A NUMERICAL APPROACH TO EVALUATE THE INFLUENCE OF THE PIEZOELECTRIC LAYER POSITION ON THE EFFECTIVE PROPERTIES OF A SMART LAMINATED COMPOSITE

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Abstract. *This study aims to evaluate the influence of the piezoelectric layer position on the shear effective coefficients of the constitutive tensor of smart layered composites subject to delamination. Thus, based on the concept of Representative Volume Element (RVE), this work carried out a case study of a nine-layer unit cell in the meso-scale. The delamination effect is represented by degrading the properties of the interface added between specific layers. The effective coefficients are calculated by the Finite Element Method (FEM) using the $ABAQUS^{TM}$ numerical simulation software. Through a computational procedure based on the Python language, the calculations of these homogenized properties of RVE are automated. Finally, the macro-scale results obtained via FEM were analyzed, and it is shown that the piezoelectric layer position has a low influence on the homogenized shear coefficients.*

1. INTRODUCTION

Smart composite systems have been widely studied due to their potential for application in engineering. The piezoelectric materials present in these structures produce an electrical response by applying mechanical stresses, as well as the reverse effect, a case in which an applied electric field causes a mechanical deformation [1]. This effect allows applications such as real-time monitoring, suppression of vibration and noise, shape control, and precise positioning [2]. Besides, the excellent characteristics of composite materials such as high directional strength, significant weight savings, and high corrosion resistance highlight the advantages of using this type of material. However, different types of failure mechanisms are observed in this class of composite materials. At the micro-scale level, for example, it is observed debonding between fiber and matrix. At the macro-scale level, it is common for the delamination between layers to lose much of their structural integrity. These problems call the attention of several researchers to carry out a micromechanical analysis to evaluate the structural properties, considering the damage effects in the interface between fiber and matrix [3-9].

As previously stated, the integrity of the interfaces between two adjacent layers also plays a critical role in determining the mechanical behavior of a composite material. Several contributions are found on papers [10-13], where delamination mechanisms are modeled through computational multi-scale analysis. On the other hand, this work proposes an extension to smart laminated composites, driving several analyses to evaluate the influence of the piezoelectric layer position of the shear effective coefficients.

2. PROBLEM FORMULATION AND METHODOLOGY

2.1. Effective proprieties for smart laminated composite

The linear effective properties of a homogenized piezoelectric material define the relation between the averages of field variables, such as stress $\{\bar{T}\}$, strain $\{\bar{S}\}$, electric field $\{\bar{E}\}$, and electric displacement $\{\bar{D}\}$. Thus, its constitutive relation can be written as

$$\begin{Bmatrix} \{\bar{T}\} \\ \{\bar{D}\} \end{Bmatrix} = \begin{bmatrix} [C]^E_{eff} & -[e]_{eff} \\ [e]^t_{eff} & [\varepsilon]^S_{eff} \end{bmatrix} \begin{Bmatrix} \{\bar{S}\} \\ \{\bar{E}\} \end{Bmatrix}. \quad (1)$$

The fourth-order linear elasticity tensor, the third-order linear piezoelectric coupling tensor, and the second-order linear dielectric tensor are written on a matrix form, denoted by $[C]^E_{eff}$, $[e]_{eff}$, $[\varepsilon]^S_{eff}$, respectively (the superscripts "E" and "S" denote constant electric and strain fields). Due to the transversely isotropic nature of this type of material, the constitutive matrix is represented by 11 independent coefficients. Since statistical homogeneity implies that the body averages and the RVE averages are the same [14], the average stress, electric field, strain, and electrical displacement field are defined by the homogenization operators as

$$\bar{T}_{ij} = \frac{1}{|V|}\int_V T_{ij}dV, \quad \bar{E}_k = \frac{1}{|V|}\int_V E_k dV, \quad \bar{S}_{kl} = \frac{1}{|V|}\int_V S_{kl}dV, \quad \bar{D}_i = \frac{1}{|V|}\int_V D_i dV, \quad (2)$$

where $|V|$ is the volume of the RVE, and T_{ij}, E_k, S_{kl}, and D_i are the respective variables within the RVE.

