RT-MAE 9842

USE OF THE EXPONENTIATED-WEIBULL DISTRIBUION FOR BATHTUB-SHAPED FAILURE RATES

by

Cancho, G.V.; Bolfarine, H.
and
Achcar, J.A.

Palavras-Chave: Bayesian analysis, Lifetime data, Gibbs sampling, Metropolis algorithm.

Classificação AMS: 62G05, 62J05
(AMS Classification)

- Dezembro de 1998 -

USE OF THE EXPONENTIATED-WEIBULL DISTRIBUION FOR BATHTUB-SHAPED FAILURE RATES

Cancho, G.V. ; Bolfarine, H.

IME, University of São Paulo, C.P.66281, 05389-970.

São Paulo, S.P., Brazil.

and Achcar, J.A.

ICMSC, University of São Paulo, C.P.668, 13560-970.

São Carlos, S.P., Brazil.

SUMMARY

The main object of this paper is to compare several used models for fitting bathtub data. The approach used is bayesian and based on the MCMC methodology in conjuntion with the Bayes factor. Of the models considered, the exponentiated-Weibull models seems to present the best fit.

Key word: Bayesian analysis, Lifetime data, Gibbs sampling, Metropolis algorithm.

1 INTRODUCTION

Standard lifetime distributions usually impose very strong restrictions on the data as is well illustrated by their inability to produce bathtub curves, and thus to adequately interprete date with this character. Some distributions were introduced to model this kind of data, as the generalized gamma distribution proposed by Stacy(1962), the generalized F distribution proposed by Prentice (1975), the IDB distribution proposed by Hjort (1980) and the exponential-power family proposed by Smith and Bain (1975). A good review of these models is presented in Rajarshi and Rajarshi (1988).

Standard classical inference procedures can have strong difficulties for these models, especially with censored data (see for example, Rajarshi and Rajarshi, 1980; Lawless, 1982; and Hjort, 1980). A new class of models to be used for this kind of data is introduced by Mudholkar (1995).

Mudholkar (1995) proposes the use of the exponentiated-Weibull distribution which is a family of distributions that generalizes the weibull distribution. The main object of paper is to compare several models used to model bathtub data. The exponentiated-Weibull distribuiton is compararred with the IDB family, the power exponential distribution and the mixture of gamma distributions. The approach used for fitting the models is the Bayesian approach based on MCMC methodology. As seen, most of the models require using the Metropolis-Hasting within Gibbs algorithm and thus their implementation is somewhat involved. Model choice is implemented by using the Bayes factor approach. The implementation of a simulation study and the analysis of a real data set seems to indicate that the exponentiated-Weibull model is the one that presents the best fit. Section 2 reviews some of the most common models used with bathtub data. In Section 3 the bayesian approach based on MCMC methodology is presented for fitting the models considered in Section 2. Section 4 reviews model choice by using the Bayes factor methodology. Finally Section 5 illustrates the approach with real and simulated data sets, which seems to indicate that the best fit is presented by the exponentiated-Weibull model.

2 SOME USEFUL MODELS

In this section, we introduce some of the distributions most commonly used to model bathtub shape for the hazard function of the lifetime data. We do not discuss classical inference for the models considered since it will required procedures based on large sample results.

2.1 THE EXPONENTIATED-WEIBULL FAMILY

The probability density function for the exponentiated-Weibull distribuiton is given by

$$f(t) = \frac{\alpha\,\theta}{\sigma}[1 - \exp(-(t/\sigma)^{\alpha})]^{\theta-1} \times \exp(-(t/\sigma)^{\alpha})(t/\sigma)^{\alpha-1}, \quad 0 < t < \infty \tag{1}$$

where $t > 0$, $\alpha > 0$ and $\theta > 0$ are shape parameters and $\sigma > 0$ is a scale parameter.

This family of distribution includes the exponential distribution when $\alpha = 1$, $\theta = 1$ and the Weibull distribution when $\theta = 1$.

The survival function is given by

$$S(t) = 1 - [1 - \exp(-(t/\sigma)^{\alpha})]^{\theta}. \tag{2}$$

and the hazard function is given by

$$h(t) = \frac{\alpha\theta[1 - \exp(-(t/\sigma))]^{\theta-1}\exp(-(t/\sigma)^{\alpha})(t/\sigma)^{\alpha-1}}{\sigma[1 - (1 - \exp(-(t/\sigma)^{\alpha}))^{\theta}]} \tag{3}$$

The great fexibilidade of this model to fit survival dada, is given by the diferent forms that the hazard function (3) can take, that is,

(i) If $\alpha \geq 1$ and $\alpha\theta \geq 1$, we have a monotone increasing hazard function;

(ii) If $\alpha \leq 1$ and $\alpha\theta \leq 1$, we have a monotone decreasing function;

(iii) If $\alpha > 1$ and $\alpha\theta < 1$, we have a bathtub form for the hazard function;

(iv) If $\alpha < 1$ and $\alpha\sigma > 1$, we have a unimodal hazard function.

2.2 THE IDB FAMILY

Hjort (1980) intruduces a distribution that can describe increasing (I), decreasing (D), constant and bathtub-shaped (B) failure rates. This motivates the working name, IDB distribution.

The survival function of the IDB distribution is given by,

$$S(t) = \frac{\exp\{-\delta t^2/2\}}{(1+\beta t)^{\theta/\beta+1}} \tag{4}$$

where $\delta > 0$, $\theta > 0$ e $\beta > 0$.

The probability density function is given by

$$f(t) = -S'(t) = \frac{(1+\beta t)\delta t + \theta}{(1+\beta t)^{\theta/\beta+1}} e^{-\delta t^2/2}, \quad 0 < t < \infty. \tag{5}$$

The hazard function is given by,

$$h(t) = \delta t + \frac{\theta}{1+\beta t}. \tag{6}$$

Special cases are given by,

(i) If $\theta = 0$, we have a Rayleigh ditribution;

(ii) If $\delta = \beta = 0$, we have an exponential distribuiton;

(iii) If $\delta = 0$, we have a decreasing failure rate;

(iv) If $\delta \geq \theta\beta$, we have a increasing failure rate;

(v) If $0 < \delta < \theta\beta$, we have a bathtub curve.

2.3 THE POWER-EXPONENTIAL FAMILY

Smith and Bain (1975,1976) introduce a family of distributions called the power-exponential family of distributions with two parameters as an alternative to model lifetime data with a bathtub rate function. The probability density function is given by:

$$f(t) = \beta\alpha^{-\beta}t^{\beta-1}\exp\left(1 - e^{(\frac{t}{\alpha})^\beta} + (\frac{t}{\alpha})^\beta\right), \tag{7}$$

where $t > 0$, $\alpha > 0$ and $\beta > 0$.

The survival function is given by,

$$S(t) = \exp\left(1 - e^{(\frac{t}{\alpha})^{\beta}}\right), \tag{8}$$

and the hazard function is given by,

$$h(t) = \alpha^{-\beta}\beta t^{\beta-1}\exp\left((\frac{t}{\alpha})^{\beta}\right). \tag{9}$$

The hazard function (9) has bathub form if $0 < \beta < 1$.

