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Inventory Perishability in the Stochastic Flexible Production and Procurement Lot-Sizing Problem

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ABSTRACT

In this paper, we address the 2-Stage Stochastic Production and Procurement Lot-Sizing Problem with Perishable Inventory (2S-PPLSP), in which demand is uncertain. The model is flexible enough to allow production decisions to be made after demand realisation, while the raw material procurement planning must be made before it. In our setting, products are perishable, thus they must be discarded if kept in inventory beyond their shelf-life. This setting is found in practical applications, especially in the food industry in which products can deteriorate quickly. We present two formulations for the 2S-PPLSP, and discuss how the optimal recourse policy of traditional lot-sizing models can lead to planned inventory losses to reduce inventory volume. We then propose an alternative recourse policy to prevent these losses without deteriorating demand fulfilment neither adding excessive costs.

KEYWORDS. Lot-Sizing, Stochastic Programming, Perishability, Recourse

1. Introduction

This paper addresses the 2-Stage Stochastic Production and Procurement Lot-Sizing Problem with Perishable Inventory (2S-PPLSP). The main objective of the 2S-PPLSP is to integrate production (i.e. how much and when to produce) and raw material procurement (i.e., how much is purchased and from which suppliers), in order to find more cost-efficient solutions that fulfil an uncertain demand. The integration of these two planning stages has several economical benefits and is applicable in different manufacturing industries [Cunha et al., 2018; Tomazella et al., 2023].

The literature on the integrated, and also on general lot-sizing problems, often assumes that inventory can be kept for a unlimited number of periods. However, when put in practice, this assumption can lead to infeasible solutions if inventory is perishable [Wei et al., 2019; Acevedo-Ojeda et al., 2020]. For example, the inventory can deteriorate over time or it has an expiration date after which it loses its function [Amorim et al., 2013; Coelho and Laporte, 2014].

In a deterministic setting, in which the demands are perfectly known in advance, the optimal solution minimises losses due to perishability. In this particular problem there are no constraints that may cause these losses, such as minimum lot sizes (that force minimum production rates), so no inventory is lost. These models, however, do not account for variations in the demand, neither provide any means to react to these variations, which can result in excessive backlogging penalties (if demand is higher than expected) or excessive inventories (if demand is lower).

In the 2S-PPLSP, we take demand uncertainty into consideration by allowing some flexibility into the production process. The concept of flexibility [King and Wallace, 2012] is a property of a model that indicates how it deals with uncertainty. A more flexible model allows recourse actions to be taken so to optimise the solution after the actual demand is known. To model the 2S-PPLSP, we use a 2-stage stochastic programming [Bookbinder and Tan, 1988], which divides the problem into **first-** (before demand realisation) and **second-stage** (after) decisions.

Here, the production process is flexible enough so *how much* is produced can be decided according to the demand (second-stage). However, procurement and setup decisions are first-stage. The motivation for this setting is the fact that procurement decisions involve third-party suppliers, delivery lead-times and long term contracts, which end up making these decisions less flexible. Also, setting up production often involves contracts with workers and availability of resources, thus the decision of in which periods the production occurs (regardless of the quantities) are less flexible too.

Our 2S-PPLSP incorporates perishability in product inventory, while raw materials are considered not-perishable. This setting is especially relevant when applied to the food industry [Wei et al., 2019], where ingredients are purchased from suppliers and can be preserved for longer periods of time in stock (usually more than the periods covered by a short-term planning horizon), but the fresh products have a limited life-span, and cannot be stored too long without spoiling. Production flexibility is also applicable in this context, since the producer is able to react to the actual demand and plan their process accordingly.

While a 2-stage stochastic programming model is able to find efficient, sometimes near-optimal solutions for the problem, the recourse decisions might lead to a situation that is not desirable from a managerial point of view: intentionally letting inventory expire as part of the optimal recourse. This happens due to the nature of the model, that opts to minimise costs and has a penalty associated with inventory disposal, which often does not consider some intangible aspects of losing inventory. When demand is lower than expected, the raw material that was initially purchased would sit in stock for too long, resulting in higher costs. Therefore, the recourse second-stage model orders an excessive production (more than the realised demand) so this material is consumed. After some periods, these extra products are discarded, essentially removing the raw materials from stock.

