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Vanishing of homology groups, Ricci estimate for submanifolds and applications

Antonio Carlos Asperti
and
Ezio de Araújo Costa

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Antonio Carlos Asperti - IME-USP
Ezio de Araújo Costa - IME-USP and IM-UFBa

Abstract

In this paper we obtain an estimate for the Ricci curvature and a criterion for the vanishing of the homology of compact submanifolds of spheres and Euclidean spaces. This criterion depends on the results of Lawson and Simons [LS], Leung [Le2] and Xin [X] on stable currents. As consequences, we obtain an extension of a theorem of Lawson and Simons and a topological version of a theorem of Alencar and do Carmo [A C], and Xu [Xu], on hypersurfaces in spheres with constant mean curvature.

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1 Introduction

In [LS], Lawson and Simons obtained a criterion for the vanishing of the homology groups of compact submanifolds of spheres. Latter on and using similar techniques, Leung [Le2] and Xin [X], where able to extend the results in [LS] to compact submanifolds of Euclidean spaces. Leung also obtained, in [Le1], an estimate for the Ricci curvature of minimal submanifolds of spheres and combined this with the results in [LS] to obtain information on the topology of such submanifolds.

In this paper we follow closely the approach in [Le1]. Firstly we obtain an estimate for the Ricci curvature of submanifolds of a space form which improves Leung estimates in [Le1] and [Le3]. Next we obtain a criterion, based in the results of [LS], [Le2] and [X], for the vanishing of the homology of compact submanifolds of Euclidean spheres or spaces. Then we combine these results to study the geometry and the topology of such submanifolds.

To state our results, let us fix some notation. We will denote by $f : M^n \to Q_c^{n+m}$ an isometric immersion of a connected n-dimensional Riemannian manifold M^n into a complete, simply connected $(n+m)$-dimensional manifold Q_c^{n+m} with constant sectional curvature c, where $n \geq 2$ and $m \geq 1$. Let $\vec{H}, H$ and S denote the mean curvature vector of the immersion, its norm and the square of the lenght of the second fundamental form, respectivelly. As usual we will denote by T_xM the tangent space of M^n and by $N_1(x)$ the first normal space of the immersion at $x \in M^n$. We will make use of the following convention: if $x \in M^n$ is such that $\vec{H}(x) \neq 0$, then $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n$ will denote the eigenvalues of the Weingarten operator A_{ξ_1}, in the direction $\xi_1 = \dfrac{1}{H}\vec{H}(x)$; if $\vec{H}(x) = 0$, just take $\lambda_i = 0,\ 1 \leq i \leq n$, and ξ_1 any unity vector normal to M^n at x. Recall that the immersion is quasi-umbilical at x if there exists an orthonormal frame $\xi_1, ..., \xi_m$ of the normal space $T_xM^\perp$, such that each Weingarten operator A_{ξ_β}, $1 \leq \beta \leq m$, has an eigenvalue of multiplicity at least $n-1$; the immersion is quasi-umbilical if it is quasi-umbilical at every x in M^n.

With this convention, we can state the mentioned estimate for the Ricci curvature of submanifolds and also a sufficient condition for a submanifold M^n of Q_c^{n+m} be a conformally flat submanifold with normal curvature tensor $R^\perp = 0$.

Theorem 1.1 *Let $f : M^n \to Q_c^{n+m}$ be an isometric immersion. For every $x \in M^n$ and $v \in T_xM$, with $||v|| = 1$, we have:*

$$Ric(v) \geq \left(\frac{n-1}{n}\right)(nc - S) + nH^2 + (n-2)H \ < A_{\xi_1}v, v > . \quad (1)$$

(a) *If $n \geq 3$ and (1) is an equality for some unity $v \in T_xM$, then:*
(a1) *f is quasi-umbilical at x, $R^\perp(x) = 0$ and* dim $N_1(x) \leq 2$. *Moreover, if $\vec{H}(x) = 0$ then* dim $N_1(x) \leq 1$;
(a2) *For any unity $\xi \in T_xM^\perp$ we have that $A_\xi v = \lambda v$, where λ has multiplicity 1 or n.*

(b) *Let $n \geq 3$. If for each x in M^n there exits an unity vector v in T_xM such that (1) is an equality, then M^n is conformally flat.*

Simons [S], Lawson [L] and Chern, do Carmo and Kobayashi [C CK], contributed for the classification of the compact minimal submanifolds M^n of the unity sphere S^{n+1}, with $S \leq n$. A still open problem is the complete classification of these submanifolds when the ambient space is the unity sphere S^{n+m}, $m \geq 2$. The following corollary is a partial answer to this question.

Corollary 1.2 *Let M^n, $n \geq 3$, be a compact minimal submanifold of S^{n+m}. If $S \leq n$ on M^n and the fundamental group of M^n is infinite, then M^n is a Clifford torus $S^1_{c_1} \times S^{n-1}_{c_2}$ in a totally geodesic $S^{n+1} \subset S^{n+m}$, where $\frac{1}{c_1} + \frac{1}{c_2} = 1$.*

We state now a criterion for the vanishing of the homology groups of compact submanifolds and also other topological results.

Theorem 1.3 *Let $f : M^n \to Q^{n+m}_c$ be an isometric immersion, where M^n is compact and $c \geq 0$.*

(a) *If for some integer p* such that $2 \leq p \leq n/2$ we have that

$$S < \frac{n^2H^2}{n-p} + \frac{n(n-2p)H\lambda_1}{n-p} + nc \tag{2}$$

on M^n, then the k-homology group $H_k(N, \mathbf{Z}) = 0$, for $p \leq k \leq n-p$.

(b) *If*

$$S < \frac{n^2H^2}{n-1} + \frac{n(n-2)H\lambda_1}{n-1} + nc \tag{3}$$

on M^n, then the fundamental group $\pi_1(M)$ of M^n is finite and the universal covering $\tilde{M}^n$ of M^n is compact. Moreover,
(b1) *If $n = 2$, then M^n is diffeomorphic either to the sphere S^2 or to the real projective space $\mathbb{R}P^2$, according to M^2 is orientable or not;*
(b2) *If $n = 3$ and $\pi_1(M) = \{0\}$, then M^3 is diffeomorphic to the sphere S^3;*

(b3) *If $n \geq 4$ and M^n is orientable when u is even, then M^n is homeomorphic to the sphere S^n.*

When M^n is complete, we have the following version of 1.3(b). We observe that the main theorem in [SX] is a consequence of the next result.

