RT-MAE-8907

THE BAYESIAN APPROACH TO ESS

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Classificação AMS: 62C10, 62N10
(AMS Classification)

- Março 1989 -

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Key Words and Phrases: Autopsy; Bayesian Design; Bayesian Inference; Defect Density; Early and Reliability Failures; Environmental Stress Screening; Exchangeability; Influence Diagrams; Minimal Expected Cost; Mixture Models; Non-Response; Optimal Duration; Propriety of Priors; Quality Control; Screening Strength; Yield.

ABSTRACT

This paper presents a Bayesian derivation of (economically optimal) stress screen durations in ESS - Environmental Stress Screening. Indicators of the quality of a Screen of any duration are derived. The posterior density for the rate of early failures of the production process is also obtained. This allows the user to update his opinion about the quality of the process.

1. INTRODUCTION

Environmental Stress Screening (ESS) has been used , for instance , to purge populations from parts having hidden latent defects not detectable by quality control. This is done by submitting the parts to stress ("compression" of time) which will precipitate "early failures". Defective parts are eliminated from the population and the surviving parts have a much smaller proportion of defective items. The economical benefits of such procedures are clear , since the cost of stressing items is usually much smaller than the cost of having failures occurring , instead , after equipment is fielded.

An excellent introduction to the problem is Perlstein, Littlefield and Bazovsky (1987), where there is a derivation of stress screen durations which will leave the failure rate of the surviving parts within a prechosen distance of the failure rate of the "good" parts. Here , the determination of (optimal) durations is approached from a Bayesian point of view. In addition to possessing philosophical advantages , the Bayesian approach pays more attention to economic considerations (being embedded in a decision-theoretic framework) and is more realistic than the classical-inspired plans which are conditional on unknown (yet assumed to be non-random !) quantities.

The ESS experiment also provides data that the engineer may use to learn about the production process. In the last part of this paper, we develop Bayesian inference about the "rate of early failures".

2. NOTATION AND ASSUMPTIONS

We are considering a lot of N parts having a proportion p of substandard parts. These substandard parts have hidden latent defects which will very probably provoke the "early failures". They can not be detected by quality control (even if applied 100%) because they appear to be normal "good" parts. This (parts purging) situation is essentially the setup in Perlstein, Littlefield and Bazovsky (1987). Environmental Stress Screening can also (and should) be applied to populations of assembled parts or of finished products. The items failing the screen are then repaired or improved (instead of replaced - in the parts case) . Such models ("complex systems") are discussed in Perlstein, Littlefield and Bazovsky (1988) . In the present paper we are restricted to the parts populations case , although the same Bayesian approach is applied to the complex systems situation without any further conceptual complication.

The failure rate of the substandard parts under stress is λ_e , while the failure rate of the good parts under stress is λ_g. We have of course $\lambda_e > \lambda_g$. There are two points which should be carefully considered in this modelling. First , the constant failure rates assumption which is usually just a mathematical simplification , plays a crucial role in ESS. The assumption that the failure times are exponentially distributed guarantees that the parts surviving the stress screen will not be aged by the stressing . The user of a stress screen plan should be attentive to the fact that if the exponential (or perhaps a decreasing failure rate) assumption is not adequate , he might end up having a lot of (mostly good) parts surviving the screen which will, nevertheless, be aged.

The other point that should be mentioned is the quantification of stress levels. In Perlstein, Littlefield and Bazovsky (1987) there are reminders of the fact that the stress level should be kept under a " threshold which could precipitate failure modes that would never occur in normal operation or which could damage the part " (Section 2.2 of that Reference). In addition to this assumption of "non-damaging" stress levels , it is assumed that the stress levels used can be translated into a factor of acceleration of time . In fact , the optimal duration of the stress screen turns out to be - in both Bayesian and non-Bayesian derivations - the inverse of the acceleration of time factor multiplied by acceleration-invariant constants. (See formula (13) in Perlstein, Littlefield and Bazovsky (1987) and Section 3 below). In practice , the user of these plans needs to be able to express the stress level (which depends on the environment , type of equipment , etc...) into a constant factor of acceleration of time. We will use this assumption and denote such a factor by l. We then have $\lambda_g = l\lambda_g'$ and $\lambda_e = l\lambda_e'$ where λ_g' and λ_e' denote the failure rates under normal operating conditions.