While this recourse action may be optimal according to the model, it not only deteriorates sustainability-related indicators, but it also wastes material that: **i)** had purchasing costs associated them, which are not usually accounted for when disposing products; **ii)** would otherwise be use in production after the planning horizon, eliminating the need to purchase them again.

In this context, alongside with two formulations for the 2P-PPLSP, we propose a recourse policy that limits the second-stage production to the realised demand, and evaluate how it compares to the optimal solution. This comparison is done by simulating the solutions over the same large set of demand scenarios, so we have reliable expected values for all associated costs, demand fulfillment and inventory lost.

The rest of this paper is divided as follows: Section 2 presents the notation and two Mixed Integer Problem formulations for the 2S-PPLSP, and Section 3 discusses how inventory is lost to perishability and the recourse policies for the 2SP-PPLSP. Section 4 describes the computational experiments and the results, and lastly, Section 5 summarises the main findings of this paper and suggests future research.

2. Mathematical Model

The 2S-PPLSP consists of a two-level lot-sizing problem, in which the production level is a capacitated lot-sizing problem with setup times [Pochet and Wolsey, 2006] and the procurement level is a supplier selection problem [Basnet and Leung, 2005]. In this section we present two formulations for the 2SP-PPLSP, one using the classic *Inventory Age* (IA) variables [Coelho and Laporte, 2014], and a *Facility Location* (FL), which is also referred to as *Transportation*, reformulation [Krarup and Bilde, 1977]. Uncertainty is modeled via 2-stage stochastic programming, in which the demand is approximated using a finite set of scenario with size Ω. Each scenario ω has a probability of occurring pr_ω, with $\sum_\omega pr_\omega = 1$. In both IA and FL formulations, the first-stage model is the same, so they differ on how the second-stage is formulated. The main notation used is presented in the next tables.

Indices, sets and parameters

$j \in \{1, \dots, J\}$	Products
$f \in \{1, \dots, F\}$	Raw materials
$s \in \{1, \dots, S\}$	Suppliers
$t \in \{1, \dots, T\}$	Periods
$d_{jt\omega}$	Demand for product j in period t in scenario ω
p_{jt}	Production cost of product j in period t
a_{jt}	Production time of product j in period t
s_{jt}	Setup cost associated with product j in period t
b_{jt}	Setup time associated with product j in period t
h^+_{jt}	Holding cost of product j in period t
h^-_{jt}	Backlogging cost of product j period t
v_j	Shelf-life of product j
h^d_{jt}	Disposal cost of product j in period t
cap_t	Production capacity in period t
$\overline{p}_{fst}$	Price of material f from supplier s in period t
o_{st}	Order cost from supplier s in period t
h_{ft}	Holding cost of raw material f in period t
r_{fj}	Quantity of material f used to produce one unit of product j

First-Stage Variables

Q_{fst}	Units of raw material f purchased from supplier s in period t
S_{st}	1 if there is a purchase from supplier s in period t; 0 otherwise

Y_{jt} 1 if product j is produced in period t; 0 otherwise

For the IA formulation [Coelho and Laporte, 2014], we use variable $X_{jt\omega}$ to indicate the amount produced in the second-stage, and include the index g in the inventory variables to denote the age of units in stock. The second-stage variables of the IA formulation are:

Second-Stage Variables in the IA formulation

$X_{jt\omega}$ Units of product j produced in period t in scenario ω
$I^g_{jt\omega}$ Inventory of product j with age g at the end of period t in scenario ω
$W^g_{jt\omega}$ Units of product j with age g used to fulfil the demand in period t in scenario ω
$I^-_{jt\omega}$ Backlog of product j at the end of period t in scenario ω
$R_{jt\omega}$ Units of product j lost due to perishability in period t in scenario ω
$I_{ft\omega}$ Inventory of material f at the end of period t in scenario ω

The first model, using IA variables, is posed as follows.