Using the FEM discretization, the average values can be calculated by

$$\bar{T}_{ij} = \frac{1}{|V|}\sum_{n=1}^{nel} T_{ij}^{(n)}\, V^{(n)}, \qquad \bar{S}_{kl} = \frac{1}{|V|}\sum_{n=1}^{nel} S_{kl}^{(n)}\, V^{(n)}, \quad (3)$$
$$\bar{E}_k = \frac{1}{|V|}\sum_{n=1}^{nel} E_k^{(n)}\, V^{(n)}, \qquad \bar{D}_i = \frac{1}{|V|}\sum_{n=1}^{nel} D_i^{(n)}\, V^{(n)}$$

where nel is the number of elements of the complete RVE, V^n is the volume of the n^{th} element, and $T_{ij}^{(n)}$, $S_{kl}^{(n)}$, $E_k^{(n)}$, $and\ D_i^{(n)}$ are the respective tensors evaluated in the n^{th} element.

2.2. Representative Volume Element (RVE)

Smart composite materials can be treated as a periodic arrangement of unit cells. If used to represent the properties of each phase of the heterogeneous medium, this unit cell is called Representative Volume Element (RVE). Through finite element discretization, the RVE can be modeled using solid elements. To ensure the continuity between adjacent RVE's (*i.e.* no gaps or interpenetration), periodic boundary conditions must be applied [8]. In this study, the homogenized piezoelectric properties used in the meso-scale were obtained from a micro-scale analysis in Brito-Santana *et al.* [9]. The RVE model in the meso-scale used in this work was previously presented by Brito-Santana *et al.* [12].

Figure 1(a) shows in detail the frontal view of the nine-layer meso-scale RVE. The meso-scale RVE is formed by eight Structural Composite Layers (SCL) and a Piezoelectric Layer (PL). A Delaminated Interface (DI) is modeled between the first and second layers. Figure 1(b) represents the periodic smart composite structure formed by an arrangement of this RVE. The micro-scale RVE is represented by Fig. 1(c), where phase 1 and 2 correspond to the matrix and the fiber of the smart composite, respectively.

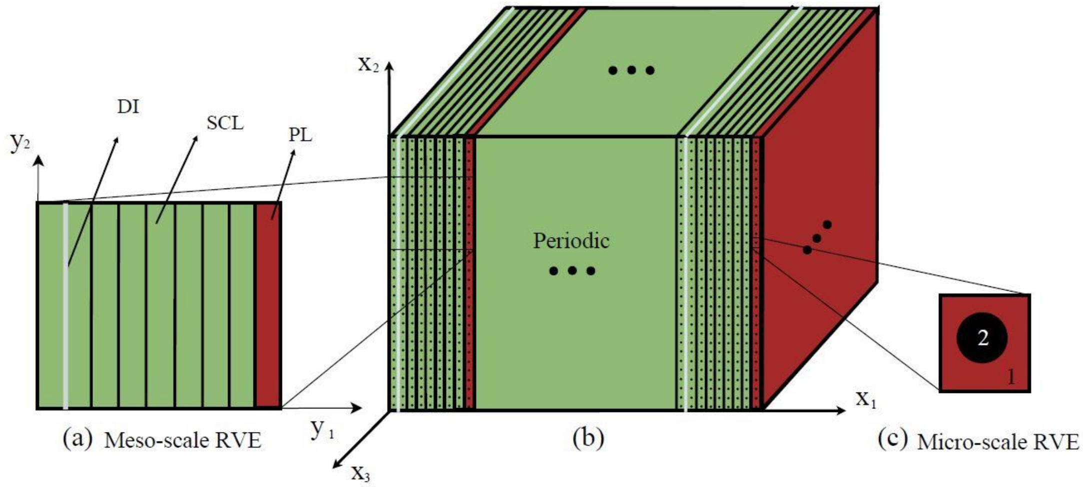

Figure 1 - (a) Nine layered meso-scale RVE, in y_1, y_2, and y_3 system coordinate, composed of a DI (Delaminated Interface), eight Structural Composite Layers (SCL), and a Piezoelectric Layer (PL). (b) Periodic smart composite structure. (c) Micro-scale RVE, where 2 is the fiber, and 1 is the matrix.

