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Exotics from QCD Sum Rules

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Abstract

Many new exotic states in the charmonium mass region were discovered in the last decades by BaBar, Belle, CLEO-c, CDF, DØ, BESIII, LHCb and CMS Collaborations. We use the QCD Sum Rule approach to study some of these states.

Keywords: QCD sum rules, Hadron and Quark masses, QCD condensates.

1. Introduction

In the last two decades several new and unexpected observations on hadron states, the X, Y, Z states, came from a variety of facilities and the future holds more surprises for us, thanks to new-generation experiments. Understanding these signals and determining the properties of these states requires an enormous theoretical effort and many theoretical reviews on these states, using different theoretical approaches, are already available [1–7]. The first of these new charmonium states and the most well studied is the $X(3872)$. It was first observed in 2003 by the Belle Collaboration [8, 9], and has been confirmed by other five Collaborations: BaBar [10], CDF [11–13], DØ [14], LHCb [15, 16] and CMS [17].

The more recent of these states was observed last year by the LHCb Collaboration [18]: the double charmed state T_{cc}^{+}. It was the most expected among the theoretical predictions for the exotic states and the observed mass is in agreement with the theoretical predictions. The references for the theoretical predictions can be found in [18].

One possible theoretical framework to study the exotic states is the method of the QCD sum rules (QCDSR). It was first introduced by Shifman, Vainshtein and Zakharov [19] for the study of the mesons. The sum rule method was latter extended to baryons by Ioffe [20] and Chung *et al.* [21]. Since then the QCDSR technique has been applied to study numerous hadronic properties with various flavor content and has been discussed in many reviews [22–25] emphasizing different aspects of the method.

The starting point of the QCDSR approach is the consideration of the correlation function:

$$\Pi(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|T[j(x) j^\dagger(0)]|0\rangle \,, \tag{1}$$

where $j(x)$ is a current which has the quantum numbers of the hadron we want to study.

The method is based in the evaluation of the correlation function in two different ways: at the quark level in terms of quark and gluon fields; at the hadronic level by introducing hadron parameters. Assuming that there is an interval in momentum for which the QCD side and the Phenomenological one are equivalent, we can compare both sides and extract the hadronic parameter we are interested with.

In what follows we present some results of the QCDSR calculations on the exotic states presented in Table 1. For more details we refer the reader to our reviews on the subject [1, 2, 6].

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Table 1: Exotic states observed in the last years considered in this talk.

state	Production mode	Ref.
$X(3872)$	$B \to K(\pi^+\pi^- J/\psi)$	[8]
$T_{cc}^+(3875)$	$p\,p \to (D^0 D^0 \pi^+) + \cdots$	[18]
$Z_c^+(3900)$	$e^+\,e^- \to \pi^-(\pi^+ J/\psi)$	[26]
$Z_{cs}^-(3985)$	$e^+\,e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$	[27]

2. X, T_{cc} and Z states

2.1. $X(3872)$

The LHCb Collaboration has determined the $X(3872)$ quantum numbers to be $J^{PC} = 1^{++}$, with more than 8σ significance [16]. Supposing that the $X(3872)$ is a four-quark state and since the X mass is very close to the $D^{*0}D^0$ threshold, there are two possible configurations for it: a molecular $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ bound state with a small binding energy, or a compact tetraquark state.

The first QCDSR calculation for the $X(3872)$, considered as a four-quark state, was done in ref. [29] using a $J^{PC} = 1^{++}$ tetraquark current in a diquark-antidiquark configuration:

$$j_\mu^{(q,di)} = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\Big[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) + (q_a^T C\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)\Big]. \quad (2)$$

The obtained mass is $M_X = (3.92 \pm 0.13)$ GeV [29].