$$\min \quad \sum_j \sum_t s_{jt} \cdot Y_{jt} + \sum_f \sum_s \sum_t p_{fst} \cdot Q_{fst} + \sum_s \sum_t o_{st} \cdot S_{st} + \sum_\omega p_\omega \cdot (\sum_j \sum_t (\sum_{g=1}^{v_j} h_{jt} \cdot I^g_{jt\omega} + h^-_{jt} \cdot I^-_{jt\omega} + h^d_{jt} \cdot R_{jt\omega} + p_{jt} \cdot X_{jt\omega}) + \sum_f \sum_t h_{ft} \cdot I_{ft\omega}) \tag{1}$$

$$s.t. \sum_{g=1}^{v_j} I^g_{j(t-1)\omega} + X_{jt\omega} = \sum_{g=0}^{v_j} W^g_{jt\omega} + \sum_{g=1}^{v_j} I^g_{jt\omega} + R_{jt\omega} \qquad \forall j,t,\omega \tag{2}$$

$$X_{jt\omega} = W^0_{jt\omega} + I^1_{jt\omega} \qquad \forall j,t,\omega \tag{3}$$

$$I^g_{j(t-1)\omega} = W^g_{jt\omega} + I^{g+1}_{jt\omega} \qquad \forall j,t,1 \leq g < v_j,\omega \tag{4}$$

$$I^{v_j}_{j(t-1)\omega} = W^{v_j}_{jt\omega} + R_{jt\omega} \qquad \forall j,t,\omega \tag{5}$$

$$\sum_{g=0}^{v_j} W^g_{jt\omega} + I^-_{jt\omega} = d_{jt\omega} + I^-_{j(t-1)\omega} \qquad \forall j,t,\omega \tag{6}$$

$$\sum_j (a_{jt} \cdot X_{jt\omega} + b_{jt} \cdot Y_{jt}) \leq cap_t \qquad \forall t,\omega \tag{7}$$

$$X_{jt\omega} \leq M_{jt\omega} \cdot Y_{jt} \qquad \forall j,t,\omega \tag{8}$$

$$I_{f(t-1)\omega} + \sum_s Q_{fst} = I_{ft\omega} + \sum_j (a_{fj} \cdot X_{jt\omega}) \qquad \forall f,t,\omega \tag{9}$$

$$Q_{fst} \leq M_{ft} \cdot S_{st} \qquad \forall f,s,t \tag{10}$$

$$X_{jt\omega}, I^g_{jt\omega}, W^g_{jt\omega}, R_{jt\omega}, I^-_{jt\omega}, I_{ft\omega} \geq 0 \qquad \forall j,f,t,g,\omega \tag{11}$$

$$Q_{fst} \geq 0 \qquad \forall f,s,t \tag{12}$$

$$Y_{jt}, S_{st} \in \{0,1\} \qquad \forall j,t,s \tag{13}$$

Objective function (1) consists of first-stage costs (production setups, raw material purchasing and suppler order) and the expected second-stage costs considering the scenario realisations (product inventory, backlogging, product disposal, production and raw material inventory).

Constraints (2) model the product inventory balance. Constraints (3) set the total produces in a period equal to the amount used to fulfil demand immediately plus the amount that is moved to inventory with age 1. Similarly, Constraints (4) model the inventory ageing process, such that current inventory of age g is either used to fulfil demand or kept in inventory with updated age $g+1$. Constraints (5) then enforce the perishability restrictions, since inventory with age $v_j + 1$ cannot be kept and must be discarded. Lastly, Constraints (6) model demand fulfillment, including the backlogs. Machine capacity constraints are enforced in (7), and Constraints (8) allow production only in the same periods that setups occur. Here the big-M is calculated as the minimum of either: the total realised demand on the planning horizon; or the maximum quantity that can be produced in that period. Raw material inventory flow is modelled using Constraints (9), and (10) enforce an order to be charged whenever there is a purchase with a supplier. Lastly, Constraints (11)-(13) define the domain of all variables.

We now introduce the Facility Location (FL) reformulation, first proposed by Krarup and Bilde [1977] for the lot-sizing problem. This reformulation separate the $X_{jt\omega}$ from the previous model into t variables that indicate when an unit of a product is produced and when it is used to fulfil the demand. Computationally-wise, this reformulation is tighter than the previous, due to the smaller big-M values that can be used here. The fact that the $X_{jtk\omega}$ variables implicitly tell how much periods a product stays in stock allows us to easily enforce the perishability constraints without the need of additional variables.