2.4 MIXTURES OF THE GAMMA FAMILY

Glasser (1980) introduce a mixture of gamma distributions with the same scale paramters to model lifetime data with a hazard function with a bathtub shape. In this model, the probability density function is given by,

$$f(t; \alpha_1\alpha_2, \beta, p) = pf_1(t; \alpha_1, \beta) + (1-p)f_2(t; \alpha_2, \beta) \tag{10}$$

where $0 < p < 1$, and

$$f_j(t; \alpha_j, \beta) = \frac{\beta^{\alpha_j}}{\Gamma(\alpha_j)} t^{\alpha_j - 1} e^{\beta t}, \tag{11}$$

where $\alpha_j > 0$, $\beta > 0$, $j = 1, 2$.

The survival function is given by,

$$S(t; \alpha_1, \alpha_2, \beta, p) = pS_1(t; \alpha_1, \beta) + (1-p)S_2(t; \alpha_2, \beta), \tag{12}$$

where $S_j(t; \alpha_j, \beta)$ is the survival function to the j^{th} component, $j = 1, 2$.

For this model, the hazard function is given a bathtub shape if,

(a) $\alpha_1 > 1$, $\alpha_2 = 1$; f_1 is increasing and f_2 exponential;

(b) $\alpha_1 > 1$, $\alpha_2 < 1$.

3 BAYESIAN INFERENCE

We assume that the lifetime are independently distributed, and also independent from the censoring mechanism. Considering right-censored lifetime data, we observe $T_i = \min(T_i^0, C_i)$, where T_i^0 is the lifetime for the i^{th} individual and C_i is the censoring time for the i^{th} individual, $i = 1, \dots, n$. In this case the likelihood function for θ, a parameter vector of dimension p is given by

$$L(\theta) = \prod_{i \in F} f(t_i; \theta) \prod_{i \in C} S(t_i; \theta). \tag{13}$$

where F denotes the set of noncensored observations, C denotes de set of censored observations, $f(.)$ e $S(.)$ are the probability density function and survival function, respectively.

Considering a prior distribuiton $\pi(\theta)$ the posterior distribution for θ is given by

$$\pi(\theta | D) = \frac{L(\theta)\pi(\theta)}{\int L(\theta)\pi(\theta) d\theta} \tag{14}$$

where D denotes the data set. Typically the posterior density (14) is hard to deal with analytically, as it happens with the models described in the previous sections. Thus, to obtain treatable expressions for the marginal posterior densities, we will have to use MCMC techniques. A brief description of the methodology is as follows.

The main idea behind Markov Chain Monte Carlo (MCMC) is to build up a Makovian process whose stationary distribuiton (with densisty f) is the one of interest. The process is then iterated for a sufficiently large time t and a sample on size m of this process is then a sample of f. Typically, two ways of selecting this sample can be considered. The first considers the last value generated form the Markovian process as the first element of the sample. The Markovian process is then (independently) reiterated until m of such sequences are obtained and the last element of each sequence is considereded, forming then (iid) samples of size m from f. Another approach consists in obtaining a large sequence from the Markovian process (say, 30000), disregarding part of this sequence (corresponding to process convergence) and the remaining part of the sequence as the sample to be used. We call attention to the fact that this sample is not an iid sample. Among the MCMC methods, the most wel known is the Gibbs sampler, introduced in Bayesian inference by Geman ad Geman (1984), when studing problems related to image processing. It became a more popular Bayesian inference procedure after the paper by Smith and Gelfand (1990), where several applications of the approach are considered. For a posterior density f depending on k variables $X_1, \ldots, X_k$, the process is built up on the conditional densities

$$p(X_j|X_i; i \neq j = 1, \ldots, k), \tag{15}$$

according to the following scheme. Giving starting values $x_1^0, \ldots, x_k^0$, generate a value x_1^1 from the distribution with density $p(X_1|X_2 = x_2^0, \ldots X_k = x_k^0)$, generate x_2^1 from the density $p(X_2|X_1 = x_1^1, X_3 = x_3^0, \ldots, X_k = x_k^0)$ and so on until a value x_k^1 is generated from the density $p(X_k|X_1 = x_1^1, \ldots, X_{k-1} = x_{k-1})$ This completes the first cycle of the sampler. The process then goes to a second cycle, with $(x_1^1, \ldots, x_k^1)$ as starting values. After t of such cycles, a sample $(x_1^t, \ldots, x_k^t)$ of the density f is obtained. Following this process, by reiterating the process m times a sample of size m is obtained from f. From this sample, the marginal densities of the varibles X_j can be estmated by constructing histograms, for example. Moreover, estimates of the posterior mean, variance and quantiles can be easily obtained by computing the corresponding sample mean, variance and quantiles that is, all the inference can be based on descriptive aspects of the generated sample. Another possibility is to compute the following estimate for the density of X_j:

$$\hat{f}_{X_j}(x_j) = \frac{1}{m} \sum_{t=1}^{m} p(x_j|x_1^t, \ldots, x_{j-1}^t, x_{j+1}, \ldots, x_k^t). \tag{16}$$

As emphasized in Gelfand and Smith (1990), this density typically presents better inference then simply considering descriptive inference from the marginal posterior densities. The estimator based on (16) is known as the Rao-Blackwellized version of the histograms, having smaller mean squared error than the histogram as an estimator of f.

The above algorithm can be implemented in any computer language, as for example, FORTRAN or C. However, in some less complex situations, a specific software (BUGS) to Gibbs sampling implementation developed by Thomas, Spielgharter and Gilks (1992) can be used. When the marginal posterior in (15) are hard to deal with it may be required to use the Metropolis-Hasting's approach. The models considered in the paper cannot be dealt with by using BUGS and so they require specific coding which is implemented by using the IML subroutinas in SAS.

3.1 THE EXPONENTIATED-WEIBULL MODEL

Considering data with nocensored observations, the likelihood function (13) is given by,

$$L(\alpha,\theta,\sigma) \propto \alpha^n\theta^n\sigma^{-n\alpha}\exp\left\{-\sum_{i=1}^{n}(\frac{t_i}{\sigma})^\alpha\right\}\prod_{i=1}^{n}t_i^{\alpha-1}(1-\exp\{-(\frac{t_i}{\sigma})^\alpha\})^{\theta-1}. \quad (17)$$

For a bayesian analysis, we assume the following prior densities for α, θ and σ:

$$\begin{aligned} \alpha &\sim \Gamma(a_1,b_1), \text{ with } a_1 \text{ and } b_1 \text{ known} \\ \theta &\sim \Gamma(a_2,b_2), \text{ with } a_2 \text{ and } b_2 \text{ known} \\ \sigma &\sim \Gamma(a_3,b_3), \text{ with } a_3 \text{ and } b_3 \text{ know.} \end{aligned} \quad (18)$$

where $\Gamma(a_i,b_i)$ denotes a gamma distribuiton with mean $\frac{a_i}{b_i}$ and variance $\frac{a_i}{b_i^2}$. We further assume independece among the parameters.