A molecular $D\bar{D}^*$ current:

$$j_\mu^{(q,mol)}(x) = \frac{1}{\sqrt{2}}\Big[\Big(\bar{q}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma_\mu q_b(x)\Big) - \Big(\bar{q}_a(x)\gamma_\mu c_a(x)\bar{c}_b(x)\gamma_5 q_b(x)\Big)\Big], \quad (3)$$

was used in ref. [30] and the mass obtained is $M_X = (3.87 \pm 0.07)$ GeV. Both masses are in good agreement with the experimental mass. As a matter of fact, in ref. [31] it was show that, using double ratio QCDSR, the differences between tetraquark and molecular currents are smaller than 0.01%.

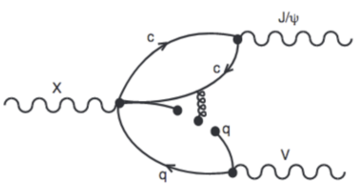

Figure 1: CC diagram which contributes to the OPE side of the sum rule. V represents a vector meson like ρ or ω.

The first calculation for the decay width of the $X(3872)$ using the QCDSR approach was done in ref. [32]. In the case of the $X \to J/\psi\pi^+\pi^-$ decay, where the $\pi^+\pi^-$ can be considered as the decay of the ρ meson, the generic decay diagram in terms of quarks has two "petals", one associated with the J/ψ and the other with the vector meson. Among the possible diagrams, there are two classes: diagrams with no gluon exchange between the petals and diagrams with gluon exchange between the petals. If there is no gluon exchange between the petals this means that there is no color exchange between the two final mesons in the decay and, therefore, the final state containing two color singlets was already present in the initial state. To avoid this problem we consider in the OPE side only the diagrams with non-trivial color structure, as the one shown in Fig. 1. This type of diagram represents the case where the $X(3872)$ is a genuine four-quark state with a complicated color structure. These diagrams are called color-conected (CC).

In ref. [32] it was shown that, considering only the color-connect diagrams, the obtained width is $\Gamma(X \to J/\psi(n\pi)) = (0.7 \pm 0.2)$ MeV, in a very good agreement with the experimental width.

2.2. $T_{cc}^+(3875)$

The first prediction for a tetraquark state with quark content $[cc][\bar{u}\bar{d}]$ and $I, J^P = 0, 1^+$ quantum numbers was done in 1986 in ref. [33], and the first time that it was called T_{cc} was in ref. [34]. As already noted in refs. [33, 34], the T_{cc} state cannot decay strongly or electromagnetically into two D mesons in the S wave due to angular momentum conservation nor in P wave due to parity conservation. Therefore, the prediction was that if its mass were below the DD^* threshold, this decay would also be forbidden, and the T_{cc}^+ state would be very narrow. Indeed, the state observed in 2021 by the LHCb Collaboration [18] is very narrow with a mass just bellow the D^0D^{*+} mass threshold at approximately 3875 MeV. The observed mass and width are $M = (3874.817 \pm 0.061)$ MeV and $\Gamma = (0.41 \pm 0.17)$ MeV [18]. As pointed out in Sec. 1, there were many theoretical predictions for the T_{cc} state using different theoretical approaches. The first prediction using the QCDSR approach was done in ref. [35] using an axial diquark-antidiquark current given by:

$$j_\mu = \frac{i}{2}\epsilon_{abc}\epsilon_{dec}[(c_a^T C\gamma_\mu c_b)(\bar{u}_d\gamma_5 C\bar{d}_e^T) = i[c_a^T C\gamma_\mu c_b][\bar{u}_a\gamma_5 C\bar{d}_b^T]\,. \quad (4)$$

This current represents well the most attractive configuration expected with two heavy quarks since the most attractive light antidiquark is expected to be in the color triplet, flavor anti-symmetric and spin 0 channel: a "good-diquark" as called by Wilczek [36].

The authors in ref. [35] found that the results are not very sensitive to the value of the charm quark mass, neither to the value of the condensates. The QCDSR predictions for the T_{cc} meson mass was:

$$M_{T_{cc}} = (4.0 \pm 0.2)\ \text{GeV}, \tag{5}$$

in a very good agreement with the experimental result.

