RT-MAE 9708

BARTLETT AND BARTLETT-TYPE CORRECTIONS FOR TESTING LINEAR RESTRICTIONS

by

Reinaldo B. Arellano-Valle, Silvia L.P. Ferrari
and
Francisco Cribari-Neto

Palavras-Chave: Bartlett correction; Bartlett-type correction; Chi-squared distribution; Edgeworth expansion; Likelihood ratio test; Linear restrictions; Regression models; Score test.

Classificação AMS: 62F05, 62J12, 62J05
(AMS Classification)

- Maio de 1997 -

Bartlett and Bartlett–type corrections for testing linear restrictions

BY REINALDO B. ARELLANO–VALLE

Departamento de Estadística, Pontificia Universidad Católica de Chile
Casilla 306, Santiago 22, Chile

SILVIA L.P. FERRARI

Departamento de Estatística, Universidade de São Paulo
Caixa Postal 66281, São Paulo/SP, 05315-970, Brazil

AND FRANCISCO CRIBARI–NETO

Department of Economics, Southern Illinois University
Carbondale IL, 62901-4515, USA

SUMMARY

This paper shows how to extend a number of published results on Bartlett and Bartlett-type corrections to likelihood ratio and score test for the test of linear restrictions in regression models.

Some key words: Bartlett correction; Bartlett-type correction; Chi-squared distribution; Edgeworth expansion; Likelihood ratio test; Linear restrictions; Regression models; Score test.

1. INTRODUCTION

We consider a regression model where an n-vector of random variables $Y = (Y_1, \ldots, Y_n)'$ whose mean (or location parameter) $\mu = (\mu_1, \ldots, \mu_n)'$ is related to an unknown p-vector of parameters $\beta = (\beta_1, \ldots, \beta_p)'$ through a monotonic, twice continuously differentiable function $d(\mu) = \eta = X\beta$, where X is a full column rank $n \times p$ known matrix of covariates. We allow the distribution of Y to depend on a scalar or vector unknown dispersion parameter, ϕ^{-1}. This class of models includes a number of important models, such as linear normal regression models, generalised linear models, linear regression models with t-distributed errors, linear heteroskedastic regression models, etc.

Bartlett corrections to likelihood ratio tests (Lawley, 1956) and Bartlett-type corrections to score tests (Cordeiro & Ferrari, 1991) have been proposed for a number of models that fall into the class of models considered here; see Cribari–Neto & Cordeiro (1996). Such corrections were derived for the test of the composite null hypothesis H_0: $\beta_1 = \beta_1^{(0)}$ against a two-sided alternative hypothesis, where $\beta = (\beta_1', \beta_2')'$. Here, β_1 contains the first q elements of β, β_2 contains the remaing $p - q$ elements, and $\beta_1^{(0)}$ is a known q-vector. For example, corrections to the likelihood ratio statistic in the class of generalised linear models were obtained by Cordeiro (1983, 1987) using this null hypothesis, and similar corrections to the score test were derived by Cordeiro, Ferrari & Paula (1993) and by Cribari–Neto & Ferrari (1995);

corrections to both statistics in models where the errors are t-distributed were obtained by Ferrari & Arellano–Vale (1996). We show how to extend results obtained for the test of the null hypothesis H$_0$: $\beta_1 = \beta_1^{(0)}$ to the more general null hypothesis H$_0$: $R_1\beta = \gamma_1^{(0)}$, where R_1 is a $q \times p$ matrix of rank q and $\gamma_1^{(0)}$ is a known q-vector. The usefulness of our result stems from the fact that practitioners are usually interested in testing linear restrictions on the parameters, especially when working with regression models. For example, economists usually test for the homogeneity and symmetry properties of large demand systems using large sample tests, and the formulation of these hypothesis involves linear restrictions on some of the regression parameters; see Cribari–Neto & Zarkos (1995, 1997).

2. The main result

Let the matrix R_1 be partitioned as $R_1 = [R_{11}, R_{12}]$, where R_{11} and R_{12} are matrices of dimension $q \times q$ and $q \times (p-q)$, respectively, and assume that $\text{rank}(R_{11}) = q$. The design matrix X is partitioned as $X = [X_1, X_2]$ following the partition of β. Define $\gamma_1 = R_1\beta$ and $\gamma_2 = R_2\beta$, where $R_2 = [0, I_{p-q}]$, I_{p-q} denoting the identity matrix of order $p-q$, and then

$$R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ 0 & I_{p-q} \end{bmatrix}.$$

It is clear that R is nonsingular since $|R| = |R_{11}| \neq 0$, and the inverse of R is given by

$$R^{-1} = \begin{bmatrix} R_{11}^{-1} & -R_{11}^{-1}R_{12} \\ 0 & I_{p-q} \end{bmatrix}.$$

It then follows that the reparameterisation from $\beta = (\beta_1', \beta_2')'$ to $\gamma = (\gamma_1', \gamma_2')'$ is one-to-one since $\gamma = R\beta$ if and only if $\beta = R^{-1}\gamma$.