The joint posterior for α, θ and σ is given by,

$$\begin{aligned} \pi(\alpha,\theta,\sigma|D) &\propto \alpha^{n+a_1-1}\theta^{n+a_2-1}\sigma^{-n\alpha+a_3-1}\exp\left\{-\sum_{i=1}^{n}(\frac{t_i}{\sigma})^\alpha-b_1\alpha-b_2\theta-b_3\sigma\right\} \\ & \prod_{i=1}^{n}t_i^{\alpha-1}(1-\exp\{-(\frac{t_i}{\sigma})^\alpha\})^{\theta-1} \end{aligned} \quad (19)$$

The conditional posterior densities for the Gibbs algorithm are given by

$$\begin{aligned} \pi(\alpha|\theta,\sigma,D) &\propto \alpha^{n+a_1-1}\exp\left\{-\sum_{i=1}^{n}(\frac{t_i}{\sigma})^\alpha-b_1\alpha\right\}\prod_{i=1}^{n}t_i^{\alpha-1}(1-\exp\{-(\frac{t_i}{\sigma})^\alpha\})^{\theta-1}, \\ \pi(\theta|\alpha,\sigma,D) &\propto \theta^{n+a_2-1}e^{-b_2\theta}\prod_{i=1}^{n}t_i^{\alpha-1}(1-\exp\{-(\frac{t_i}{\sigma})^\alpha\})^{\theta-1}, \\ \pi(\sigma|\alpha,\theta,D) &\propto \theta^{-n\sigma+a_3-1}\exp\left\{-\sum_{i=1}^{n}(\frac{t_i}{\sigma})^\alpha-b_3\sigma\right\}\prod_{i=1}^{n}t_i^{\alpha-1}(1-\exp\{-(\frac{t_i}{\sigma})^\alpha\})^{\theta-1}. \end{aligned} \quad (20)$$

Observe that we need to use the Metropolis-Hasting algorithm to generate the variables α, θ and σ from the conditional posterior density.

3.2 THE IDB MODEL

The likelihood function for δ, β and θ is given by,

$$L_n(\delta,\beta,\theta) \;=\; \exp\left\{-\frac{\delta\sum_{i=1}^{n} t_i^2}{2}\right\}\prod_{i=1}^{n}\frac{(1+\beta t)\delta t_i-\theta}{(1+\beta t_i)^{\theta/\beta+1}}. \tag{21}$$

Assuming independence among the parameters, considerr the following prior densities for δ, β and θ:

$$\begin{aligned}\delta &\sim \Gamma(c_1,d_1);\ \text{with } c_1,\ c_1 \text{ known;}\\ \beta &\sim \Gamma(c_2,d_2);\ \text{with } c_2,\ d_2 \text{ known;}\\ \theta &\sim \Gamma(c_3,d_3);\ \text{with } c_3,\ d_3 \text{ known.}\end{aligned} \tag{22}$$

The joint posterior distribution for δ, β and θ is given by,

$$\pi(\alpha,\theta,\sigma|D) \;\propto\; \delta^{c_1-1}\beta^{c_2-1}\theta^{c_3-1}\exp\left\{-d_1\delta-\frac{\delta\sum_{i=1}^{n} t_i^2}{2}-d_2\beta-d_3\theta\right\}\Psi_n(\delta,\beta,\theta), \tag{23}$$

where $\Psi_n(\delta,\beta,\theta)=\prod_{i=1}^{n}\frac{(1+\beta t)\delta t_i-\theta}{(1+\beta t_i)^{\theta/\beta+1}}$.

It can be shown that the complete marginal densities required for implementing the Metropolis-Hasting algorithm are given by

$$\begin{aligned}\pi(\delta|\beta,\theta,D) &\propto \delta^{c_1-1}\exp\left\{-d_1\delta-\frac{\delta\sum_{i=1}^{n} t_i^2}{2}\right\}\Psi_n(\delta,\beta,\theta),\\ \pi(\beta|\delta,\theta,D) &\propto \beta^{c_2-1}e^{-d_2\beta}\Psi_n(\delta,\beta,\theta),\\ \pi(\theta|\delta,\beta,D) &\propto \theta^{a_3-1}e^{-d_3\theta}\Psi_n(\delta,\beta,\theta).\end{aligned} \tag{24}$$

3.3 THE POWER-EXPONENTIAL MODEL

From (7), the likelihood function for α and β is given by

$$L(\alpha,\beta)=\beta^n\alpha^{-n\beta}\prod_{i=1}^{n}t_i^{\beta-1}\exp\left\{n+\sum_{i=1}^{n}(\frac{t_i}{\alpha})^\beta-\sum_{i=1}^{n}e^{(\frac{t_i}{\alpha})^\beta}\right\}. \tag{25}$$

Assuming the prior densities,

$$\begin{aligned} \alpha &\sim \Gamma(e_1, f_1) \text{ with } e_1 \text{ and } f_1 \text{ known}, \\ \beta &\sim \Gamma(e_2, f_2) \text{ with } e_2 \text{ and } f_2 \text{ known}, \end{aligned} \tag{26}$$

it follows that the joint posterior distribution for α and β is given by,

$$\pi(\alpha, \beta|D) \propto \beta^{n+e_2-1}\alpha^{-n\beta+e_1-1}\prod_{i=1}^{n} t_i^{\beta-1}\exp\left\{\sum_{i=1}^{n}(\frac{t_i}{\alpha})^{\beta} - \sum_{i=1}^{n} e^{(\frac{t_i}{\alpha})^{\beta}} - f_1\alpha - f_2\beta\right\}. \tag{27}$$

From (27), we obtain the conditional posterior densities for the Gibbs algorithm, wich are given by

$$\begin{aligned} \pi(\alpha|\beta, D) &\propto \alpha^{-n\beta+e_1-1}\exp\left\{\sum_{i=1}^{n}(\frac{t_i}{\alpha})^{\beta} - \sum_{i=1}^{n} e^{(\frac{t_i}{\alpha})^{\beta}} - f_1\alpha\right\}, \\ \pi(\beta|\alpha, D) &\propto \beta^{n+e_2-1}\prod_{i=1}^{n} t_i^{\beta-1}\exp\left\{\sum_{i=1}^{n}(\frac{t_i}{\alpha})^{\beta} - \sum_{i=1}^{n} e^{(\frac{t_i}{\alpha})^{\beta}} - f_2\beta\right\}. \end{aligned} \tag{28}$$

3.4 MIXTURE OF GAMMA DISTRIBUTIONS

The likelihood function for the mixture of gamma distributions given in (10) (see Dempster et al. 1977) is given by,

$$L(\alpha_1, \alpha_2, \beta, p) = \prod_{i=1}^{n} [pf(t_i; \alpha_1\beta)]^{z_{i1}} [(1-p)f(t_i; \alpha_2\beta)]^{1-z_{i1}}, \tag{29}$$

where z_{i1} is an indicator function ($z_{i1} = 1(0)$ if t_i belongs to the first (second) component of the mixture model).

