Revisiting the triangulation method for pointing to supernova and failed supernova with neutrinos

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In view of the advent of large-scale neutrino detectors such as IceCube, the future Hyper-Kamiokande and the ones proposed for the Laguna project in Europe, we reexamine the determination of the directional position of a Galactic supernova by means of its neutrinos using the triangulation method. We study the dependence of the pointing accuracy on the arrival-time resolution of supernova neutrinos at different detector locations. For a failed supernova, we expect better results due to the abrupt termination of the neutrino emission which allows one to measure the arrival time with higher precision. We found that for the time resolution of ± 2 (4) ms, the supernova can be located with a precision of $\sim 5°$ (10°) on the declination and of $\sim 8°$ (15°) on the right ascension angle if we combine the observations from detectors at four different sites.

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I. INTRODUCTION

The observation of neutrinos coming from the next Galactic supernova (SN) driven by gravitational core collapse (hereafter, SN implies the one caused by the gravitational collapse) is expected to provide very interesting information on the dynamics of the process, namely, how these stars explode and form black holes (BH); see, for instance, Refs. [1,2]. Moreover, it may also shed light on some unknown neutrino properties such as the neutrino mass ordering; see, e.g., [3,4].

Since neutrinos can break free from the dense region of the star from which photons cannot escape, they will be the first messengers from the sky to inform us of the occurrence of the gravitational collapse. Indeed, it might be possible that the next Galactic SN cannot be located by optical observations due to obscuration. If so, observing neutrinos may be the only way to access its direction in the sky, apart from the possible simultaneous detection of gravitational waves [5].

The possibility of determining the direction of a Galactic SN by merely using its neutrinos has been investigated in the past [6–12]. Most of the authors considered neutrino electron elastic scattering events in a water Cherenkov detector in order to determine the SN direction [6,8,10,11]. According to Ref. [11], for a SN at 10 kpc, the pointing accuracy is $\sim 8°$ at 95% C.L. if the Super-Kamiokande detector is considered. This can be further improved to $\sim 3°$ if gadolinium is added to water [13], allowing us to tag neutrinos from the inverse beta decay background. A megaton size water Cherenkov detector using this technique may be able to increase the pointing precision to $\sim 1°$ [11].

On the other hand, the method of the arrival-time triangulation, previously discussed in Refs. [6–8], was readily dismissed due to the low precision on the arrival time of SN neutrinos expected mainly because the available detectors at that time were too small to register enough statistics for such a purpose.

We are now, however, entering a new era of large-scale detectors with IceCube currently working in the South Pole [14], the proposals of Hyper-Kamiokande in Japan [15], and of the European detectors which will be built in the Pyhäsalmi mine in Finland [16]. In view of this new trend, it is timely to revisit the usefulness of neutrino triangulation using big detectors in different continents, as suggested in Ref. [17].

According to [17], the IceCube detector can determine the arrival time of SN neutrinos with an uncertainty of ± 3.5 ms at 95% C.L. In the case of a so-called failed SN, where a black hole is formed while the neutrino flux is still measurably high [18], one expects the neutrino signal to terminate abruptly. As this sharp transition is expected to take place in $\lesssim 0.5$ ms [19] the end point of the neutrino spectrum can also be used for triangulation.

Since the observation of the arrival of SN neutrinos by various detectors will be a valuable tool to alert astronomers about the occurrence of the star collapse [20] allowing them to observe the light curve as early as possible or the formation of a black hole (in the case of a failed SN), it is important to explore different approaches to reconstruct the location of the SN as well as the failed SN in the sky.

II. TRIANGULATION METHOD

The distribution of SN in the Milky Way is expected to be concentrated in the Galactic disc. For the sake of

