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On global linear instability mechanisms of flow around airfoils at low Reynolds number and high angle of attack

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Abstract

The existence of modal and non-modal linear three-dimensional global instabilities of spanwise-homogeneous low-Reynolds number laminar incompressible flow around two-dimensional airfoils is documented. Work has commenced to examine the relative significance of such instabilities as a function of airfoil thickness and camber at conditions close to stall: the symmetric NACA0009 and NACA0015 airfoils as well as the cambered NACA4415 are used for these analyses at angles of attack $15 \leq \text{AoA} \leq 20$. At all conditions examined, the combination of the Re and AoA parameters is chosen such that steady two-dimensional flow over the airfoil is obtained. Four independent codes, one based on finite-volume, one on finite-element and two on spectral-element spatial discretization have been used to cross-validate the base flow and instability analysis results presented, all delivering consistent predictions. In line with previous findings on the NACA0015 airfoil, two classes of three-dimensional stationary and traveling global eigenmodes are recovered. Contrary to what was previously reported, the traveling wake-mode instability dominates that pertaining to the stationary three-dimensional flow eigenmode. Furthermore, neither traveling nor stationary unstable three-dimensional modes have been found in the NACA0015 airfoil, prior to the two-dimensional wake mode undergoing a Hopf bifurcation. Consequently, amplified three-dimensional structures akin to the stall cells reported in earlier work have not been observed in the present analyses. Finally, relatively strong transient energy growth is shown here for the first time for flows around airfoils, as known from earlier analyses of flow around bluff bodies. In the context of non-modal instability too, strongest energy amplification is found to be associated with the two-dimensional flow and converts, via the Orr mechanism, optimal initial conditions to wake-mode perturbations.

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1. Introduction

Rapid progress in algorithms for the solution of the multi-dimensional eigenvalue and initial-value problems governing linear (global) instability of flows around relatively complex geometries permit addressing questions of increasing relevance to aeronautics applications. The first such efforts regarding global instability of flow around the NACA0012 airfoil were reported by Theofilis & Sherwin¹, followed by the work of Theofilis *et al.*². Encouraged

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by the capability of the analysis to identify, for the first time, linear instability in the wake of the airfoil as a global eigenmode, without resorting to the approximations of weakly-nonparallel flow used prior to those analyses, work by Abdessemed *et al.*³ was performed in order to identify linear global instability mechanisms in a Low Pressure Turbine (LPT) passage. Interestingly, four possible classes of linearly amplified global modes were found: (i) three-dimensional (BiGlobal) instability of the steady two-dimensional flow followed by (ii) two-dimensional unsteadiness of the wake, ensuing as linear amplification of the two-dimensional BiGlobal wake eigenmode earlier identified in the wake of the NACA0012 airfoil¹. The resulting two-dimensional time-periodic flow was also found to be (iii) linearly unstable to three-dimensional Floquet eigenmodes akin to those discovered in the wake of the circular cylinder by Barkley and Henderson⁴. However, the significance of all of these three linear modal mechanisms was put in perspective by the result of the (iv) first transient growth (TG) analysis performed in a global linear instability context in the same work of Abdessemed *et al.*³, according to which strong transient energy amplification of several orders of magnitude was identified in the LPT passage. The latter finding gave rise to the subsequent demonstration of TG in the wake of the circular cylinder⁵ and the further quantification of this phenomenon in the LPT passage in the work of Sharma *et al.*⁶.

Work to quantify linear instability mechanisms in spanwise homogeneous flow around two-dimensional airfoils continued with the analyses of Kitsios *et al.*⁷ and Rodríguez and Theofilis⁸, both of which were performed in a modal framework. The latter work provided a topological description of the results obtained in the former regarding three-dimensional BiGlobal instability of steady two-dimensional massively separated flow in the wake of a NACA0015 airfoil at $Re = 200, AoA = 18^\circ$. Reconstruction of the three-dimensional total flowfield ensuing modal linear amplification of the three-dimensional stationary perturbation associated with the steady laminar separation bubble⁹, superposed upon the nominally two-dimensional steady base flow, delivered surface streamlines patterns reminiscent of the well-documented *Stall Cells* appearing on airfoils at flight conditions close to stall. However, work by¹⁰ on the same airfoil but different Reynolds number and angle of attack, as well as the results of Tsiloufas *et al.*¹¹ in the wake of the NACA0012 airfoil, asserted that the first linear instability mechanism to be encountered has been that found in the wake of the cylinder, namely a three-dimensional Floquet mode superposed upon the unsteady two-dimensional base flow behind the airfoil. Motivation thus exists to revisit this problem and quantify the relative importance of the possible linear instability mechanisms as a function of flow parameters. In addition, no TG analysis of flow around two-dimensional airfoils has been performed to-date, in order to assess the relative importance of modal and non-modal linear (global) instability mechanisms.