2.3. Periodic boundary conditions and delamination parameters

As previously mentioned, the piezoelectric laminated composite is considered a periodic heterogeneous medium in the smaller scales. In this context, periodic conditions on the RVE boundary are given by Suquet [15]. It can be noted that for any regular RVE model, the displacements of pairs of opposite faces are described as

$$u_i^{K^+} = \bar{S}_{ij} y_j^{K^+} + v_i^{K^+}, \qquad u_i^{K^-} = \bar{S}_{ij} y_j^{K^-} + v_i^{K^-}, \tag{4}$$

where v_i is the periodic part of the displacement components (local fluctuation) on the boundary surfaces, the indices i and j denote the global three-dimensional coordinate directions in a range from 1 to 3. The index K^+ means along the positive y_j direction and K^- means along the negative y_j direction on the corresponding surfaces A^-/A^+, B^-/B^+, and C^-/C^+ (Fig. 2). The local fluctuations $v_i^{K^+}$ and $v_i^{K^-}$ around the average macroscopic value are identical on two opposing faces due to periodic conditions of RVE. Thus, isolating the local fluctuations in Eq. (4) and equaling them,

$$u_i^{K^+} - u_i^{K^-} = \bar{S}_{ij}\left(y_j^{K^+} - y_j^{K^-}\right) = \bar{S}_{ij} \Delta \mathrm{y}_j^{\mathrm{K}}. \tag{5}$$

In this particular RVE, the difference $\Delta y_j^K = y_j^{K^+} - y_j^{K^-}$ is constant for each pair of the parallel boundary surfaces. Since the $\bar{S}_{ij}$ components are known, the right side of Eq. (5) becomes constant. Similarly, the periodic boundary condition for electric potential is given by the applied macroscopic electric field condition

$$\Phi^{K^+} - \Phi^{K^-} = -\bar{E}_i\left(y_j^{K^+} - y_j^{K^-}\right) = -\bar{E}_i \Delta \mathrm{y}_j^{\mathrm{K}}. \tag{6}$$

These equations can be applied in the numerical model as RVE boundary conditions. The methodology used can be found in the work of De Medeiros [16]. Six analyzes are needed to find the

11 effective coefficients. However, in this study, only shear effective coefficients (C_{66}^{eff}, C_{44}^{eff}, and e_{15}^{eff}) were analyzed, so only three analyses are needed. The interface is degraded according to a parameter defined as "θ", as presented by Brito-Santana *et al.* [12]. Considering $\theta = 1$, the consecutive layers are subjected to full separation. The case in which $\theta = 0$, implies on perfect contact between them. Thus, to approach a real delamination situation, the elements that make up the interface must have their isotropic elastic coefficients degraded.

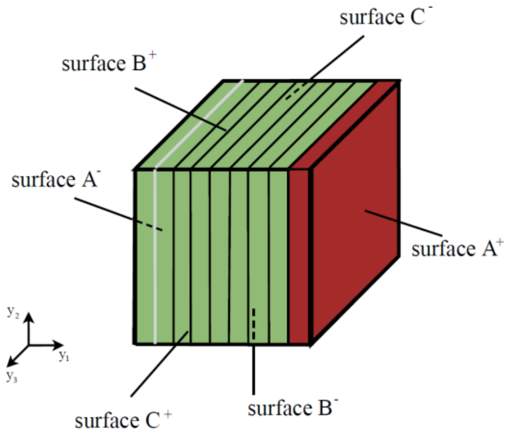

Figure 2 - Representation of all surfaces of the nine-layered meso-scale RVE.

3. ANALYSIS AND DISCUSSION OF RESULTS

Considering the numerical analysis, it is used the 20-node tri-quadratic isoparametric hexahedral piezoelectric elements, named C3D20E in the $ABAQUS^{TM}$ software. The C3D20E is a mixed finite element that has 27 integration points (3 × 3 × 3), containing three degrees of freedom of translational displacement and one another of electric potential in each node. In the analyses, a mesh refinement study was carried out to evaluate the convergence of the numerical results. The mesh convergence analyzes were performed based on the values of the effective coefficients. After performing the analyzes for all coefficients, the final mesh presented approximately 5500 elements showed good convergence. Figure 3 illustrates the mesh chosen for the analysis.

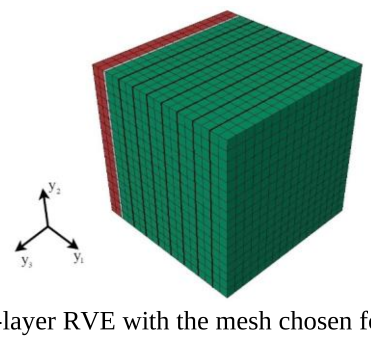

Figure 3 – Nine-layer RVE with the mesh chosen for the analyzes.