Corollary 1.4 *Let M^n be a complete submanifold of $Q_c^{n+m}, c \geq 0$, such that M^n is orientable when n is even. If $\sup(S-T) < 0$, where T is the right side of* (3), *then M^n is compact and the same conclusions of* 1.3(b) *are valid.*

In [A C] and [Xu] was proved that, if M^n is a compact hypersurface of S^{n+1} with constant mean curvature $H \neq 0$ such that $S \leq C(H)$ on M^n, then M^n is umbilical or isometric to a torus $S^1_{c_1} \times S^{n-1}_{c_2}$, where $C(H)$ is given in (4) below. In our next result, we remove the condition H=constant and obtain a topological-geometrical version of this statement, in any codimension.

Theorem 1.5 *Let $f: M^n \to S^{n+m}$ be an isometric immersion, where M is compact. Let $S \leq C(H)$ on M^n, where*

$$C(H) = n + \frac{n^3H^2}{2(n-1)} - \frac{n(n-2)}{2(n-1)}\sqrt{n^2H^2 + 4(n-1)} \tag{4}$$

Then the Ricci curvature of M^n is nonnegative and we have only two possibilities, (a) *and* (b)*:*

(a) *$\pi_1(M)$ is finite. In this case we have:*
(a1) *If $n = 2$, then M^2 is diffeomorphic to S^2 or to $\mathbb{R}P^2$;*
(a2) *If $n = 3$ and $\pi_1(M) = \{0\}$, then M^3 is diffeomorphic to S^3;*
(a3) *Let $n \geq 4$ and assume that $H \neq 0$ when $S = C(H)$. If M^n is orientable when n is even, then M^n is homeomorphic to S^n.*

(b) *$\pi_1(M)$ is infinite. In this case, $S = C(H)$ on M^n and:*
(b1) *If $n = 2$, then M^2 is flat;*
(b2) *If $n \geq 3$, assume that M^n is orientable. Then the codimension m of f can be reduced to* 1, *the mean curvature H is constant and f has two constant principal curvatures, μ_1 and μ_2, with multiplicities* 1 *and $n-1$, respectivelly, such that $\mu_1.\mu_2 = -1$. Consequentely, M^n is isometric to a torus $S^1_{c_1} \times S^{n-1}_{c_2}$ in S^{n+1}, where $c_i = 1 + \mu_i^2$, $i = 1, 2$.*

Note that the pinching $C(H)$ for S in Theorem 1.5 depends on H (and n) and then depends on a specific immersion. A pinching constant depending only of n was firstly obtained by Lawson and Simons in [LS]. There is proved that if M^n is a compact submanifold of S^{n+m} such that $n \geq 5$ and $S < 2\sqrt{n-1}$ on M^n, then M^n is homeomorphic to S^n. Related to this, recently, Hou [H_0] proved the following rigidity result: let M^n be a compact submanifold of S^{n+m} with non-zero parallel mean curvature vector. If $n \geq 8$ or $m \leq 2$ and $S \leq 2\sqrt{n-1}$ on M^n, then the codimension can be reduced to 1 and M^n is either umbilical or isometric to $S^1_{c_1} \times S^{n-1}_{c_2}$, where $c_1 = 1 + \sqrt{n-1}$ and $c_2 = 1 + 1/\sqrt{n-1}$. The next corollary is an extension of the theorem in [LS] and a topological-geometrical version of the result in [H_0].

Corollary 1.6 *Let M^n be a compact submanifold of S^{n+m} such that $S \leq 2\sqrt{n-1}$ on M^n. Then the Ricci curvature of M^n is nonnegative and we have only two possibilities* (a) *and* (b)*:*

(a) *There exists a point x in M^n such that* $\mathrm{Ric}\,(v) > 0$, *for all unity vector v in T_xM. In this case M^n admits a metric of strictely positive Ricci curvature and*
(a1) *If $n = 2$, then M^n is diffeomorphic to S^2 or to $\mathbb{R}P^2$;*
(a2) *If $n = 3$, then M^3 is orientable with $H_2(M, \mathbf{Z}) = \{0\}$ and, if $\pi(M) = \{0\}$, then M^3 is diffeomorphic to S^3;*
(a3) *If $n \geq 4$ and M^n is orientable when n is even, then M^n is homeomorphic to S^n.*

(b) *For each point x in M^n, there exists an unity vector v in T_xM such that* $\mathrm{Ric}\,(v) = 0$. *In this case $S = 2\sqrt{n-1}$ and $n^2H^2 = n\sqrt{n-1} - 2(n-1)$ on M^n and*
(b1) *If $n = 2$, then M^2 is flat and minimal. In particular, if $m = 1$, M^2 is isometric to a torus $S^1_{\frac{1}{2}} \times S^1_{\frac{1}{2}}$.*
(b2) *If $n \geq 3$, the codimension m can be reduced to 1 and, if M^n is orientable, M^n is isometric to the torus $S^1_{c_1} \times S^{n-1}_{c_2}$, where $c_1 = 1 + \sqrt{n-1}$, $c_2 = 1 + \dfrac{1}{\sqrt{n-1}}$.*

We observe that the result in (a1) was obtained by Wei [W].

2 Notations and preliminary lemmas

Let $M = M^n$, $n \geq 2$, be a connected n-dimensional Riemannian manifold. We denote by $<,>$ the metric and by $|| \ ||$ the respective norm. If R denotes the curvature tensor of M, then the Ricci tensor (at $x \in M$) is defined by

$$Ric(v,w) = \sum_{i=1}^{n} < R(v_i, v)w, v_i >,$$

where v, w are in the tangent space T_xM of M at x, and $\{v_i\}_{i=1}^{n}$ is any orthonormal basis of T_xM. The Ricci curvature $Ric(v)$ in the unity direction $v \in T_xM$ and the scalar curvature τ of M in x are given respectively by

$$Ric(v) =< Qv, v >, \ \tau = trQ, \tag{5}$$

where $Q : T_xM \to T_xM$ is given by

$$< Qv, w >= Ric(v,w). \tag{6}$$

Let $f : M^n \to Q_c^{n+m}$, $m \geq 1$, be an isometric immersion, where Q_c^{n+m} is a complete, simply connected $(n+m)$-dimensional manifold with constant sectional curvature c. For each $x \in M^n, (T_xM)^{\perp}$ will denote the normal space of f at x, and $\alpha : T_xM \times T_xM \to (T_xM)^{\perp}$ will denote the second fundamental form of f at x.

