Vibration control of a rotor by active hybrid bearing and the Udwadia-Kalaba methodology

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Abstract

The Udwadia-Kalaba methodology is a possible way of explicitly obtaining the equations of motion of constrained systems, thus allowing to estimate the necessary forces in the constraint to keep the system in a given motion. Such forces can be used for control. Previous results show that the application of such control strategy is equivalent to the application of a known boundary condition to the structure in the point of application of control forces. In this work, one investigates numerically the application of the Udwadia-Kalaba methodology in the control of lateral vibrations of a rigid rotor supported by active bearings. The active bearing is composed of a hydrodynamic tilting-pad bearing with embedded electromagnetic actuators. Hence, the mathematical modeling of the system comprises the dynamics of the hydrodynamic bearing and the non-linear dynamics of the electromagnetic actuators. The numerical results show the feasibility of using the Udwadia-Kalaba methodology to reduce vibration amplitude of the rotor. The reduction of vibration amplitude reached 11% at the critical speed with a maximum power consumption of 100 mW, thus showing that the Udwadia-Kalaba methodology represents a minimum energy solution for system control.

1 Introduction

Rotating machinery is widely used in industry and it represents an important link of the production chain. Hence, by improving the performance of the rotating machinery, one can significantly improve productivity. One of the ways of improving the performance of rotating machinery is controlling and keeping the vibration of the rotating parts within acceptable and safe limits. In that way, many different strategies have already been studied in literature.

In the case of the lateral vibration of shafts, active bearings represent a possible strategy for vibration attenuation and control. By acting on the interface between the shaft (rotating part) and the casing (stationary part), active bearings can significantly change the dynamics of the rotating system. The most thoroughly studied active bearing is the magnetic bearing [1], but other design solutions can also be found in literature: the hydraulic chamber bearing [2], the active squeeze film damper [3], the actively lubricated bearing [4], the active bearing with piezo stacks [5], and the active hydrodynamic bearing with embedded electromagnetic actuators [6]. In all these cases, one must implement a feedback control strategy, being the classical PD (proportional-derivative) and the PID (proportional-integral-derivative) controllers those ones most widely adopted. However, there are successful examples of implementation in literature of other strategies, like the LQR (Linear Quadratic Regulator), the H_∞ controller, and fuzzy logic controllers, also including system uncertainties in the controller design (robust control).

The Udwadia-Kalaba methodology represents a different kind of control strategy because it is based on the concept of system constraining. The methodology was originally conceived to find the equations of motion of constrained systems [7], thus allowing to estimate the necessary constraint forces that kept a system in a given motion. However, it was easy to see the further applicability of the methodology to system control, and the development of the methodology for system control was straightforward [8]. Results showed that the application of such control strategy is equivalent to the application of a known boundary condition to the structure in the point of application of control forces [9].

In the present work, one investigates numerically the application of the Udwadia-Kalaba methodology in the control of lateral vibrations of a rigid rotor supported by active bearings. The active bearing in study is composed of a hydrodynamic tilting-pad bearing with embedded electromagnetic actuators (active hybrid bearing). Hence, the mathematical modeling of the system comprises the dynamics of the hydrodynamic bearing and the non-linear dynamics of the electromagnetic actuators. The numerical results show the feasibility of using the Udwadia-Kalaba methodology to reduce vibration amplitude of the rotor. The reduction of vibration amplitude reached 11% at the critical speed with a maximum power consumption of 100 mW, thus showing that the Udwadia-Kalaba methodology represents a minimum energy solution for system control.

2 Mathematical modelling of the active hybrid bearing

The active hybrid bearing in study is a hydrodynamic tilting-pad bearing composed of four pads with embedded electromagnetic actuators (Fig. 1). The rotating shaft is supported by the hydrodynamic forces that arise in the oil in the pad-shaft interface. The electromagnetic actuators are only used to control lateral motion of the shaft, thus not representing a support mechanism but a control mechanism (shaft is supported without action of the actuators).