Let T be the optimal duration for the screen (under a given stress level l) and t denote time. We will now introduce the notation and discuss the costs involved in stressing and stopping the screen. Let c_1 represent the cost of having a substandard part escaping the screen and c_2 represent the cost of having a good part destroyed by the stress screen. The costs c_1 and c_2 are "decision" costs in the sense that they describe the cost of wrong "decisions" regarding a part. Since the major concern in ESS is to purify the population of parts , the cost c_1 is usually much larger than c_2. If one defines a substandard part as a part failing the screening (such a definition can be inappropriate, particularly if the duration of the screening is large) , then actually $c_2 = 0$. The "stressing" - as opposed to "decision" - costs are now considered. Let c_3 be the cost of stressing and failing a part, and c_4 be the cost of stressing and releasing a part. We are assuming, for simplicity, that c_3 and c_4 depend neither on t nor on the quality of the part. These assumptions are of course not suitable in many situations. For example, the cost of stressing can be modeled as a linear function of time of actual stress of the part until failure or release, etc... Under these more realistic assumptions, the adaptation of the derivation presented in this paper is straightforward.

The total cost depends on λ_g, λ_e and on p, the *parameters* of this model. The Bayesian engineer has probability distributions ("prior" distributions) which describe his uncertainty about the value of unknown parameters. The prior distributions are his personal opinions about the parameters and should be based on his expertise. The design problem to be solved is the determination of T in such a way that the expected total cost with respect to the prior distribution is minimized.

Once data have been observed, the opinion of the Bayesian is updated. This process replaces the prior distribution by the "posterior" distribution. Standard references on the Bayesian approach are de Finetti (1974), Savage (1954) and de Groot (1970). We will consider families of prior densities which are convenient and large enough to accommodate different opinions the engineer might have.(See also Section 6 below).

We suggest the use of the (absolutely continuous) Beta family of prior densities on (0,1) for p. The Beta family is indexed by positive numbers a and b and the Beta(a,b) prior density on (0,1) is given by

$$f(p) = (\Gamma(a+b)/(\Gamma(a)\Gamma(b)))\, p^{a-1}(1-p)^{b-1}$$

The uniform density on (0,1) is the particular case $a=b=1$. The expected value of a Beta distribution is given by $E(p) = a/(a+b)$ and its variance by $V(p) = ab(a+b)^{-2}(a+b+1)^{-1}$.

We will assume a joint prior density for $(p, \lambda_g, \lambda_e)$ given by

$$f(p, \lambda_g, \lambda_e) = f(p)\, \theta\tau e^{-\lambda_g(\theta-\tau)} e^{-\lambda_e \tau} ,$$

for $0 < \lambda_g < \lambda_e$. This is equivalent to assuming p independent of (λ_g, λ_e) , a Beta (a,b) density for p , an Exponential(θ) density for λ_g , and a conditional Shifted by λ_g Exponential (τ) density for λ_e , given λ_g . It follows $E(\lambda_g) = \theta^{-1}$, $E(\lambda_e \mid \lambda_g) = \lambda_g + \tau^{-1}$, and $E(\lambda_e) = (\theta+\tau)/(\theta\tau)$. The engineer will elicit values a , b , θ and τ and it will typically be the case in ESS that $a < b$ and $\theta > \tau$.

In the practical situations where λ_g is much smaller than λ_e , the value of θ will be chosen much larger than the value of τ . The conditonal exponential density for λ_e then becomes , when compared to the prior exponential density for λ_g , practically flat . The prior uncertainty about λ_g can be expressed through prior densities that make use of the knowledge about production process standards , as contained in publications as the "Military Standards" Series. On the other hand, the user might express the relatively much larger "ignorance" about λ_e through an almost flat prior density which is nevertheless proper. In addition to satisfying coherence requirements, proper priors can be very helpful when deriving marginal posterior densities - see Section 5 for examples.