Assuming prior independence, considerer the following prior densities:

$$\begin{aligned} p &\sim \text{beta}(\gamma_1, \gamma_2) \text{ with } \gamma_1 \text{ and } \gamma_2 \text{ known}, \\ \alpha_j &\sim \Gamma(1, \lambda_j) \text{ with } \lambda_j \text{ known } j = 1, 2, \\ \beta &\sim \Gamma(\beta_1, \beta_2) \;\; \beta_1 \; and \; \beta_2 \text{ known}. \end{aligned} \tag{30}$$

where beta(δ_1, δ_2) denotes a beta distributions with parameters δ_1 and δ_2. Thus, the joint posterior distribution for α_1, α_2, β and p is given by,

$$\pi(\alpha_1, \alpha_2, \beta, P|D, \underline{z}) \propto p^{z_{.1}+\gamma_1-1}(1-p)^{n-z_{.1}+\gamma_2-1}\beta^{\alpha_1 z_{.1}+\alpha_2(n-z_{.1})+\beta_1-1} \times \exp\left\{-\beta(\sum_{i=1}^{n} t_i + \beta_2) - \lambda_1\alpha_1 - \lambda_2\alpha_2\right\} \frac{\left(\prod_{i=1}^{n} t_i^{z_{i1}}\right)^{\alpha_1}}{[\Gamma(\alpha_1)]^{z_{.1}}} \times \frac{\left(\prod_{i=1}^{n} t_i^{1-z_{i1}}\right)^{\alpha_2}}{[\Gamma(\alpha_1)]^{n-z_{.1}}}, \tag{31}$$

where $z_{.1} = \sum_{i=1}^{n} z_{i1}$.

The conditional posterior densities for the Gibbs algorithm are given by,

$$\begin{aligned}
\pi(z_{i1}|\alpha_1, \alpha_2, \beta, D) &\sim \text{bernoulli}\left(\frac{pf(t_i;\alpha_1,\beta)}{pf(t_i;\alpha_1,\beta)+(1-p)f(t_i;\alpha_2,\beta)}\right), \\
\pi(p|\alpha_1, \alpha_2, \beta, \underline{z}, D) &\sim \text{Beta}\,(z_{.1}+\gamma_1, n-z_{.1}+\gamma_2), \\
\pi(\beta|\alpha_1, \alpha_2, p, \underline{z}, D) &\sim \Gamma\left(\alpha_1 z_{.1} + \alpha_2(n-z_{.1}) + \beta_1, \beta_2 + \sum_{i=1}^{n} t_i\right), \\
\pi(\alpha_1|\alpha_2, \beta, p, \underline{z}, D) &\propto \exp\{-\lambda_1\alpha_1\} \frac{\beta^{\alpha_1 z_{.1}}\left(\prod_{i=1}^{n} t_i^{z_{i1}}\right)^{\alpha_1}}{[\Gamma(\alpha_1)]^{z_{.1}}}, \\
\pi(\alpha_2|\alpha_1, \beta, p, \underline{z}, D) &\propto \exp\{-\lambda_2\alpha_2\} \frac{\beta^{\alpha_2(n-z_{.1})}\left(\prod_{i=1}^{n} t_i^{1-z_{i1}}\right)^{\alpha_2}}{[\Gamma(\alpha_2)]^{n-z_{.1}}}.
\end{aligned} \tag{32}$$

Notice that the last two conditional densities require Metropolis-Hasting steps for the implementations of the MCMC methodology.

4 MODEL SELECTION

Model determination is a fundamental issue in statistics. The literature on model assessment or checking and model selection presents many approaches, beginning with the Bayes factor approach. Several modifications of Bayes factors are presented in the literature (see for example, Aitkin, 1981; Berger and Perichi, 1996; or Spiegelhalter and Smith 1982). Geisser and Eddy (1979) took a predictive approach based on cross validation methods to obtain pseudo-Bayes factors.

Consider a choice betwen two parametric models denoted by the joint density $f(\underline{t};\boldsymbol{\theta}_i, M_i)$ or the likelihood function $L(\boldsymbol{\theta}_i;\underline{t}, M_i)$, $i = 1,2$. Suppose that w_i is the prior probability of selecting the model M_i, $i = 1,2$ and $f(\underline{t}|M_i)$ is the predictive distribution for model M_i, i.e.,

$$f(\underline{t}|M_i) = \int f(\underline{t}|\boldsymbol{\theta}_i, M_i)\pi(\boldsymbol{\theta}_i|M_i)d\boldsymbol{\theta}_i, \tag{33}$$

where $\pi(\boldsymbol{\theta}_i|M_i)$ is the prior under model M_i. If $\underline{t}_0$ denotes the observed data, then we choose the model yielding the larger $w_i f(\underline{t}_0|M_i)$.

Often we set $w_i = 0.5$, $i = 1,2$ and compute the Bayes factor of M_1 with respect to M_2 as

$$B_{12} = \frac{f(\underline{t}_0|M_1)}{f(\underline{t}_0|M_2)}. \tag{34}$$

The predictive distribution (33) can be approximated by its Monte Carlo estimate using the S generated samples from the prior $\pi(\boldsymbol{\theta}_i|M_i)$, that is,

$$\hat{f}(\underline{t}|M_i) = \frac{1}{S}\sum_{s=1}^{S} f(\underline{t}||\boldsymbol{\theta}_i^{(s)}, M_i), \tag{35}$$

where $\boldsymbol{\theta}_i^{(s)}$ denotes the vector for $\boldsymbol{\theta}_i$ drawn in the s^{th} sample.

Some modifications of the estimate (35) for the preditive density are proposed in the literature (see for example, Newton and Raftery, 1994; or Gelfand and Dey, 1994).

For model selection we also could consider the conditional predictive ordinate (CPO) (see for example, Gelfand and Dey, 1994; or Gelfand, Dey and Chang, 1992), given by

$$f(t_r|\underline{t}, M_i) = \int f(t_r|\boldsymbol{\theta}_i, \underline{t}_{(r)}M_i)\pi(\boldsymbol{\theta}_i|\underline{t}_{(r)}, M_i)d\boldsymbol{\theta}_i \tag{36}$$

where $\underline{t}_{(i)} = (t_1, \ldots, t_{r-1}, t_{r+1}, \ldots, t_n)$.

Using the generated Gibbs samples , (36) can be approximated by its Monte Carlo estimate,

$$\hat{f}(t_r|t_{(r)}, M_i) = \frac{1}{S}\sum_{s=1}^{S} f(t_r|\boldsymbol{\theta}_i^{(s)}, \underline{t}_{(r)}M_i) \tag{37}$$

where $\boldsymbol{\theta}_i^{(s)}$ denotes the vector for $\boldsymbol{\theta}_i$ drawn in the s^{th} Gibbs sample.

We can use the obtained estimates $c_r(l) = \hat{f}(t_r|t_{(r)}, M_l)$ in model selection. In this way we considerer plots of $c_r(l)$ versus r $(r = 1, 2, \ldots, n)$ for diferent models; large values (in average) indicates the better model.

An alternative is to choose the model for wich $c(l) = \prod_{r=1}^{n} c_r(l)$ is maximum (l indexes models).

Geisser and Eddy (1979) sugests that the product of the predictive densities $\prod_{r=1}^{n} f(t_r|\underline{t}_{(r)}, M_l)$ could be used in model selection as a relative indicator. If we have two models M_1 and M_2, we have the ratio,

$$\frac{\prod_{r=1}^{n} f(t_r|\underline{t}_{(r)}, M_1)}{\prod_{r=1}^{n} f(t_r|\underline{t}_{(r)}, M_2)} \tag{38}$$

as an approximation to the Bayes factor.

5 SOME EXAMPLES

5.1 EXAMPLE 1

To illustrated the approach developed in the previous sections we consider the data set illustrated in Arset (1987). The data describe survival times for 50 indistrial devices put on life test at time zero.

