagrangian (11) in terms of two vector backgrounds, C^ν and C^ν_A:

$$\mathcal{L}^{(odd)}_{(1)} = \frac{1}{2} g'_1 \bar{\psi}_{\nu_l}\gamma^\mu \psi_{\nu_l} B_{\mu\nu} C^\nu - \frac{1}{2} g'_1 \bar{\psi}_{\nu_l}\gamma^\mu\gamma_5 \psi_{\nu_l} B_{\mu\nu} C^\nu_A + \frac{3}{2} g'_1 \bar{\psi}_l \gamma^\mu \psi_l B_{\mu\nu} C^\nu + \frac{1}{2} g'_1 \bar{\psi}_l\gamma^\mu\gamma_5\psi_l B_{\mu\nu} C^\nu_A. \tag{12}$$

For purpose of better investigation, one explicitly analyzes the lepton (l) content of Eq. (12), writing

$$\mathcal{L}^{(odd)}_{(1)l} = \frac{1}{2} g'_1 \Big[3\bar{\psi}_l\gamma^0 B_{0i} C^i \psi_l + \bar{\psi}_l\gamma^0\gamma_5 B_{0i} C^i_A \psi_l + 3\bar{\psi}_l\gamma^i B_{ij} C^j\psi_l + \bar{\psi}_l\gamma^i\gamma_5 B_{ij} C^j_A\psi_l + 3\bar{\psi}_l\gamma^i B_{i0} C^0 \psi_l + \bar{\psi}_l\gamma^i\gamma_5 B_{i0} C^0_A \psi_l\Big]. \tag{13}$$

The lepton Lagrangian term $\bar{\psi}_l\gamma^i\gamma_5\psi_l B_{i0} C^0_A$ is the unique one compatible with the EDM signature, as shown in Table 1, since it contains the pieces

$$\left(\bar{\psi}_l\gamma^0\Sigma^i\psi_l\right) B_{0i} C^0_A, \tag{14}$$

where $B_{0i} = E^i + \tilde{E}^i$ could yield electric and electroweak EDM, with $\tilde{E}^i$ representing the weak electric field, as shown in (19). This analysis also holds for the neutrino terms in Lagrangian (11), where the EDM-like term is $\left(\bar{\psi}_{\nu_l}\gamma^0\Sigma^i\psi_{\nu_l}\right) B_{0i} C^0$. The presence of the γ^0, however, prevents the EDM behavior, since it avoids the resemblance to the EDM Lagrangian $\left(\bar{\psi}_l \Sigma^j E^j \psi_l\right)$. The γ^0 factor disappears in the corresponding Hamiltonian form, yielding the relativistic interaction pieces, $\Sigma^j E^j$, $\Sigma^j \tilde{E}^j$, which are undetectable (at first order) in accordance with the Schiff's theorem [4,43]. This occurs with the Hamiltonian interactions stemming from Eq. (14),

$$\psi_l^\dagger \Sigma^i E^i C^0_A \psi_l, \;\; \psi_l^\dagger \Sigma^i Z_{0i} C^0_A \psi_l, \tag{15}$$

implying the absence of EDM physics. Here, we have used

$$\sigma^{0j} = i\alpha^j, \;\; \sigma^{ij} = \epsilon_{ijk}\Sigma^k, \tag{16}$$

$$\alpha^j\gamma_5 = \Sigma^j, \;\; \alpha^j = \Sigma^j\gamma_5, \tag{17}$$

$$F_{0j} = E^j, \;\; F_{mn} = \epsilon_{mnp} B^p, \tag{18}$$

$$Z_{0j} = \tilde{E}^j, \;\; Z_{mn} = \epsilon_{mnp}\tilde{B}^p. \tag{19}$$

We can also investigate MDM behavior of leptons and neutrinos. The closest MDM term in Lagrangian (12) is

$$\bar{\psi}_l\gamma^i\gamma_5 B_{ij} C^j_A \psi_l, \tag{20}$$

where $B_{ij} = F_{ij} + Z_{ij}$. Considering the definitions (18) and (19), this term involves the usual (B^k) and weak ($\tilde{B}^k$) magnetic fields,

$$\bar{\psi}_l\gamma^0 (\mathcal{C}_A)_{ik}\Sigma^i B^k\psi_l, \;\; \bar{\psi}_l\gamma^0 (\mathcal{C}_A)_{ik}\Sigma^i \tilde{B}^k\psi_l, \tag{21}$$

in a non conventional way (it couples the spin to a "rotated" LV background structure, $(\mathcal{C}_A)_{ik} = \epsilon_{jik} C^j_A$. As shown in Table 1, the coefficient C^j_A does not exhibit the exact signature of a MDM interaction, due the presence of γ^0. Thus, it does not generate usual MDM or WMDM. For experimental purposes, as this tensor background has no diagonal components, $\mathcal{C}_{ii} = 0$, the contributions (21) could only be probed with a magnetic field orthogonal to the spin, as discussed in Refs. [42,44]. The same conclusions hold for the neutrino counterparts.