3. OPTIMIZING THE STRESS SCREENING DURATION

There is a proportion p of substandard parts in the lot of size N. But the inspection of any part does not reveal whether it is substandard or not. This fact makes all parts look similar and entails a judgment of exchangeability of the parts with respect to quality and behavior under the Screening Stress experiment. In particular, for any part in the lot, the engineer's probability that it is substandard is $E(p) = a/(a+b)$, where E stands for integration with respect to the Beta(a,b) prior for p.

The conditional cost (per part) of a screen of duration t at stress level l is therefore easily derived as

$$p\ [\ (1-e^{-\lambda_s t})c_3 + e^{-\lambda_s t}(c_1+c_4)\ ]\ +\ (1-p)\ [\ (1-e^{-\lambda_g t})(c_2+c_3) + e^{-\lambda_g t}c_4\ ]$$

To be strict, the above conditional (on t and on the parameters) cost is the expected - with respect to its lifetime - total screening cost of a part. The assumptions of exponentiality of the lifetime distributions (discussed in the previous Section) are used in the derivation of the expression above, since

$$P(\,X > x \mid p, \lambda_g, \lambda_s) = pe^{-\lambda_s x} + (1-p)e^{-\lambda_g x}$$

where X is the lifetime of a part from the lot. The integration over the sample space for X is correct for the Bayesian, since there is no violation of the Likelihood Principle (Berger and Wolpert (1984)) when the decision (choice of stress duration) has to be made before the observation $\min(x,T)$ becomes available. This ("preposterior" integration) is of course a common feature in *design* decision problems.

The whole optimization problem can be easily visualized by means of an influence diagram (Barlow and Pereira(1987)). Figure 2 of the Appendix is the influence diagram for the problem of finding the duration T - the "decision" node - minimizing the expected value of the cost - the "value" node - with respect to the relevant random quantities - the "random" nodes. These are , respectively , the box node , the diamond node, and the circle nodes. The influence diagram is for a single part , since the exchangeability of the parts implies its sufficiency , i.e. , the expected cost for the lot is N times the expected cost per part.

After rearranging the terms we obtain the cost per part expressed as

$$c_3 + c_2(1-p) + (c_1+c_4-c_3)pe^{-\lambda_e t} + (c_4-c_2-c_3)(1-p)e^{-\lambda_g t} .$$

The dependence of the conditional cost on the stress level l is expressed through the failure rates which are average numbers of failures per unit of time under stress . (See Section 2).

The last expression for the conditional cost can be easily integrated with respect to the joint prior $f(p,\lambda_g,\lambda_e)$. We will then have the expected cost per part (in the engineer's opinion) or the *risk* of a screen plan of duration t and stress level l , denoted by $R(t,l)$. The risk obviously depends also on the cost structure of the experiment , but we will omit this from the notation . By using the joint prior $f(p,\lambda_g,\lambda_e)$ considered in the previous Section , one obtains

$$R(t,l)=c_3+c_2b(a+b)^{-1}+(c_1+c_4-c_3)a(a+b)^{-1}\theta\tau(\tau+t)^{-1}(\theta+t)^{-1}+(c_4-c_2-c_3)b(a+b)^{-1}\theta(\theta+t)^{-1}$$

In order to obtain the optimal T (i.e., the value of t minimizing the risk or expected cost) one can minimize $R(t,l)$ by elementary differentiation methods. Let us set

$$K = \tau^{-1}(b/a)[(c_4-c_2-c_3)/(c_3-c_1-c_4)] \quad ;$$

If K is either negative or zero , the optimal T will typically be equal to 0 (no screening) or to ∞ (screening until failure of all parts). We will examine the more interesting situation where K is strictly positive and finite. Under this assumption, one obtains

$$T = K^{-1}[1-K\tau+(1-K(\tau-\theta))^{1/2}]$$

as the optimal duration for the screen. If the value of T in the expression above is negative, then the optimal decision is of course to not stress the population. Notice that T can be written in the form $T = l^{-1}D$, with D being a constant invariant to acceleration of time, that is , D has the same value for all l. The minimal (Bayes' risk) total expected cost of the screening experiment of optimal duration T is $R(T,l)$ multiplied by N , by the exchangeability assumption.