The present contribution addresses modal and non-modal linear instability of nominally steady two-dimensional flow around three airfoil profiles, the NACA0009, NACA0015 and NACA4415, in an attempt to unravel the linear perturbations leading laminar flow to turbulence and understand the effect of curvature on the underlying physical mechanisms. The multiparametric nature of the problem suggests that a relatively sparse discretization of the parameter space can be addressed around values of physical significance. The Reynolds number is chosen to be sufficiently low, such that, at a given angle of attack, steady laminar two-dimensional flow results around a given airfoil. The angle of attack is chosen at values around stall, when massive separation in the form of a closed recirculation bubble can be observed in the base flow, in the neighborhood of the trailing edge of the airfoil. BiGlobal modal and non-modal analysis is performed, in which the spanwise periodicity length, L_z , is one of the problem parameters; the associated wavenumber $\beta = 2\pi/L_z$ is varied from $\beta = 0$, corresponding to two-dimensional flow, to large values at which the flow is strongly stable. The open source code *nektar++* and *FreeFEM++*, respectively based on spectral and finite-element spatial discretization are used for both base flow calculations, as well as for modal and non-modal analyses. In addition, base flows have been obtained using the finite-volume open source code OpenFOAM.

2. Two-dimensional flows

Steady laminar base states around the three above-mentioned airfoils are obtained first. In all cases a sharp trailing-edge airfoil closure has been used, based on earlier evidence obtained by He *et al.*¹² as regards the effect of different airfoil closures in the NACA0015 airfoil. A range of angles of attack AoA and Reynolds numbers have been addressed and results are shown in Table 1. The general tendency observed is that an increase of Reynolds number leads to two-dimensional unsteadiness, later to be verified in the global instability analyses as linear amplification of the $\beta = 0$ BiGlobal eigenmode, while thickening of the airfoil profile at the same angle of attack, as well as introducing camber

at the same thickness lowers the Reynolds number value at which unsteadiness occurs. An example of the separation bubble for the steady flow in NACA0015 for angle of attack 18° and Reynolds 200 is presented in Figure 1 (a) in terms of the streamwise basic flow velocity components. Figure 1 (b) shows transverse velocity of two-dimensional the least stable wake mode for this base flow.

Table 1. Threshold of unsteadiness obtained with simulations in OpenFOAM. S stands for Steady and Us stands for Unsteady.

	NACA 0009				NACA 0015				NACA 4415			
	Re				Re				Re			
AoA	100	200	300	400	100	200	300	400	100	200	300	400
20°	S	Us	Us	Us	S	Us	Us	Us	S	Us	Us	Us
15°	S	S	S	Us	S	S	S	Us	S	S	Us	Us
10°	S	S	S	S	S	S	S	S	S	S	S	S
5°	S	S	S	S	S	S	S	S	S	S	S	S

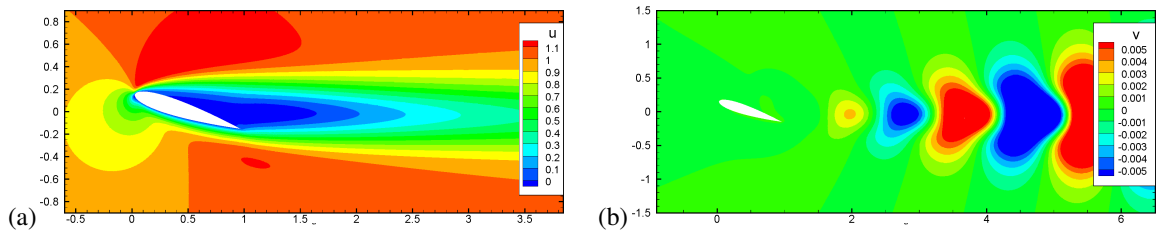


Fig. 1. NACA0015 at AoA 18° and Reynolds 200: (a) Steady flow using the spectral-element method. (b) Leading two-dimensional mode from stability analysis of the steady flow at $\beta = 0$.

