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A NOTE ON AGE REPLACEMENT POLICY UNDER A SYSTEM SIGNATURE POINT PROCESS REPRESENTATION

by

Vanderlei da Costa Bueno

Palavras-Chaves: Coherent systems, age replacement policy, infinitesimal-look ahead stopping rule, smooth semi-martingales, signature point process.

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A note on age replacement policy under a system signature point process representation

Vanderlei da Costa Bueno,
Institute of Mathematics and Statistics,
São Paulo University,
Cx Postal 66281 - 05311-970, São Paulo, Brazil.

Summary We search the optimal replacement age policy observing the component lifetimes of a coherent system under its point process signature representation.

Keywords: Coherent systems, age replacement policy, infinitesimal-look-ahead stopping rule, smooth semi-martingales, .

1.Introduction

The lifetime of a coherent system, as in Barlow and Proschan (1981), is described by a positive random variable S. Upon failure the system is immediately replaced by an equivalent one and the process repeat itself. A preventive replacement can be carried out before failure. In a basic age replacement policy each replacement incurs a cost of $c > 0$ and each failure adds a penalty cost of $k > 0$.

For an age replacement policy, an replacement age $a > 0$ is fixed for each system at which a preventive replacement takes place. If S_i, $i = 1, 2, ...$ are the successive lifetimes of the system, then $\tau_i = S_i \wedge a$ denotes the operation time of the i-th system and equals the i-th cycle length. The lifetime S_i are assumed to be independent and identically distributed with distribution function F.

The costs for one cycle are described by the stochastic process $(Z_t)_{t\geq 0}$, $Z_t = c + k1_{\{S\leq t\}}$. The total cost per unit time up to time t is

$$C_t = \frac{1}{t}\sum_{i=1}^{N_t} Z_{\tau_i},$$

where $(N_t)_{t\geq 0}$, is the renewal counting process generated by τ_i and $Z_\tau = c + k1_{\{S\leq\tau\}}$ is the incurred costs in one cycle. From renewal theory we know that

$$C = \lim_{t\to\infty} E[C_t] = \frac{E[Z_\tau]}{E[\tau]}.$$

The goal is to find the replacement age that minimizes this long run average cost per unit time. Inserting the cost function $Z_t = c + k1_{\{S\leq t\}}$ we get

$$C_a = \frac{c + kF(a)}{\int_0^a (1 - F(t))dt}$$

and we are looking for the age a^* which minimizes this quantity.

The replacement policies can be strongly connected to the following stopping problem: Minimize

$$C_\tau = \frac{E[Z_\tau]}{E[X_\tau]},$$

in a suitable class of stopping time related to a filtration $(\Im_t)_{t\geq 0}$ which represents our observations, in the probability space $(\Omega, \Im, P)$, assumed to fulfill the usual conditions of right continuity and completeness. The stochastic processes Z_t and X_t are observable in $(\Im_t)_{t\geq 0}$, that is, Z_t and X_t are $\Im_t$-measurable.

In particular, if we consider the σ-algebra

$$\Im_t = \sigma\{1_{\{S>s\}}, 0 \leq s \leq t\}$$

that is, at any time t we known whether the system works or not, the processes $Z_t = c+k1_{\{S\leq t\}}$ and $X_t = t$ are $\Im_t$-measurable and the $\Im_t$-stopping times are of the form $\tau = t\wedge S$. Then we have $E[Z_\tau] = c+k1_{\{S\leq\tau\}}$ and $E[X(\tau)] = E[\tau]$ which reduce the stopping problem to the basic age replacement policy above.

In this work we intend to consider a coherent system in its signature point process representation under a complete information level, observing its component lifetimes. In this context we resumes to solve the basic age replacement optimal stopping problem using an infinitesimal-look-ahead stopping rule.

2 The Signature Marked Point Process.

2.1 The mathematical details.

In our general setup, we consider the vector $(T_1, ..., T_n)$ of n component lifetimes of a coherent system with lifetime S, which are finite and positive random variables defined in a complete probability space $(\Omega, \Im, P)$, with $P(T_i \neq T_j) = 1$, for all $i \neq j, i, j$ in $C = \{1, ..., n\}$, the index set of components. The component lifetimes can be dependent but simultaneous failures are ruled out. As in Barlow and Proschan (1981), the system lifetime and its components can be related by the series parallel decomposition:

$$S = \phi(\mathbf{T}) = \min_{1 \leq j \leq k} \max_{i \in K_j} T_i,$$

where $K_j, 1 \leq j \leq k$ are minimal cut sets, that is, a minimal set of components whose joint failure causes the system fail.