We now derive "goodness" measures of a Screen of duration t. The probability that a substandard part will escape from the Screening is

$$E(e^{-\lambda_e t}) = \theta\tau/[(\tau+t)(\theta+t)]$$

where E stands for integration with respect to the marginal prior (of λ_e). The expected number of substandard parts that will escape from the Screening, also called the Remaining Defect Density, is therefore

$$D_R(t) = N\ E(p)\ \theta\tau/[(\tau+t)(\theta+t)] \ = \ N\ [a/(a+b)]\ \theta\tau/[(\tau+t)(\theta+t)]$$

where E now stands for integration with respect to the marginal prior (of p). Notice also the use of the assumption of prior independence between p and λ_e.

The probability that a substandard part will *not* escape from the Screen is sometimes called the Screening Strength (SS) and we have

$$SS(t) = 1 - \theta\tau/[(\tau+t)(\theta+t)]$$

On the other hand, the probability that a good part will survive the Screening is

$$E(e^{-\lambda_s t}) = \theta/(\theta+t)$$

and the expected number of good parts remaining in the lot after the Screen is

$$N\ \mathrm{E}(1-p)\,\theta/(\theta+t) = N\ [b/(a+b)]\ \theta/(\theta+t)\,.$$

Another measure of interest is the **Yield** , defined as the prior probability of having zero substandard parts remaining in the lot after the Screen. The Yield is derived as

$$Y(t) = \mathrm{E}([1-e^{-\lambda_e t}]p^N) = \mathrm{E}([p(1-e^{-\lambda_e t})+(1-p)]^N) =$$

$$= \theta\tau[\Gamma(a+b)/\Gamma(a)] \sum_{j=0}^{N} \binom{N}{j} (-1)^{N-j}\ \Gamma(a+N-j)/[\Gamma(a+b+N-j)(\tau+(N-j)t)(\theta+(N-j)t)]$$

$$= \theta\tau[\Gamma(a+b)/(\Gamma(a)\Gamma(b)\Gamma(a+b+N))] \quad \times$$

$$\times \quad \sum_{j=0}^{N}\sum_{i=0}^{j} \binom{N}{j}\binom{j}{i} (-1)^{j-i}\ \Gamma(a+j)\Gamma(b+N-j)/[(\tau+(j-i)t)(\theta+(j-i)t)]$$

The approximation above is obtained by assuming a prior Beta(a,b) density for a *propensity* p in place of the proportion p. Such "infinite population" models are discussed in Section 5. The last identity above then results from the equivalence between a Beta(a,b) assumption for the propensity p and a Beta-Binomial(N,a,b) assumption for the number of substandard parts initially present in the lot: by using the equivalence, we obtain

$\mathrm{E}([p(1-e^{-\lambda_e t})+(1-p)]^N) = \sum_{j=0}^{N} q_j \mathrm{E}[(1-e^{-\lambda_e t})^j]$, where q_j are the Beta-Binomial probability function values.

Since it is very difficult to solve any of the expressions above for t , we obtain the following approximation, which is a Poisson term for zero occurrences

$$Y(t) = e^{-\theta\tau[Na/(a+b)]/[(\theta+t)(\tau+t)]} = e^{-D_R(t)}$$

The parameter of the approximating Poisson distribution is the Remaining Defect Density $D_R(t)$. Notice that $D_R(t) = (1-\mathrm{SS}(t))D_{IN}$, where D_{IN} is the expected number of substandard parts before the Screen, or the Incoming Defect Density $[Na/(a+b)]$, while $1-SS(t) = \theta\tau/[(\theta+t)(\tau+t)]$.

4. A NUMERICAL EXAMPLE

We will examine the same numerical example of section 5 of Perlstein, Littlefield and Bazovsky (1987). They consider a population of 3750 electronic parts and derive a duration of 215 hours in order to have a 96% "power screen" value. Let us now consider a cost structure given by $c_1 = 100$, $c_2 = 20$, $c_3 = 1$, and $c_4 = 0.01$. Suppose the engineer chooses $a = 1$, $b = 3999$ and (knowing l) , $\theta = 2(10^6)$ and $\tau = 67$. We obtain $K \doteq 12.6535$ and $T = 330.64$ hours. The expected cost when using this duration is 0.0179 per part or 67.07 for the whole lot. The expected cost if using a duration of 215 hours is (in the Bayesian engineer's opinion) 0.0184 per part or 68.94 for the whole lot. The expected cost per part if no stressing at all is performed ($t = 0$) is 0.0350 - 0.01 = 0.0250 or 93.75 for the lot. Notice that c_4 is to be subtracted from $R(0,l)$ in order to obtain the correct risk at $t = 0$ (there is no stressing cost when $t = 0$) . The relative stability of the expected cost - per part - for small and moderate durations is caused , in this particular example , by the assumptions of an extremely small proportion of defectives in the lot (as expressed by the prior Beta values of a and b) and independence of c_3 and c_4 from t. (See Section 2). The graph of $R(t,l)$ as a function of t is in the Appendix.