3.2. Rank-3 CPT-odd NM couplings

We now examine a rank−3 dimension-five nonminimal coupling in Lagrangian of the GSW model. It can be written as

$$\mathcal{L}^{(odd)}_{(3)} = -g'_2 Y_L \bar{L}_l\left(\gamma^\mu B^{\alpha\beta} H_{\mu\alpha\beta}\right) L_l - g'_2 Y_R \bar{R}_l\left(\gamma^\mu B^{\alpha\beta} H_{\mu\alpha\beta}\right) R_l, \tag{22}$$

where $H_{\mu\alpha\beta}$ is the LV background tensor, with the supposed symmetry $H_{\mu\alpha\beta} = -H_{\mu\beta\alpha}$, and $Y_L=-1$, $Y_R=-2$, so that

$$\mathcal{L}^{(odd)}_{(3)} = g'_2 \bar{L}_l\left(\gamma^\mu B^{\alpha\beta} H_{\mu\alpha\beta}\right) L_l + 2g'_2 \bar{R}_l\left(\gamma^\mu B^{\alpha\beta} H_{\mu\alpha\beta}\right) R_l. \tag{23}$$

This EW Lagrangian can be written in terms of the lepton and neutrino pieces, $\mathcal{L}^{(odd)}_{(3)} = \mathcal{L}^{(odd)}_{(3)l} + \mathcal{L}^{(odd)}_{(3)\nu}$, given as

$$\mathcal{L}^{(odd)}_{(3)l} = \frac{g'_2}{2}\bar{\psi}_l\left[3\gamma^\mu H_{\mu\alpha\beta}B^{\alpha\beta} + \gamma^\mu\gamma_5 B^{\alpha\beta}(H_A)_{\mu\alpha\beta}\right]\psi_l, \tag{24}$$

$$\mathcal{L}^{(odd)}_{(3)\nu} = \frac{g'_2}{2}\bar{\psi}_{\nu_l}\left[\gamma^\mu H_{\mu\alpha\beta}B^{\alpha\beta} - \gamma^\mu\gamma_5 B^{\alpha\beta}(H_A)_{\mu\alpha\beta}\right]\psi_{\nu_l}, \tag{25}$$

where we have introduced the rank-3 background $(H_A)_{\mu\alpha\beta}$ for the coupling involving γ_5, as we have done in Eq. (12). In order to verify the possibility of EDM generation for leptons, we investigate the tensor structure of lepton Lagrangian (24) that can be expressed as

$$\mathcal{L}^{(odd)}_{(3)l} = g'_2\Big[\bar{\psi}_l\gamma^0 B^{0i} H_{00i}\psi_l + \bar{\psi}_l\gamma^0 B^{ij} H_{0ij}\psi_l + \bar{\psi}_l\gamma^i B^{0j} H_{i0j}\psi_l + \bar{\psi}_l\gamma^i B^{jk} H_{ijk}\psi_l + \bar{\psi}_l\gamma^0\gamma_5 B^{0i}(H_A)_{00i}\psi_l + \bar{\psi}_l\gamma^0\gamma_5 B^{ij}(H_A)_{0ij}\psi_l + \bar{\psi}_l\gamma^i\gamma_5 B^{0j}(H_A)_{i0j}\psi_l + \bar{\psi}_l\gamma^i\gamma_5 B^{jk}(H_A)_{ijk}\psi_l\Big]. \tag{26}$$

In Eq. (26), we see that the term, $\bar{\psi}_l\gamma^i\gamma_5 B^{0j}(H_A)_{i0j}\psi_l$, is the unique that has EDM signature, as shown in Table 2. This piece can be written as

Table 2
Classification under C, P and T for the CPT-odd rank-3 nonminimal couplings of Lagrangian (26).

Coupling	$g_2' H_{00i}$	$g_2' H_{0ij}$	$g_2' H_{i0j}$	$g_2' H_{ijk}$
P	−	+	+	−
C	+	+	−	+
T	+	−	+	+
Coupling	$g_2' (H_A)_{00i}$	$g_2' (H_A)_{0ij}$	$g_2' (H_A)_{i0j}$	$g_2' (H_A)_{ijk}$
P	+	+	−	+
C	−	−	−	−
T	+	+	−	+

$$\bar{\psi}_l \gamma^0 \Sigma^i B_{0j} (H_A)_{i0j} \psi_l, \tag{27}$$

in which B_{0j} contains the electric and weak electric counterparts. Analogously to the rank−1 CPT-odd NM couplings, the presence of the γ^0 avoids the EDM behavior. As it occurs for the rank-1 case, the Table 2 shows that the couplings of Lagrangian (26) do not possess MDM behavior.

There are another possibilities of hermitian rank-3 nonminimal couplings. An example is

$$\begin{aligned}\tilde{\mathcal{L}}_{(3)l}^{(odd)} &= g_3' \bar{\psi}_l (\gamma^\alpha B^{\beta\nu} - \gamma^\beta B^{\alpha\nu}) \psi_l \bar{H}_{\nu\alpha\beta} \\ &+ g_3' \bar{\psi}_l (\gamma^\alpha B^{\beta\nu} - \gamma^\beta B^{\alpha\nu}) \gamma_5 \psi_l \left(\bar{H}_A\right)_{\nu\alpha\beta}.\end{aligned} \tag{28}$$

These couplings do not generate EDM behavior nor possess the correct MDM signature, and will no longer be examined. Note, however, that Lagrangian (28) can be obtained from the lepton Lagrangian (23) using the following transformation:

$$H_{\mu\alpha\beta} = \bar{H}_{\beta\mu\alpha} - \bar{H}_{\beta\alpha\mu}, \;\; (H_A)_{\mu\alpha\beta} = \left(\bar{H}_A\right)_{\beta\mu\alpha} - \left(\bar{H}_A\right)_{\beta\alpha\mu}. \tag{29}$$