If $\{\xi_\beta\}_{\beta=1}^{m}$ is any orthonormal basis of $(T_xM)^{\perp}$, then the Weingarten operator A_{ξ_β}, in the normal direction ξ_β, is defined by

$$< A_{\xi_\beta} v, w > = < \alpha(v,w), \ \xi_\beta >, \ v, w \in T_xM.$$

The mean curvature vector $\vec{H} = \vec{H}(x)$ at x and its norm are defined by

$$\vec{H} = \frac{1}{n} \sum_{\beta=1}^{m} (trA_{\xi_\beta})\xi_\beta, \ H = ||\vec{H}||. \tag{7}$$

The square of the lenght of the second fundamental form of f at x is defined by

$$S = \sum_{\beta=1}^{m} trA_{\xi_\beta}^2. \tag{8}$$

We then have the following relations:

$$\sum_{\beta=1}^{m} A^2_{\xi_\beta} - \sum_{\beta=1}^{m} (trA_{\xi_\beta})A_{\xi_\beta} = -Q + (n-1)cI, \tag{9}$$

where $I : T_xM \to T_xM$ is the identity map, and

$$S = -\tau + n^2H^2 + n(n-1)c. \tag{10}$$

We now present three lemmas which will be used in the next sections.

Lemma 2.1 *Let V be a real vector space of dimension $n \geq 2$ with an inner product $<,>$ and respective norm $\| \ \|$. Let $A : V \to V$ be a symmetric linear map and let H be such that $trA = nH$. If $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n$ are the eingenvalues of A, then for any unity $v \in V$ we have*
(a) $< A^2v, v > \leq \dfrac{n-1}{n}[trA^2 - nH^2] + 2H < Av, v > -H^2$. *If $n \geq 3$ and the equality occurs in (a), for some unity v, then $Av = \lambda_j v$ where j is such that $|\lambda_j - H| = max\{|\lambda_i - H|, i = 1, ..., n\}$. Also $\lambda_k = \lambda_l$ for all $k, l \neq j$, and $Aw = \lambda_k w$, $k \neq j$, for any w orthogonal to v.*
(b) $< Av, v > \geq \ \lambda_1 \geq H - \sqrt{\frac{n-1}{n}[trA^2 - nH^2]}$. *Moreover, if $\lambda_1 = H - \sqrt{\frac{n-1}{n}[trA^2 - nH^2]}$, then $\lambda_i = \lambda_2$ for $i \geq 2$.*

Proof Let $\{v_i\}_{i=1}^n$ be an orthonormal basis of eigenvectors of A, where $Av_i = \lambda_i v_i$, for all i. Assume first that $trA = nH = 0$ and let λ_j be such that $\lambda_j^2 = max\{\lambda_i^2, i = 1, ..., n\}$. Since $\lambda_j = -\sum_{i\neq j} \lambda_i$, then

$$\lambda_j^2 = (\sum_{i\neq j} \lambda_i)^2 \leq (n-1) \sum_{i\neq j} \lambda_i^2 \tag{11}$$

and

$$\lambda_j^2 \leq \left(\frac{n-1}{n}\right) trA^2.$$

If v is an unity vector, it is clear that

$$< Av, v > \leq \ \lambda_j^2 \leq \left(\frac{n-1}{n}\right) trA^2.$$

and then the first part of (a) follows for $H = 0$. Now assume that $n \geq 3$ and let $v = \sum\limits_{i=1}^{n} a_i v_i$ be an unity vector such that $< A^2v, v > = \left(\frac{n-1}{n}\right) trA^2$. Then $< A^2v, v > = \lambda_j^2 = \left(\frac{n-1}{n}\right) trA^2$ and (11) is an equality. Since

$$(\sum_{i \neq j} \lambda_j)^2 = (n-1) \sum_{i \neq j} \lambda_i^2 - \sum_{\substack{k,l \neq j \\ k<l}} (\lambda_k - \lambda_l)^2$$

we have by (11) that $\lambda_k = \lambda_l$ for all $k, l \neq j$. Also for any $k \neq j$ we have

$$\lambda_j^2 =< A^2v, v > = \sum_{i=1}^{n} a_i^2 \lambda_i^2 = \lambda_k^2 (1 - a_j)^2 + a_j^2 \lambda_j^2,$$

that is, $\lambda_j^2(1 - a_j^2) = \lambda_k^2(1 - a_j^2)$. If $a_j^2 = 1$, then $a_i = 0$ for $i \neq j$ and $v = \pm v_j$. If $a_j^2 \neq 1$, it follows that $\lambda_i^2 = \lambda_j^2$ for all $i = 1, ..., n$ and by the equality in (11), this implies that $\lambda_j^2 = (n-1)\lambda_j^2$. Since $n \geq 3$, $\lambda_i = 0$ for all $i = 1, ..., n$. In any case, we have $Av = \lambda_j v$. Now let w be orthogonal to v. If $v = \pm v_j$, then $w = \sum\limits_{i \neq j} b_i\ v_i$ and so $Aw = \lambda_k w$, $\forall k \neq j$; otherwise every $\lambda_i = 0$ and then $Aw = 0$. For the part (b), note that $< Av, v > \geq \lambda_1$. Since $\lambda_1 \leq H = 0$ and $\lambda_1^2 \leq \left(\frac{n-1}{n}\right) trA^2$, then

$$\lambda_1 \geq -\sqrt{\left(\frac{n-1}{n}\right) trA^2}.$$

which is the desired result (b) for $H = 0$.

Suppose now that $H \neq 0$ and let $B = A - HI$, where $I : V \to V$ is the identity map. Then $trB = 0$, $B^2 = A^2 - 2HA + H^2 I$ and $trB^2 = trA^2 - nH^2$. The result (a) follows immediatelly by applying the case $H = 0$ to B. Since $< Av, v > \geq \lambda_1$, and $H \geq \lambda_1$, choose $v = v_1$ in (a) for $B = A - HI$. Then

$$(\lambda_1 - H)^2 \leq (\frac{n-1}{n})[trA^2 - nH],$$

and the first part of (b) follows for $H \neq 0$. For the proof of the second part of (b), note that $(n-1)\sum(b_i)^2 = (\sum b_i)^2$, where $b_i = \lambda_i - H$ and $i \geq 2$. Then $\lambda_i - H = \lambda_2 - H$ for all $i \geq 2$. This completes the proof.