Next, let $V = XR^{-1} = [X_1R_{11}^{-1}, X_2 - X_1R_{11}^{-1}R_{12}] = [V_1, V_2]$, where $V_1 = X_1R_{11}^{-1}$ and $V_2 = X_2 - X_1R_{11}^{-1}R_{12}$ are matrices of dimension $n \times q$ and $n \times (p-q)$, respectively. It then follows that

$$\eta = X\beta = XR^{-1}\gamma = V\gamma = V_1\gamma_1 + V_2\gamma_2.$$

Hence, in the new parameterisation, $d(\mu) = \eta = V_1\gamma_1 + V_2\gamma_2$. Therefore, testing H$_0$: $R_1\beta = \gamma_1^{(0)}$ for the original model is equivalent to testing H$_0$: $\gamma_1 = \gamma_1^{(0)}$ in the reparameterised model. Since the likelihood ratio and score tests are invariant under reparameterisation, it follows that the Bartlett and Bartlett-type corrections for the linear restriction of interest in the original model can be obtained in the traditional way for the null hypothesis H$_0$: $\gamma_1 = \gamma_1^{(0)}$ considering the transformed model. This then implies that the formulae available in the literature for Bartlett and Bartlett-type corrections of hypotheses that do not impose linear restrictions on the parameters, which typically involve X, remain valid when X is replaced by $V = XR^{-1}$. In other words, the formulae for the corrections for the test of the null hypothesis H$_0$: $R_1\beta = \gamma_1^{(0)}$ are the same as the ones for the test of H$_0$: $\beta_1 = \beta_1^{(0)}$ with X replaced by V.

Bartlett and Bartlett-type corrections to likelihood and score tests in normal linear homoskedastic models typically do not involve X, and hence our result implies that the available corrections (Cordeiro, 1987; Cribari–Neto & Ferrari, 1995) can be used to test linear restrictions without any reparameterisation.

Some remarks are in order. The argument outlined above is based on the assumption that $\text{rank}(R_{11}) = q$. However, it is important to note that it is always possible to force the matrix R_{11} to have full rank by reordering the columns of R_1 which corresponds to reordering the elements of the parameter vector β.

It is also important to note that Bartlett and Bartlett-type corrections to likelihood ratio and score statistics in regression models usually only depend on X through the $n \times n$ matrices $Z = X(X'WX)^{-1}X'$ and $Z_2 = X_2(X_2'WX_2)^{-1}X_2'$, where W is a matrix of weights which can depend on unknown parameters; see, for instance, Cordeiro (1983, 1987), Cordeiro, Ferrari & Paula (1993) and Cribari–Neto & Ferrari (1995). Since Z does not change when we replace X by XR^{-1}, it then follows that the corrections are only affected through Z_2. Indeed, if we denote the matrix obtained from Z with X replaced by V as Z^*, then $Z^* = XR^{-1}\{(R^{-1})'X'WXR^{-1}\}^{-1}(R^{-1})'X' = XR^{-1}R(X'WX)^{-1}R'(R^{-1})'X' = X(X'WX)^{-1}X' = Z$.

3. Examples

Two examples are now introduced to illustrate our result. Let b denote the quantity demanded of beer, p_b the price of beer, p_l the price of other liquor, p_r the price of all other remaining goods and services, and m income. Let $Y = \log(b)$ with $\text{E}(Y) = \mu$ and use a logarithmic functional form for the demand equation to get (Griffiths, Hill & Judge, 1993, §11.1)

$$\mu = \beta_1 + \beta_2 \log(p_b) + \beta_3 \log(p_l) + \beta_4 \log(p_r) + \beta_5 \log(m).$$

An important hypothesis is that there is no money illusion, that is, we expect that if all prices and income go up by the same proportion there should be no change in the quantity demanded of beer. That is, if we multiply all prices and income by a positive constant δ, $\mu = \beta_1 + \beta_2 \log(\delta p_b) + \beta_3 \log(\delta p_l) + \beta_4 \log(\delta p_r) + \beta_5 \log(\delta m) = \beta_1 + \beta_2 \log(p_b) + \beta_3 \log(p_l) + \beta_4 \log(p_r) + \beta_5 \log(m) + (\beta_2 + \beta_3 + \beta_4 + \beta_5) \log(\delta)$. Therefore, the hypothesis of no money illusion can be tested by testing H_0: $\beta_2 + \beta_3 + \beta_4 + \beta_5 = 0$ in a regression where the model matrix is $X = [1 \;\log(p_b)\; \log(p_r)\; \log(m)]$. Assuming that Y is normally distributed and using the result in our paper, one can then use the Bartlett correction to the likelihood ratio statistic for this test derived by Cordeiro (1987) or the Bartlett-type correction to the score statistic obtained by Cribari–Neto & Ferrari (1995).