If the optimal duration $T = 330$ is used , the expected number of good parts surviving the test is

$$3750 \times (3999/4000) \times 2(10^6)/[2(10^6)+330] = 3748.44$$

and the expected number of defective parts surviving the test is

$$3750 \times (1/4000) \times 2(10^6) \times 67/[\{2(10^6)+330\}\{67+330\}] = 0.16$$

There is no precise way of comparing the Bayesian approach to the "power screen" approach of Perlstein, Littlefield and Bazovsky (1987) , since they do not consider any cost structure in their numerical example. In addition , their (non-Bayesian) approach is conditioned on the values of the parameters.

5. STATISTICS : THE POSTERIOR DENSITY

In the previous Sections, we used the Bayesian approach to decision making in the problem of determining the optimal duration of a Stress Screening Test. It was assumed that the user has the figures of a cost structure and values determining the joint prior density for the three parameters involved in the problem. Now, we will consider the use of data : In fact , an ESS experiment provides (once it is performed) data consisting of failure times and the number of truncations at the duration. The genuine Bayesian obtains a joint posterior (to the data) density which is his updated (by the data) opinion about the parameters. He will use it as the prior in the derivation of the duration of a possible second Screen in the future. Alternatively, he might use the updated opinion in order to suggest changes in the production process. Conceptually, the production process could be "fine tuned" up to the point where future Screenings would not be necessary anymore. This is of course an idealistic goal, but it illustrates the dynamics of Bayesian statistical control : the Screening experiment purges the lot; in addition to this original aim, it provides data which are informative about the production process; changes (suggested by the revised opinion) in the production process might improve it, making future Screenings shorter (and less expensive).

However, there are computational difficulties in the derivation and use of such joint posteriors - they are not as simple as the joint prior that we presented in Section 2. The problem has been approached by Bernardo and Girón (1987) who consider a very particular case where the only parameter is p , the other two assumed to be known. They point out the inexistence of conjugate densities for this kind of model and present some approximation (to the posterior) methods. The general problem of the practical difficulty of deriving multidimensional posterior densities has been recently discussed by several authors. See for instance Kass, Kadane and Tiesney(1987). The problem of using data provided by the screen has also been discussed, from a non-Bayesian point of view, by Mendenhall and Hader (1958) who suggest an iterative method to solve simultaneous equations for maximum likelihood estimates of the parameters. However, they assume that all failed parts have their quality revealed; Perlstein and Bazovsky (1988) and Rider (1961) use the classical method of moments for estimation of the parameters. However, they assume no truncation of the observations. In addition , the classical method of moments very often provides negative estimates for the failure rates. An efficient sequential design of ESS screens should make use of the (truncated) data arising from the screens. Such a procedure would not need any estimation (in addition to the screening) experiment and would be better derived as a Bayesian sequential design procedure.

We suggest a "restriction to interest" approach: we will slightly summarize the data, sacrificing some of the information obtained about the failure rates, in order to obtain a computable marginal posterior density for p. It is important to realize that the original model is being embedded in a larger one: specifically, the proportion *per se* of substandard parts in the original lot is not anymore a parameter of interest. Actually, after the Screening this proportion is drastically reduced (we hope !). At this point, what we are interested on is the proportion of substandard parts in the next lot. So, we are now considering a model where the production process generates parts which are substandard with propensity p and from which the first lot was actually a sample of size N (with a proportion of substandard parts not anymore necessarily equal to p). This modelling arises as a consequence of an exchangeability judgment the user has about the parts with respect to their quality.

We will now derive the (joint, first) posterior density. The full data provided by the experiment are a list of failure times and the number of surviving parts.