In ref. [37] the authors have studied, using double ratio QCDSR, the ratio between the masses of the $X(3872)$ and the T_{cc}, assuming that they are respectively described by a $D\bar{D}^*$ and a DD^* molecular currents. They found that the masses of these two states are almost degenerate. Using the $X(3872)$ experimental mass as input they have obtained:

$$M_{T_{cc}} = (3872.2 \pm 39.5)\ \text{MeV}, \tag{6}$$

in an excelent agreement with the experimental result.

It is straightforward to extend the analysis done for the T_{cc} to the the bottom sector. The prediction for the T_{bb} mass from ref. [35] is:

$$M_{T_{bb}} = (10.2 \pm 0.3)\ \text{GeV}\,, \tag{7}$$

while from ref. [37] is:

$$M_{T_{bb}} = (10.14 \pm 0.10)\ \text{GeV}\,, \tag{8}$$

in a very good agreement between them and with the results in ref. [34].

2.3. $Z_c^+(3900)$

In March 2013 the BESIII Collaboration reported the observation of a charmonium charged state called $Z_c^+(3900)$ [26]. Almost at the same time the Belle Collaboration also reported the observaion of the same state in the $M(\pi^\pm J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi\pi^+\pi^-$ decay channel [28].

From a QCDSR calculation assuming $SU(2)$ symmetry, the mass for the Z_c^+ coincides with the one obtained for the $X(3872)$. However, the $Z_c^+(3900)$ has a much larger decay width as compared with the $X(3872)$ width. Indeed, the $Z_c^+(3900)$ decay width is in the range 40 – 50 MeV, while the $X(3872)$ width is smaller than 1.2 MeV. According to the experimental observations, the $Z_c^+(3900)$ decays into $J/\psi\,\pi^+$ with a relatively large decay width. This is unexpected for a $D^* - \bar{D}$ molecular state, in which the distance between the D^* and the $\bar{D}$ is large. In Ref. [38] this fact was taken as an indication that the Z_c^+ is a compact object, which may be better understood as a quark cluster, such as a tetraquark. The quantum numbers for the neutral state in the isospin multiplet was assumed to be $I^G(J^{PC}) = 1^+(1^{+-})$. Therefore, the current used in ref. [38] to evaluate the decay widths in the vertices $Z_c^+(3900)J/\psi\pi^+$, $Z_c^+(3900)\eta_c\rho^+$, $Z_c^+(3900)D^+\bar{D}^{*0}$ and $Z_c^+(3900)\bar{D}^0D^{*+}$, was:

$$\begin{aligned} j_\mu &= \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\Big[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) \\ &- (q_a^T C\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)\Big]\,. \end{aligned} \tag{9}$$

Since the $Z_c^+(3900)$ is expected to be a genuine tetraquark state with a non-trivial color structure, in ref. [38] only color-connected diagrams, as the one in Fig. 1, were considered. The obtained decay widths, are given in Table 2.

Table 2: $Z_c^+(3900)$ decay widths in different channels

Vertex	decay width (MeV)
$Z_c^+(3900)J/\psi\pi^+$	29.1 ± 8.2
$Z_c^+(3900)\eta_c\rho^+$	27.5 ± 8.5
$Z_c^+(3900)D^+\bar{D}^{*0}$	3.2 ± 0.7
$Z_c^+(3900)\bar{D}^0D^{*+}$	3.2 ± 0.7

The obtained total width for the $Z_c^+(3900)$ was:

$$\Gamma_{Z_c^+} = (63.0 \pm 18.1)\ \text{MeV},$$

in good agreement with the two experimental values: $\Gamma = (46 \pm 22)$ MeV from BESIII [26], and $\Gamma = (63 \pm 35)$ MeV from BELLE [28].