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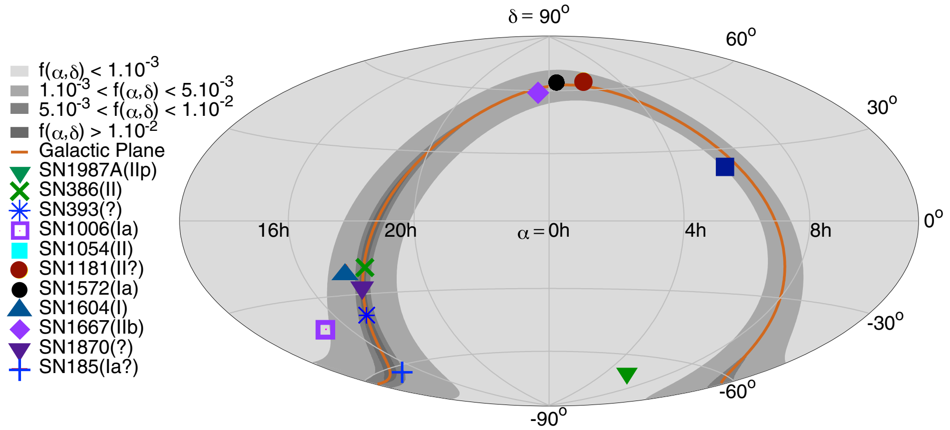

FIG. 1 (color online). Expected SN probability distribution $f(\alpha, \delta)$ based on the model considered in Ref. [21], shown in the plane of the equatorial coordinates α-δ using the Hammer projection. Different contrast of the colors reflects the difference in probabilities as indicated in the legend. The position of the Galactic plane in the sky is indicated by the red curve. The location of the historical Galactic SN explosions are also shown with their type, when known, in parentheses.

discussion, let us consider the same SN distribution considered in Refs. [21,22]. In Fig. 1 we show the expected SN distribution $f(\alpha, \delta)$, in the plane of equatorial coordinates α-δ where α and δ are, respectively, right ascension and declination, and $f(\alpha, \delta) d\alpha \cos\delta d\delta$ corresponds to the probability to find a SN in the sky in the interval between $(\alpha, \alpha + d\alpha)$ and $(\delta, \delta + d\delta)$. The distribution function $f(\alpha, \delta)$ is normalized, as in [21], such that $\int d\alpha \int \cos\delta d\delta f(\alpha, \delta) = 1$ with α and δ given in radian. In this figure, we also show the location of the historical Galactic SN and SN1987A explosions.

Let us consider two arbitrary detector sites $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ on the Earth and define the displacement vector as $\boldsymbol{d}_{ij} \equiv \boldsymbol{x}_i - \boldsymbol{x}_j$, and denote the SN direction in the sky by the unit vector $\boldsymbol{n}$. Then the difference of the arrival time of SN neutrino signals between two detectors, $\Delta t_{ij} \equiv t_i - t_j$, is given by

$$\Delta t_{ij} = \boldsymbol{d}_{ij} \cdot \boldsymbol{n}/c, \tag{1}$$

where c is the speed of light in vacuum. Here we will ignore the possible time delay due to the neutrino mass which can be estimated as

$$\Delta t_{\rm mass} \simeq 0.6 \left[\frac{D}{10\ \rm kpc}\right] \left[\frac{m_\nu}{0.1\ \rm eV} \frac{10\ \rm MeV}{E}\right]^2 \rm ms, \tag{2}$$

where D is the distance to the SN and m_ν is the neutrino mass.

In this work, for the purpose of illustration of this method, we consider up to four different detector sites on Earth, namely, Kamioka, the South Pole, Pyhäsalmi, and the Agua Negra Deep Experiment Site (ANDES) [23,24] (see also [25]). This is because four is the minimum number of detector positions needed to uniquely determine the SN location, as we will see below. If we add more detector sites such as Gran Sasso and Sudbury, the results would be improved. We note that there is no strong dependence of the results as long as we select four detector locations which are well separated from each other.

We note that ANDES is the first deep underground laboratory in the Southern Hemisphere, which could be constructed in the Agua Negra tunnels that will link Argentina and Chile under the Andes, the world's longest mountain range. The potential of a neutrino detector at the ANDES location for the observation of SN neutrinos as well as of geoneutrinos is discussed in Ref. [22].

In Fig. 2 we show the solution of Eq. (1) for the case where the SN occurs in the Galactic center, given by $\alpha = 17^{\rm h}42^{\rm m}27^{\rm s}$ and $\delta = -28°55'$, for various different combinations of the four detector sites mentioned above. For definiteness, it was assumed that the SN neutrinos arrived at the Earth on March 20th of 2000 at 12:00 UTC (coordinated universal time) but it is straightforward to change this condition.