A crucial assumption here is that autopsy on all the failed parts will be performed, with some of them failing to respond (that is, not revealing whether they were of substandard quality or not). This enables one to use the methodology of Basu and Pereira(1982) to handle data showing non-response.

Once the autopsies are performed, we summarize the data as:

x = number of failed parts that were (revealed by autopsy) substandard

y = number of failed parts that were (revealed by autopsy) good

z = number of failed parts with quality not revealed by the autopsy

$m = N-(x+y+z)$ number of surviving parts

Notice that the actual failure times are "collapsed".

A (factorable) nuisance parameter has to be now introduced: let α denote the probability that a failed part does not respond to the autopsy. Once again, the introduction of α in the model depends on a judgment of exchangeability of the failed parts with respect to responsiveness (see de Finetti (1937) ; Lindley and Novick(1981) on exchangeability).

Let us recall the prior density introduced in Section 2:

$$f(p,\lambda_g,\lambda_e) = (\Gamma(a+b)/(\Gamma(a)\Gamma(b)))\, p^{a-1}(1-p)^{b-1}\theta\tau e^{-\lambda_g(\theta-\tau)} e^{-\lambda_e\tau} ,$$

for $0 < p < 1$ and $0 < \lambda_g < \lambda_e$. We will use Bayes' theorem in order to obtain the joint posterior $f(p,\lambda_g,\lambda_e \mid x,y,z,m)$ (and the marginal $f(p \mid x,y,z,m)$). The probabilistic dependences among parameters and data can be easily visualized in the influence diagram for this inference problem - see Figure 3 in the Appendix. All the operations performed below correspond to reversions and removals of arcs in the influence diagram , aiming the "final" diagram , which has node "data" as the only predecessor of node p.

The likelihood $L(\alpha , p , \lambda_g , \lambda_e \mid x,y,z,m)$ is proportional to the multinomial probability term :

$$L(\alpha , p , \lambda_g , \lambda_e \mid x,y,z,m) = [p(1-e^{-\lambda_e T})(1-\alpha)]^x [(1-p)(1-e^{-\lambda_g T})(1-\alpha)]^y$$

$$[\alpha p(1-e^{-\lambda_e T}) + \alpha(1-p)(1-e^{-\lambda_g T})]^z [pe^{-\lambda_e T} + (1-p)e^{-\lambda_g T}]^m$$

Assuming that α is a priori independent of (p,λ_g,λ_e), one derives (for any prior density $h(\alpha)$) the posterior density $f(p \mid x,y,z,m)$ by first using Bayes' theorem (C denotes the proportionality constant) to obtain the joint posterior density

$$f(\alpha,p,\lambda_g,\lambda_e \mid x,y,z,m) = C\ h(\alpha) f(p,\lambda_g,\lambda_e) L(\alpha , p , \lambda_g , \lambda_e \mid x,y,z,m)$$

Successive applications of Newton's binomial then yield:

$$f(\alpha,p,\lambda_g,\lambda_e \mid x,y,z,m) = C\ [h(\alpha)\alpha^z(1-\alpha)^{x+y}]$$

$$p^{a+x-1}(1-p)^{b+y-1} \sum_{i=0}^{x}\sum_{j=0}^{y}\sum_{l=0}^{m}\sum_{k=0}^{z}\sum_{r=0}^{k}\sum_{s=0}^{z-k} \binom{x}{i}\binom{y}{j}\binom{m}{l}\binom{z}{k}\binom{k}{r}\binom{z-k}{s}(-1)^{x+y+z-i-j-r-s}$$

$$p^{k+l}(1-p)^{m-l+z-k} e^{-\lambda_e T(x-i+l+k-r+\tau/T)} e^{-\lambda_g T(y-j+m-l+z-k-s+(\theta-\tau)/T)}$$