3. Numerical aspects of the instability analyses performed

3.1. Time-stepping approach

Instability analyses have been performed using a time-stepper technique⁴, as implemented in two versions of the nektar open-source code. The velocity boundary conditions that were imposed on this system are perturbation velocity $\mathbf{u}' = 0$ on boundaries where Dirichlet conditions are specified for the base flow, and $\partial \mathbf{u}' / \partial n = 0$ on boundaries where Neumann conditions are specified for the base flow (outflow downstream). The search was limited to the leading traveling and stationary eigenvalues, for which a Krylov space with dimension of 30 has been used and the convergence criterion is residual 10^{-5} . Since the base flow is steady and it is a time-stepper technique, we choose to develop the perturbation for a non-dimensional time interval of 1 using the same time step size used for the base flow calculation.

3.2. Matrix-forming approach

In order to cross-validate results delivered by the time-stepping approach, and also in view of the fact that the earlier instability analysis of the NACA0015 airfoil^{7,8} has been performed using a matrix-forming methodology, a new matrix-forming code was written, based on the FreeFEM++¹³ open-source software. The spatial discretization is performed using a Taylor-Hood element on triangular elements. The domain size is $(-32 \leq x/c \leq 40$ and $-32 \leq y/c \leq 32)$ with following boundary conditions: zero velocity on airfoil, $\mathbf{u} = 1\hat{i}$ upstream and sides, and outflow $p - Re^{-1} \partial \mathbf{u} / \partial n = 0$. The steady base flow is calculated from the steady Navier-Stokes and continuity equations using Newton iteration technique, and it is iterated until $\|\Delta \mathbf{u}\|_2 / \|\mathbf{u}\|_2 < 10^{-12}$. For the stability analysis, sparse matrices

associated with the discretized three-dimensional perturbation equations are formed using FreeFEM++. Then the eigenproblem is set with a in-house code and calculated using Arpack. The boundary conditions for the perturbation equation are $\mathbf{u}' = 0$ on boundaries where it is specified velocity on the base flow and $p' - Re^{-1}\partial\mathbf{u}'/\partial n = 0$ at outflow.

4. Results

4.1. Modal global linear instability analysis

Results have been obtained for all three airfoils, however discussion here is focused on those pertaining to the NACA0015 airfoil, since they are qualitatively representative of the physical instability mechanisms on all three airfoils. The eigenvalue problem pertaining to steady laminar basic flow at $Re = 200$, $AoA = 18^\circ$ has been solved in the wavenumber range $0 \leq \beta \leq 10$.

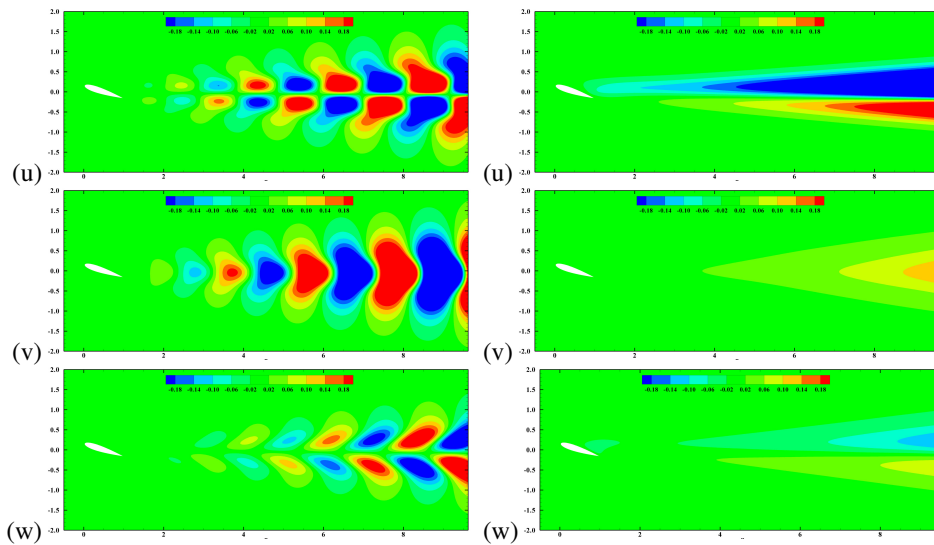


Fig. 2. Amplitude functions of the least-damped traveling (left column) and stationary (right column) perturbation velocity components at $\beta = 1$ and $\beta = 3$, respectively