Table 1: *Survival times for the 50 devices*

0.1	0.2	1	1	1	1	1	2	3	6	7	11	12	18	18	18	18	18	21
32	36	40	45	46	47	50	55	60	63	63	67	67	67	67	72	75	79	82
82	83	84	84	84	85	85	85	85	85	86	86							

To analyse the survival data of table (1), we first assume the exponentiated-Weibull ditributions (1). Considering the prior densities for α, θ and σ given in (18) with $a_1 = 8.6$, $b_1 = 1.83$, $a_2 = 7.58$, $b_2 = 51.98$, $a_3 = 26491.6$ and $b_3 = 290.251$, we generated 10 separate Gibbs chains each of wich ran for 5000 iterations, and we monitored the convergence of the

Gibbs samples using the Gelman and Rubin (1992) method that uses the analysis of variance technique to determine if further iterations ared needed. For each parameter, we consider iterations $2525^{th}, 2550^{th}, \ldots, 5000^{th}$ wich for 10 chains yeilds a sample of size 1000. In the table 2, we report posterior summaries for the parameters, and in figure 1 we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. We also have in table 2, the estimated potential scale reductions $\hat{R}$ (see Gelman and Rubin, 1992) for all the parameters. In this case, the considered number of iterations appears to be sufficient for approximate convergence ($\sqrt{\hat{R}} < 1.1$ for all parameters).

Table 2: *Posterior summaries for exponentiated-Weibull model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α	2.6444	2.6446	0.27758	(2.081139;3.16199)	1.002173
θ	0.20422	0.20432	0.05406	(0.10314;0.317079)	1.002107
σ	91.016	91.005	0.547375	(89.9555;92.0941)	1.0028216

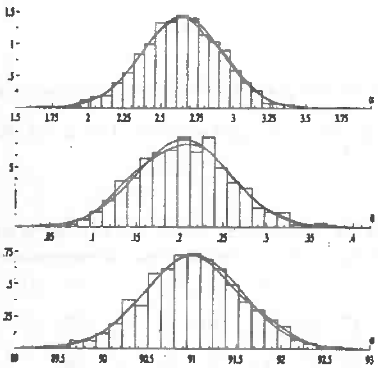

Figure 1: Approximate marginal posterior densities for α, θ e σ of the exponentiated-Weibull model

Considering the IBD model (5) and the prior distributions (1) with $c_1 = 33.30$, $d_1 =$

73669.89, $c_2 = 0.8775$, $d_2 = 0.2944$, $c_3 = 1.4832$ and $d_3 = 7.315$, we generated 20 separate Gibbs chains each of wich ran for 2000 iterations. For each parameter we considered iterations $1010^{th}, 1020^{th}, \ldots, 2000^{th}$, which for 20 chains yeilds a sample of size 1000. In table 3, we have the obtained posterior summaries for the parameters and in figure 2, we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. We also have in the table 3, the estimated potential scale reductions $\hat{R}$ for all the parameters, wich indicates that all chains converged.

Table 3: *Posterior summary for the IDB model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
δ	0.00030	0.00030	0.00005	(0.000229;0.000407)	1.006153
β	0.72470	0.5024	0.8999	(0.0271;2.527)	1.0000402
θ	0.18837	0.16352	0.12462	(0.0247;0.48762)	1.0002546

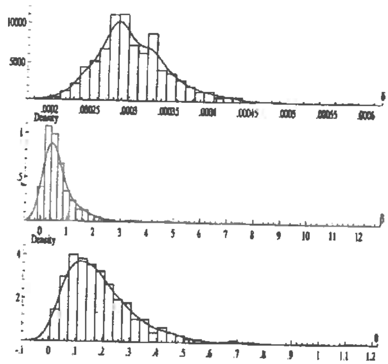

Figure 2: Approximate marginal posterior densities for δ, β e θ of IDB model

Considering now, the power-exponential model (7) and the prior densities (26) with $e_1 = 113.33$, $f_1 = 1.07$, $e_2 = 28.58$ and $f_2 = 53.4$, we generated 8 separate Gibbs chains each of wich ran for 50000 iterations and considered iterations $25025^{th}, 25050^{th}, \ldots, 50000^{th}$, which for 8 chains yeilds a sample of size 8000. In the table 4, we have the obtained posterior summaries for the parameters and in figure 3, we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. For all parameters, we observe (see table 4), $\sqrt{\hat{R}} < 1.1$, indicating approximate convergence

Table 4: *Posterior summary for the power-exponential model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α	76.498	76.4919	2.991	(68.8193;81.3473)	1.000109
β	2.166	2.166	0.315	(1.18404;2.64272)	1.009575

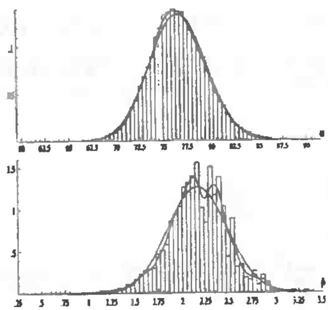

Figure 3: Approximate marginal posterior densities for α e β do of power-exponential model

Considering finally, the gamma mixtures given in (10), and the prior densities (30) with $\gamma_1 = 0.5$, $\gamma = 0.5$, $\lambda_j = 0.000001$ $(j = 1, 2)$, $\beta_1 = 1$ and $\beta_2 = 0.00002$, we generated 20 separate Gibbs chains each of wich ran for 5000 iterations. For each parameter we considered iterations $2550^{th}, 2600^{th}, \ldots, 5000^{th}$, which for 20 chains yeilds a sample of size 1000. In table 5, we have the obtained posterior summaries, and in figure 4, we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. We also observe approximate convergence, since the esimated potential scale reductions $\hat{R}$ are close one for all parameters.

From the generated Gibbs samples, we have in table 6, the pseudo-Bayes fator (38) for

Table 5: *Posterior summary for mixtures of gamma*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α_1	14.542	12.494	10.405	(1.33710;39.6366)	1.000312
α_2	0.91499	0.89006	1.19612	(0.68565;1.32152)	1.000304
β	0.02148	0.02030	0.00655	(0.01271;0.04058)	1.085947
p	0.02672	0.00601	0.09164	(0.000012;0.0318169)	1.021205

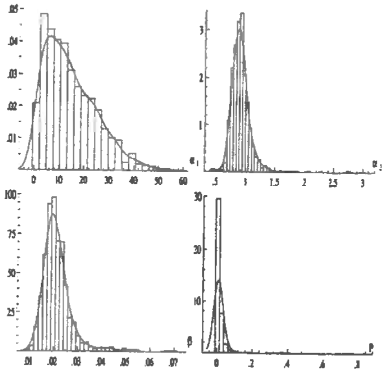

Figure 4: Approximate marginal posterior densities for α_1, α_2, β and p for the mixture of gammas

choosing of the best model for the lifetime data of table 2. In figure 5, we plot of the CPO ratios for each model with respect to the others against the observations. Values of the CPO ratio greater than one indicates preference for the first model. For example, in figure 5, the plot for the exponentiated-Weibull model versus power-exponential model indicates 28 out of 50 observations supporting the exponentiated-Weibull model over power-exponential model. Similarly, 38 observations support exponentiated-Weibull model over the IDB model, 33 observations support the exponentiated-Weibull model over the mixtures of gamma model, 38 observations support the IDB model over the power-exponential model, 32 observations support the power-exponential model over the mixtures of gamma model, and 39 observations support the mixtures of gamma model over the IDB model. Thus, in conclusion, this criteria says that the exponentiated-Weibull model is the one that presents the best fit for the survival time data of table 2.