By recalling the (propriety of prior) fact

$$\int_0^\infty\int_{\lambda_g}^\infty e^{-\lambda_s(\theta-\tau)}e^{-\lambda_s\tau}d\lambda_s d\lambda_g = (\theta\tau)^{-1} ,$$

one can easily integrate out λ_g , λ_s (and α) , obtaining

$$f(p \mid x,y,z,m) = C\sum_* G(i,j,l,k,r,s)\, p^{a+x+k+l-1}(1-p)^{b+y+m-l+z-k-1}$$

where $\sum_*$ denotes $\sum_{i=0}^{x}\sum_{j=0}^{y}\sum_{l=0}^{m}\sum_{k=0}^{z}\sum_{r=0}^{k}\sum_{s=0}^{z-k}$

and $G(i,j,l,k,r,s) = \binom{x}{i}\binom{y}{j}\binom{m}{l}\binom{z}{k}\binom{k}{r}\binom{z-k}{s}(-1)^{x+y+z-i-j-r-s}$ /

/ $[(x-i+l+k-r+\tau/T)(y-j+m+z-s+\theta/T+x-i-r)]$

Finally, by recalling (propriety of prior again) that:

$$\int_0^1 p^{a-1}(1-p)^{b-1}dp = \Gamma(a)\Gamma(b)/\Gamma(a+b) = \mathrm{B}(a,b)$$

we can solve for C , thus determining the computable analytic expression of the posterior density of p :

$$1/C = \sum_* G(i,j,l,k,r,s)\mathrm{B}(a+x+k+l,b+y+m-l+z-k)$$

Analogous computations provide the posterior moments of p and integration of the posterior density $f(p \mid \text{Data})$ determines credibility intervals for p.

6. CONCLUSION

The existing literature for planning, monitoring and controlling of ESS experiments is, to the authors' knowledge, entirely non-Bayesian. This paper presents a Bayesian treatment to the ESS problem which possesses the usual advantages that the Bayesian formulation has over classical/frequentists methods.

The duration design problem is solved in an economically optimal way. The difficulty of obtaining an exact *separation* - i.e. a tractable posterior $f(p\ ,\lambda_g\ ,\lambda_e\ \mid \text{Data}\ )$ - reflects the general computational problems of handling non-conjugate multidimensional posterior densities. These kind of computational problems are being gradually solved, as hardware and software (and numerical methods) are being developed and as the Bayesian approach is taking over the whole field of Applied Statistics and Decision-Making. We avoided this problem by summarizing the data and using non-responsive data techniques which proved to be successful in the medical research area (Pereira and Barlow(1989)). This enabled us to obtain a useful analytic onedimensional posterior probability density for p. Probability (or Credibility) intervals and Bayesian point estimates of p can therefore be easily constructed. We believe this paper indicates the correct way of handling also the various more complex ESS situations which exist , as the ESS of assembled parts or of finished products.