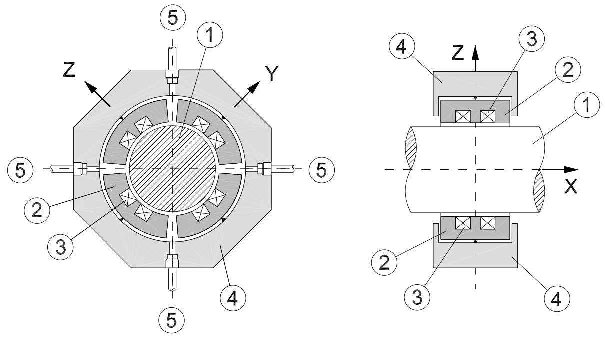


Figure 1: Tilting-pad hydrodynamic bearing with embedded electromagnetic actuators: 1) shaft, 2) tilting-pad, 3) electromagnetic actuator, 4) bearing casing, 5) oil inlet.

The dynamics of the shaft supported by this kind of bearing is represented by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{s}} + \mathbf{D}\dot{\mathbf{s}} + \mathbf{K}\mathbf{s} = \mathbf{f}_d + \mathbf{f}_a \tag{1}$$

where $\mathbf{M}$ is the inertia matrix of the shaft, $\mathbf{D}$ is the damping matrix of the oil film (linearized coefficients), $\mathbf{K}$ is the stiffness matrix of the oil film (linearized coefficients), $\mathbf{s}$ is the vector of shaft lateral displacements (y, z), $\mathbf{f}_d$ is the vector of disturbance forces, and $\mathbf{f}_a$ is the vector of control forces (electromagnetic actuators).

The hydrodynamic forces in the pad-shaft interface is modeled by linearized dynamic coefficients of stiffness and damping, represented by matrices $\mathbf{K}$ and $\mathbf{D}$. According to [10], the dynamic coefficients of a four pad tilting-pad hydrodynamic bearing in a load-between-pad configuration (present case) is a function of the Sommerfeld number of the bearing as shown in Figs. 2(a) and 2(b). The Sommerfeld number is an adimensional number that characterizes the bearing, and it can be related to the eccentricity of the shaft as shown in Fig. 3. The eccentricity of the shaft is the ratio between the shaft total lateral displacement $(\sqrt{y^2 + z^2})$ and the bearing clearance.

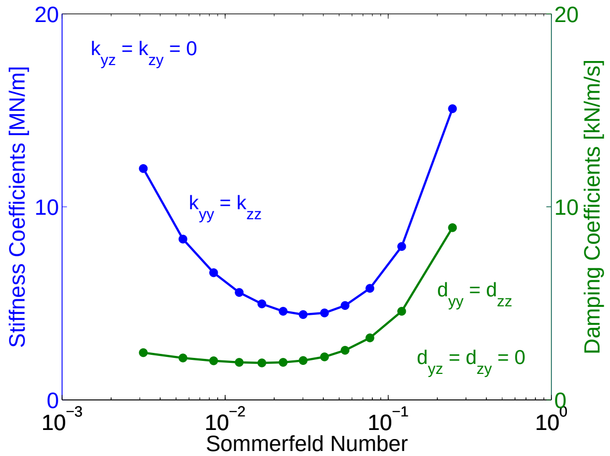


Figure 2: Equivalent dynamic coefficients of the oil film in the bearing in study: length/diameter = 1.0, pre-load factor = 0.75, pad aperture angle = 80^o [10].