As known from earlier analyses^{7,8} the least damped members of the eigenspectrum contain both traveling and stationary disturbances. Figure 2 presents the amplitude functions of the perturbation velocity components at $\beta = 1$ and $\beta = 3$, respectively corresponding to the wake-mode and the stationary mode instabilities. Results being linear, they have been scaled with the respective largest perturbation velocity magnitude values. These are $-0.78 \leq \hat{u}(x, y) \leq 0.40$, $-0.93 \leq \hat{v}(x, y) \leq 1$ and $-0.40 \leq \hat{w}(x, y) \leq 0.41$ for the wake mode and $-1.00 \leq \hat{u}(x, y) \leq 0.80$, $-0.002 \leq \hat{v}(x, y) \leq 0.152$ and $-0.15 \leq \hat{w}(x, y) \leq 0.10$ for the stationary three-dimensional eigenmode.

However, unlike the earlier reported results,^{7,8} only *stable* eigenvalues have been encountered at all β values examined. Here we find that at $\beta = 1$ ^{7,8} the leading mode for low wavenumbers (long wavelengths) is unsteady, while at high wavenumbers the least damped mode corresponds to a stationary linear global instability. At all conditions examined, $0 \leq \beta \leq 10$, the least-damped eigenmode corresponds to the wake-mode instability, the stationary three-dimensional eigenmode being substantially more damped than the wake mode. No unstable modes were found under these flow conditions, as can be seen in the results of Figure 3(a). The integrity of these results was verified in a twofold manner. First, in the time-stepping context two versions of the nektar code were employed, using different meshing algorithms, domain extensions ($-15 \leq x/c \leq 30$ and $-15 \leq y/c \leq 15$) and polynomial degrees ($p = 5, 7$ and 9); they produced the results shown in Fig. 3(a). Second, the matrix-forming code described earlier was used to analyze the same base flow and delivered results that are consistent with those obtained by the time-stepping methodology, as seen in Figure 3(b).

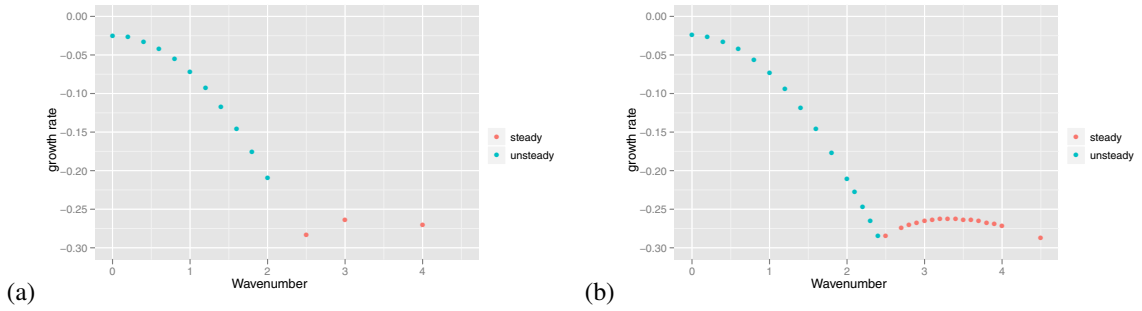


Fig. 3. Comparison of time-stepping (*left*) and matrix forming (*right*) results at $Re = 220$.

Figure 4(a) shows instability analysis results in the Reynolds number range $200 \leq Re \leq 230$, the highest Reynolds number value being slightly smaller than the unsteadiness threshold. At $\beta = 0$ these results are in agreement with the threshold of unsteadiness obtained through two-dimensional numerical simulations of flow around NACA0015 at AoA 18° , while three-dimensional instability analyses performed at $\beta \neq 0$ show that the Reynolds number parameter has a quantitative but not a qualitative influence on the relative importance of the three-dimensional wake and stationary eigenmodes. Finally, the non-dimensional frequency of the wake mode is also seen in Figure 4(b) to be practically independent of the spanwise wavenumber or the Reynolds number parameters.