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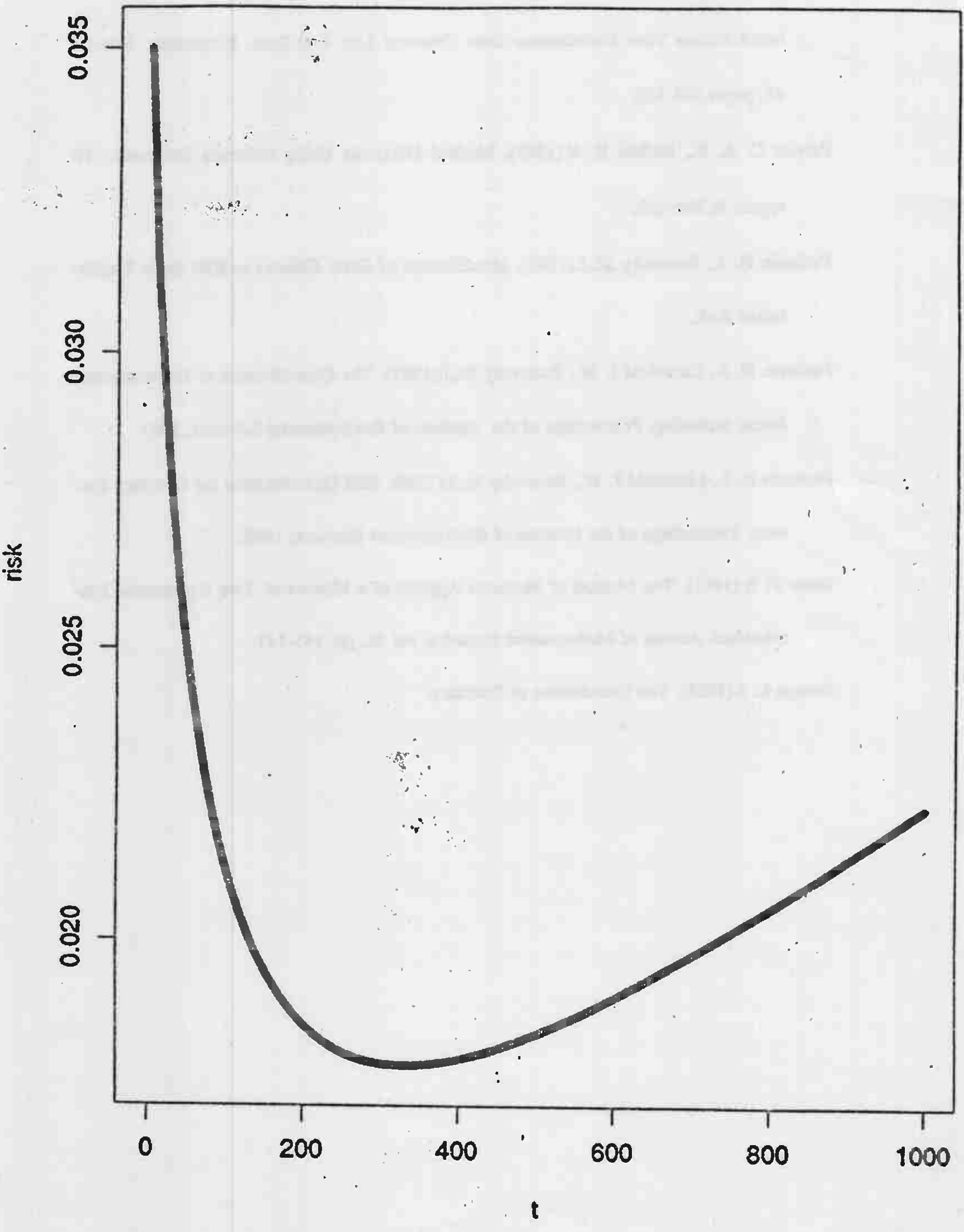
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0.035
0.030
0.025
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0
200
400
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800
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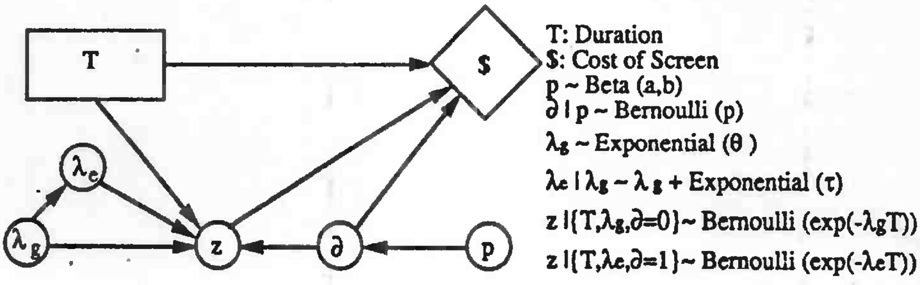

Figure 2: Decision Problem Influence Diagram

$$q_1 = (1 - \alpha)p(1 - \exp(-\lambda_e T))$$

$$q_2 = (1 - \alpha)(1 - p)(1 - \exp(-\lambda_g T))$$

$$q_3 = \alpha\{p(1 - \exp(-\lambda_e T)) + (1 - p))(1 - \exp(-\lambda_g T))\}$$

$$q_4 = p\exp(-\lambda_e T) + (1 - p)\exp(-\lambda_g T)$$

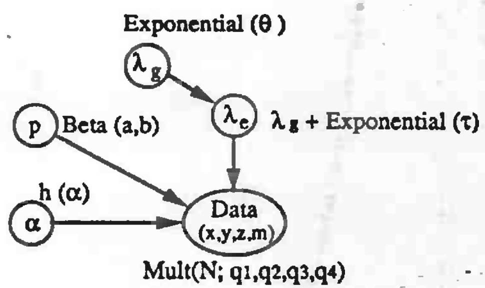

Figure 3: Inference Problem Influence Diagram

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