Note that all analyzes were performed for an RVE with the delamination located between the first and the second layer, while the piezoelectric layer varies its position within the model. After choosing the mesh, the model was subjected to analysis by the FEM. The computational homogenization procedure was performed based on a Python language routine. Thus, the three shear effective coefficients of RVE for each of the proposed configurations were found. The material properties are presented in Tab. 1.

Table 1 - Mechanical properties of the orthotropic structural composite layers (unidirectional carbon fiber embedded in an epoxy matrix) [10], isotropic interface properties (epoxy) [10], and piezoelectric layer properties [9].

Structural Composite Layers		**Piezoelectric Layer**				**Interface**	
E_L^{el}	127 GPa	C_{11}	9.296 GPa	e_{13}	-0.204 C/m²	Perfect contact ($\theta = 0$)	
E_T^{el}	10 GPa	C_{12}	4.328 GPa	e_{15}	0.015 C/m²	E^{el}	5.033 GPa
ν_{TL}	0.34	C_{13}	5.320 GPa	e_{33}	9.771 C/m²	ν	0.4
ν_{LT}	0.306	C_{33}	31.667 GPa	ε_{11}	0.241 nF/m	Delaminated ($\theta = 1$)	
G_{TL}	5.4 GPa	C_{44}	1.776 GPa	ε_{33}	3.842 nF/m	E^{el}	50.33 Pa
G_{LT}	3.05 GPa	C_{66}	1.354 GPa			ν	0.4E-8

* E_L^{el} and E_T^{el} are the modulus of elasticity, G_{LT} and G_{TL} are the shear module, and ν_{LT} and ν_{TL} are the Poisson's ratio.

3.1. Case study

The case study covered in this work is the nine-layer RVE. The stacking chosen for this model was of the symmetrical type [Piezo*(0)/interface/0/45/-45/90/90/-45/45/0], where "*" means that the piezoelectric layer varies its position between the layers of the RVE and the interface is intact or delaminated. For each piezoelectric layer position, three analyzes are necessary to obtain the three desired shear effective coefficients. Therefore, for the nine-layer RVE, 54 analyzes were performed (27 for intact interface and 27 for delaminated interface). Figure 4 shows the shear effective coefficients found for the nine-layer RVE for each position of the piezoelectric layer as a function of the degree of delamination (θ). It is possible to notice that the elastic coefficient (C_{66}^{eff}), related to shear strain $\bar{S}_{12}$, tends to zero when $\theta = 1$. This behavior was expected due to the loss in the stiffness caused by the delaminated interface in this direction. The coefficients C_{44}^{eff} and e_{15}^{eff} present a very small variation regarding the degradation of the interface, because of the direction of the shearing applied.

Through Fig. 5, it is possible to observe the variations of each coefficient as a function of the piezoelectric layer position (note that the values are normalized). Thus, $R_{C_{44}^{eff}}$, $R_{C_{66}^{eff}}$, and $R_{e_{15}^{eff}}$ represent the ratio between the effective coefficient analyzed and its maximum value obtained in each configuration (intact or delaminated).