There are still other possibilities of dimension-5 CPT-odd couplings with rank-3 tensors, as revealed in the general investigation by Kostelecky & Ding [38],

$$a^{(5)\mu\alpha\beta} \bar{\psi} \gamma_\mu i D_{(\alpha} i D_{\beta)} \psi, \tag{30}$$

$$b^{(5)\mu\alpha\beta} \bar{\psi} \gamma_\mu \gamma_5 i D_{(\alpha} i D_{\beta)} \psi, \tag{31}$$

$$H^{(5)\mu\nu\alpha\beta} \bar{\psi} \sigma_{\mu\nu} i D_{(\alpha} i D_{\beta)} \psi, \tag{32}$$

where $D_\beta = i\partial_\beta - qA_\beta$. As these couplings do not involve the field strength tensor, $F_{\alpha\beta}$, they can not yield EDM nor MDM contributions, being of no interest for such a purpose.

4. *CPT*-even dimension-five NM LV electroweak couplings

In this section, we analyze CPT-even dimension-five nonminimal couplings composed of rank-2 and rank-4 tensors, which generate EDM and MDM behavior.

4.1. Rank−2 nonminimal coupling

The EDM Lagrangian terms should have the form presented in Eq. (1). Initially, the idea could be to propose a form in terms of a covariant derivative into the interaction Lagrangian (9). In the hermitian form, we first propose a non axial (without γ_5) modified covariant derivative,

$$\mathcal{D}_\mu = D_\mu - \frac{i}{2}\lambda_1 \left(T_{\mu\nu} B^{\nu\beta} - T^{\beta\nu} B_{\nu\mu}\right) \gamma_\beta, \tag{33}$$

based on the pattern first analyzed in Ref. [42]. Replacing this covariant derivative in the EW chiral Lagrangian structure for left-handed leptons, $\bar{L}_l \gamma^\mu i D_\mu L_l$, we obtain

$$\mathcal{L} = \bar{L}_l \gamma^\mu i \left[-\frac{i}{2}\lambda_1 \left(T_{\mu\nu} B^{\nu\beta} - T^{\beta\nu} B_{\nu\mu}\right) \gamma_\beta\right] L_l. \tag{34}$$

Using the identity, $\gamma^\mu \gamma_\beta = \left(\delta^\mu{}_\beta - i\sigma^\mu{}_\beta\right)$, it becomes

$$\mathcal{L} = -i\lambda_1 \bar{L}_l \left[\sigma^{\mu\beta} \left(T_{\mu\nu} B^\nu{}_\beta\right)\right] L_l, \tag{35}$$

where it was neglected a term of the form $\bar{L}_l[\left(T_{\beta\nu} B^{\nu\beta}\right)]L_l$, since it does not contain any gamma matrices nor spin components. Now it is necessary to remark that this nonminimal coupling is not properly communicated to the Lagrangian pieces of leptons and neutrinos. Indeed, we notice that

$$\left(\frac{1 \pm \gamma_5}{2}\right) X \left(\frac{1 \mp \gamma_5}{2}\right) = 0, \tag{36}$$

if the operator X contains an even number of gamma matrices, which includes $X = \sigma^{\mu\beta}$ as a special case. Otherwise, if the operator X possesses an odd number of gamma matrices, the quantity in Eq. (36) is not null, in principle. Thus, Lagrangian (35) yields a null contribution; the same holds for the right-handed fermions:

$$\bar{L}_l \left[\sigma^{\mu\beta} \left(T_{\mu\nu} B^\nu{}_\beta\right)\right] L_l = \bar{R}_l \left[\sigma^{\mu\beta} \left(T_{\mu\nu} B^\nu{}_\beta\right)\right] R_l = 0. \tag{37}$$

In order to circumvent this difficulty, we can propose $U(1)$ CPT-even NM couplings directly on the neutrino and lepton Lagrangian spinors:

$$\mathcal{L}_{(2)l}^{(even)} = \lambda_l \bar{\psi}_l \left[\sigma^{\mu\beta} T_{\mu\nu} B^\nu{}_\beta - i\sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^\nu{}_\beta\right] \psi_l, \tag{38}$$

$$\mathcal{L}_{(2)\nu}^{(even)} = \lambda_{\nu_l} \bar{\psi}_{\nu_l} \left[\sigma^{\mu\beta} T_{\mu\nu} B^\nu{}_\beta - i\sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^\nu{}_\beta\right] \psi_{\nu_l}, \tag{39}$$

where the imaginary factor was introduced with the matrix γ_5 in order to assure hermiticity. The leptons' NM couplings in Eq. (38) exhibit a "non axial" and an "axial" (with γ_5) interaction pieces:

$$\mathcal{L}_{(2)l(T)}^{(even)} = \lambda_l \bar{\psi}_l \sigma^{\mu\beta} \left(T_{\mu\nu} B^\nu{}_\beta\right) \psi_l, \tag{40}$$

$$\mathcal{L}_{(2)l(A)}^{(even)} = i\lambda_l \bar{\psi}_l \left(\sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^\nu{}_\beta\right) \psi_l, \tag{41}$$

where the label (T) refers to the tensor $T_{\mu\nu}$ and the label (A) refers to the "axial" tensor $\gamma_5 R_{\mu\nu}$ coupling. Such couplings are represented by two distinct tensors, $T_{\mu\nu}$ and $R_{\mu\nu}$, to stress that the interactions with and without γ_5 are physically different, in principle.