Second-Stage Variables in the FL formulation

$X_{jtk\omega}$	Units of product j produced in period t to fulfil the demand of period k in scenario ω
$R_{jt\omega}$	Units of product j produced in period t lost due to perishability in scenario ω

In $X_{jtk\omega}$, the t index ranges from 1 to $T+1$, with $X_{j(T+1)k\omega}$ referring to the demand of period k that is unfulfilled by the end of the planning horizon (final backlog). $R_{jt\omega}$ is used to account for scenarios in which there excess production occurs, and the k index is not necessary, since the loss always occur after v_j periods (if $t+v_j > T$, this variable, in theory, represents leftover product inventory at the end of the planning horizon). Note that variables $I_{ft\omega}$ are still used to model the raw material inventory flow. An experiment was done using a formulation using facility location variables for raw materials, and the results showed that this formulation had inferior results than IA and FL, so it was left out of this article. To simplify the notation used in the model, we also introduce the auxiliary variable $P_{jt\omega}$ to represent the total of j produced in t in scenario ω.

The costs associated with $X_{jtk\omega}$ (c_{jtk}) and $R_{jt\omega}$ (r_{jt}) are calculated using Equations (14)-(15).

$$c_{jtk} = \begin{cases} p_{jt} + \sum_{l=k}^{t} h^{-}_{jl} & \forall j, t, k < t \\ p_{jt} & \forall j, t, k = t \\ p_{jt} + \sum_{l=t}^{k} h_{jl} & \forall j, t, k > t \\ \sum_{l=k}^{T+1} h^{-}_{jl} & \forall j, k, t = T+1 \end{cases} \tag{14}$$

$$r_{jt} = \begin{cases} p_{jt} + \sum_{l=t}^{t+v_j} h_{jl} + h^{d}_{j(t+v_j)} & \forall j, t \le T - v_j \\ p_{jt} + \sum_{l=t}^{T} h_{jl} & \forall j, t > T - v_j \end{cases} \tag{15}$$

The FL model is posed as follows:

$$\begin{aligned} \min \quad & \sum_j \sum_t s_{jt} \cdot Y_{jt} + \sum_f \sum_s \sum_t p_{fst} \cdot Q_{fst} + \sum_s \sum_t o_{st} \cdot S_{st} + \\ & \sum_\omega p_\omega \cdot (\sum_j \sum_{t=1}^{T+1} \sum_k c_{jtk} \cdot X_{jtk\omega} + \sum_j \sum_t r_{jt} \cdot R_{jt\omega} + \sum_f \sum_t h_{ft} \cdot I_{ft\omega}) \end{aligned} \tag{16}$$

$$s.t. \quad (10), (12), (13)$$

$$\sum_{t=\max(k-v_j,1)}^{T+1} X_{jtk\omega} = d_{jk\omega} \qquad \forall j, k, \omega \tag{17}$$

$$P_{jt\omega} = \sum_k X_{jtk\omega} + R_{jt\omega} \qquad \forall j, t, \omega \tag{18}$$

$$\sum_j (a_{jt} \cdot P_{jt\omega} + b_{jt} \cdot Y_{jt}) \leq cap_t \qquad \forall t, \omega \tag{19}$$

$$X_{jtk\omega} \leq d_{jk\omega} \cdot Y_{jt} \qquad \forall j, t, k, \omega \tag{20}$$

$$R_{jt\omega} \leq M_{jt\omega} \cdot Y_{jt} \qquad \forall j, t, \omega \tag{21}$$

$$I_{f(t-1)\omega} + \sum_s Q_{fst} = I_{ft\omega} + \sum_j (a_{fj} \cdot P_{jt\omega}) \qquad \forall f, t, \omega \tag{22}$$

$$X_{jtk\omega} \geq 0 \qquad \forall j, t \leq T+1, k \leq t + v_j, \omega \tag{23}$$

$$X_{jtk\omega} = 0 \qquad \forall j, t, t + v_j < k, \omega \tag{24}$$

$$R_{jt\omega}, I_{ft\omega} \geq 0 \qquad \forall j, t, f\omega \tag{25}$$

Objective function (16) keeps the same first-stage total costs from (1) as well as the raw materials inventory costs. All the remaining second-stage costs are replaced with the two terms associated with the new variables. Constraints (17) guaranteed that demand in period k is fulfilled on time by products within their shelf-lives (produced at most v_j periods earlier), or fulfilled later as backlogged demand. Constraints (18) define the auxiliary variable $P_{jt\omega}$, which is used in Constraints (19), that limit production and setups to the machine capacity. Constraints (20)-(21) allow production to happen only in periods in which a setup has been planned according to the first-stage. Note that in (20) we can use a tighter big-M ($d_{jk\omega}$), improving the linear relaxation of the model. Constraints (22) model raw material inventory flow. Constraints (23) and (24) set the domain of the $X_{jtk\omega}$ variables. Note that we can enforce the perishability constraints here as well by setting them to zero whenever demand fulfillment (k) would occur after more than v_j periods after production (t). Lastly, Constraints (25) set the domain of the other second-stage variables.