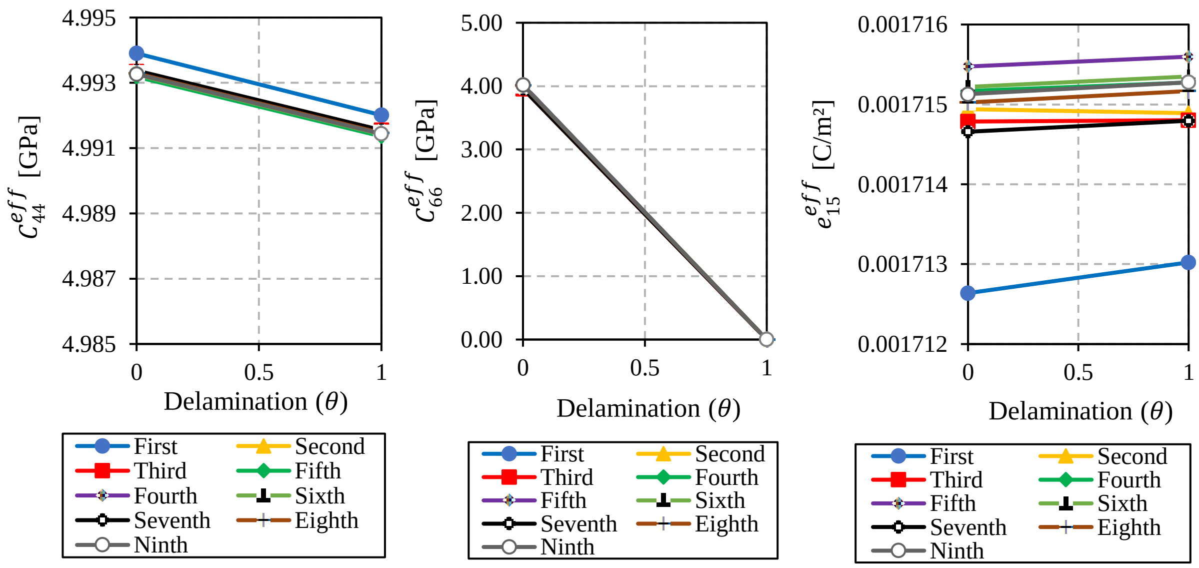

Figure 4 – C_{44}^{eff}, C_{66}^{eff} and e_{15}^{eff} as a function of the degree of delamination (θ) for each position of the piezoelectric layer of the nine-layer RVE.

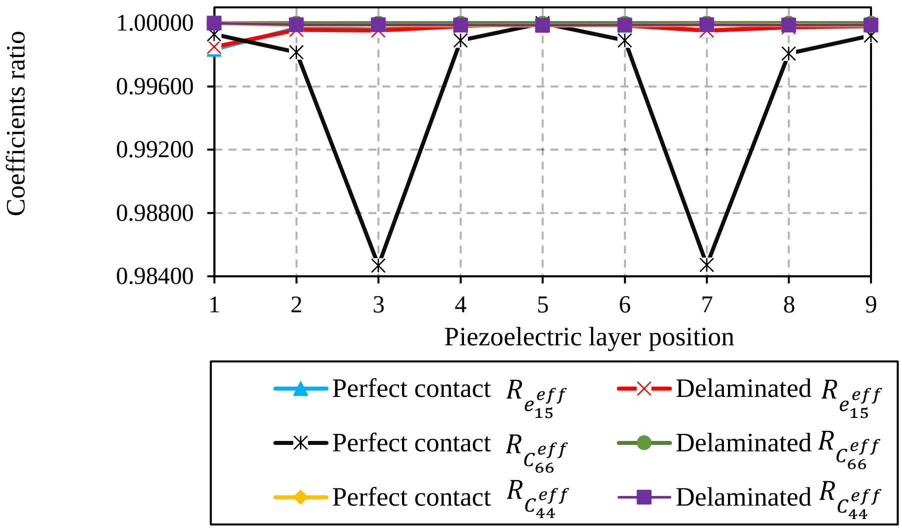

Figure 5 – Normalized shear effective coefficients ($R_{C_{44}^{eff}}$, $R_{C_{66}^{eff}}$ and $R_{e_{15}^{eff}}$) as a function of the piezoelectric layer position for the nine-layer RVE (with and without delamination).

The relative variation between the highest and lowest value of each effective coefficient due to the piezoelectric layer position with and without delamination was calculated. Regarding the maximum relative difference, it is observed that the coefficients C_{44}^{eff} and e_{15}^{eff} showed a variation of less than 0.20% due to the piezoelectric layer position. On the other hand, the coefficient C_{66}^{eff} has a variation of 1.53% due to the piezoelectric layer position.

4. CONCLUSION

Given the wide interest in systems that use the piezoelectric composite materials for monitoring and acting in real-time on structures, this work analyzed the RVE piezoelectric layer position influence on macro-scale shear effective coefficients in a piezoelectric composite. From the analyzes performed, it is possible to notice that the position of the piezoelectric layer in the RVE has little influence on the value of the effective shear coefficients C_{44}^{eff} and e_{15}^{eff}. Note that there was a variation of 0.17% without delamination ($\theta = 0$) and 0.15% with delamination ($\theta = 1$) for the coefficient e_{15}^{eff} and a variation of 0.014% without delamination ($\theta = 0$) and 0.013% with delamination ($\theta = 1$) for the coefficient C_{44}^{eff}. However, it is noteworthy that for the effective coefficient C_{66}^{eff} the position of the piezoelectric layer has a greater influence, so that for the RVE without delamination this coefficient varies by about 1.50% and when the RVE presents the delamination the coefficient tends to zero. Thus, it was possible to conclude that the piezoelectric layer position presents some change in the effective coefficients of the smart homogenized composite.

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