A Galois group functor for the category of Special Groups

Hugo Luiz Mariano

[1] University of São Paulo, IME-USP, hugomar@ime.usp.br

The relationship between Galois groups of fields with orderings and quadratic forms, established by the works of Artin-Schreier (1920's) and Witt (late 1930's) are reinforced by a seminal paper of John Milnor ([5], 1971) through the definition of a (mod 2) k-theory graded ring that "interpolates" the graded Witt ring and the cohomology ring of fields: the three graded rings constructions determine functors from the category of fields where 2 is invertible that, almost tree decades later, are proved to be naturally isomorphic by the work of Voevodsky with co-authors.

Since the 1980's, have appeared many abstract approaches to the algebraic theory of quadratic forms over fields that are essentially equivalent (or dually equivalent): between them we emphasize the (first-order) theory of special groups developed by Dickmann-Miraglia. The notions of (graded) Witt rings and k-theory are extended to the category of Special groups with remarkable pay-offs on questions on quadratic forms over fields ([2], [3]).

In the present ongoing work we extended to (well-behaved) Special Groups the work of J. Mináč and Spira ([6]) that describes a (pro-2)-group of a field extension that encodes the quadratic form theory of a given field F: in [1] it is shown that its associated cohomology ring is contains a copy of the cohomology ring of the field F. Our construction, a contravariant functor $G \in SG \ \mapsto \ Gal(G) \in Pro-2-groups$, is essentially given by generators and relations of profinite-2-groups. We prove that such profinite groups $Gal(G)$ encodes the space of orders of the special group G and provide a criteria to detect when G is formally real or not. We calculate the Galois groups associated to some of the main constructions of special groups like quotients, directed colimits and extensions. The next step is to develop and understand the associated cohomology ring, relating it with k-theory functor of the special groups, and apply such machinery to questions on Special Groups theory.

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References

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