From this plot, we can see that for a given combination of two detector sites, the SN location can be constrained, as expected, to a closed curve in the sky. It is also possible to see that if we have three different detector sites, we can restrict the possible SN positions to only two locations in the sky. For example, the curves for the Kamioka-South Pole and Pyhäsalmi-South Pole combinations intersect in two locations, the true location of the SN as well as a *fake* solution. If we have detectors at four different sites, it is possible to eliminate the fake solutions and single out the true location in the sky, as shown in [7].

In practice, however, due to the finite resolution of the SN neutrino arrival-time measurement, we can only establish the SN direction with limited precision. The accuracy of the determination of θ, the angle between the SN direction and the axis connecting two given detectors, can be roughly estimated as

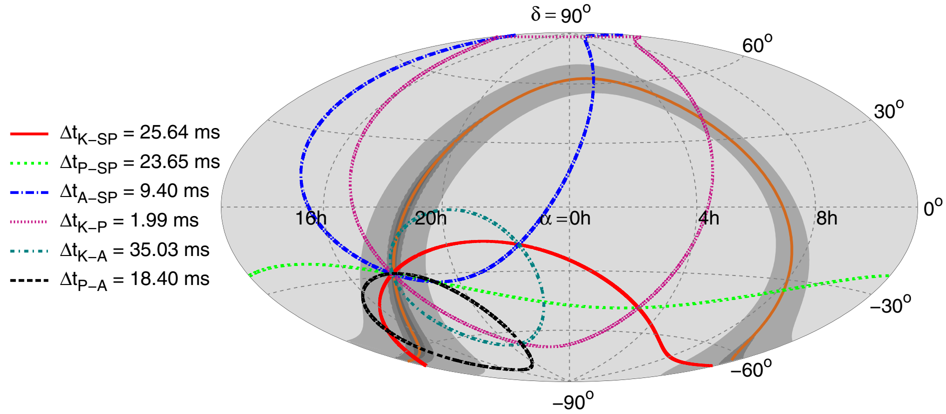

FIG. 2 (color online). Possible solutions for the SN direction (α, δ) consistent with a certain difference of the arrival time determined by the combinations of detectors located at two different sites. Here the true (input) position of the SN is assumed to be the Galactic center, with $\alpha = 17^{\rm h}42^{\rm m}27^{\rm s}$ and $\delta = -28°55'$. It is assumed that SN neutrinos are detected at the Earth on the vernal point on March 20th, 2000 at 12:00 UTC. We consider four sites: Kamioka, Pyhäsalmi, ANDES, and the South Pole, indicated by the labels K, P, A and SP, respectively.

$$\delta(\cos\theta) \sim \frac{c\,\delta(\Delta t_{ij})}{d_{ij}}. \tag{3}$$

Let us try to estimate the precision of the arrival time of the SN neutrino signal following the discussion given in Ref. [8]. Let us consider the case where the neutrino event rate $N(t)$ at a given detector, which is proportional to the SN neutrino flux, increases (decreases) before (after) $t = t_0$ exponentially as follows:

$$N(t) = \begin{cases} \propto \exp\left[+\frac{(t-t_0)}{\tau_1}\right] & (t < t_0) \\ \propto \exp\left[-\frac{(t-t_0)}{\tau_2}\right] & (t > t_0), \end{cases} \tag{4}$$

where we set $\tau_1 = 30$ ms, $\tau_2 = 3$ s following Ref. [8], and t_0 corresponds to the peak of the event rate. Note that τ_1 (τ_2) characterizes the time scale of the rising (decaying) part of the time profile of the SN neutrino flux or the event rate. The behavior of the event rate as a function of time is shown schematically in Fig. 2 of Ref. [8].

Under this assumption, very roughly speaking, the accuracy of the determination of the arrival time of the SN neutrino signal at a given detector, $\delta t_{\rm arrival}$, can be estimated as [8]

$$\delta t_{\rm arrival} \sim \frac{\tau_1\tau_2}{\sqrt{N}} \sim \frac{\tau_1}{\sqrt{N_1}}, \tag{5}$$

where N_1 is the number of events in the rising part of the SN neutrino pulse, given as $N_1 \sim N(\tau_1/\tau_2)$, and N is the total number of events. We note that when the event rate is characterized by Eq. (4) with $\tau_1 \ll \tau_2$, typically the fraction of events relevant for the determination of $\delta t_{\rm arrival}$ is only $\sim$a few%.