3. Recourse in the 2SP-PPLSP

The second-stage of the 2SP-PPLSP consists of a linear lot-sizing problem, in which all setups have been defined on the first-stage, and production is limited not only by these designated setups and machine capacity, but also by how much raw material is available in stock (already established by first-stage procurement decisions). Therefore, the optimal recourse for each scenario is the solution of this linear problem, in which production-related and raw material inventory keeping costs are minimised.

In scenarios with demand near or higher than the expected values, the optimal recourse solution would produce to guarantee maximum demand fulfillment (minimising backlogs and inventory costs) while adhering to the productive and perishability constraints.

However, in scenarios with lower demand, there is a raw material surplus, which ends up resulting in high inventory keeping costs. Per definition, these raw materials have infinite shelf-lives, and could remain in stock until the end of the planning horizon, but there is a possibility of removing these materials from the balance: using them in production and letting these products expire past their shelf-lives. While this practice is not a desirable from managerial and sustainable points-of-view, in an optimal recourse solution, this will happen when is economically viable to do so.

Considering that raw materials can be used for different products and that multiple materials need to be discarded at once, it is not straightforward (without solving the linear second-stage model) to tell when the recourse solution will have these planned losses. It is possible, however, to estimate when they could occur. Let $H_j^f = \sum_f r_{fj} \cdot h_f$ be the holding costs of all the raw materials used in the production of product j for a single period. Assuming that all costs are time-invariant, the cost of disposing the raw material needed to produce one unit of j is $p_j + v_j \cdot h_j + h_j^d$ (production, holding the product for the length of its shelf-life and disposal). In this calculation, setups are not included because we assumed that this excess production would happen in a period with a setup already assigned in the first-stage (otherwise, it would become too expensive to do so). Assuming $p_j = 0$ and $h_j = H_j^f$, the excess production happens if this surplus of raw material would be otherwise kept for $n \geq v_j + \lceil \frac{h_j^d}{H_j^f} \rceil$ periods. It is important to note that this is an illustrative example and uses some simplifications (such as the p_j and h_j ones) and the fact that no other products will use those raw materials, which makes this estimate for n a loose lower bound.

This overproduction needs to be addressed for three reasons: **i)** knowing that production planning is done in a rolling horizon context, any raw material left at the end of planning horizon can be used in a future period, eliminating the need of purchasing it again; **ii)** while in terms of the 2SP-PPLSP objective function, losing the production intentionally might result in lower costs, the disposal cost of a product (h_j^d) does not account for the procurement-related costs (the price paid for the units of raw material being lost), thus creating an inconsistency between the first- and second-stage solutions (the material is purchased and then discarded); **iii)** there are intangible costs and implications of these recourse decisions, such as key performance indicators of inventory lost, carbon emissions related to this excess production and waste generation.

In order to address this issue and prohibit product disposal, these losses can be avoided by adding Constraints (26) to the models, which guarantee that, for each scenario, the amount produced during the planning horizon will be at most equal to the total demand.

$$R_{jt\omega} = 0 \quad \forall j, t, \omega \tag{26}$$

This same constraint can be used in the second-stage subproblem to evaluate the first-stage solution of the original model using a recourse policy that prevents inventory losses as well. This policy is referred to as Limited Production Recourse (LPR), while the original policy (without these constraints) is referred to as Optimal Production Recourse (OPR).

To evaluate how the solutions behave under these constraints, three approaches will be compared:

- Solving the 2-stage model as originally presented and evaluating it using OPR.
- Solving the 2-stage model as originally presented and evaluating it using LPR.
- Solving the 2-stage model with Constraints (26) and evaluating it using LPR.

4. Computational Experiments and Results

All models were implemented in Python and solved using Gurobi 10.0.0, with a time limit of 3600 seconds. The experimentation was done using the Euler Cluster, located in ICMC, São Carlos. Each node of the cluster contains 2 Intel Xeon E5-2680v2 processors each, 10 cores, 2.8GHz and 128GB DDR3 1866MHz RAM.