Lemma 2.2 *Let M^n, $n \geq 4$, be a connected, compact n-dimensional Riemannian manifold such that M^n is orientable if n is even, and let $\tilde{M}^n$ be the universal covering of M^n. If $\pi_1(M)$ is finite and $H_p(M, \mathbf{Z}) \simeq H_p(\tilde{M}, \mathbf{Z}) = \{0\}$ for all $p = 2, ..., n-2$, then M^n is homeomorphic to a sphere S^n.*

Proof. Firstly we observe that M^n is orientable if n is odd. In fact, if not, then $H_n(M, \mathbf{Z}) = \{0\}$, see Corollary 7.12 of [B]. But the Euler characteristic $\chi(M)$ of M^n is zero and also $\chi(M) = b_0 - b_1 + ... + b_{n-1} - b_n$, where $bi =$ rank $H_i(M, \mathbf{Z})$. Since $\pi_i(M)$ is finite, $b_1 = 0$ and then $\chi(M) = 1 + b_{n-1} \geq 1$, a contradiction. Now the torsion part of $H_1(M, \mathbf{Z})$ is $H_1(M, \mathbf{Z})$, because it is finite. But by the universal coefficient theorem, see [B, p.282], the cohomology group $H^i(M, \mathbf{Z})$ is isomorphic to $F_i \oplus T_{i-1}$, where F_1 and T_i are the free and torsion parts of $H_i(M, \mathbf{Z})$. By Poincaré duality, $H_1(M, \mathbf{Z})$ is isomorphic to $H^{n-1}(M, \mathbf{Z})$ and so $H_1(M, \mathbf{Z}) = \{0\}$. Again by the universal coefficient theorem, $H^1(M, \mathbf{Z}) = \{0\}$ and, by Poincaré duality, $H_{n-1}(M, \mathbf{Z}) = \{0\}$. Then M^n is a homology sphere and the same arguments applied to $\tilde{M}^n$ tell us that $\tilde{M}^n$ is a homology sphere. Since $\pi_1(\tilde{M}) = \{0\}$, by standards arguments using the Hurewicz isomorphism theorem and Whitehead theorem, see [Sp, p.398], we conclude that $\tilde{M}^n$ is indeed a homotopy sphere. By the generalized Poincaré conjecture for $n \geq 4$, $\tilde{M}^n$ is homeomorphic to a sphere. Then we have a homology sphere M^n which is covered by a sphere $\tilde{M}^n$ and so, by a theorem of Sjerve [Sj], $\pi_1(M) = \{0\}$ and hence M^n also homeomorphic to a sphere. This concludes the proof.

Lemma 2.3 *Let V be a real vector space of dimension $n \geq 2$ with inner product $<,>$ and let $A : V \to V$ be a symmetric linear map with $trA = nH$. Let p be a positive integer such that $p \leq n/2$ and let $\{v_1, ..., v_p, v_{p+1}, ..., v_n\}$ be an orthonormal basis of V. Denote by*

$$\Theta = \begin{cases} trA^2 - (n+1)H^2 - (n-2)H < Av_1, v_1 >, & \text{if } p = 1, \\ \dfrac{p(n-p)}{n} trA^2 - pnH^2 - (n-2p)H \sum_{i=1}^{p} < Av_1, v_1 >, & \text{if } p > 1 \end{cases} . \quad (12)$$

Then

$$(\sum_i < Av_i, v_i >)^2 - nH \sum_i < Av_i, v_i > + 2\sum_{i,k} < Av_i, v_k >^2 \leq \Theta, \quad (13)$$

where, $i = 1, ..., p$ and $k = p+1, ..., n$.

Proof. Since A is symmetric, clearly

$$trA^2 \geq \sum_i < Av_i, v_i >^2 + \sum_k < Av_k, v_k >^2 + 2\sum_{i,k} < Av_i, v_k >^2 . \quad (14)$$

Assume first that $trA = nH = 0$. If $p = 1$, we have

$$trA^2 \geq < Av_1, v_1 >^2 + 2\sum_k < Av_1, v_k >^2$$

and the lemma follows for $H = 0$ and $p = 1$. If $2 \leq p \leq n/2$, then $p(n-p) \geq n$. On the other hand, we have that

$$trA^2 \geq \frac{1}{p}[\sum_i < Av_i, v_i >]^2 + \frac{1}{n-p}[\sum_k < Av_k, v_k >]^2 + 2\sum_{i,k} < Av_i, v_k >^2 .$$

Since $trA = \sum_i < Av_i, v_i > + \sum_k < Av_k, v_k >= 0$, then from the above inequalities it follows that

$$\frac{p(n-p)}{n} trA^2 \geq [\sum_i < Av_i, v_i >]^2 + 2\sum_{i,k} < Av_i, v_k >^2,$$

which is the desired result (13) for $H = 0$ and $p > 1$. For $H \neq 0$, let $B = A - HI$, where $I : V \to V$ is the identity map. Then $trB = 0$, $B^2 = A^2 - 2HA + A^2 I$ and $trB^2 = trA^2 - nH^2$. The result (13) follows immediately by applying the case $H = 0$ to B.

3 Proofs of Theorem 1.1 and Corollary 1.2

The following theorem, which was proved by Lawson and Simon [LS] in the case $c > 0$ and, independently, by Leung [$Le2$], Wei [W] and Xin [X] in the case $c = 0$, is essential in the proof of Theorem 1.1.

Theorem 3.1 *Let M^n be a compact manifold isometrically immersed in Q_c^{n+m}, $c \geq 0$. Denote by α the second fundamental form of the immersion and let p and q be positive integers such that $p + q = n$. Suppose that at each point x of M^n and for all orthonormal basis $\{v_1, ..., v_p, v_{p+1}, ..., v_n\}$ of T_xM, the following condition is valid.*

$$\Phi := \sum_{i=1}^{p} \sum_{k=p+1}^{n} [2||\alpha(v_i, v_k)||^2 - < \alpha(v_i, v_i), \ \alpha(v_k, v_k) >] \ < \ p.q.c \quad (15)$$

Then $H_p(M,\mathbf{Z}) = H_q(M,\mathbf{Z}) = \{0\}$ *and* $\pi_1(M) = \{0\}$ *if* $p = 1$.