However the evolution of components in time define a marked point process given through the failure times and the corresponding marks.

We denote by $T_{(1)} < T_{(2)} < ... < T_{(n)}$ the ordered lifetimes $T_1, T_2, ..., T_n$, as they appear in time and by $X_i = \{j : T_{(i)} = T_j\}$ the corresponding marks. As a convention we set $T_{(n+1)} = T_{(n+2)} = ... = \infty$ and $X_{n+1} = X_{n+2} = ... = e$ where e is a fictitious mark not in C, the index set of the components. Therefore the sequence $(T_n, X_n)_{n \geq 1}$ defines a marked point process.

The mathematical description of our observations, the complete information level, is given by a family of sub σ-algebras of $\Im$, denoted by $(\Im_t)_{t \geq 0}$, where

$$\Im_t = \sigma\{1_{\{T_{(i)} > s\}}, X_i = j, 1 \leq i \leq n, j \in C, 0 < s \leq t\},$$

satisfies the Dellacherie conditions of right continuity and completeness.

Intuitively, at each time t the observer knows if the event $\{T_{(i)} \leq t, X_i = j\}$ have either occurred or not and if it had, he knows exactly the value $T_{(i)}$ and the mark X_i. Follows that the component and the system lifetimes are $\Im_t$ stopping times.

We consider the lifetimes $T_{(i),j}$ defined by the failure event $\{T_{(i)}, X_i = j\}$ with their sub-distribution function $F_{(i),j}(t) = P(T_{(i),j} \leq t) = P(T_{(i)} \leq t, X_i = j)$ suitable standardized.

In what follows we assume that relations between random variables and measurable sets, respectively, always hold with probability one, which means that the term P-a.s., is suppressed.

The marked point $N_t((i),j) = 1_{\{T_{(i)}\leq t, X_i=j\}}$ is an $\Im_t$-sub-martingale, that is, $T_{(i),j}$ is $\Im_t$-measurable and $E[N_t((i),j)|\Im_s] \geq N_s((i),j)$ for all $0 \leq s \leq t$.

From Doob-Meyer decomposition, there exists a unique $\Im_t$-predictable process (see appendix A.1), $(A_t((i),j)_{t\geq 0}$, called $\Im_t$-compensator of $N_t((i),j)$, with $A_0((i),j) = 0$ and such that $M_t((i),j) = N_t((i),j) - A_t((i),j)$ is a zero mean uniformly integrable $\Im_t$-martingale. We assume that $T_i, 1 \leq i \leq n$ are totally inaccessible $\Im_t$-stopping time and, under this assumption, $A_t((i),j)$ is continuous. In certain sense, an absolutely continuous lifetime is totally inaccessible (see Appendix A.1). Resuming we assume a general lifetime model for $T_{(i),j}$ represented by the smooth $\Im_t$-semi-martingale:

$$1_{\{T_{(i),j}\leq t\}} = \int_0^t 1_{\{T_{(i),j}>s\}}\lambda_s((i),j)ds + M_t((i),j).$$

The process $(\lambda_t((i),j))_{t\geq 0}$ is called the intensity process of the semi-martingale representation and generalizes the classical notion of hazard rate. Intuitively indicates the prominence for failure, on the basis of all observations available up to, but not including, the present. As $N_t((i),j)$ can only count on the time interval $(T_{(i-1)}, T_{(i)}]$, the corresponding compensator differential $\lambda_t((i),j)$ must vanish outside this interval.

Note that, to count the $i-th$ failure we let $N_t((i)) = \Sigma_{j\geq 1}N_t((i),j)$ with $\Im_t$-compensator process $A_t((i)) = \Sigma_{j\geq 1}A_t((i),j)$. $N_t(j) = \Sigma_{i\geq 1}N_t((i),j)$, counts the component failure and it has $\Im_t$-compensator process $A_t(j) = \Sigma_{i\geq 1}A_t((i),j)$.