The lepton first piece can be explicitly written as

$$\begin{aligned}\mathcal{L}_{(T)}^l &= \cos\theta \bar{\psi}_l \left[i\lambda_l \alpha^i T_{00} E^i + i\lambda_l \epsilon_{aip} \alpha^i T_{0a} B^p\right. \\ &- i\lambda_l \alpha^i T_{ij} E^j + \lambda_l \mathcal{T}_{jk} \Sigma^k E^j \\ &\left.+ \lambda_l T_{ii} \Sigma^k B^k - \lambda_l T_{ik} \Sigma^k B^i\right] \psi_l - \sin\theta \bar{\psi}_l \left[i\lambda_l \alpha^i T_{00} \tilde{E}^i\right. \\ &+ i\lambda_l \alpha^i T_{0a} \epsilon_{aik} \tilde{B}^k - i\lambda_l T_{ij} \alpha^i \tilde{E}^j \\ &\left.+ \lambda_l \mathcal{T}_{jk} \Sigma^k \tilde{E}^j + \lambda_l T_{ii} \Sigma^k \tilde{B}^k - \lambda_l T_{ik} \Sigma^k \tilde{B}^i\right] \psi_l,\end{aligned} \tag{42}$$

where we have used the conventions (16), (17), (18) and (19). Such an expression provides "rotated" EDM and weak EDM contributions:

$$\mathcal{L}_{(T)EDM}^l = \lambda_l \cos\theta \bar{\psi}_l \left(\mathcal{T}_{jk} \Sigma^k E^j\right) \psi_l, \tag{43}$$

$$\mathcal{L}_{(T)WEDM}^l = -\lambda_l \sin\theta \bar{\psi}_l \left(\mathcal{T}_{jk} \Sigma^k \tilde{E}^j\right) \psi_l, \tag{44}$$

having as counterpart the following Hamiltonian contributions:

$$\mathcal{H}_{EDM}^l = -\lambda_l \cos\theta \psi_l^\dagger \gamma^0 \left(\mathcal{T}_{jk} \Sigma^k E^j\right) \psi_l, \tag{45}$$

$$\mathcal{H}_{WEDM}^l = \lambda_l \sin\theta \psi_l^\dagger \gamma^0 \left(\mathcal{T}_{jk} \Sigma^k \tilde{E}^j\right) \psi_l, \tag{46}$$

with the γ^0 factor circumventing the Schiff theorem [43] and assuring the effective EDM character. The EDM signature is also revealed by the behavior of these couplings under C, P and T operations, as shown in Table 4. Here, $\mathcal{T}_{jk}$ is a "rotated" background redefined as

$$\mathcal{T}_{jk} = \epsilon_{ijk} T_{i0}, \tag{47}$$

that allows to write the interactions in a more compact way. In expression (42), we also identify MDM and weak MDM (WMDM) interactions for leptons associated with the Lagrangian terms:

$$\mathcal{L}^l_{(T)(MDM)} = \lambda_l (\cos\theta) \left[\bar{\psi}_l \left(T\Sigma^k B^k\right)\psi_l - \bar{\psi}_l \left(T_{ik}\Sigma^k B^i\right)\psi_l\right], \tag{48}$$

$$\mathcal{L}^l_{(T)(WMDM)} = \lambda_l (\sin\theta) \left[-\bar{\psi}_l \left(T\Sigma^k \tilde{B}^k\right)\psi_l + \bar{\psi}_l \left(T_{ik}\Sigma^k \tilde{B}^i\right)\psi_l\right], \tag{49}$$

where $T = T_{ii} = \mathrm{Tr}(T_{ii})$ is the trace of the $T_{\mu\nu}$ spatial sector. Analogously, using Eqs. (16), (17), (18), (19), the same analysis for the second lepton piece (41) yields

$$\begin{aligned}\mathcal{L}^l_{(A)} &= \cos\theta\,\bar{\psi}_l \Big[\lambda_l R_{00}\Sigma^i E^i + \lambda_l \mathcal{R}_{ik}\Sigma^i B^k - \lambda_l R_{ij}\Sigma^i E^j \\ &- i\lambda_l \epsilon_{ijk}\alpha^k R_{i0}E^j - i\lambda_l \alpha^k R_{ii} B^k + i\lambda_l \alpha^k R_{ik} B^i\Big]\bar{\psi}_l \\ &- \sin\theta\,\bar{\psi}_l \Big[\lambda_l R_{00}\Sigma^i \tilde{E}^i + \lambda_l \mathcal{R}_{ik}\Sigma^i \tilde{B}^k - \lambda_l R_{ij}\Sigma^i \tilde{E}^j \\ &- i\lambda_l \epsilon_{ijk}\alpha^k R_{i0}\tilde{E}^j - i\lambda_l \alpha^k R_{ii} \tilde{B}^k + i\lambda_l \alpha^k R_{ik} \tilde{B}^i\Big]\psi_l,\end{aligned} \tag{50}$$

where $\mathcal{R}_{jk} = \epsilon_{ijk} R_{i0}$. Such an expression provides two direct Lorentz-violating EDM contributions for leptons:

$$\mathcal{L}^l_{(A)(EDM)} = \lambda_l \cos\theta \left[\bar{\psi}_l \left(R_{00}\Sigma^i E^i\right)\psi_l - \bar{\psi}_l \left(R_{ij}\Sigma^i E^j\right)\psi_l\right], \tag{51}$$

and two direct Lorentz-violating weak EDM (weak dipole moment) pieces:

$$\mathcal{L}^l_{(A)(WEDM)} = \lambda_l \sin\theta \left[-\bar{\psi}_l \left(R_{00}\Sigma^i \tilde{E}^i\right)\psi_l + \bar{\psi}_l \left(R_{ij}\Sigma^i \tilde{E}^j\right)\psi_l\right]. \tag{52}$$