Table 6: *Pseudo Bayes factor*

Models	PSFB
exponentiated-Weibull vs power-exponential	32.83
exponentiateted-Weibull vs IDB	17.61
exponentiated-Weibull vs mixture gamma	620.75
power-exponential vs IDB	0.54
power-exponential vs mixture gamma	18.91
IDB vs mixture gamma	35.26

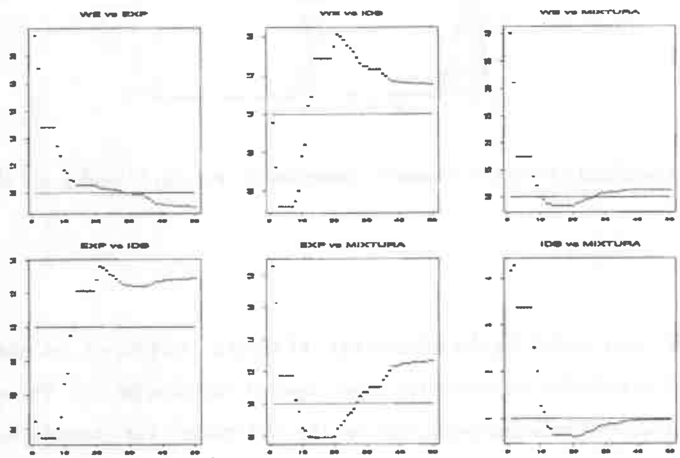

Figure 5: ratio CPO for different models versus *observations*,

5.2 EXAMPLE 2

In table 7, we have 50 observations generated from density (1) with parameters $\alpha = 2$, $\theta = 0.3$ and $\sigma = 1$, not including censored data.

Table 7: *Generated data from density (1)*

0.17087	0.67164	0.044448	1.0409	0.14450	0.44826	0.77785	0.0048173	0.0032595	0.27430
0.0015956	0.48105	0.66123	1.8675	0.22152	0.63347	0.012058	0.41785	1.9351	0.57678
0.67180	0.15366	1.0742	0.46588	0.038705	0.72372	0.17963	1.4175	0.48098	0.36706
0.64136	0.13995	1.4367	0.34256	0.29463	0.27305	0.060425	0.15204	1.0947	0.53263
1.2511	0.43501	0.30608	0.40391	0.025192	0.39053	0.42179	1.0422	0.0048170	1.3440

Considering the exponentiated-Weibull model (1), and the prior distribution (18) with $a_1 = 0.4$, $b_1 = 0.2$, $a_2 = 0.6$, $b_2 = 0.18$, $a_3 = 0.001$ and $b_3 = 0.001$. We generated 20 separate Gibbs chains each of wich ran for 2000 iterations, and we monitored the convergence of the Gibbs samples using the Gelman and Rubin (1992) method that uses the analysis of variance technique to determine if further iterations ared needed. For each parameter, we consider iterations $10010^{th}, 1020^{th}, \ldots, 2000^{th}$ wich for 20 chains yeilds a sample of size 2000. In the table 8, we report posterior summaries for the parameters, and in figure 6 we have the approximate marginal posterior densities considering the $S = 2000$ Gibbs samples. We also have in table 8, the estimated potential scale reductions $\hat{R}$ (see Gelman and Rubin, 1992) for all the parameters. In this case, the considered number of iterations appears to be sufficient for approximate convergence ($\sqrt{\hat{R}} < 1.1$ for all parameters).

Table 8: *Posterior summaries for exponentiated-Weibull model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α	1.7279	1.7054	0.3570	(1.0938 ; 2.4497)	1.000382
θ	0.4098	0.3996	0.12686	(0.1822 ; 0.6866)	1.001075
σ	0.9944	0.9889	0.1753	(0.6783; 1.3637)	1.001018

Considering the IBD model (5) and the prior distributions (1) with $c_1 = 7.76$, $d_1 =$

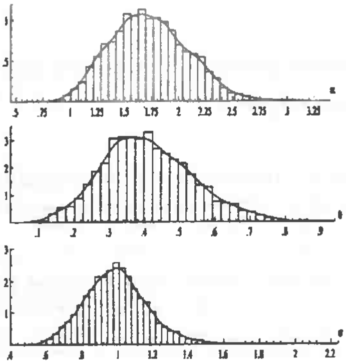

Figure 6: Approximate marginal posterior densities for α, θ e σ of the exponentiated-Weibull model

3.94, $c_2 = 34.68$, $d_2 = 2.63$, $c_3 = 6.75$ and $d_3 = 1.5$, we generated 20 separate Gibbs chains each of wich ran for 2000 iterations. For each parameter we considered iterations $1010^{th}, 1020^{th}, \ldots, 2000^{th}$, which for 20 chains yeilds a sample of size 2000. In table 3, we have the obtained posterior summaries for the parameters and in figure 7, we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. We also have in the table 9, the estimated potential scale reductions $\hat{R}$ for all the parameters, wich indicates that all chains converged.

Table 9: *Posterior summary for the IDB model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
δ	2.0098	1.9954	0.4110	(1.24363;2.8898)	1.000107
β	13.283	13.197	2.247	(9.3060 ; 182471)	1.0000402
θ	3.8369	3.7369	0.9027	(2.3219 ; 5.8996)	1.007290

Considering now, the power-exponential model (7) and the prior densities (26) with $e_1 = 4.58$, $f_1 = 4.78$, $e_2 = 4.161$ and $f_2 = 6.451$, we generated 20 separate Gibbs chains each of wich

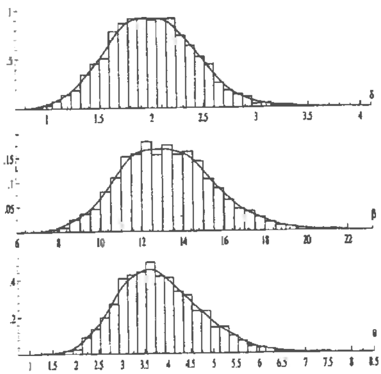

Figure 7: Approximate marginal posterior densities for δ, β e θ of IDB model

ran for 5000 iterations and considered iterations $2525^{th}, 2550^{th}, \ldots, 5000^{th}$, which for 20 chains yeilds a sample of size 2000. In the table 10, we have the obtained posterior summaries for the parameters and in figure 8, we have the approximate marginal posterior densities considering the $S = 2000$ Gibbs samples. For all parameters, we observe (see table 10), $\sqrt{\hat{R}} < 1.1$, indicating approximate convergence

Table 10: *Posterior summary for the power-exponential model*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α	0.9828	0.9674	0.16199	(0.7169 ; 1.3393)	1.000081
β	0.6575	0.6419	0.20657	(0.2969 ; 1.1467)	1.0005173

Considering finally, the gamma mixtures given in (10), and the prior densities (30) with $\gamma_1 = 1$, $\gamma = 1$, $\lambda_j = 0.000001$ $(j = 1, 2)$, $\beta_1 = 1$ and $\beta_2 = 0.00001$, we generated 20 separate Gibbs chains each of wich ran for 5000 iterations. For each parameter we considered iterations $2550^{th}, 2600^{th}, \ldots, 5000^{th}$, which for 20 chains yeilds a sample of size 1000. In table 11, we have the obtained posterior summaries, and in figure 9, we have the approximate marginal posterior densities considering the $S = 1000$ Gibbs samples. We also observe approximate convergence,

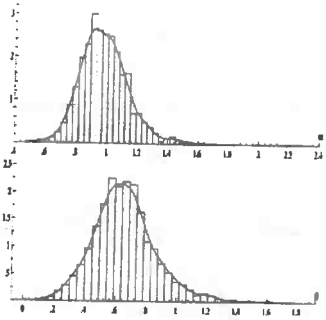

Figure 8: Approximate marginal posterior densities for α e β do of power-exponential model

since the esimated potential scale reductions $\hat{R}$ are close one for all parameters.