As our reference SN model, we consider the same one considered in Ref. [22]. We assume that the total energy released by neutrinos is 3×10^{53} erg, equally divided by six species of neutrinos and antineutrinos. We further assume that the SN neutrino spectra are given by the parametrization obtained by the Garching group [26–28],

$$F^0_{\nu_\alpha}(E) = \frac{1}{4\pi D^2}\frac{\Phi_{\nu_\alpha}}{\langle E_{\nu_\alpha}\rangle}\frac{\beta_\alpha^{\beta_\alpha}}{\Gamma(\beta_\alpha)}\left[\frac{E}{\langle E_{\nu_\alpha}\rangle}\right]^{\beta_\alpha-1} \times \exp\left[-\beta_\alpha\frac{E}{\langle E_{\nu_\alpha}\rangle}\right], \tag{6}$$

where D is the distance to the SN, Φ_{ν_α} is the total number of ν_α emitted, $\langle E_{\nu_\alpha}\rangle$ is the average energy of ν_α, and β_α is a parameter which describes the deviation from a thermal spectrum (pinching effect) that can be taken to be $\sim$2–4; $\Gamma(\beta_\alpha)$ is the gamma function. As in Ref. [22], we set $\beta_\alpha = 4$ for all flavors, $\langle E_{\nu_e}\rangle = 12$ MeV, $\langle E_{\bar\nu_e}\rangle = 15$ MeV, and $\langle E_{\nu_x}\rangle = 18$ MeV. Here ν_x implies any non-electron neutrino.

Because of oscillations, the $\bar\nu_e$ SN neutrino spectrum, for example, to be observed at the Earth gets modified as [3]

$$F^{\rm obs}_{\bar\nu_e}(E) = \bar p F^0_{\bar\nu_e}(E) + (1-\bar p)F^0_{\nu_x}(E), \tag{7}$$

where $\bar p$ is the survival probability of $\bar\nu_e$. For definiteness and simplicity, we consider the neutrino mass hierarchy to be normal and ignore any possible effects which could come from shock waves (see, e.g., [29]) and/or nonlinear collective effects (see, e.g., [30]). In this case, to a good approximation [3], we can set $\bar p = \cos^2\theta_{21} = 0.69$ as in [22].

We then compute the number of events, N, N_1, and estimate the expected uncertainty on the arrival time $\delta t_{\rm arrival}$. In this work we consider six detectors, two existing ones, Super-Kamiokande and IceCube, one in construction, SNO+, and three others which have been proposed, Hyper-Kamiokande [15], LENA [31], and ANDES [22]. For the Hyper-Kamiokande detector, for

TABLE I. Estimated number of events for a SN at 10 kpc from the Earth, as well as the expected precision on the arrival time of the SN signal for the existing detector, IceCube, Super-Kamiokande (denoted as Super-K), as well as the proposed neutrino detectors, SNO+ (in construction), LENA, Hyper-Kamiokande (denoted as Hyper-K), and ANDES.

Detector	Fiducial Mass (kt)	N	N_1	$\delta t_{\text{arrival}}$ (ms)
Super-K	32	8.0×10^3	80	3.4
Hyper-K	740	1.9×10^5	1.9×10^3	0.7
SNO+	0.8	400	4	15
LENA	44	1.8×10^4	1.8×10^2	2.7
ANDES	3	1.2×10^3	12	8.7
IceCube	$\sim 10^3$	$\sim 10^6$	$\sim 10^4$	0.3

SN neutrino observations, we consider the total inner volume of 740 kt. For IceCube, we take the numbers estimated by the IceCube Collaboration [32] for a progenitor of 20 $M_\odot$. The number of useful neutrino induced Cherenkov photons to be recorded by the entire IceCube detector is $\sim 10^6$. For the Water Cherenkov detectors, Super-Kamiokande and Hyper-Kamiokande, we consider the inverse beta decay $\bar{\nu}_e + p \to n + e^+$, and the elastic scattering $\nu_\alpha + e^- \to \nu_\alpha + e^-$, whereas for the liquid scintillators, SNO + , LENA, and ANDES, we consider the inverse beta decay and the proton neutrino elastic scattering, $\nu_\alpha + p \to \nu_\alpha + p$.