Proof of Theorem 1.1 Let $x \in M^n$. If $\vec{H}(x) = 0$, let $\xi_1, ..., \xi_m$ be an orthonormal basis of $T_xM^\perp$ such that $\xi_1 = \frac{1}{H}\vec{H}(x)$. Since $trA_{\xi_1} = nH$ and $trA_{\xi_\beta} = 0$ for $\beta \geq 2$, it follows from (9) that

$$\sum_{\beta=2}^{m} < A_{\xi_\beta}^2 v, v > + < A_{\xi_1} v, v > -nH < A_{\xi_1} v, v >=$$
$$= -Ric(v) + (n-1)c. \tag{16}$$

If $\vec{H}(x) = 0$, let $\{\xi_\beta\}_{\beta=1}^m$ be any orthonormal basis of $(T_xM)^\perp$ and then (16) also follows from (9). Now applying Lemma 2.1 to each A_{ξ_β} in (16), we get (1).

(a) Suppose that $n \geq 3$ and that (1) is an equality for some unity v in T_xM. By (8) and (16) we have

$$\sum_{\beta=2}^{m}[< A_{\xi_\beta}^2 v, v > -(\frac{n-1}{n})trA_{\xi_\beta}^2] +$$
$$+[< A_{\xi_1}^2 v, v > -(\frac{n-1}{n})trA_{\xi_1}^2 - 2H < A_{\xi_1} v, v > +nH^2] = 0. \tag{17}$$

Also by Lemma 2.1(a), we see that in (17), the expressions between the brackets are null. Therefore, again by 2.1(a), v is an eigenvector for all A_{ξ_β} and, for any w orthogonal to v is T_xM, we have $A_{\xi_\beta} w = \lambda_\beta w$. That is, f, is quasi-umbilical at x and also $R_x^\perp = 0$. Let $v_1 = v, v_2, ..., v_n$ be an orthonormal basis of T_xM such that $A_{\xi_\beta} v_1 = \mu_\beta v_1$ and $A_{\xi_\beta} v_i = \lambda_\beta v_i$ for all $i \geq 2$. The first normal space $N_1(x)$ is spanned by $\alpha(v_i, v_i), i = 1, ..., n$. For $i \geq 2$ we have that $\alpha(v_i, v_i) = \sum_{\beta=1}^{m} \lambda_\beta \xi_\beta = \xi$ and, for $i = 1$, $\alpha(v_1, v_1) = \sum_{\beta=1}^{m} \mu_\beta \xi_\beta = \xi_0$. This shows that the dimension of $N_1(x)$ is at most two. In particular, if $\vec{H}(x) = 0$ then $(n-1)\xi + \xi_0 = 0$ and the dimension of $N_1(x)$ is at most one. This proves (a1). For (a2), let $\xi = \sum_\beta a_\beta \xi_\beta$ be an unity vector in $T_xM^\perp$. Then $A_\xi v = \lambda v$, where $\lambda = \sum_\beta a_\beta \mu_\beta$ and the multiplicity of λ is 1 or n, if the quasi-umbilical x is actually umbilical.

(b) Let $n \geq 3$. If $n \geq 4$, then (b) follows from (a) and from a result of [*CY*] on conformally flat submanifolds. If $n = 3$, by a well known characterization of conformally flat manifolds, see [D, p.108], we have to show that the tensor $\gamma(X) = Q(X) - \frac{\tau X}{4}$ satisfies the Codazzi's equation $(\nabla_X\gamma)(Y) = (\nabla_Y\gamma)(X), X, Y \in TM$. Given x in M^3, since $R^\perp = 0$ there exists a connected open set V around x in M^3, where we can take an normal orthonormal frame field, $\xi_1, ..., \xi_m$ such that $\nabla^\perp\xi_\beta = 0, \beta \geq 1$.

By Theorem 1(a), there exists an orthonormal basis $\{X_1, X_2, X_3\}$ of T_xM such that $A_{\xi_\beta}X_1 = \mu_\beta X_1$ and $A_{\xi_\beta}X_i = \lambda_\beta X_i$, $\beta \geq 1$, $i = 2, 3$. Observe that $trA_{\xi_\beta} = \mu_\beta + 2\lambda_\beta$ and $\tau = Ric(X_1) + 2Ric(X_i), i = 2, 3$. Then by (9) we obtain that $Ric(X_1) = 2c + 2\sum_\beta \lambda_\beta\mu_\beta$ and

$$Ric(X_i) = 2c + \sum_\beta(\lambda_\beta\mu_\beta + \lambda_\beta^2),\ i = 2, 3, \beta \geq 1.$$

Taking these to the definition of γ, gives

$$\gamma(X_1) = (\frac{c}{2} + \sum_\beta \lambda_\beta(\mu_\beta - \frac{\lambda_\beta}{2})X_1,\ \gamma(x_i) = (\frac{c}{2} + \frac{1}{2}\sum_\lambda x_\beta^2)X_i,\ i = 2, 3.$$

Therefore $\gamma = \frac{c}{2}I + \sum_\beta \lambda_\beta(A_{\xi_\beta} - \frac{\lambda_\beta}{2}I)$.

Assume now $m = 1$ (the case $m \geq 2$ is similar) and consider $A = \{y \in V : \lambda_1 = \mu_1\}$ and $B = \{y \in V : \lambda_1 \neq \mu_1\}$ (for the general m, we have to consider 2^m of such subsets.) Clearly $int(A) \cup B$ is open and dense in V and, in particular, λ_1 and μ_1 are smooth in this set. Now using the Codazzi's equations for each H_{ξ_β} in $int(A) \cup B$, it follows that γ satisfies the Codazzi's equations in $in(A) \cup B$ and therefore in V. This proves that M^3 is conformally flat.

Proof of Corollary 1.2 Let M^n be a compact and minimal submanifold of S^{n+m} such that $S \leq n$ on M^n and with infinite $\pi_1(M)$. By (1) we have that M^n has everywhere nonnegative Ricci curvature and we claim that for all x in M^n, there exists an unity v in T_xM such that $\mathrm{Ric}(v) = 0$. In fact, if there exists x in M^n such that $\mathrm{Ric}(v) > 0$ for all unity v in T_xM, then by Aubin's theorem [A, p.397], M^n has a metric with strictely positive Ricci curvature and by the Bonnet-Myers' theorem, $\pi_1(M)$ is finite, a contradiction which proves our claim. Then by (1) and 1.1(a1), $S = n$, $R^\perp = 0$ and the dimension of N_1 is 1 (because $S \neq 0$) on M^n. Since M^n is minimal in S^{n+m} with $R^\perp = 0$ and $\dim N_1 \equiv 0$, by a result of Dajczer [D, Theorem 4.4], N_1 is a 1-dimensional parallel subbundle of the normal bundle and, by a well known result of Erbacher, the codimension m can be reduced to 1. We are then left with a compact minimal submanifold M^n of S^{n+1} with $S = n$. The corollary now follows from the results of [C CK] or [L].