Table 11: *Posterior summary for mixtures of gamma*

	Mean	Median	S.D	95% credible interval	$\hat{R}$
α_1	3.5629	2.9961	3.3662	(0.2050 ; 10.0671)	1.000312
α_2	0.4548	0.4416	0.0958	(0.3412 ; 0.6667)	1.000304
β	2.1423	2.2023	0.1023	(1.2383 ; 4.1721)	1.085947
p	0.0308	0.0054	0.10229	(0.000013 ; 0.4247)	1.021205

As in Subsection 5.1, in figure 10 we plot the log of the CPO ratios for diferents models with respect to others against the observations number. Positive values of log CPO ratios indicate the preference of the first model with respect to the other. In figure 10, the exponetiated-Weibull model versus IDB model plot indicates that 41 out of 50 observations supportting the exponentiated-Weibull model over IDB model. Similarly, 47 observations support exponentiated-Weibull model over the power-exponential model, 40 observations support the exponentiated-Weibull model over the mixtures of gamma model, 28 observations support the IDB model over the power-exponential model, 35 observations support the power-exponential model over the mixtures of gamma model, and 36 observations support the mixtures of gamma

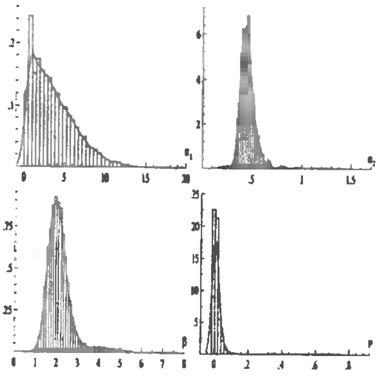

Figure 9: Approximate marginal posterior densities for α_1, α_2, β and p for the mixture of gammas

model over the IDB model. So here this criterio says that the exponentiated-Weibull model is the best fit for the data.

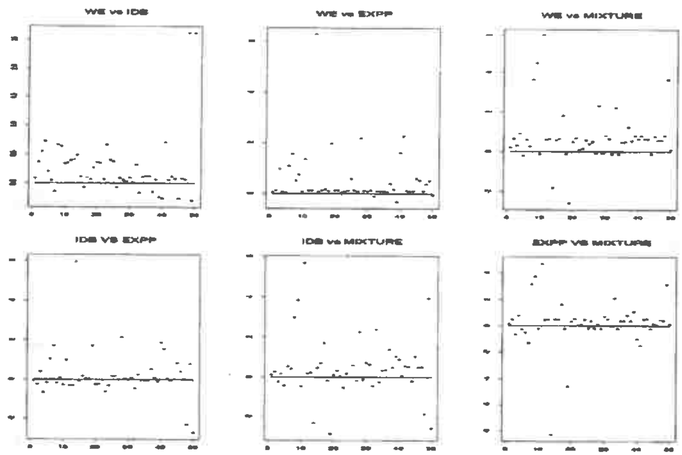

Figure 10: log ratio CPO for different models versus observations,

The above conclusions is also supported by the log of the pseudo-Bayes fators (38) given in the table 12

Table 12: *log of the pseudo-Bayes factor*

Models	PSFB
exponentiated-Weibull vs IDB	13.9908
exponentiated-Weibull vs power-exponential	26.62271
exponentiated-Weibull vs mixture gamma	38.0926
IDB vs power-exponential	12.63263
IDB vs mixture gamma	24.10018
power-exponential vs mixture gamma	11.46755

References

[1] Arset, M. V. (1987). "How to identify bathtub hazard rate", *IEEE Trans. Reliability*, **36**, 106-108.

[2] Gelfand, A.E. and Smith, A.F.M. (1990). Samplig-based approaches to calculating marginal densities. *J. Amer. Statist. Assoc.*, **85**, 398-409.

[3] Gelman, A. and Rubin, D.B. (1992). Inference from iterative simulating using multiple sequence (with discussion). *Statistical Science*, **7**,457-511.

[4] Geman, S., and Geman, D. (1984). Stochastic ralaxation, Gibbs distributions and the bayesian restoration of images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, **6**, 881-889.

[5] Glasser, R.E. (1980). Bathtub and related failure rate characterizations. *J. Amer. Statist. Assoc.*, **75**, 667-672.

[6] Hjort , U. (1980). A realiability distributions with increasing, decreasing, constant and bathtub failure rates. *Technometrics*, **22**, 99-107.

[7] Mudholkar, G.S., Srivastava, D.K. and Friemer, M. (1995). The exponentiated Weibull family: a reanalysis of the bus-motor-failure data. *Technometrics*, **37**,436-445.

[8] Printice, R.L. (1975). Discrimation among some parametric models. *Biometrika*, **61**, 539-544.

[9] Rajarshi, S. and Rajarshi, M.B (1988). Bathtub distributions: a review. *Commun. Statist.-Theory Meth.*, **17(8)**, 2597-2621.

[10] Stacy, E. W. (1962). A generalization of the gamma distributions. *Ann. Mathe. statist.* , **33** 1187-1192.

[11] Smith, A.F.M. and Roberts, G.O. (1993). Bayesian computation via the Gibbs sampler and related Markov Chain Monte Carlo Methods, *J. Roy. Statist. Soc., Ser.* **B, 55**, 3-23.

[12] Smith, R. M. and Bain, L. J. (1975) An exponential power life-testing distribution. *Commun. in Statist.*, **469-481**.

[13] Smith, R. M. and Bain, L. J. (1976). correlation-type goodness of fit sattistics with censored sampling. *Commun. in Statist.*, **469-481**.