We show our results in Table I where for a given detector and fiducial mass, the total number of events N, the number

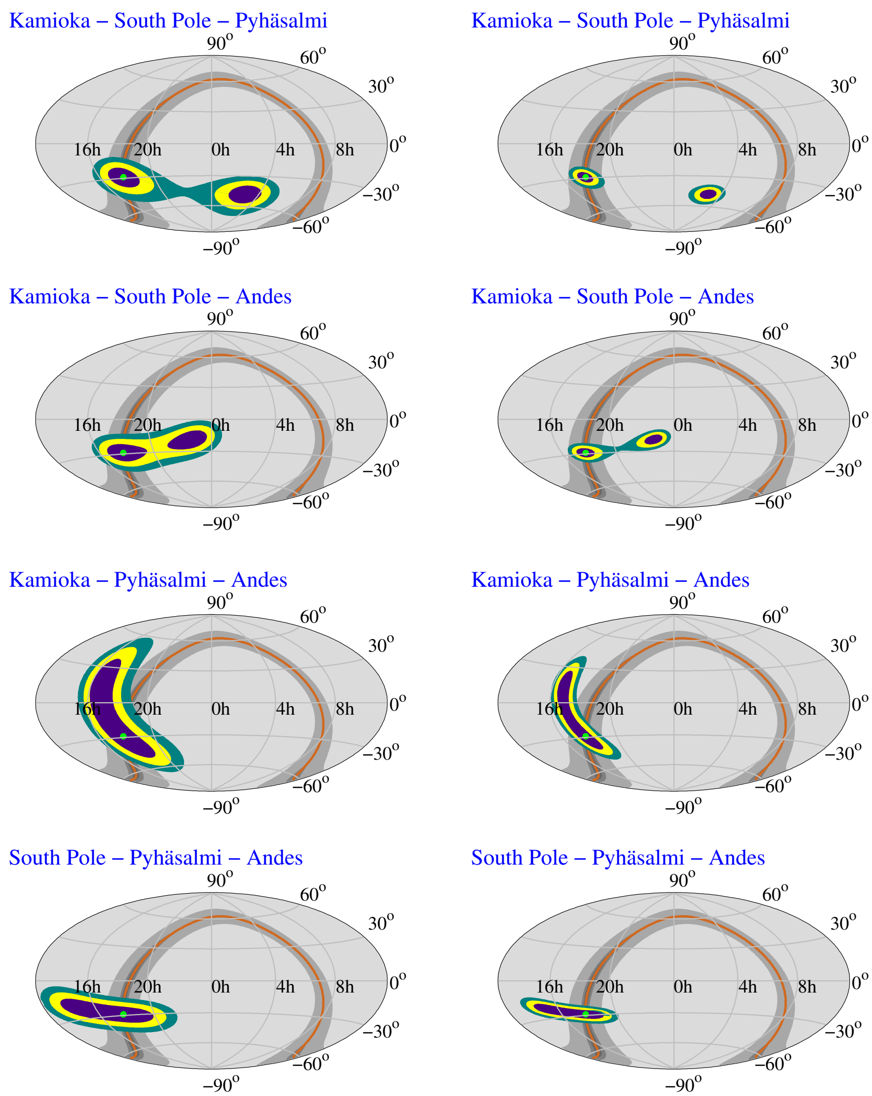

FIG. 3 (color online). Cases where a set of three detectors is considered each time to determine the position of the Galactic SN: Kamioka-South Pole-Pyhäsalmi (first row), Kamioka-South Pole-ANDES (second row), Kamioka-Pyhäsalmi-ANDES (third row), and South Pole-Pyhäsalmi-ANDES (fourth row). The colors purple, yellow, and green indicate, respectively, the regions allowed at 1, 2, and 3σ C.L.. Here the uncertainty in the time difference measurement between two detectors is assumed to be ± 4 ms (left panels) ± 2 ms (right panels).

of events in the rising part of the SN signal N_1, and the arrival-time uncertainty $\delta t_{\rm arrival}$, are shown. Our results for IceCube can be compared with the ones obtained in [17].