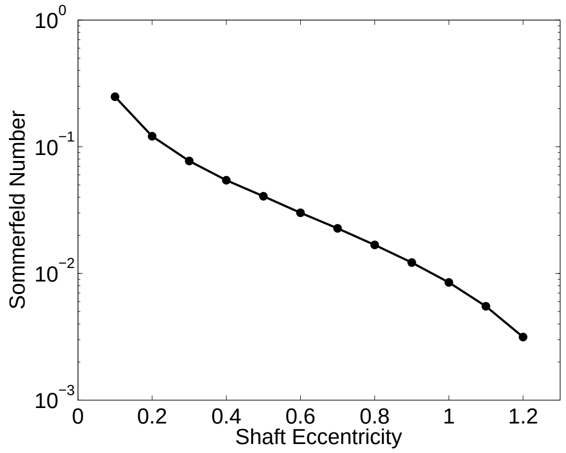


Figure 3: Sommerfeld number as a function of shaft eccentricity for the bearing in study: length/diameter = 1.0, pre-load factor = 0.75, pad aperture angle = 80^o [10].

The force of the electromagnetic actuators is defined as a function of the electric current in the coil and of the distance between the shaft and the actuator. Such function can be related to the electric voltage applied to the actuators by a non-linear model:

$$F_a = \left(\frac{j\omega a_1 + a_0}{j\omega b_1 + b_0}\right)^2 \frac{v^2}{d^2} \tag{2}$$

where ω is frequency, v is the electric voltage applied to the actuator, d is the distance between the actuator and the shaft, and the coefficients a_1, a_0, b_1, and b_0 were obtained experimentally [6], given in Table 1.

parameter	value
a_1	5.33×10^{-5}
a_0	4.91×10^{-5}
b_1	2.6
b_0	0.095

Table 1: Coefficients of the model of the electromagnetic actuators.

Hence, considering that the shaft mass is 21.5 Kg, one can integrate eq.(1) to obtain the shaft lateral displacements in Y and Z directions, subjected to a disturbance (e.g. unbalance forces) and considering the dynamics of the oil film and of the actuators.

3 The Udwadia-Kalaba methodology

As mentioned before, the Udwadia-Kalaba methodology allows the estimation of the necessary forces to keep the system in a given motion. Consider that the lateral motion of the shaft supported by the hybrid bearing (shaft orbit) must be limited to a maximum radius R_{max}. Hence, the motion of the shaft is constrained by:

$$\varphi = y^2 + (z + \delta_{st})^2 - R_{max}^2 \tag{3}$$

where φ represents a holonomic constraint, and δ_{st} is the vertical static deflection of the shaft due to gravity.

By differentiating this constraint equation, one has:

$$\dot{\varphi} = y\dot{y} + (z + \delta_{st})\dot{z} = 0 \tag{4}$$

$$\ddot{\varphi} = \dot{y}^2 + y\ddot{y} + \dot{z}^2 + (z + \delta_{st})\ddot{z} = 0 \tag{5}$$

These new constraint equations can be rearranged in the matrix form:

$$\mathbf{A}\ddot{\mathbf{s}} = \left[\begin{array}{cc} y & z + \delta_{st} \end{array}\right] \left\{\begin{array}{c} \ddot{y} \\ \ddot{z} \end{array}\right\} = -\dot{y}^2 - \dot{z}^2 - \sigma\dot{\varphi} - \kappa\varphi = b \tag{6}$$

where σ and κ are weighing factors of the linear combination of the constraint equations (4) and (5).

According to [7], the necessary forces to keep the system in the given constrained motion is given by:

$$\mathbf{f}_c = \left\{\begin{array}{c} F_{cY} \\ F_{cZ} \end{array}\right\} = -\mathbf{M}^{1/2}\left(\mathbf{A}\mathbf{M}^{-1/2}\right)^{+}(\mathbf{A}\mathbf{a} - b) \tag{7}$$

where the superscript $+$ refers to the Moore-Penrose pseudo-inverse, and $\mathbf{a}$ is the vector of feedback accelerations given by an output matrix $\mathbf{C}$: $\mathbf{a} = \mathbf{C}\ddot{\mathbf{s}}$.