ÚLTIMOS RELATÓRIOS TÉCNICOS PUBLICADOS

9801 - **GUIOL, H.** Some properties of K-step exclusion processes. 1998. 21p. (RT-MAE-9801)

9802 - **ARTES, R., JØRSENSEN, B.** Longitudinal data estimating equations for dispersion models. 1998. 18p. (RT-MAE-9802)

9803 - **BORGES, W.S.; HO, L.L.** On capability index for non-nornal processes. 1998. 13p. (RT-MAE-9803)

9804 - **BRANCO, M.D. , BOLFARINE, H., IGLESIAS, P., ARELLANO-VALLE, R.** Bayesian analysis of the calibration problem under elliptical distributions. 1998. 15p. (RT-MAE-9804)

9805 - **IGLESIAS, P., MATÚS, F., PEREIRA, C.A.B., TANAKA, N.I.** On finite sequences conditionally uniform given minima and maxima. 1998. 13p. (RT-MAE-9805)

9806 - **IGLESIAS, P., PEREIRA, C.A.B., TANAKA, N.I.** Characterizations of multivariate spherical distribution in l_∞ norm. 1998. 19p. (RT-MAE-9806)

9807 - **GIAMPAOLI, V., SINGER, J.M.** A note on the comparison of two normal populations with restricted means. 1998. 12p. (RT-MAE-9807)

9808 - **BUENO, V.C.** A note on conditional reliability importance of components. 1998. 07p. (RT-MAE-9808)

9809 - **GIMENEZ, P., BOLFARINE, H.** Consistent estimation in comparative calibration models. 1998. 19p. (RT-MAE-9809)

9810 - **ARTES, R., PAULA, G.A., RANVAUD, R.** Estudo da direção tomada por pombos através de equações de estimação para dados circulares longitudinais. 1998. 16p. (RT-MAE-9810)

9811 - **LIMA, A.C.P., SEM, P.K.** Time-Dependent Coefficients in a Multi-Event Model for Survival Analysis. 1998. 21p. (RT-MAE-9811)

9812 - **KOLEV, N., MINKOVA, L.** Over-and underdispersed models for ruin probabilities. 1998. 34p. (RT-MAE-9812)

9813 - **KOLEV, N.** Negative Binomial Cross-Classifications. 1998. 16p. (RT-MAE-9813)

9814- **BAKEVA, V., GEORGIEVA, M., KOLEV, N.** Two Characterizations of the Geometric Distribution Related to an Unreliable Geo/G_D/1 System. 1998. 11p. (RT-MAE-9814)

9815 - **KOLEV, N., MINKOVA, L.** Some basic Inflated-Parameter Discrete Distributions. 1998. 19p. (RT-MAE-9815)

9816 - **ARTES, R., PAULA, G.A., RANVAUD, R.** Strategy for the analysis of circular longitudinal data based on generalized estimating equations. 1998. 15p. (RT-MAE-9816)

9817 - **PAULA, G.A., ARTES, R.** A Score-Type Test to Assess Overdispersion in Linear Logistic Models. 1998. 17p. (RT-MAE-9817)

9818 - **LOSCHI, R.H., IGLESIAS, P.L., WECHSLER, S.** Unpredictability and Probability Updating. 1998. 11p. (RT-MAE-9818)

9819 - **KOLEV, N., MINKOVA, L.** New Over-/Under- Dispersed Class of Inflated-Parameter Discrete Probability Distributions. 1998. 10p. (RT-MAE-9819)

9820 - **KOLEV, N., MINKOVA, L.** Inflated-Parameter Family of Generalized Power Series Distributions. 1998. 11p. (RT-MAE-9820)

9821 - **AUBIN, E.C.Q., STANGENHAUS, G.** A Robust Estimation of Growth Curves. 1998. 7p.(RT-MAE-9821)

9822 - **BORGES, W., HO, L. L.,** A Fraction Non-Conforming Based Capability Index. 1998. 13p. (RT-MAE-9822)

9823 - **GALEA-ROJAS, M., BOLFARINE, H.** Hypothesis Testing in Functional Comparative Calibration Models. 1998. 13p. (RT-MAE-9823)

9824 - **FONTES, L. R. G., ISOPI, M., NEWMAN, C. M.** Chaotic Time Dependence in a Disordered Spin System. 1998. 23p. (RT-MAE-9824)

9825 - **FERRARI, P. A., FONTES, L. R. G., VARES, M. E.** The Asymmetric Simple Exclusion Model With Multiple Shocks. 1998. 24p. (RT-MAE-9825)

9826 - **CABRAL, C. R. B., PAULA, G. A., BOLFARINE, H.** One-Sided Tests for the Analysis of Pretest-Posttest Data. 1998. 12p. (RT-MAE-9826)

9827 - **GASCO, L., BOLFARINE, H., ARELLANO-VALLE, R. B.** Asymptotic properties of the sample mean vector and sample covariance matrix with application in elliptical multiplicative measurement error models. 1998. 17p. (RT-MAE-9827)

9828 - **FERRARI, S. L. P., CRIBARI-NETO, F.** On the robustness of analytical and bootstrap corrections to score tests in regression models. 1998. 11p. (RT-MAE-9828)

9829 - **PERES, C. A., ANDRE, C. D. S., NARULA, S. C.** Whyn the observations within a treatment of a random-effects model are correlated? 1998. 8p. (RT-MAE-9829)

9830 - **FONTES, L.R., MATHIEU, P., PICCO, P.** On the averaged dynamics of the random field curie-weiss model. 1998. 27p. (RT-MAE-9830)

9831 - **ANDRÉ, C.D.S., ELIAN, S.N., TAVARES, R.** Coeficientes de determinação na regressão de mínima soma dos erros absolutos e sua utilização em problemas de seleção de variáveis. 1998. 14p. (RT-MAE-9831)

9832 - **SOUZA, F.A.M., PAULA, G.A.** Deviance residuals for an angular response. 1998. 13p. (RT-MAE-9832)

9833 - **SOLER, J.M.P., SINGER, J.M.** Optimal covariance adjustment in growth curve models. 1998. 13p. (RT-MAE-9833)

9834 - **BOTTER, D.A., PERES, A.A.Q., ZANDONADE, E.** A longitudinal study of mortality and air pollution for São Paulo, Brazil. 1998. 16p. (RT-MAE-9834)

9835 - **CRIBARI-NETO, F., FERRARI, S.L.P.** Monotonic improved critical values for econometric chi-squared asymptotic criteria. 1998. 12p. (RT-MAE-9835)

9836 - **CORDEIRO, G.M., FERRARI, S.L.P., URIBE-OPAZO, M.A., VASCONCELLOS, K.L.P.** Corrected Maximum Likelihood estimation in a class of symmetric nonlinear regression models[1]. 1998. 14p. (RT-MAE-9836)

9837 - **BELITSKY, V., FERRARI, P.A.** Invariant measures and convergence for cellular automaton 184 and related processes. 1998. 19p. (RT-MAE-9837)

9838 - **CASSANDRO, M., COLLET, P., GALVES, A., GALVES, CHARLOTTE.** A Statistical-Physics Aprroach to Language Acquisition and Language Change. 1998. 15p. (RT-MAE-9838)

9839 - **ANDRADE, D.F., VALLE, R.C.** Uma introdução à teoria da resposta ao item : conceitos e aplicações. 1998. 15p. (RT-MAE-9839)

9840 - **BUENO, V.C.** Component importance in a random environment. 1998. 6p. (RT-MAE-9840)

9841 - **OGLIARI, P.J., ANDRADE, D.F.** Analysing longitudinal data via nonlinear models in randomized block designs. 1998. 18p. (RT-MAE-9841)

The complete list of "Relatórios do Departamento de Estatística", IME-USP, will be sent upon request.

Departamento de Estatística
IME-USP
Caixa Postal 66.281
05315-970 - São Paulo, Brasil