According to Ref. [17], based on Monte Carlo studies, IceCube can reconstruct the signal onset of the SN neutrinos with a resolution of $\delta t_{\rm arrival} = 1.7$ ms at 1σ C.L. for the SN signal consistent with $\tau_1 = 50$ ms. This value is about a factor of 6 worse than what we obtained in Table I for IceCube. We observe that the difference can be partially explained by the fact that Ref. [17] considered a larger τ_1 than us and also the number of N_1 in the first 30 ms in Ref. [17] is smaller ($\sim 6 \times 10^3$) than what we considered here. If we simply consider $\tau_1 = 50$ ms and $N_1 \sim 6000$, we would obtain $\delta t_{\rm arrival} \sim 0.6$ ms which is still smaller than that obtained in [17]. So it is probably safe to assume that the values obtained in Table I could vary within a factor of 2 or so, depending on how one estimates.

For the case where the edge is really sharp or if the decaying time of the SN signal is considered to be zero, roughly corresponding to the case of a failed SN with the formation of a BH, the uncertainty on the arrival time is given by the inverse of the event rate before the cutoff of the SN signal [8],

$$\delta t^{\rm BH}_{\rm arrival} \sim \frac{\tau}{N}, \tag{8}$$

where τ is the duration of the signal and N is the total number of observed events to be obtained before the abrupt termination of the neutrino flux. According to [18], the duration of the SN signal before the BH formation is $\sim O(1)$ s. In this case, for all the detectors considered in Table I, except for SNO +, $\delta t_{\rm arrival}$ is less than 1 ms, and even for the smallest detector, SNO +, we expect that $\delta t^{\rm BH}_{\rm arrival} \sim \tau/N \sim 1/400 = 2.5$ ms. We note, however, that due to the uncertainty associated with the formation of the black hole, which is about 0.5 ms [19], $\delta t^{\rm BH}_{\rm arrival}$ cannot be smaller than 0.5 ms.

III. COMBINED ANALYSIS

In this section, we discuss the results of our combined analysis by considering observations of SN neutrinos at three and four different detector sites on the Earth.

We define our χ^2 function as follows:

$$\chi^2 = \sum_{i,j} \left[\frac{\Delta t^{\rm obs}_{ij}(\alpha_0, \delta_0) - \Delta t^{\rm theo}_{ij}(\alpha, \delta)}{\sigma_{\Delta t}} \right]^2, \tag{9}$$

where $\Delta t^{\rm obs}_{ij}(\alpha_0, \delta_0)$ is the arrival-time difference of SN neutrinos to be observed (expected) for the input (true) SN location in the sky (α_0, δ_0) for the combination of ith and jth detector sites on the Earth whereas $\Delta t^{\rm theo}_{ij}(\alpha, \delta)$ is the theoretically expected one for a given SN location (α, δ). $\sigma_{\Delta t}$ is the assumed time resolution. Note that by construction, the best fit values (α, δ) obtained by our χ^2 analysis are the solution of Eq. (1) for an input value of $\Delta t^{\rm obs}_{ij}$.

In Fig. 3 we show for the same input SN location at the Galactic center used in Fig. 2 what would be the angular resolution for (α, δ) that would result from a combination of arrival-time differences registered by three different detectors. For definiteness and simplicity, we assume, for the combination of two detectors, that the arrival-time difference resolution can be ± 4 ms (left panels) and ± 2 ms (right panels).

As expected, for all the combinations we considered, we obtained two solutions at different locations in the sky, the true solution and the fake one. We note the true and fake allowed regions are connected at 1σ C.L. for the cases shown in the lower four panels in Fig. 3. Though we have two solutions, in practice, the one that lays in the region of the Galactic disc has a greater probability of being the true one. The fake solution can be eliminated by considering a fourth detector location as we can see below.