There are rotated lepton MDM and weak MDM contributions as well:

$$\mathcal{L}^l_{(A)(MDM)} = \lambda_l \cos\theta\,\bar{\psi}_l \left(\mathcal{R}_{ik}\Sigma^i B^k\right)\psi_l, \tag{53}$$

$$\mathcal{L}^l_{(A)(WMDM)} = \lambda_l \sin\theta\,\bar{\psi}_l \left(\mathcal{R}_{ik}\Sigma^i \tilde{B}^k\right)\psi_l. \tag{54}$$

All these terms are shown in Table 3, in accordance with the EDM, WEDM, MDM and WMDM associated interaction.

Both Lagrangian pieces (40) and (41), $\mathcal{L}^l_{(T)}$ and $\mathcal{L}^l_{(A)}$, contain terms representing EDM, WEDM, MDM and WMDM contributions, that can be combined as follows:

$$\mathcal{L}^l_{(2)(EDM)} = \lambda_l \cos\theta\,\bar{\psi}_l \left[\left(\mathcal{T}_{jk}\Sigma^k E^j\right) + R_{00}\Sigma^i E^i - R_{ij}\Sigma^i E^j\right]\psi_l, \tag{55}$$

$$\mathcal{L}^l_{(2)WEDM} = -\lambda_l \sin\theta\,\bar{\psi}_l \left[\mathcal{T}_{jk}\Sigma^k \tilde{E}^j - R_{00}\Sigma^i \tilde{E}^i + R_{ij}\Sigma^i \tilde{E}^j\right]\psi_l, \tag{56}$$

Table 3
EDM, WEDM, MDM and WMDM contributions to the Hamiltonian of the lepton nonminimal coupling (38).

EDM	WEDM
$-\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(R_{00}\Sigma^i E^i\right)\psi_l$	$\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(R_{00}\Sigma^i \tilde{E}^i\right)\psi_l$
$-\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(R_{jk}\Sigma^k E^j\right)\psi_l$	$\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(R_{jk}\Sigma^k \tilde{E}^j\right)\psi_l$
$\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(\mathcal{T}_{ij}\Sigma^i E^j\right)\psi_l$	$-\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(\mathcal{T}_{ij}\Sigma^i \tilde{E}^j\right)\psi_l$
MDM	**WMDM**
$-\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(T\Sigma^k B^k\right)\psi_l$	$\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(T\Sigma^k \tilde{B}^k\right)\psi_l$
$-\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(T_{ik}\Sigma^i B^k\right)\psi_l$	$\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(T_{ik}\Sigma^i \tilde{B}^k\right)\psi_l$
$\lambda_l \cos\theta\,\psi_l^\dagger \gamma^0 \left(\mathcal{R}_{ik}\Sigma^k B^i\right)\psi_l$	$-\lambda_l \sin\theta\,\psi_l^\dagger \gamma^0 \left(\mathcal{R}_{ik}\Sigma^k \tilde{B}^i\right)\psi_l$

Table 4
Complete classification under C, P, T for the CPT-even nonminimal couplings of Table 3, showing the EDM signature for $\lambda_l \mathcal{T}_{jk}$, $\lambda_l R_{00}$, $\lambda_l R_{ij}$, and the MDM behavior for $\lambda_l T$, $\lambda_l T_{ij}$, $\lambda_l \mathcal{R}_{ij}$.

Coupling	$\lambda_l T$	$\lambda_l T_{ij}$	$\lambda_l \mathcal{T}_{jk}$	$\lambda_l R_{00}$	$\lambda_l R_{ij}$	$\lambda_l \mathcal{R}_{ij}$
P	+	+	−	−	−	+
C	+	+	+	+	+	+
T	+	+	−	−	−	+

$$\mathcal{L}^l_{(2)(MDM)} = \lambda_l (\cos\theta)\,\bar{\psi}_l \left[T\Sigma^k B^k - \bar{\psi}_l T_{ik}\Sigma^k B^i + \mathcal{R}_{ik}\Sigma^i B^k\right]\psi_l, \tag{57}$$

$$\mathcal{L}^l_{(2)(WMDM)} = \lambda_l (\sin\theta)\,\bar{\psi}_l \left[-\left(T\Sigma^k \tilde{B}^k\right) + \left(T_{ik}\Sigma^k \tilde{B}^i\right) + \mathcal{R}_{ik}\Sigma^i \tilde{B}^k\right]\psi_l. \tag{58}$$

In typical EDM experiments for atoms or molecules, the system is subjected to the action of constant electric and magnetic fields aligned in some axis in space. In general, the spin of the atoms is polarized in the field direction, in such a way it is natural to measure the interaction $\mathbf{\Sigma}\cdot\mathbf{E}$. This kind of experiment could be used as prototype for new measurement proposals, where the spin is orthogonal to the applied field, reflecting spin-field interactions as $\Sigma^i E^j$, analogue to $\mathbf{\Sigma}\times\mathbf{E}$, as theoretically discussed in Refs. [42], [44]. The high energy scattering processes that provide upper bounds on the tau lepton MDM and EDM, however, do not work with a similar idea of constant polarized electromagnetic fields. In this sense, the theoretical route used to constrain "rotated" or orthogonal spin responses [42], [44] will not be used here, preventing the constraining on the coefficients $\mathcal{R}_{ik}$, $\mathcal{T}_{jk}$.