A total of 72 instances were used for the experimentation, generated based on the instances from Tempelmeier and Buschkühl [2009] and Cárdenas-Barrón et al. [2021]. These instances have $J = \{1, 3, 5\}$ products, $F = 10$ raw materials, $S = 5$ suppliers, $T = 12$ periods.

Raw material purchasing prices were generated from $U[1, 5]$ while order costs had two ranges,low ($U[1000, 2000]$) and $high$ ($U[10000, 12000]$). The parameter r_{fj}, that says how many units of f are used to produce one unit of j, was generated from $U[1, 3]$. Raw material holding costs are continuous values from $U(0.5, 1)$, and product holding costs are calculated as $h^+_{jt} = \sum_f r_{fj} \cdot h_{ft} + 1$. Product backlogging and disposal costs are 50 and 2 times, respectively, their holding costs. The shelf-lives of all products are $v_j = 2$. Unitary production costs and times are both equal to 1 and setup times are randomly selected from $\{5, 10, 20\}$.

Product demand was taken from a database of real sales data from a Brazilian manufacturer, and was altered for secrecy purposes (although it remains faithful to the original values). The mean expected demands (d_{jt}) range from 50 to 1500, and the scenarios used an uncertainty level of 50%, so $d_{jt\omega}$ is generated from an uniform distribution, $U[0.5 \cdot d_{jt}, 1.5 \cdot d_{jt}]$. Setup costs and machine capacity are calculated using the formulation presented in Tempelmeier and Buschkühl [2009], which considers the mean demands, *time-between-orders* (TBO) and capacity factor (CF).

With the exception of the demands, all other parameters are time-invariant, having the same value for all periods. In total, three instances were generated for the 24 possible combinations of products ($J \in \{1, 3, 5\}$), order costs (low and $high$), TBO ($TBO \in \{1, 4\}$), and CF ($CF \in \{1.5, 2\}$). These instances are available at `github.com/caiotmz/thesis`.

4.1. General results

For the first experimentation, we tested both formulations with four different number of scenarios, $\Omega \in \{25, 50, 100, 250\}$. The main results the two proposed formulations are summarised in Table 5. It is possible to see that the FL reformulation improves the results of the models with up to 100 scenarios, with overall shorter solve times, comparable optimality gaps and more optimal solutions found. With 250 scenarios, the IA formulation has better results since the FL models start to get prohibitively large, and cannot be properly solved.

The main advantage of the FL reformulation is an improvement on the linear relaxation, gained with the replacement of Constraints (8) with (20), that use a tighter big-M. Indeed, an experimentation using instances with 25 scenarios showed that the linear relaxation value is increased in 1.6%, 6.0% and 8.3% for instances with 1, 3 and 5 products, respectively. From now on, the solutions of the FL reformulation are used for the analysis, due to its better reported results.

Table 5: General results of the proposed models: average computational time (in seconds), average optimality gap and number of optimal solutions found (out of 72).

	Inventory Age			Facility Location		
Ω	Time	Gap	Opt.	Time	Gap	Opt.
25	1175	0.60%	54	1157	0.43%	54
50	1647	1.07%	45	1527	0.98%	48
100	2280	2.17%	34	2126	2.29%	42
250	3081	5.17%	19	3116	11.62%	19

4.2. Value of the Stochastic Solution

Table 5 shows that it is possible to obtain high quality solutions by solving models with 100 scenarios. However, the optimality gaps of the stochastic solutions alone do not indicate which Ω is the most suitable, and how the solutions compare to each other.

In this section we compare the first-stage solutions by sampling them over the same set of scenarios, with size 10000. The metrics taken from this evaluation are the evaluated costs (sum of both first- and second-stage costs), the expected percentages of product disposal and unfulfilled demand. The solution obtained with the four values of Ω are evaluated, along with the solution from the deterministic model (solved using the expected demand). The results are summarised in Table 6.