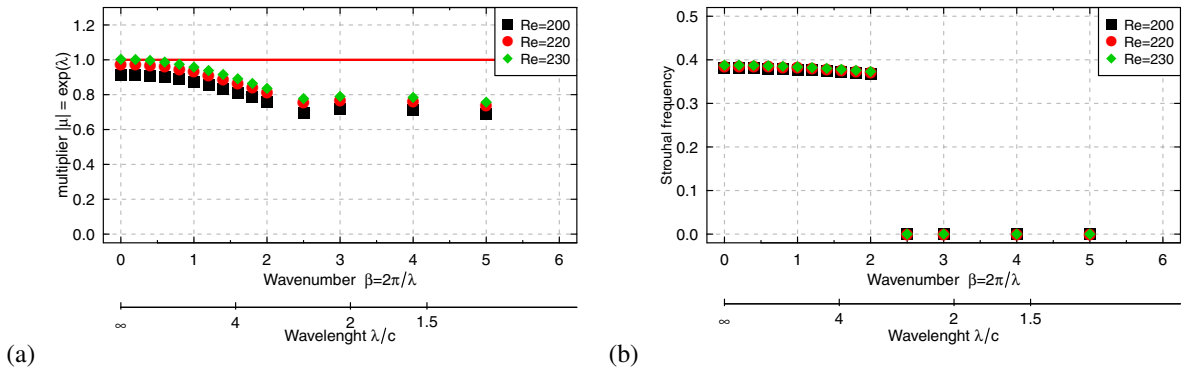


Fig. 4. Stability analysis of the flow around NACA0015 at AoA 18° and Reynolds 200: (a) Modulus of multiplier $\mu = \exp(\lambda)$, λ is the leading eigenvalue, as function of spanwise wavenumber β . the red line is the marginal stability threshold. (b) Strouhal frequencies of the leading eigenvalues with respect to spanwise wavenumbers β .

4.2. Non-modal global linear instability analysis

The steady base flow of the NACA0015 airfoil at the same parameters, $Re = 200$, $AoA = 18^\circ$, is analyzed next with respect to its capacity to sustain transient energy growth. The same range of spanwise wavenumbers, $0 \leq \beta \leq 10$, is monitored, and the direct-adjoint iteration procedure¹⁴ is performed for short-time parameter, τ , values, such that the maximum of the growth function $G(\tau)$ is well defined in all β results. The energy gain function dependence on τ is shown in Figure 5 at spanwise wavenumber values $0 \leq \beta \leq 10$. Higher β values were not analyzed, since it is clear from the results that the largest transient energy growth is attained for two-dimensional perturbations ($\beta = 0$) superposed upon the steady two-dimensional base flow. Such linear optimals reach a maximum of $G(\tau \approx 30) \approx 10^4$. At the parameter $\beta = 1$, for which the earlier modal analyses have been performed, the maximum $G(\tau \approx 20) > 10^3$.

The optimal initial conditions at $\beta = 1$ are shown at different values of the time parameter in Fig.6: all optimal initial conditions evolve into the wake mode identified by the modal analysis.

Such large transient energy amplification values are known from the related earlier works of Abdessemed *et al.*³ and Sharma *et al.*⁶ in the wake of a Low Pressure Turbine blade cascade, as well as from the analogous analysis of Abdessemed *et al.*⁵ in the circular cylinder wake. While the relevance of the strong transient growth in the cascade to the present open flow results may be questioned on account of the transverse periodicity of the domain analyzed, the results of Fig.6 are entirely consistent with those in the wake of the cylinder: two-dimensional perturbations experience the strongest transient growth, while the relevance of this scenario is diminishing for increasingly large wavenumbers / short spanwise domain lengths.

