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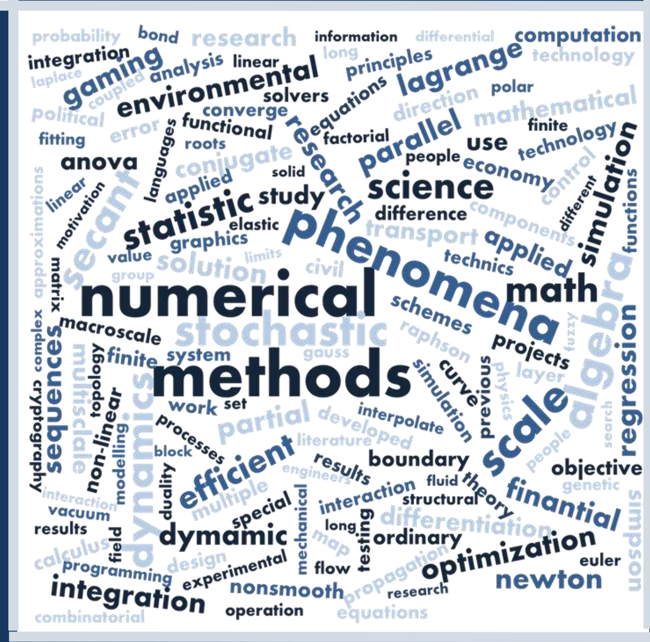

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Mean-of-order-p value-at-risk estimation: a Monte-Carlo comparison

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Abstract

The risk of a big loss that occurs rarely is a primordial parameter of extreme events. A possible and common indicator of such a risk is the *value-at-risk* (VaR), i.e. the size of the loss that occurred with a small probability, q. For any unknown cumulative distribution function F underlying a possibly weakly dependent and stationary available sample, and denoting by $F^{\leftarrow}(y) := \inf\{x : F(x) \geq y\}$ the generalized inverse function of F, we are thus dealing with a (high) quantile, $\chi_{1-q} \equiv \mathrm{VaR}_q := F^{\leftarrow}(1-q)$. With n denoting the size of the available sample, we often have $nq \leq 1$, and this justifies theoretically the assumption that $q = q_n \to 0,$ as $n \to \infty$. We thus want to extrapolate beyond the sample, being then in the area of statistical *extreme value theory* (EVT). Since in real applications in the areas of biostatistics, finance, insurance and statistical quality control, among others, one often encounters heavy right-tails, we shall assume that, for some $\xi > 0$, the *right-tail function* (RTF) satisfies the condition $\overline{F}(x) := 1 - F(x) \sim c\, x^{-1/\xi}$, as $x \to \infty$, for some positive constant c, where the notation $a(y) \sim b(y)$ means that $a(y)/b(y) \to 1$, as $y \to \infty$. The parameter ξ is a positive version of the general *extreme value index* (EVI), the primary parameter of extreme (and large) events. For heavy right-tailed or Paretian-type models, and with Q standing for quantile, Weissman ([1]) proposed the following semi-parametric VaR-estimator,

$$\mathrm{Q}^{(q)}_{\hat{\xi}}(k) := X_{n-k:n}\,(k/(nq))^{\hat{\xi}} =: X_{n-k:n}\; r_n^{\hat{\xi}}, \quad r_n \equiv r_n(k;q) = k/(nq),$$

where $X_{n-k:n}$ is the $(k+1)$-th upper order statistic and $\hat{\xi}$ is any consistent estimator for ξ. This is obviously an *asymptotic* estimator, in the sense that it provides useful estimates when the sample size n is high. For heavy RTFs, the classical EVI-estimator, usually the one which is used for a semi-parametric quantile estimation, is the Hill estimator $\hat{\xi} = \hat{\xi}(k) =: \mathrm{H}(k)$ ([2]), the average of the log-excesses, $V_{ik} := \ln(X_{n-i+1:n}/X_{n-k:n}) =: \ln U_{ik}$, $1 \le i \le k < n$. Since the Hill estimator is the logarithm of the *geometric mean* (or *mean-of-order*-0) of U_{ik}, Brilhante *et al.* ([3]) considered as basic statistics the Hölder's *mean-of-order-p* (MO_p) of U_{ik}, $1 \le i \le k$, $p \in \mathbb{R}_0^+$. More generally, Caeiro *et al.* ([4]) worked with $p \in \mathbb{R}$ and a class of MO_p EVI-estimators, $\mathrm{H}_p(k)$, which can be used for the VaR_q-estimator, through the class $\mathrm{Q}^{(q)}_{\mathrm{H}_p}(k)$. The MO_p EVI-estimators can often have a high asymptotic bias, and bias reduction has recently been a vivid topic of research in the area of statistical EVT. On the basis of partially reduced-bias ([5]) and reduced-bias ([6]) H_p EVI-estimators, respectively denoted by $\mathrm{PRB}_p(k)$ and $\mathrm{CH}_p(k)$, it is thus sensible to work with $\mathrm{Q}^{(q)}_{\mathrm{PRB}_p}(k)$ (already considered in [7]) and with the new VaR_q-estimators $\mathrm{Q}^{(q)}_{\mathrm{CH}_p}(k)$. After a brief reference to the asymptotic properties of these new VaR-estimators, we proceed to an overall comparison of VaR-estimators, through Monte-Carlo simulation techniques.

Keywords: heavy right-tails, Bias reduction, Monte-Carlo simulation, semi-parametric estimation, statistics of extremes, value-at-risk estimation.

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