In Fig. 4 we show the case where four different detector locations are considered for the same SN input location in the Galactic center for the time resolution of ± 4 ms (left panel) and ± 2 ms (right panel). With four detectors at different sites, it is possible to single out the true location

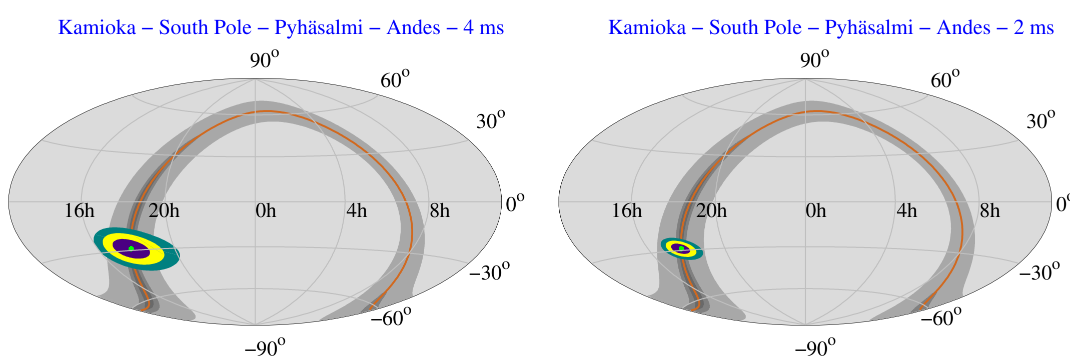

FIG. 4 (color online). Allowed regions at 1, 2, and 3σ C.L. compatible with the combinations of the arrival-time differences assuming four detector sites: Kamioka, the South Pole, Pyhäsalmi, and ANDES. We assumed the SN to be at the Galactic center and that the uncertainty in the time difference measurement between two detectors to be ± 4 ms (left panel) and ± 2 ms (right panel).

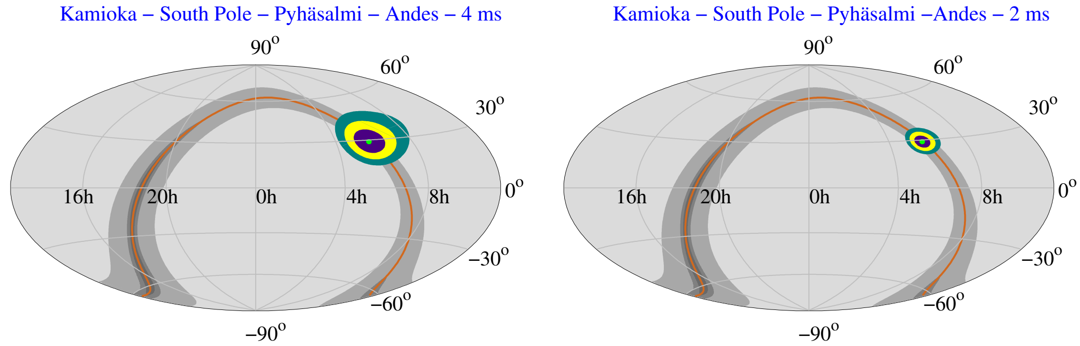

FIG. 5 (color online). Same as Fig. 4 but for a SN explosion that occurred at a location opposite to the Galactic center.

of the SN. In Fig. 5 we show similar plots for the case of different SN input location, opposite to the Galactic center, $\alpha = 5^{\rm h}42^{\rm m}27^{\rm s}$ and $\delta = 28°55'$. From these plots we can conclude that the expected precision is $\Delta(\alpha) \sim 15°(8°)$ and $\Delta(\delta) \sim 10°(5°)$ for the time resolution of ± 4 ms (2 ms).

Let us make a brief summary of our results on how the pointing accuracy depends on the number of detectors used in the analysis. If we consider only two detectors, in general, the region compatible with the location of the SN in the sky is quite large, as we can easily guess from Fig. 2 although this plot corresponds to the case of no arrival-time uncertainty or $\delta t_{\rm arrival} = 0$. From two to three detectors, the reduction of the region compatible with the SN location is quite sizable; we can see this by comparing Figs. 2 and 3. From three to four detectors, roughly speaking, the region is again reduced by about one half, as we can see by comparing Figs. 3–5.

We note, however, that even if we consider four detectors, the allowed angular ranges of the SN location in the sky are much larger than the typical field of view of an optical telescope. Nevertheless, this is better than no information at all and especially useful in the case where the SN cannot be located by optical observation, because of dust, or, in the case of a failed SN, because it is accompanied by a BH formation.