In order to keep correspondence between the present approach and the conventional electric and magnetic dipoles, upper limits will be stated on the isotropic coefficients R_{00} and trace coefficients of space sectors, T and R, of the tensors $T_{\mu\nu}$ and $R_{\mu\nu}$, associated with usual EDM, WEDM, MDM and WMDM interactions. For the EDM coupling (55), for instance, only $R_{00}\Sigma^i E^i$ and the diagonal terms of $R_{ij}\Sigma^i E^j$ can be reduced to the usual EDM form. In this sense, proposing a total isotropic parametrization for the diagonal components, $R_{ij} = (1/3)R\delta_{ij}$, $T_{ij} = (1/3)T\delta_{ij}$, the interactions (55), (56), (57), (58) are rewritten as

$$\mathcal{L}^l_{(2)(EDM)} = \lambda_l \cos\theta\,\bar{\psi}_l \left[\left(R_{00} - \frac{1}{3}R\right)\Sigma^i E^i\right]\psi_l, \tag{59}$$

$$\mathcal{L}^l_{(2)WEDM} = -\lambda_l \sin\theta\,\bar{\psi}_l \left[\left(R_{00} - \frac{1}{3}R\right)\Sigma^i \tilde{E}^i\right]\psi_l, \tag{60}$$

$$\mathcal{L}^l_{(2)(MDM)} = \lambda_l \cos\theta\,\bar{\psi}_l \left[\frac{2}{3}T\Sigma^k B^k\right]\psi_l, \tag{61}$$

$$\mathcal{L}^{l}_{(2)(WMDM)} = -\lambda_l \sin\theta \bar{\psi}_l \left[\frac{2}{3} T \Sigma^k \tilde{B}^k\right] \psi_l, \tag{62}$$

where $R = R_{ii} = \mathrm{Tr}(R_{ii})$ is the trace of the $R_{\mu\nu}$ space sector, and $\mathcal{T}_{jj} = 0$ due to definition (47). The same holds for $\mathcal{R}_{jj} = 0$. Notice that R_{00} and R engender indistinguishable EDM and WEDM contributions.

The tau lepton WMDM and WEDM data are obtained from electron-positron scattering [13,14], [58,59]. Due its large mass, the tau lepton is the one that presents the most significant electroweak contributions on the dipole moments. It is known that larger dipole magnitudes do not yield the tightest upper limits. Yet, the tau data will be used to constrain weak moments since the WEDM and WMDM experimental data for the electron and muon are not clearly available. Using the upper bound for the tau lepton WEDM [59], the element (60) leads to

$$\left|\lambda_\tau (\sin\theta)\left(R_{00} - \frac{1}{3}R\right)\right| < 1.2\times 10^{-17}\, e\cdot \mathrm{cm}, \tag{63}$$

$$\left|\lambda_\tau \left(R_{00} - \frac{1}{3}R\right)\right| < 4\times 10^{-4}\ (\mathrm{GeV})^{-1}, \tag{64}$$

where we used $\sin\theta = 0.48$. Tau WMDM experimental upper bounds [59], $\alpha_w < 1\times 10^{-3}$, considered with the tau Bohr magneton factor, see Lagrangian (1), also reads

$$\frac{e}{2m_\tau}\alpha_w < 9\times 10^{-5}\,(\mathrm{GeV})^{-1}, \tag{65}$$

and can be used to constrain the WMDM interaction (62), that is, $\sin\theta\,|\lambda_\tau 2T/3| < 4\times 10^{-5}\ (\mathrm{GeV})^{-1}$, or

$$|\lambda_\tau T| < 3\times 10^{-4}\,(\mathrm{GeV})^{-1}. \tag{66}$$

In order to constrain the EDM couplings, we should use the electron EDM measurements, which represent the smallest EDM limit ever established, $d_e < 1.1\times 10^{-29}\, e\cdot\mathrm{cm}$ [8]. The interaction (59) is bounded as

$$\left|\lambda_e \left(R_{00} - \frac{1}{3}R\right)\right| < 2\times 10^{-16}\ (\mathrm{GeV})^{-1}, \tag{67}$$

where we have used $\cos\theta = 0.88$.