2.4. $Z_{cs}^-(3985)$

The first QCDSR prediction for a state with $J^P = 1^+$ and quark content $c\bar{s}q\bar{c}$, where q represents a u or d quark, was done in 2008 [30]. The authors in ref. [30] have considered the molecular current:

$$\begin{aligned} j_\mu(x) &= \frac{1}{\sqrt{2}}\Big[\Big(\bar{s}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma_\mu q_b(x)\Big) \\ &- \Big(\bar{s}_a(x)\gamma_\mu c_a(x)\bar{c}_b(x)\gamma_5 q_b(x)\Big)\Big], \end{aligned} \tag{10}$$

and the predicted mass for such a state was:

$$M_{Z_{cs}} = (3.97 \pm 0.08)\ \text{GeV}\,, \tag{11}$$

which is very close to the $D_s^+\bar{D}^{*0}$ threshold at 3.976 GeV. In a subsequent work [39], the method of QCDSR was used to study hadronic decays of $Z_{cs}^\pm$, considering the Z_{cs} as a tetraquark state, similar to what was done for the $Z_c^\pm(3900)$ state in ref. [38]. Therefore, the considered interpolating field for Z_{cs}^+ was:

$$j_\alpha = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(u_a^T C\gamma_5 c_b)(\bar{s}_d\gamma_\alpha C\bar{c}_e^T) - (u_a^T C\gamma_\alpha c_b)(\bar{s}_d\gamma_5 C\bar{c}_e^T)] \,, \quad (12)$$

The authors in ref. [39] have considered four decay channels: $Z_{cs}^+ \to J/\psi\, K^+$, $Z_{cs}^+ \to \eta_c\, K^{*+}$, $Z_{cs}^+ \to \bar{D}^{*0}\, D_s^+$ and $Z_{cs}^+ \to \bar{D}^0\, D_s^{*+}$. To assure that the non-trivial color structure of the current in Eq. (12) is maintained in the QCDSR calculation, in the OPE side it was considered only the diagrams with non-trivial color structure, the CC diagrams, as in refs. [32, 38]. The obtained decay widths, are given in Table 3.

Table 3: Decay widths of the Z_{cs}^+ in different channels.

Vertex	decay width (MeV)
$Z_{cs}^+ J/\psi K^+$	11.2 ± 3.5
$Z_{cs}^+ \eta_c K^{*+}$	10.8 ± 6.2
$Z_{cs}^+ D_s^+ \bar{D}^{*0}$	1.5 ± 1.5
$Z_{cs}^+ \bar{D}^0 D_s^{*+}$	1.4 ± 1.4

Considering these four decay channels the obtained total width is:

$$\Gamma_{Z_{cs}} = (24.9 \pm 12.6) \ \text{GeV}. \quad (13)$$

The $Z_{cs}^-(3985)$ state was finally observed in 2020 by the BESIII Collaboration [27] with a mass and decay width of $M = (3982.5 \pm 2.2)$ MeV and $\Gamma = (12.8 \pm 4.7)$ MeV, which are in agreement with the QCDSR predictions in Eqs. (11) and (13).

3. Conclusions

In conclusion, we have reported the masses and decay widths of the exotic states $X(3872)$, $T_{cc}^+(3875)$, $Z_c^+(3900)$ and $Z_{cs}^-(3985)$, using the QCDSR approach. In the case of the $X(3872)$ we found that it can be well described as a four-quark state with a tetraquark or a molecular current. The predictions for the T_{cc} mass are in very good agreement with the observed mass also for a tetraquark or a molecular current. In the case of the $Z_c^+(3900)$, its mass and decay width can be well described supposing the Z_c^+ to be a compact tetraquark state. Finally, the predictions for the $Z_{cs}^-(3985)$ mass and decay width are in complete agreement with the observed values. These results show that the QCDSR method is robust and reliable.

The most important message from the experimental program carried out by the BaBar, Belle, CLEO-c, CDF, DØ, BESIII, LHCb and CMS Collaborations is that definitely there is something really new happening in the hadronic spectroscopy. The program started in 2003 with the observation of the $X(3872)$ by the Belle Collaboration and the observation of the $T_{cc}^+(3875)$ in 2021 by the LHCb Collaboration reinforced our belief that we are observing multiquark states.

In the next years it is important: i) from the experimental side to determine the quantum numbers of all obsereved exotic states and eliminate the suspicion that some of them could be mere threshold effects and not real particles. ii) from the theoretical side to focus on the determination of the internal quark structure of these states.

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