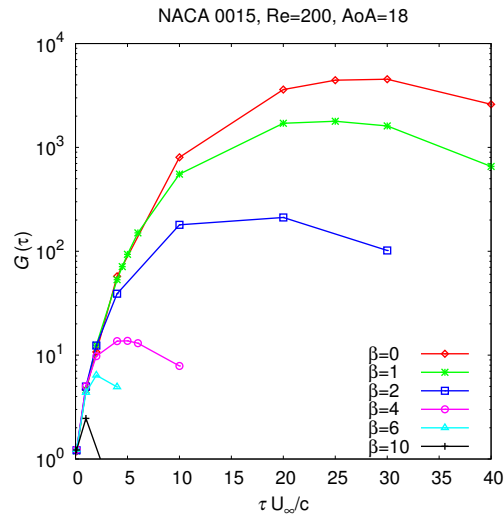


Fig. 5. The changes of maximum amplitude $G(\tau)$ at different wavenumbers β .

Finally, the question of surface streamline patterns arising from the discovered transient growth mechanism at short times is addressed. Figure 7 shows isolines of the $\hat{w}(x, y, z)$ is plotted at $\tau = 0.1$. No signs of streamwise vortex formation arising from this scenario can be appreciated neither in these, nor in analogous results obtained at different times, but not shown here.

5. Summary

Modal and non-modal three-dimensional global linear instability analyses of spanwise homogeneous incompressible two-dimensional flow around symmetric and cambered airfoils at high angles of attack are being revisited. The present work reports the unsteadiness boundaries of two-dimensional basic flows around the NACA009, NACA0015 and NACA4415 airfoils. In addition, instability results for the NACA0015 airfoil at parameters previously analyzed by modal linear global stability theory are presented, using two independent and well-validated codes. The two classes of three-dimensional global modes, stationary and traveling, identified in earlier global modal instability work, have been confirmed by the present results. However, the relative importance of the two classes of modal disturbances has been found to be the opposite of that previously reported: at all conditions examined, $200 \leq Re \leq 230, AoA = 18^\circ, 0 \leq \beta \leq 10$, the traveling wake mode dominates over the stationary modal disturbance. Moreover, up to the limit of unsteadiness at these flow parameters, $Re \approx 230$, no unstable global perturbation has been identified. A corollary of this finding is the inability to detect the stall cell structures previously reported as being the result of linear amplification of the three-dimensional stationary global eigenmode⁸.