The evaluated costs (column 2 - *Ev. cost*) show that the high optimality gaps obtained with 250 scenarios translate into poor solutions, given the higher costs when compared to the models with fewer scenarios. The best solutions were obtained with 50 and 100 scenarios. Interestingly, using 100 scenarios results in solutions with less % of unfulfilled demand, but slightly more costly and with more inventory lost. This shows a tendency of a more conservative solution, in which higher production rates are planned and more raw materials are bought. In scenarios with lower demand, these materials are eventually disposed through overproduction.

The solutions of the deterministic model are poor in terms of evaluated costs and demand fulfillment, since the solution is optimised for the mean demands, thus there is no protection against higher values. This results in solutions with excessive backlogging penalties and a % of unfulfilled demand that is 8 times higher than those from the 2-stage stochastic models, justifying the use of the latter.

Table 6: Evaluated results of the stochastic solutions.

Ω	Ev. cost	% disposal	% unf. demand
25	675718	5.82%	0.49%
50	659398	6.80%	0.35%
100	665916	6.91%	0.32%
250	774029	7.15%	0.33%
Deterministic	1252128	1.13%	2.64%

4.3. Recourse Policies

We now discuss how the proposed alternatives for preventing these losses affect the solutions. Using the three approaches proposed in Section 3, the first-stage solutions (obtained by solving the models with 100 scenarios) were sampled over the same set of scenarios, and the results are summarised in Table 7.

When using the optimal recourse policy (OPR), 7% of the total production is expected to be lost due to perishability. This reduces inventory costs of the non-perishable materials over time, achieving the minimum evaluated total costs.

When the LPR policy is applied to the same solutions, limiting the production to the demand, inventory losses are totally avoided, accompanied by an increase of 4% in the expected costs. This increase is due to the additional raw material inventory keeping costs, which nearly double, while some other costs (production, product inventory and disposal) are reduced. Using the LPR policy also increase the value of the final raw material inventory (at the end of the planning horizon) in 4.5 times. While this comes as a cost in the objective function, we remind that this inventory can also be used for production in later periods, adding inventory value to a future application of the model.

Solving the model with Constraints (26) to limit the second-stage production, instead of using them as recourse policy only, results in slightly cheaper solutions, with no losses too, but a worse % of unfulfilled demand. This happens because, in this model, fewer quantities of raw materials are purchased to prevent excessive inventory levels, constraining the second-stage production and causing more stock-outs. Therefore, even though these solutions are more efficient in terms of total costs, the worse demand fulfilment might make them solutions less desirable in from a practical point of view.

Table 7: Comparison of the recourse policies.

Recourse policy	OPR	LPR	LPR
Production limited by Constraints (26)			✓
Evaluated solution costs	665916	691943	685009
Total raw material inventory costs	64216	116563	104063
Final raw material inventory costs	1517	6949	6378
Total product inventory costs	31246	18542	17935
Final product inventory costs	0	0	0
Disposal costs	12696	0	0
% of production lost	6.91%	0.00%	0.00%
% of unfulfilled demand	0.32%	0.32%	0.41%

5. Conclusions

In this paper, we have discussed the 2S-PPLSP, a problem integrating production and procurement decisions in which demand is uncertain and product inventory is perishable. The model is flexible in the sense that production quantities can be decided after demand realisation, although raw material procurement must be made beforehand.

This problem was modeled using 2-stage stochastic programming, and two formulations were presented. The experiments showed that the reformulation using facility location variables was slightly more efficient, due to its tighter linear relaxation. An evaluation of the stochastic solution, comparing it to the solution of the deterministic model, showed the value of incorporating the uncertainty into the model, resulting in more efficient solutions in terms of demand fulfillment.

We point out, however, that the optimal recourse policy for the 2P-PPLSP results in planned inventory losses to reduce raw material inventory excess, in case demand is lower than expected. In order to address this issue, we propose an alternative recourse policy for the model, in which we limit the production to the demand. This policy was applied in two ways: as an evaluation policy for the model solution, and incorporated into the model itself. Our results show that the former approach manages to eliminate losses while keeping the optimal demand fulfilment, and provide enough information (in terms of costs and inventory levels) to aid the decision maker in deciding which approach to take.

As for future research, we suggest a sensibility analysis, evaluating how the models and the recourse policies behave under different disposal costs (when compared to product inventory and backlogging costs) and shorter/longer shelf-lives. We also suggest research on the proposed formulations in order to verify the impact of shelf-lives in their efficiency, and in which cases the Facility Location reformulation is more efficient than the formulation using Inventory Age variables.

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