2.2 The signature marked point process.

The behavior of the stochastic process $P(S > t|\Im_t)$, as the information flows continuously in time was presented by Bueno,V.C., at the 26-th European Conference on Operational Research, Rome, Italy, 2013. Resuming:

Theorem 2.2.1 Let $T_1, T_2, ..., T_n$ be the component lifetimes of a coherent system with lifetime T. Then,

$$P(S \leq t|\Im_t) = \sum_{k,j=1}^{n} 1_{\{S=T_{(k),j}\}}1_{\{T_{(k),j}\leq t\}}.$$

Proof From the total probability rule we have $P(S \le t|\Im_t) =$

$$\sum_{k,j=1}^{n} P(\{S \le t\} \cap \{S = T_{(k),j}|\Im_t) = \sum_{k,j=1}^{n} E[1_{\{S=T_{(k),j}\}} 1_{\{T_{(k),j} \le t\}}|\Im_t].$$

As S and $T_{(k),j}$ are $\Im_t$-stopping time and it is well known that the event $\{S = T_{(k),j}\} \in \Im_{T_{(k),j}}$ where

$$\Im_{T_{(k),j}} = \{A \in \Im_\infty : A \cap \{T_{(k),j} \le t\} \in \Im_t, \forall t \ge 0\},$$

we conclude that $\{S = T_{(k),j}\} \cap \{T_{(k),j} \le t\}$ is $\Im_t$-measurable.

Therefore $P(S \le t|\Im_t) =$

$$\sum_{k,j=1}^{n} E[1_{\{S=T_{(k),j}\}} 1_{\{T_{(k),j} \le t\}}|\Im_t] = \sum_{k,j=1}^{n} 1_{\{S=T_{(k),j}\}} 1_{\{T_{(k),j} \le t\}}.$$

The above decomposition allows us to define the signature point process at component level.

Definition 2.2.2: The vector $(1_{\{S=T_{(k),j}\}}, 1 \le k, j \le n)$ is defined as the marked point signature point process of the system ϕ with lifetime S.

Remark 2.2.3

We can calculate the system reliability as

$$P(S \le t) = E[P(S \le t|\Im_t)] =$$

$$E[\sum_{k,j=1}^{n} 1_{\{S=T_{(k),j}\}} 1_{\{T_{(k),j} \le t\}}] = \sum_{k,j=1}^{n} P(\{S = T_{(k),j}\} \cap \{T_{(k),j} \le t\}).$$

If the component lifetimes are independent and identically distributed we have,

$$P(S \le t) = \sum_{k,j=1}^{n} P(S = T_{(k),j}) P(T_{(k),j} \le t)$$

recovering the classical result as in Samaniego (1985).

To calculate the $\Im_t$-compensator of $1_{\{S \le t\}}$, where S is the system lifetime we consider the smooth semi-martingale representation in Section 2.1.

Corollary 2.2.4 Let $T_1, T_2, ..., T_n$, be the components lifetimes of a coherent system with lifetime T. Then, the $\Im_t$-submartingale $P(S \leq t|\Im_t)$, has the $\Im_t$-compensator

$$\sum_{k,j=1}^{n} \int_0^t 1_{\{T_{(k),j} > s\}} 1_{\{S=T_{(k),j}\}} \lambda_s((k),j)ds.$$

Proof

We consider the process

$$1_{\{S=T_{(k),j}\}}(w,s) = 1_{\{S=T_{(k),j}\}}(w).$$

As $T_{(k),j} \leq S$, for all k, j, it is left continuous and $\Im_t$-predictable. Therefore

$$\int_0^t 1_{\{S=T_{(k),j}\}}(s)dM_s((k),j)$$

is an $\Im_t$-martingale.

As a finite sum of $\Im_t$-martingales is an $\Im_t$-martingale, we have

$$\sum_{k,j=1}^{n} \int_0^t 1_{\{S=T_{(k),j}\}} d1_{\{T_{(k),j} \leq s\}} - \sum_{k,j=1}^{n} \int_0^t 1_{\{T_{(k),j} > s\}} 1_{\{S=T_{(k),j}\}} \lambda_s((k),j)ds =$$

$$\sum_{k,j=1}^{n} \int_0^t 1_{\{S=T_{(k),j}\}} dM_s((k),j)$$

is an $\Im_t$-martingale. As the compensator is unique we finish the proof.