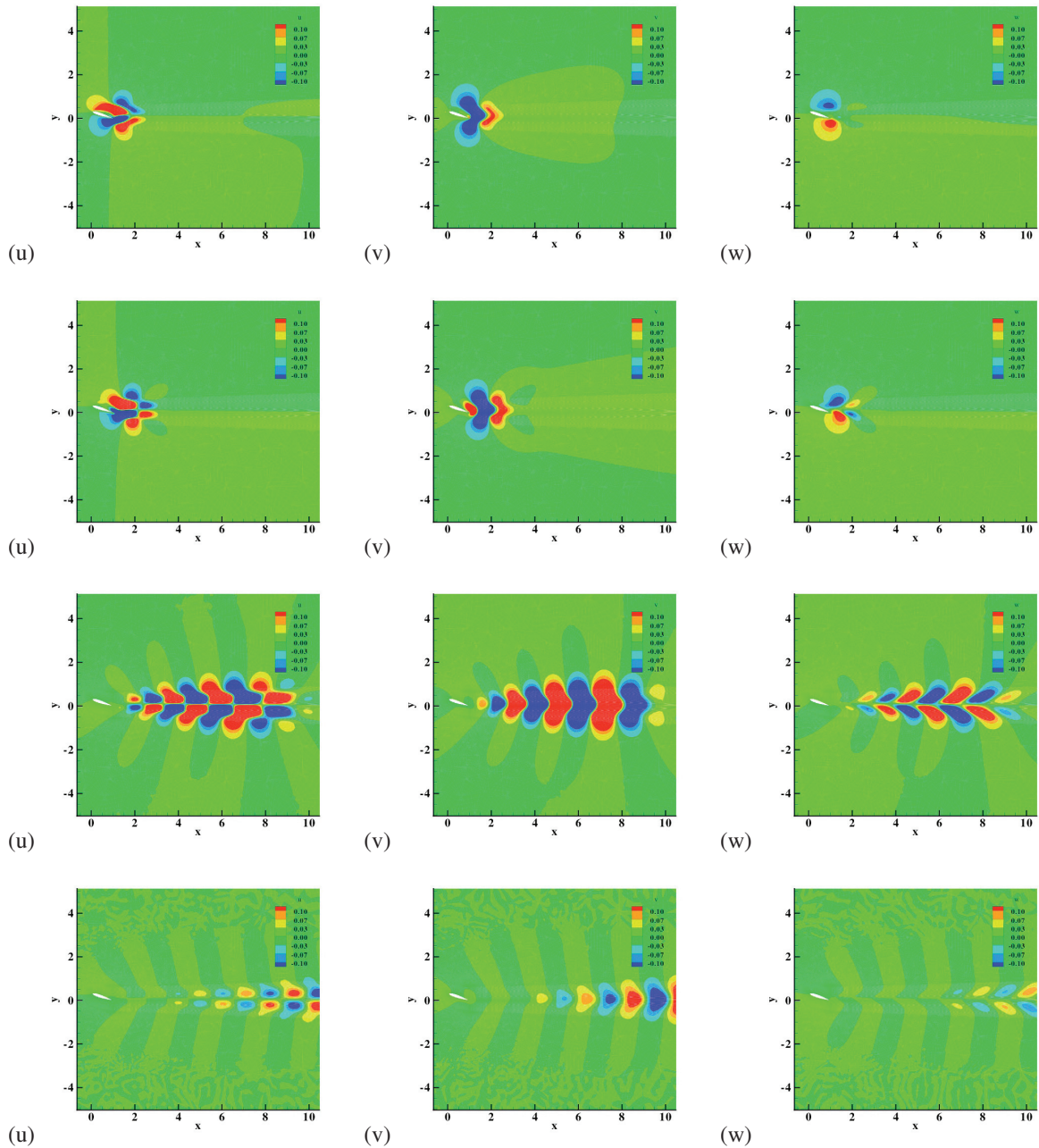


Fig. 6. Transient growth stability analysis of the flow around NACA0015 at AoA 18° and Reynolds 200: From top to bottom: Optimal perturbations at $\tau = 0.1, 1, 10$ and $\tau = 30$; from left to right: velocity components of u, v and $w, \beta = 1$.

The present results lend support to earlier findings of Brehm and Fasel¹⁰ on the NACA0015 and of Tsiloufas *et al.*¹¹ on the NACA0012 airfoil as regards the bluff-body scenario of three-dimensionalization in the wake of two-dimensional airfoils at high angle of attack: unsteady two-dimensional flow becomes three-dimensionally unstable

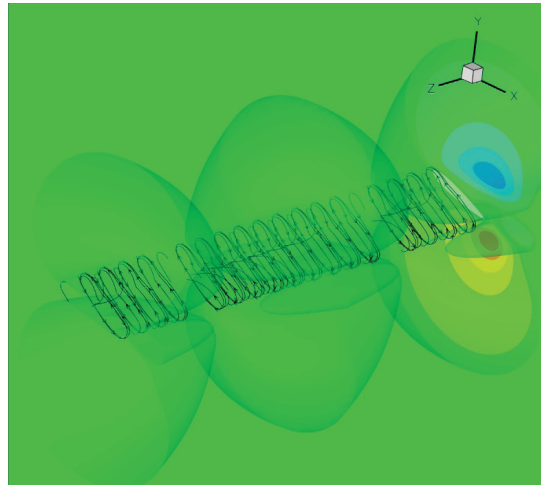


Fig. 7. Near-wall streamlines on the NACA0015 airfoil at $Re = 200$, $AoA = 18^\circ$, $\beta = 1$, arising from the TG mechanism at $\tau = 0.1$

through linear modal amplification of three-dimensional Floquet modes, much like the transition path in the wake of the circular cylinder.

Finally, non-modal analysis of the NACA0015 airfoil at $Re = 200$, $AoA = 18^\circ$ has identified the optimal initial linear perturbations of this flow: strong transient energy amplification exists in the two-dimensional flow limit, progressively becoming less significant as the spanwise periodicity length shortens. The effect of transient growth is to convert these optimal initial conditions to the traveling wake mode perturbation identified in the modal analysis. The effect of the angle of attack and the airfoil camber on the findings of the present work is currently being examined and results will be reported elsewhere.

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