Concerning the electron MDM interaction,

$$\mathcal{L} = \bar{\psi}\left[g\frac{e}{2m_e}\sigma^{\mu\nu}F_{\mu\nu}\right]\psi, \tag{68}$$

precise measurements reveal that the experimental imprecision on the electron MDM is at the level of 2.8 parts in 10^{13} [16], that is, $\Delta a \le 2.8\times 10^{-13}$. This value represents the window for new contributions that stem from dimension-five interactions of the type $H' = -\mu_B g\,(\boldsymbol{S}\cdot\mathbf{B})$, in such a way that $\lambda_e(2T/3)\cos\theta < \mu_B\Delta a = 2.4\times 10^{-20}\,(\mathrm{eV})^{-1}$, implying

$$\lambda_e T < 1\times 10^{-10}\,(\mathrm{GeV})^{-1}. \tag{69}$$

As for the muon MDM physics, in a general way, the discrepancy $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} \simeq 260\times 10^{-11}$ could be ascribed to nonminimal dimension-5 couplings, in such a way it may be used to state a limit on the weak MDM interaction (62) for muons. It is also worthy to remember that 1-loop and 2-loop electroweak contributions yield $a_\mu^{EW} = 153\times 10^{-11}$ [21], close to the discrepancy magnitude. We thus have $\lambda_\mu(2T/3)\sin\theta < (\mu_B)_\mu\,\Delta a_\mu = 1\times 10^{-18}\,(\mathrm{eV})^{-1}$, attaining:

$$\lambda_\mu T < 1\times 10^{-8}\,(\mathrm{GeV})^{-1}. \tag{70}$$

Regarding the muon EDM, the main experiments are based on spin precession in muon $g-2$ storage ring at Brookhaven National Laboratory, where the EDM magnitude is restrained by the effect it produces on the precession frequency [11] as $|d_\mu| < 1.8\times 10^{-19}\, e\cdot\mathrm{cm}$. This upper limit can be used to constrain the interaction (59) for the muon:

$$\left|\lambda_\mu \left(R_{00} - \frac{1}{3}R\right)\right| < 3\times 10^{-6}\ (\mathrm{GeV})^{-1}. \tag{71}$$

The same kind of analysis holds for the neutrino NM coupling contained in Lagrangian term (39), which can be analogously separated into two pieces,

$$\mathcal{L}^{\nu}_{T} = \lambda_\nu \bar{\psi}_\nu \sigma^{\mu\beta}\left(T_{\mu\nu}B^{\nu}{}_{\beta}\right)\psi_\nu, \tag{72}$$

$$\mathcal{L}^{\nu}_{A} = -i\lambda_\nu \bar{\psi}_\nu \left(\sigma^{\mu\beta}\gamma_5 R_{\mu\nu}B^{\nu}{}_{\beta}\right)\psi_\nu. \tag{73}$$

Due to the similar structure between the lepton and neutrino NM couplings in Eqs. (38) and (39), the EDM, WEDM, MDM and WMDM Lagrangian contributions for neutrinos are, in principle, the same ones of Table 3, only by replacing $\psi_l \to \psi_\nu$ and $\bar{\psi}_l \to \bar{\psi}_\nu$.

4.2. Rank−4 dimension-five nonminimal LV electroweak couplings

In this section, we introduce, directly on the GSW model Lagrangian, the rank−4 dimension-five nonminimal LV couplings:

$$\mathcal{L}^{(even)}_{(4)} = \frac{\lambda_l}{2}\bar{\psi}_l\left[\sigma^{\mu\nu}K_{\mu\nu\alpha\beta}B^{\alpha\beta} + i\sigma^{\mu\nu}\gamma_5\bar{K}_{\mu\nu\alpha\beta}B^{\alpha\beta}\right]\psi_l + \frac{\lambda_{\nu_l}}{2}\bar{\psi}_{\nu_l}\left[\sigma^{\mu\nu}K_{\mu\nu\alpha\beta}B^{\alpha\beta} + i\sigma^{\mu\nu}\gamma_5\bar{K}_{\mu\nu\alpha\beta}B^{\alpha\beta}\right]\psi_{\nu_l}, \tag{74}$$

where the rank-4 background tensors $K_{\mu\nu\alpha\beta}$, $\bar{K}_{\mu\nu\alpha\beta}$ are antisymmetric in the two pairs:

$$K_{\mu\nu\alpha\beta} = -K_{\nu\mu\alpha\beta}, \tag{75}$$

$$K_{\mu\nu\alpha\beta} = -K_{\mu\nu\beta\alpha}. \tag{76}$$

Supposing $T_{\nu\beta} = (K)^{\alpha}{}_{\nu\alpha\beta}$ and $R_{\nu\beta} = (\bar{K})^{\alpha}{}_{\nu\alpha\beta}$, one can propose the prescription,

$$(K)_{\mu\nu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha}T_{\nu\beta} - g_{\mu\beta}T_{\nu\alpha} + g_{\nu\beta}T_{\mu\alpha} - g_{\nu\alpha}T_{\mu\beta}\right), \tag{77}$$

$$(\bar{K})_{\mu\nu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha}R_{\nu\beta} - g_{\mu\beta}R_{\nu\alpha} + g_{\nu\beta}R_{\mu\alpha} - g_{\nu\alpha}R_{\mu\beta}\right), \tag{78}$$

where the tensors $T_{\nu\beta}, R_{\nu\beta}$ are now symmetric and traceless. Replacing such a prescription in the lepton sector of the Lagrangian (74), we obtain:

$$\mathcal{L}^{(even)}_{(4)l} = \frac{\lambda_l}{2}\bar{\psi}_l\left[\sigma^{\alpha\nu}T_{\nu\beta}B_{\alpha}{}^{\beta} + i\sigma^{\alpha\nu}\gamma_5 R_{\nu\beta}B_{\alpha}{}^{\beta}\right]\psi_l. \tag{79}$$