3. Age replacement policy under a signature marked point process.

As before, S represents a coherent system lifetime.

We let $(Z_t)_{t\geq 0}$, with $Z_t = c + k1_{\{S\leq t\}}$ and $(X_t)_{t\geq 0}$, with $X_t = t$ which are real right continuous stochastic processes adapted to $\Im_t$ and such that $E[Z_S] > -\infty$ and $E[|X_S|] < \infty$. We intend to minimize the rate

$$C_T = \frac{E[Z_T]}{E[X_T]}$$

over the class of $\Im_t$-stopping time

$$C_S^{\Im_t} = \{\tau : \tau \ is \ an \ \Im_t - stopping \ time, \tau \leq S, E[Z_\tau] > -\infty, E[|X_\tau|] < \infty\},$$

that is, to find a stopping time $\sigma \in C_S^{\Im_t}$, with

$$C^* = C_\sigma = \inf\{C_\tau : \tau \in C_S^{\Im_t}\}.$$

It is well known that a smooth semimartingale representations for the processes $(Z_t)_{t\geq 0}$ and $(X_t)_{t\geq 0}$, see Appendix A.2.1, is an excellent tool to carry out the stopping problem.

Under the signature point process representation we have, from Corollary 2.2.4, the semi-martingale representation for $1_{\{S\leq t\}}$:

$$1_{\{S\leq t\}} = \sum_{k,j=1}^{n} \int_0^t 1_{\{T_{(k),j}>s\}} 1_{\{S=T_{(k),j}\}} \lambda_s((k),j)ds + \sum_{k,j=1}^{n} M_t((k),j).$$

Also $X_t = t = \int_0^t ds$.

To solve the above stopping problem is equivalent to solve the following maximization problem. Observe that the inequality $C_\tau = \frac{E[Z_\tau]}{E[X_\tau]} \geq C^*$ is equivalent to $C^* E[X_\tau] - E[Z_\tau] \leq 0$ for all $\tau \in C_S^{\Im_t}$, where the equality holds for an optimal stopping time. We have the maximization problem: Find $\sigma \in C_T^{\Im_t}$, with

$$E[Y_\sigma] = \sup\{E[Y_\tau] : \tau \in C_S^{\Im_t}\} = 0,$$

where $Y_t = C^* X_t - Z_t$ and $C^* = \inf\{C_\tau : \tau \in C_S^{\Im_t}\}$.

A smooth semi-martingale representation for Y_t for the basic age replacement policy

$$Y_t = -c + \int_0^t (C^* - k 1_{\{S>s\}} \lambda_s)ds] + M_t^*,$$

where $\lambda_s = \sum_{k,j=1}^{n} 1_{\{S=T_{(k),j}\}} 1_{\{T_{(k),j}>s\}} \lambda_s((k),j)$ and $M_t^* = \sum_{k,j=1}^{n} M_t((k),j$.

Therefore

$$E[Y_t] = E[C^* X_t - Z_t] = -c + E[\int_0^t (C^* - k1_{\{S>s\}}\lambda_s)ds]$$

To find an explicit solution of the stopping problem we adopt a condition called the monotone case:

Definition 3.1 (MON) Let $Y = (f, M)$ be an SSM. Then the following condition

$$\{f_t \leq 0\} \subset \{f_{t+h} \leq 0\}, \forall t, h \in \Re^+, \bigcup_{t \in \Re^+} \{f_t \leq 0\} = \Omega$$

is said to be the monotone case and the stopping time

$$\sigma = \inf\{t \in \Re^+ : f_t \leq 0\}$$

is called the ILA-stopping rule (infinitesimal-look-ahead).