A similar example deals with regressing (aggregate) investment on a constant, a time trend, the gross national product, (nominal) interest rate and inflation rate. If investors are only interested in the real interest rate, then equal increases in interest rates and the rate of inflation should have no effect on investment. If the coefficients associated with the nominal interest rate and with the rate of inflation in the linear predictor are β_4 and β_5, respectively, this hypothesis can be tested by testing (Greene, 1993, §6.5.2) H_0: $\beta_4 + \beta_5 = 0$, which is a linear restriction. If we assume that investment has a normal distribution, a gamma distribution or an inverse Gaussian distribution, then, again, the results in Cordeiro (1987) and Cribari-Neto & Ferrari (1995) can be applied by making use of the transformation proposed in this paper. For other examples of linear restrictions in generalised linear models and an algorithm for estimating such models subject to linear restrictions, see Nyquist (1991).

4. Simulation results

This section reports the results of a small Monte Carlo simulation experiment that illustrates the result outlined in the previous section. We consider three normal linear regression models where $\eta = \beta_1 + \sum_{j=2}^{p} \beta_j x_j$ with $p = 5, 7, 10$. The null hypothesis of interest is $H_0: \beta_3 = \beta_4$ which is tested against a two-sided alternative hypothesis using Rao's score test. The Bartlett-corrected score statistic can be written (Cordeiro & Ferrari, 1991) as $S_R^* = S_R(1 - \sum_{j=1}^{3} \gamma_j S_R^{j-1})$ where S_R is Rao's score statistic and $\gamma_1 = (A_1 - A_2 + A_3)/(12q)$, $\gamma_2 = (A_2 - 2A_3)/\{12q(q+2)\}$ and $\gamma_3 = A_3/\{12q(q+2)(q+4)\}$. It has been shown by Cribari-Neto & Ferrari (1995), that in this highly tractable case $A_1 = 12q(p-q)/n$, $A_2 = -6q(q+2)/n$ and $A_3 = 0$. Since the coefficients that define the correction in this case do not depend on X, our result implies that they can be used to test linear restrictions and no reparaterisation of the model is required. We now illustrate our result using Monte Carlo simulations. The number of replications is set at 10,000, all parameters (including ϕ) equal one, and the covariates x_2 through x_{10} are chosen as random draws from the following distributions: standard normal, t_1, t_3, t_6, uniform on (0, 1), F(3, 2), standard lognormal, exponential with mean equal to one, and χ_2^2, respectively. The covariate values were kept constant throughout the experiment. The estimated sizes (in percentages) of the score test and its Bartlett-corrected version for the nominal levels $\alpha = 0.10, 0.05, 0.01$ and sample sizes $n = 10, 20, 30, 40, 50$ are given in Table 1.

It is clear from the figures in Table 1 that the score test tends to be oversized and that the correction works well even when the sample size is quite small. These simulation results then provide a numerical illustration of our main result that corrections developed for the null hypothesis H_0: $\beta_1 = \beta_1^{(0)}$ can be used to test null hypotheses of the type H_0: $R_1\beta = \gamma^{(0)}$. When the corrections do not depend on the covariate values, as is the case here, no reparameterisation of the model is required.

Acknowledgement

The first author thanks the financial support from FONDECYT/Chile (process number 1960937), the second author thanks the financial support from CNPq/Brazil, and the third author gratefully acknowledges a research grant from the Office of Research Development at Southern Illinois University.