These couplings recover the ones involving ranking-2 tensors, already presented. Thus, if the rank-4 tensor is written as shown in expression (77), the upper bounds found in the last section hold equivalently for some components of $(K)_{\mu\nu\alpha\beta}$. For instance, $T_{jj} = (K)_{0j0j} - (K)_{njnj}$, so that the WEDM upper limit (66) reads as:

$$\lambda_\tau\left|(K)_{0j0j} - (K)_{njnj}\right| < 3\times 10^{-4}\ (\mathrm{GeV})^{-1}. \tag{80}$$

We must point out it is possible to connect the dimension-5 couplings written in terms of $K_{\mu\nu\alpha\beta}$, $\bar{K}_{\mu\nu\alpha\beta}$, to the dimension-4 coupling, $c_{\mu\nu}$, that appears in the term $\bar{\psi}c^{\mu\nu}\gamma_\mu iD_\nu\psi$, with $iD_\nu = i\partial_\nu - qA_\nu$, found in the Lagrangian (4) of Ref. [38]. In accordance with Eq. (29) of Ref. [38], a field redefinition of the type $X_V^{\mu\alpha}p_\alpha\gamma_\mu$ implemented in a nonminimal Lagrangian can induce dimension four and dimension five contributions:

Table 5
Upper bounds on EDM and WEDM contributions to leptons, in $(\mathrm{GeV})^{-1}$.

Coupling	Upper bound
tau-WEDM	$\left\|\lambda_\tau\left(R_{00}-\frac{1}{3}R\right)\right\|<4\times10^{-4}\ (\mathrm{GeV})^{-1}$
tau-WMDM	$\|\lambda_\tau T\|<3\times10^{-4}\ (\mathrm{GeV})^{-1}$
e-EDM	$\left\|\lambda_e\left(R_{00}-\frac{1}{3}R\right)\right\|<2\times10^{-16}\ (\mathrm{GeV})^{-1}$
e-MDM	$\lambda_e T<1\times10^{-10}\ (\mathrm{GeV})^{-1}$
μ-EDM	$\left\|\lambda_\mu\left(R_{00}-\frac{1}{3}R\right)\right\|<3\times10^{-6}\ (\mathrm{GeV})^{-1}$.
μ-WMDM	$\lambda_\mu T<1\times10^{-8}\ (\mathrm{GeV})^{-1}$

$$c^{\mu\alpha}=-2mX_V^{\mu\alpha}, \tag{81}$$

$$H_F^{(5)\mu\nu\alpha\beta}=2q\left[X_V^{\mu[\alpha}g^{\beta]\nu}-X_V^{\nu[\alpha}g^{\beta]\mu}\right]. \tag{82}$$

The expression for $H_F^{(5)\mu\nu\alpha\beta}$ in Eq. (82) has a similar structure to $(K)_{\mu\nu\alpha\beta}$, given in Eq. (77), with the replacement $X_V^{\mu\alpha}\to T^{\mu\alpha}$. Except for the trace of $X_V^{\mu\alpha}$, which is in principle non null, there is such a possible correspondence. Accordingly, as the bound on $T_{\mu\nu}$ can be used to impose limits on $K_{\mu\nu\alpha\beta}$ components, it can also constrain $c^{\mu\alpha}/m$, in view of Eq. (81).

5. Conclusion and final remarks

We analyzed dimension-five LV nonminimal couplings in the EW sector. The CPT-odd ones are not effective in generating EDM or MDM contributions, both in the rank-1 and rank−3 forms. Such impossibility is confirmed by the EDM-incompatible signature under C, P and T operators, as shown in Table 1. We also examined CPT-even nonminimal electroweak couplings, which generate tree-level EDM, MDM, WEDM and WMDM contributions. We firstly have introduced rank-2 dimension-five nonminimal couplings directly in the GSW model Lagrangian, using two rank-2 background tensors, $T_{\mu\nu}$ and $R_{\mu\nu}$, as presented in Lagrangians (40) and (41). We have identified the coefficients that generate EDM, MDM, WEDM and WMDM lepton contribution to the Hamiltonian.

Due to the tau's mass, electroweak effects contribute the most to the tau's dipole moments. In addition, WEDM and WMDM experimental data for the electron and muon are not clearly available. For these reasons, the tau experimental EW data were used to impose constraints on the dimension-5 nonminimal WEDM and WMDM couplings to the level of $10^{-4}\ (\mathrm{GeV})^{-1}$. In order to constrain new physics with experimental MDM or EDM data, the most restrictive upper limits are obtained from the smallest dipole magnitudes. With that in mind, we have also used the experimental data on the two other leptons in order to achieve tighter upper bounds, whenever possible, which also fulfilled the goal of constraining the NM couplings for each lepton flavor (electron, tau and muon). In this respect, muon EDM and WMDM have implied upper bounds at the level of $10^{-6}\ (\mathrm{GeV})^{-1}$ and $10^{-8}\ (\mathrm{GeV})^{-1}$, while electron MDM and EDM enhance the constraining to the level of $10^{-10}\ (\mathrm{GeV})^{-1}$ and $10^{-16}\ (\mathrm{GeV})^{-1}$, respectively. These upper bounds are shown in Table 5.

We have also proposed CPT-even nonminimal EW couplings involving a rank-4 background tensor, $K_{\mu\nu\beta\alpha}$, coupled to the $U(1)$ field strength and the leptons' (neutrinos') spinors. Using suitable relations, Eqs. (77), (78), we showed that some rank-4 couplings may become equivalent to rank-2 couplings, see Eq. (79), being bounded on the same level of constraining that holds on the corresponding rank-2 coefficients.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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