Obviously in the monotone case the process f driving the smooth semimartingale Y_t remains nonpositive if once crosses zero from above and the ILA-stopping rule σ is a candidate to solve the maximization problem. Jensen (1990), easily prove that

Theorem 3.2 Let $Y = (f, M)$ be an SSM and σ the ILA-stopping rule. Then, in the monotone case,

$$E[Y_\sigma] = \sup\{E[Y_\tau] : \tau \in C^{\Im_t}\},$$

where

$$C^{\Im_t} = \{\tau : \tau \ \ is \ \ an \ \ \Im_t \ \ stopping \ \ time, \tau < \infty, E[Y_\tau] > -\infty\}.$$

Clearly, the monotone case holds when f_s is increasing, a.s., $f_0 < C^*$ and $\lim_{s\to\infty} f_s > C^*$. However it is seem too restrictive to demand that f_s is increasing a.s.. We would like the monotone case to cover cases as the bath-tub-shaped functions, which decrease first up to same value, and increase after that value. The definition of (a,b)-increasing function allows such cases, see appendix A.2.2.

The main idea to solve the stopping problem, for the basic replacement policy, using the monotonicity condition is, instead to considering all stopping time in $C_S^{\Im_t}$ we may restrict the search for an optimal stopping time to the class of index stopping times

$$\rho_x = \inf\{s \in \Re^+ : x - k\lambda_s \leq 0\} \wedge S, \inf \emptyset = \infty, x \in \Re.$$

The optimal stopping level can be determined from $E[Y_\tau] = 0$ and coincides with C^* as in the following Theorem from Jensen (1990).

Theorem 3.3 If the process $(Y_t)_{t\geq 0}$ in its SSM representation has an intensity with (a,b) increasing paths on $(0,S]$, then

$$\sigma = \rho_{x^*}, \quad with \quad x^* = \inf\{x \in \Re : xE[\rho_x] - E[Z_{\rho_x}] \geq 0\}$$

is an optimal stopping time with $x^* = C^*$.

To clarify the procedure, without loss of generality, we consider the penalty cost $k = 1$ in the age basic replacement model. Therefore,

$$E[Y_t] = -c + E[\int_0^t (C^* - \sum_{k,j=1}^{n} \int_0^t 1_{\{T_{(k),j}>s\}} 1_{\{S=T_{(k),j}\}} \lambda_s((k),j)ds].$$

Using Theorem 3.3 we have to find the index stopping times

$$\rho_x = \inf\{s \in \Re^+ : x - \lambda_s \leq 0\} \wedge S, \inf \emptyset = \infty, x \in \Re.$$

$$\sigma = \rho_{x^*}, \quad with \quad x^* = \inf\{x \in \Re : xE[\rho_x] - E[Z_{\rho_x}] \geq 0\}.$$

We conclude that x^* is the optimal stopping time and $x^* = C^*$.

In our context

$$\lambda_s = \sum_{k,j=1}^{n} 1_{\{T_{(k),j}>s\}} 1_{\{S=T_{(k),j}\}} \lambda_s((k),j)$$

and,

$$\rho_x = \inf\{s \in \Re^+ : x - \lambda_s \leq 0\} \wedge S, \inf \emptyset = \infty, x \in \Re$$

considering values of x in $[\frac{c}{E[S]}\lambda_0, \frac{c+1}{E[S]}]$ obtained from Lemma A.2.3.

We consider n independent and identically distributed component lifetimes with exponential distribution with mean $\frac{1}{\alpha}$. Therefore

$$\lambda_s = \sum_{k,j=1}^{n} 1_{\{T_{(k),j}>s\}} 1_{\{S=T_{(k),j}\}}(n-k+1).\alpha.$$

We have to distinguish between the cases $(j-1)\alpha \leq x < j\alpha, \;\; j = 1, 2, ...n$. If $(k-1)\alpha \leq x < k\alpha$ and $T_{(k)} < S$, we have $\rho_x = T_{(k)}$, $E[Z_{T_{(k)}}] = c$ and

$$E[\rho_x] = E[T_{(k)}] = k.\binom{n}{k}\Sigma_{j=0}^{k-1}\binom{k-1}{j}(-1)^{k-1-j}\frac{1}{\alpha(n-j^2)}.$$

Therefore

$$x^* E[\rho_x] - E[Z_{\rho_x}] = 0 \Leftrightarrow x^* = \frac{c}{E[T_{(k)}]}.$$