4 Proofs of Theorem 1.3 and Corollary 1.4

Proof of Theorem 1.3 (a) Let $2 \leq p \leq n/2$, $x \in M^n$ and let $\{v_1, ..., v_p, v_{p+1}, ..., v_n\}$ be an orthonormal basis of T_xM. Let $\{\xi_\beta\}_{\beta=1}^m$ be an orthonormal basis of $T_xM^\perp$ such that $\xi_1 = \frac{1}{H}\vec{H}(x)$ if $\vec{H}(x) \neq 0$. Since $trA_{\xi_1} = nH$ and $trA_{\xi_\beta} = 0 \quad \beta \geq 2$, we have

$$\begin{aligned}
\Phi = {} & \sum_{i,k}[2||\alpha(v_i, v_k)||^2 - < \alpha(v_i, v_i),\ \alpha(v_k, v_k) >] = \\
& \sum_{i,k}\sum_{\beta\geq 1}[2 < A_{\xi_\beta}v_i, v_k >^2 - < A_{\xi_\beta}v_i, v_i >< A_{\xi_\beta}v_k, v_k >] = \\
& 2\sum_{i,k}\sum_{\beta>1} < A_{\xi_\beta}v_i, v_k >^2 + \sum_{\beta>1}[\sum_i < A_{\xi_\beta}v_i, v_i >]^2 + \\
& 2\sum_{i,k} < A_{\xi_1}v_i, v_k >^2 + [\sum_i < A_{\xi_1}v_i, v_i >]^2 - nH\sum_i < A_{\xi_1}v_i, v_i >, \qquad (18)
\end{aligned}$$

for $i = 1, ..., p$ and $k = p+1, ..., n$.. Applying Lemma 2.3 to each A_{ξ_β}, we get

$$\Phi \leq \frac{p(n-p)}{n} S - pnH^2 - (n-2p)H\sum_i < A_{\xi_1}v_i, v_i > . \qquad (19)$$

Applying Lemma 2.1(b) to (19), we get

$$\Phi \leq \frac{p(n-p)}{n} S - pnH^2 - p(n-2p)H\lambda_1,$$

and then the condition (15) of Theorem 3.1 is valid if

$$S < \frac{n^2H^2}{n-p} + \frac{n(n-2p)}{n-p}H\lambda_1 + nc$$

on M^n. On the other hand, for $p \leq k \leq n-p$, we have that

$$\frac{n^2H^2}{n-p}+\frac{n(n-2p)}{n-p}H\lambda_1+nc\leq\frac{n^2H^2}{n-k}+\frac{n(n-2k)}{n-k}H\lambda_1+nc \qquad (20)$$

and it follows from Theorema 3.1 and (20) that $H_k(M,\mathbf{Z})=\{0\}$, which proves 1.3(a).

(b) Let $x\in M^n$ and v be an unity vector in T_xM. Since $<A_{\xi_1}v,v>\geq\lambda_1$, when $H\neq 0$, by (1) we have that

$$\mathrm{Ric}(v)\geq(\frac{n-1}{n})(nc-S)+(n-2)H\lambda_1+nH^2.$$

If the condition in 1.3(b) holds on M^n, then the Ricci curvature of M^n is strictely positive and by Bonnet-Myers' theorem, $\pi_1(M)$ is finite. If $n=2$, we obtain 1.2(b1) by the Gauss-Bonnet's formula. If $n=3$, since $\pi_1(M)=\{0\}$, we have (b2) by Hamilton's theorem [H]. Now let $n\geq 4$ and M^n orientable when n is even. Since

$$\frac{n^2H^2}{n-1}+\frac{n(n-2)}{n-1}H\lambda_1+nc\leq\frac{n^2H^2}{n-2}+\frac{n(n-4)}{n-2}H\lambda_1+nc,$$

it follows from part (a) that $H_k(M,\mathbf{Z})=\{0\}$ for $2\leq k\leq n-2$. The above arguments applyed to the immersion $f\circ\pi:\tilde{M}^n\rightarrow Q_c^{n+m}$, where $\pi:\tilde{M}^n\rightarrow M^n$ is te covering map, tell us that $H_k(\tilde{M},\mathbf{Z})=\{0\}$ for $2\leq k\leq n-2$. Then (b2) is a consequence of Lemma 2.2.

Proof of Corollary 1.4 Under the hypothesis of 1.4, let $x\in M^n$ and let v be unity in T_xM. By (1) we have that

$$\begin{aligned}&\mathrm{Ric}(v)\geq\frac{n-1}{n}(nc-S)+nH^2+(n-2)H\lambda_1=\\&\frac{n-1}{n}[-S+(nc+\frac{(n^2H^2}{n-1}+\frac{(n(n-2)}{n-1}H\lambda_1)]\geq\\&\frac{n-1}{n}[-sup(S)+inf(T)]=-(\frac{n-1}{n})sup(S-T)=\delta>0.\end{aligned}$$

Then M^n is compact and the result follows from 1.3(b).