IV. CONCLUSIONS

The era of high-statistics neutrino detectors has started. IceCube is already operating in the South Pole and in a decade or so we expect to also have Hyper-Kamiokande in Japan as well as one very large neutrino detector in the future European underground laboratory in Pyhäsalmi. There is also a possibility to construct a new neutrino detector in the Southern Hemisphere at ANDES. This makes the determination of the angular position of a nearby SN by comparing the arrival time of the first SN neutrinos at these different detector locations on the Earth an interesting possibility.

The time resolution of the triangulation technique will be dominated by the smallest detector, since the precision of the reconstruction of the neutrino signal onset depends on the number of neutrinos registered by the detector [see Eqs. (5) and (8)]. We have demonstrated that, in general, one needs to combine the timing of four different detector locations in order to uniquely localize the SN using this method.

Assuming a rather optimistic, but not impossible, uncertainty on the arrival-time difference between two detectors to be $\sim \pm (2\text{–}4)$ ms, we have estimated the angular resolution of the determination of the location of a SN that could occur in the Galactic center given by four detectors located at the South Pole (IceCube), Kamioka (Super-Kamiokande or Hyper-Kamiokande), ANDES, and Pyhäsalmi (LENA, MEMPHYS, and GLACIER). We established that in this case the angular position can be known within $\sim 5°$ $(10°)$ in declination and $\sim 8°$ $(15°)$ in right ascension for the time resolution of 2 (4) ms.

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APPENDIX: DESCRIPTION OF THE EQUATORIAL COORDINATE SYSTEM

The SN location can be given in the so-called equatorial coordinate system by two angular coordinates, α, which is known as right ascension and δ, which is known as declination. Right ascension measures the angular distance eastward along the celestial equator from the vernal equinox; it is analogous to terrestrial longitude. Usually, right ascension is not given in degrees but rather in sidereal hours, minutes, and seconds. The vernal point is defined by where the celestial equator and the ecliptic intersect at $00^{\rm h}00^{\rm m}00^{\rm s}$ and longitude $0°$. By definition, the north celestial pole corresponds to $\delta = +90°$, so it is analogous to the terrestrial latitude.

This defines the unit vector $\mathbf{n_0}$, which points in the direction of propagation of the neutrinos arriving at the Earth coming from the SN as

$$\mathbf{n_0} = (n_{0x}, n_{0y}, n_{0z}), \tag{A1}$$

where

$$n_{0x} = -\cos\alpha \sin\delta, \qquad n_{0y} = -\sin\alpha\sin\delta, \qquad n_{0y} = -\cos\delta. \tag{A2}$$

Let us assume that a detector positioned at the ith site on the Earth is localized, at a certain time t, by the following vector:

$$\mathbf{x}_i = (x_i, y_i, z_i), \tag{A3}$$

with coordinates

$$x_i(t) = R_\oplus \cos\phi_i(t)\sin\theta_i, \qquad y_i(t) = R_\oplus \sin\phi_i(t)\sin\theta_i, \qquad z_i(t) = R_\oplus\cos\theta_i, \tag{A4}$$

where $R_\oplus$ is the radius of the Earth and θ_i is the latitude corresponding to the position of the detector. The angle $\phi_i(t)$ depends on time and can be given by

$$\phi_i(t) = \phi_i(0) + \omega t - \Omega T - \pi, \tag{A5}$$

where $\phi_i(0)$ is the longitude corresponding to the initial position of the detector, ω is the angular velocity of the daily rotation of the Earth, and Ω is the angular velocity corresponding to the annual rotation of the Earth around the Sun. The time t refers to the moment of the day the SN explosion occurred ($0 \le t \le 24$ h), given in terms of the UTC, whereas T, assumed to be common for all detectors, is the time elapsed after the vernal point when the detector received the SN neutrinos.

So, if we have two detectors, say, one at site 1 and the other at site 2, we can, explicitly, write the observed arrival-time difference as

$$\begin{aligned}\Delta t_{12} = (R_\oplus/c)[&(\cos\phi_1(t)\sin\theta_1 - \cos\phi_2(t)\sin\theta_2)n_{0x} \\ &+ (\sin\phi_1(t)\sin\theta_1 - \sin\phi_2(t)\sin\theta_2)n_{0x} \\ &+ (\cos\theta_1 - \cos\theta_2)n_{0z}],\end{aligned} \tag{A6}$$

which constrains the possible values of α and δ.

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