References

Cordeiro, G.M. (1983). Improved likelihood ratio statistics for generalized linear models. *Journal of the Royal*

Table 1. *Estimated rejection probabilities*

		$p = 5$		$p = 7$		$p = 10$	
n	α	score	Bartlett	score	Bartlett	score	Bartlett
	10	23.0	11.5	36.8	16.1	–	–
10	5	13.4	5.9	26.1	9.1	–	–
	1	2.4	0.7	9.0	0.9	–	–
	10	15.0	10.7	18.2	10.9	24.5	11.7
20	5	8.3	5.6	10.4	5.6	15.7	6.5
	1	1.5	0.9	2.6	1.2	5.1	1.5
	10	12.6	10.5	14.3	10.2	17.4	10.4
30	5	6.5	4.7	7.8	4.9	10.3	5.1
	1	1.1	0.8	1.6	0.9	2.5	1.0
	10	12.0	10.0	13.0	10.1	15.2	10.3
40	5	6.1	5.0	7.2	5.1	8.5	5.1
	1	1.1	0.8	1.5	1.0	2.1	1.2
	10	11.3	10.0	12.3	10.1	13.8	9.9
50	5	5.9	4.9	6.5	4.9	7.6	5.1
	1	1.2	1.1	1.4	1.1	1.7	1.0

Statistical Society B **45**, 404-13.

CORDEIRO, G.M. (1987). On the corrections to the likelihood ratio statistics. *Biometrika* **74**, 265-74.

CORDEIRO, G.M. & FERRARI, S.L.P. (1991). A modified score test statistic having chi-squared distribution to order n^{-1}. *Biometrika* **78**, 573-82.

CORDEIRO, G.M., FERRARI, S.L.P. & PAULA, G.A. (1993). Improved score tests for generalized linear models. *Journal of the Royal Statistical Society B* **55**, 661-74.

CRIBARI-NETO, F. & CORDEIRO, G.M. (1996). On Bartlett and Bartlett-type corrections. *Econometric Reviews* **15**, 339-367.

CRIBARI-NETO, F. & FERRARI, S.L.P. (1995). Second order asymptotics for score tests in generalised linear models. *Biometrika* **82**, 426-32.

CRIBARI-NETO, F. & ZARKOS, S.G. (1995). Improved test statistics for multivariate regression. *Economics Letters* **49**, 113-20.

CRIBARI-NETO, F. & ZARKOS, S.G. (1997). Finite-sample adjustments for homogeneity and symmetry tests in systems of demand equations: a Monte Carlo evaluation. *Computational Economics*, forthcoming.

FERRARI, S.L.P. & ARELLANO-VALLE, R.B. (1996). Improved likelihood ratio and score tests in linear models using the *t*-distribution. *Brazilian Journal of Probability and Statistics* **10**, 15-33.

GREENE, W.H. (1993). *Econometric Analysis*, 2nd ed. Englewood Cliffs: Prentice Hall.

GRIFFITHS, W.E., HILL, R.C. & JUDGE, G.C. (1993). *Learning and Practicing Econometrics*. New York: Wiley.

LAWLEY, D.N. (1956). A general method for approximating to the distribution of likelihood ratio criteria. *Biometrika* **71**, 233-44.

NYQUIST, H. (1991). Restricted estimation of generalized linear models. *Applied Statistics* **40**, 133-41.

ÚLTIMOS RELATÓRIOS TÉCNICOS PUBLICADOS

9701 - BOFARINE, H.; ARELLANO-VALE, R.B. Weak nondifferential measurement error models. IME-USP, 1997. 12p. (RT-MAE-9701)

9702 - FERRARI, S.L.P.; CORDEIRO, G. M.; CRIBARI-NETO, F. Higher Order Asymptotic Refinements for Score Tests in Proper Dispersion Models. IME-USP, 1997. 14p. (RT-MAE-9702)

9703 - DOREA, C.; GALVES, A.; KIRA, E.; ALENCAR, A. P. Markovian modeling of the stress contours of Brazilian and European Portuguese. IME-USP, 1997. 11p. (RT-MAE-9703)

9704 - FONTES, L.R.G.; ISOPI, M.; KOHAYAKAWA, Y.; PICCO, P. The Spectral Gap of the REM under Metropolis Dynamics. IME-USP, 1997. 24p. (RT-MAE-9704)

9705 - GIMENEZ, P.; BOLFARINE, H.; COLOSIMO, E.A. Estimation in weibull regression model with measurement error.IME-USP, 1997. 17p. (RT-MAE-9705)

9706 - GALVES, A.; GUIOL, H. Relaxation time of the one-dimensional symmetric zero range process with constant rate. IME-USP, 1997. 10p. (RT-MAE-9706)

9707 - GUIOL, H. A note about a burton keane's theorem. IME-USP, 1997. 7p. (RT-MAE-9707)

The complete list of ''Relatórios do Departamento de Estatística'', IME-USP, will be sent upon request.

Departamento de Estatística
IME-USP
Caixa Postal 66.281
05315-970 - São Paulo, Brasil