5 Proofs of Theorem 1.5 and Corollary 1.6

Proof of Theorem 1.5 Let $x\in M^n$ and let v be an unity vector in T_xM. By using (1) and Lemma 2.1(b), it is easy to see that

$$\frac{n}{n-1}\mathrm{Ric}(v) \geq A \geq B \geq C, \tag{21}$$

where

$$A = \frac{n^2H^2}{n-1} + \frac{n(n-2)}{n-1}H\lambda_1 + n - S, \tag{22}$$

$$B = n - S + 2nH^2 - \frac{n(n-2)H}{n-1}\sqrt{\frac{n-1}{n}(S_H - nH^2)}, \tag{23}$$

$$C = n - S + 2nH^2 - \frac{n(n-2)H}{n-1}\sqrt{\frac{n-1}{n}(S - nH^2)}, \tag{24}$$

where $S_H = trA_{\xi_1}^2$. We claim that $S \leq C(H)$ is equivalent to $C \geq 0$ and also $S = C(H)$ if and only if $C = 0$. In fact, writing $S_1 = S - nH^2$, we have

$$C = n + nH^2 - [\sqrt{S_1} + \frac{n(n-2)}{2(n-1)}\sqrt{\frac{n-1}{n}}]^2 + \frac{n(n-2)^2H^2}{4(n-1)} =$$

$$= (K-L)(K+L),$$

where $K = \sqrt{n + \frac{n^3H^2}{4(n-1)}}$ and $L = \sqrt{S_1} + \frac{(n-2)H}{2}\sqrt{\frac{n}{n-1}}$. Then $C \geq 0$ if and only if $K \geq L$ or

$$n + \frac{n^3H^2}{4(n-1)} \geq S_1 + \frac{n(n-2)^2H^2}{4(n-1)} + \frac{(n-2)H}{2}\sqrt{\frac{nS_1}{n-1}},$$

from which our claim follows. By (21) we then have that the Ricci cuvature of M^n is nonnegative.

(a) Let $\pi_1(M)$ be finite. Observe that $\tilde{M}^n$ is compact, in this case.
(a1) If $n = 2$, it follows from the Gauss-Binnet's formula that M^2 is diffeomorphic to S^2 or $\mathbb{R}P^2$.
(a2) If $n = 3$, assume firstly that there exists a point x in M^3 such that $Ric(v) > 0$ for all unity v in T_xM. It follows then by Aubin's theorem [A, p. 397] that M^n has a metric of strictely positive Ricci curvature and, since $\pi_1(M) = \{0\}$, M^3 is diffeomorphic to S^3 by Hamilton's theorem [H].

Suppose now that for each x in M^n, there exists an unity $v \in T_xM$ such that $Ric(v) = 0$. Since $C \geq 0$, we have by (21) that $A = B = C = 0$ and also, as is easy to see, an equality occurs in (1) of Theorem 1.1 for each $x \in M$ and some unity $v \in T_xM$. Then M^3 is conformally flat and, since $\pi_1(M) = \{0\}$, we have by a theorem of Kuiper, see [D, p.116], that M^3 is (conformally) diffeomorphic to S^3.

(a3) Let $n \geq 4$ and assume that $H \neq 0$ when $S = C(H)$. Since $A \geq B \geq C \geq 0$, see (21), we have that

$$S \leq \frac{n^2H^2}{n-1} + \frac{n(n-2)}{n-1}H\lambda + n \leq \frac{n^2H^2}{n-2} + \frac{n(n-4)}{n-2}H\lambda_1 + n \tag{25}$$

on M^n. We claim that

$$S < \frac{n^2H^2}{n-2} + \frac{n(n-4)}{n-2}H\lambda_1 + n \tag{26}$$

on M^n. In fact, if there exists a point x in M^n where (26) is an equality, then if follows from (25) that $H = 0$ or $\lambda_1 = H$ at x. It also follows from the equality in (26) that $A = B = B = C = 0$ and so $S = C(H)$, that is, $H \neq 0$. Therefore $\lambda_1 = H$ and by $A = 0$, we obtain that $S = n + 2nH^2$ at x. On the other hand, by $S = n + 2nH^2$ and $C = 0$, we obtain that $S = nH^2$ at x, a contradiction. This proves (26) and then, by Theorem 1.3(a) we have that $H_i(M, \mathbf{Z}) = \{0\}$ for $i = 2, \ldots, n-2$ and the same holds for $\tilde{M}^n$. Then (a3) follows from Lemma 2.2.

(b) If $\pi_1(M)$ is infinite then, for each x in M^n there exists an unity $v \in T_xM$ such that $Ric(v) = 0$. Otherwise by Aubin's theorem and Bonnet-Myers' theorem, we would have that $\pi_1(M)$ is finite. By (21) and $C > 0$ we have that

$$S = C(H),\ A = B = C = 0 \tag{27}$$

on M^n. Moreover, if $n \geq 3$ and $H \neq 0$, we have on M^n that

$$S = S_H,\ \lambda_1 = H - \sqrt{\frac{n-1}{n}(S_H - nH^2)}. \tag{28}$$

(b1) If $n = 2$, it is clear that M^2 is flat.
(b2) If $n \geq 3$, assume firstly that $H \neq 0$ on M^n. By (28) and Lemma 2.1(b), we can see that A_{ξ_1} has two eigenvalues λ_1, where λ_1 has multiplicity at least $n-1$. Also $N_1 = [\vec{H}]$ because $S = S_H$. Now we want to show that $\lambda_1\lambda_2 = -1$. For this, observe that if v is an unity vector in T_xM and $Ric(v) = 0$, then the equality occurs in (1) for x and v, and $A_{\xi_1}v = \lambda_1 v$. Let $v_1 = v, v_2, ..., v_n$ be an orthonormal basis of T_xM such that $A_{\xi_1}v = \lambda_1 v$ and $A_{\xi_1}v_1 = \lambda_2 v_i$ for $i \geq 2$. By the Gauss' equation we have

$$0 = Ric(v) = \sum_{i=2}^{n}(1+ < \alpha(v_1, v_1),\ \alpha(v_i, v_i) >)\ -\ ||\alpha(v_1, v_i)||^2$$

$$= (n-1)(\lambda_1\lambda_2 + 1).$$

that is, $\lambda_1\lambda_2 \equiv -1$. We claim that N_1 is parallel in the normal connection. Let $\xi_1, ..., \xi_m$ and $x_1, ..., x_n$ be local orthonormal frame fields, normal and tangent to M^n respectively, choosen in a way that $\xi_1 = \vec{H}/H$ and $A_{\xi_1}X_1 = \lambda_1 X_1,\ A_{\xi_1}X_i = \lambda_2 X_i,\ i \geq 2$. Since $\lambda_1 \neq \lambda_2$, they are smooth functions on M^n. By the Codazzi equations

$$(\nabla^{\perp}_{X_1}\alpha)(X_i, X_1) = (\nabla^{\perp}_{X_1}\alpha)(X_1, X_1),\ (\nabla^{\perp}_{X_i}\alpha)(X_i, X_i) = (\nabla^{\perp}_{X_i}\alpha)(X_1, X_i),$$

and by the fact that N_1 is spanned by $\vec{H}$, we obtain that

$$\begin{aligned} \lambda_1\nabla^{\perp}_{X_i}\xi_1 &= [(\lambda_1 - \lambda_2) < \nabla_{X_1}X_1, X_i > -X_i(\lambda_1)]\xi_1, \\ \lambda_2\nabla^{\perp}_{X_1}\xi_1 &= [(\lambda_2 - \lambda_1) < \nabla_{X_1}X_i, X_1 > -X_1(\lambda_2)]\xi_1. \end{aligned} \tag{29}$$