If $S = T_{(k)}$, $\rho_x = S$, $E[\rho_x] = E[S]$ and $E[Z_S] = c + 1$. Therefore

$$x^* E[\rho_x] - E[Z_{\rho_x}] = 0 \Leftrightarrow x^* = \frac{c+1}{E[S]}.$$

In the particular case where S is the lifetime of a parallel system of two components independent and identically distributed lifetimes with exponential distribution with parameter α. The lifetimes are related by $S = T_1 \vee T_2$.
The intensity of the signature point process for $1_{\{S \leq t\}}$ is

$$\lambda_s = \alpha 1_{\{T_{(1),1}>s\}} 1_{\{S=T_{(1),1}\}} + \alpha 1_{\{T_{(1),2}>s\}} 1_{\{S=T_{(1),2}\}}$$

Now, if $x < \alpha$ we have

$$\rho_x = \inf\{s \in \Re^+ : x - \lambda_s \leq 0\} = T_{(1),1} \wedge T_{(1),2} = T_{(1)}$$

, $E[\rho_x] = E[T_{(1)}] = \frac{1}{2.\alpha}$, and $E[Z_{T_{(1)}}] = c$.
Therefore $x.E[\rho_x] - E[Z_{\rho_x}] = 0$ results $x^* = 2\alpha c$. As $x^* < \alpha$ we have $c \leq 0,5$.

If $x > \alpha$ we have $\rho_x = \inf\{s \in \Re^+ : x - \lambda_s \leq 0\} = S$, $E[\rho_x] = E[S] = \frac{3}{2.\alpha}$ and $E[Z_S] = c + 1$. Therefore $x.E[\rho_x] - E[Z_{\rho_x}] = 0$ results $x^* = (c+1).\frac{2}{3}\alpha$. As $x^* > \alpha$ we have $c \geq 0,5$.

Appendix

A.1 An extended and positive random variable τ is an $\Im_t$-stopping time if, and only if, $\{\tau \leq t\} \in \Im_t$, for all $t \geq 0$; an $\Im_t$-stopping time τ is called predictable if an increasing sequence $(\tau_n)_{n\geq 0}$ of $\Im_t$-stopping time, $\tau_n < \tau$, exists such that $\lim_{n\to\infty} \tau_n = \tau$; an $\Im_t$-stopping time τ is totally inaccessible if $P(\tau = \sigma < \infty) = 0$ for all predictable $\Im_t$-stopping time σ. For a basis of stochastic processes see the book of Bremaud (1981).

A.2

The stopping problem.

Definition A.2.1 A stochastic process $Z = (Z_t)_{t\geq 0}$ is called a smooth semi-martingale representation (SSM) if it has a decomposition of the form

$$Z_t = Z_0 + \int_0^t f_s ds + M_t,$$

where $(f_t)_{t\geq 0}$, is a progressively measurable process with $E[\int_0^t |f_s|ds < \infty$ for all $t \in \Re$, $E[|Z_0|] < \infty$ and $(M_t)_{t\geq 0}$ an zero mean uniformly integrable $\Im_t$ martingale. We denote a SSM by $Z = (f, M)$.

Definition A.2.2 Let $a, b \in \Re \cup \{-\infty, \infty\}, a \leq b$. Then a function $f : \Re^+ \mapsto \Re$ is called (a, b)-increasing if for all $t, h \in \Re^+$,

$$f(t) \geq a, \;\; implies \;\; f(t+h) \geq \min\{f(t), b\}.$$

Roughly spoken, an (a, b)-increasing function $f(t)$ passes with increasing t the levels a, b from bellow and never falls back bellow such a level. The first step to detect the parameters a and b is to establish bounds for C^*:

Lemma A.2.3 Let $X = (g, L)$ and $Z = (f, M)$ be smooth semimartingales under the above assumptions and

$$q = \inf\{\frac{f_t(w)}{g_t(w)} : 0 \le t < S(w), w \in \Omega\} > -\infty.$$

Then

$$b_l \le C^* \le b_u$$

holds true, where the bounds are given by

$$b_u = \frac{E[Z_S]}{E[X_S]},$$

$$b_l = \begin{cases} \frac{E[Z_0 - qX_0}{E[X_S]} + q, & \text{if } E[Z_0 - qX_0] > 0; \\ \frac{E[Z_0]}{E[X_0]}, & \text{otherwise.} \end{cases}$$

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