But the left sides of (29) cannot be parallel to ξ_1 unless $\nabla^{\perp}_{X_i}\xi_1 = 0$, for $i = 1, 2, ..., n$. This proves our claim. By a well known result of Erbacher, the codimension m of f can be reduced to 1 and f can be seen as an immersion of M^n into S^{n+1} where A_{ξ_1} has two eigenvalues $\lambda_1 \neq \lambda_2$. Now by a result of Ryan [R, p.372], the distribution $T_{\lambda_2} := \{X : A_{\xi_1}X = \lambda_2 X\}$ is differentiable and involutive, and $X_i(\lambda_2) = 0$ for $i \geq 2$. Since $\lambda_1\lambda_2 = -1$, we also have that $X_i(\lambda_1) = 0$ for $i \geq 2$. Taking this to (29), we see that $< \nabla^{X_1}_{X_1}, X_i >= 0$ for $i \geq 2$ and, since $< \nabla^{X_1}_{X_1}, X_i >= 0$, this shows that the orthogonal distribution $T^{\perp}_{\lambda_2} = \{\xi_1 : A_{\xi_1}X = \lambda_1 X\}$ is totally geodesic. We then have a compact manifold M^n with nonnegative Ricci curvature and with a codimension one foliation, defined by T_{λ_2}, whose orthogonal distribution $T_{\lambda_2}^{\perp}$ is totally geodesic. It follows immediately from Corollary 2 of [BW] that T_{λ_2} is also totally geodesic and then $< \nabla_{X_i} X_i, X_1 >= 0$ for $i \geq 2$. Again by (29) we conclude that $X_1(\lambda_2) = 0$. This proves that λ_1 and λ_2 are constant on M^n. Clearly f has constant mean curvature $H \neq 0$ and $S = C(H)$. Now (b2) follows, in this case, from the theorem in [A C] or [Xu] quoted in the introduction.

Suppose now that there exists a point x_0 in M^n such that $H(x_0) = 0$. Since $A = B = C = 0$ on M^n, (1) is an equality everywhere. By Theorem 1.1, $R^{\perp} = 0$ and M^n is conformally flat. On the other hand, $S(x_0) = n$ because $H(x_0) = 0$ and then $\tau(x_0) = n(n-2)$ by (10), that is, M^n cannot be flat. In this case, since $\pi_1(M)$ is infinite, we can use the same arguments of the proof of Theorem 1 of [N, p. 259], to conclude that $\tilde{M}^n = \mathbb{R}^n \times S_{c_2}^{n-1}$, for some $c_2 > 0$. This shows that $\tilde{M}^n$ has constant scalar curvature and so M^n also, that is, $\tau \equiv n(n-2)$. Again by (10), $S = n^2H^2 + n$ on M^n and combining this with $S = C(H)$, we obtain that $H \equiv 0$, $S \equiv n$. The result (b2) now follows from Corollary 1.2, in this case.

Proof of Corollary 1.6 Let $S \leq 2\sqrt{(n-1)}$ on M^n. By (24), we have that

$$C = D \geq E \geq 0 \tag{30}$$

on M^n, where

$$\begin{aligned} D &= n - \frac{nS}{2\sqrt{n-1}} + \frac{a^2}{2\sqrt{n-1}}, \\ a &= (\sqrt{n-1}+1)\sqrt{n}H - (\sqrt{n-1}-1)\sqrt{S-nH^2}, \end{aligned} \tag{31}$$

and

$$E = n\Big(1 - \frac{S}{2\sqrt{n-1}}\Big) \tag{32}$$

Since $C \geq 0$, we have that M^n has nonnegative Ricci curvature and that $S \leq C(H)$ on M^n. We have two possibilities, (a) and (b):

(a) There exists a point x in M^n such that $Ric(v) > 0$ for all unity v in T_xM. In this case, it follows from Aubin's theorem and Bonnet-Myers' theorem that M^n has a metric os strictely positive curvature, M^n is compact and $\pi_1(M)$ is finite. If $n = 2$ or $n = 3$, then (a1) and (a2) follows from the same arguments as in 1.5(a1)(a2). For (a3), let $n \geq 4$ and observe that $H \neq 0$ when $S = C(H)$. In fact, if there exists x_0 in M^n with $S = C(H)$ and $H = 0$, then $C(x_0) = 0$ and also $D(x_0) = E(x_0) = 0$, by (30). This shows that $0 = S = 2\sqrt{n-1}$ at x_0, which is a contradiction. Then (a3) follows from 1.5(a3).

(b) For the case (b), using (30) we obtain that $A = B = C = D = E = 0$ on M^n. Then $S \equiv 2\sqrt{n-1}$ and $n^2H^2 \equiv n\sqrt{n-1} - 2(n-1)$. (b1) If $n = 2$, then M^2 is flat and $H = 0$. In particular, if $m = 1$, it follows from $[CCK]$ or $[L]$ that M^2 is isometric to $S^1_{\frac{1}{2}} \times S^1_{\frac{1}{2}}$. (b2) Let $n \geq 3$. In this case, it follows from $B = 0$ that $S = S_H$ on M^n and therefore N_1 is spanned by $\vec{H}$. We now imitate the proof of 1.5(b) to show that f can be seen as an isometric immersion of M^n into S^{n+1} with constant $H \neq 0$ and $S = 2\sqrt{n-1}$ on M^n; (b2) is now a direct consequence of the result in $[H_0]$ mentioned in the Introduction.

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Antonio Carlos Asperti
IME-USP
Rua do Matão 1010, Cidade Universitária
05508-900 - São Paulo - SP - Brazil
e-mail: asperti@ime.usp.br

Ezio Araujo Costa
IME-USP - e-mail: ezio@ime.usp.br
and
Instituto de Matemática - UFBa
Rua Ademar de Barros s/n - Campus de Ondina
40170-110 - Salvador - Ba - Brazil
e-mail: ezio@ufba.br

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Rua do Matão, 1010 - Cidade Universitária
Caixa Postal 66281 - CEP 05315-970