The forces defined in eq.(7) are the desired forces to keep the system motion under the adopted constraint. However, unless the actuator has an unitary gain, one has to consider the inverse dynamics of the actuators to achieve such desired forces. Thus, the control voltage to be applied to the i-th direction of actuation is:

$$v_i = d_i\left(\frac{j\omega b_1 + b_0}{j\omega a_1 + a_0}\right)\sqrt{F_{ci}} \tag{8}$$

Such control voltage applied to the actuators of the hybrid bearing (eq.(2)) will result in the desired control forces defined by the Udwadia-Kalaba methodology. In this case, considering that electromagnetic actuators can only pull the shaft, the control voltage obtained by eq.(8)) is divided into an original and a complementary signal. The original signal is sent to one actuator and the complementary signal is sent to the other actuator that form the electromagnetic pair in i-th direction (see Fig. 1). In addition, due to limitations of the D/A ports of a real acquisition system, one imposes the saturation limit of $\pm$10 V in the voltage signals during simulations.

4 Numerical results

The equations of the system (eqs.(1), (2), (7), and (8)) are integrated in time using MATLAB standard algorithms (`ode` family). The rotor system is accelerated from 0 to 18,000 rpm (run-up test) and shaft lateral motion is constrained to a maximum radius $R_{max} = 1$ μm. In this case, the system presents an unbalance of 300 g.mm. The results are shown in Fig. 4.

As one can see in Fig. 4, the control system keeps the shaft lateral vibrations within the constraint value of 1 μm up to the rotating speed of $\sim$6,000 rpm. Above this rotating speed, lateral vibration increases, far beyond the limit value of 1 μm. The reason for that is the adopted voltage limitation of $\pm$10 V (saturation of the D/A ports in a real acquisition system). As one can see in Fig. 5, control voltage begins to saturate after the rotating speed of $\sim$5,000 rpm, thus affecting the evolution of the control forces (Fig. 6). The control forces increase up to the rotating speed of $\sim$5,000 rpm and they are suddenly limited to $\sim$200 N (Fig. 6), thus affecting the controller performance which probably required higher forces to impose the desired constraint. Despite such limitation, the control system managed to reduce the lateral vibrations of the shaft by 11% in the critical speed of $\sim$11,000 rpm.

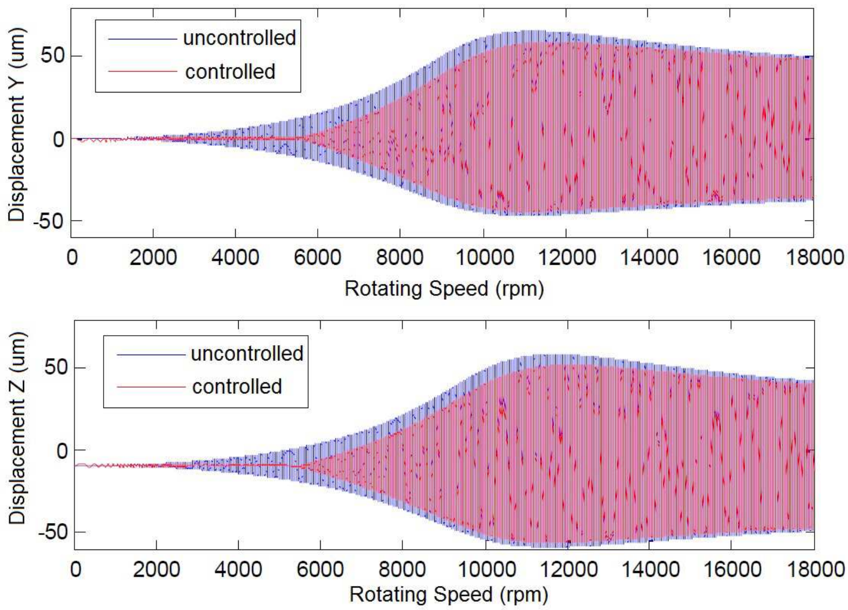


Figure 4: Shaft lateral displacement during run-up test: comparison between uncontrolled and controlled conditions with control voltage limitation of ± 10 V.

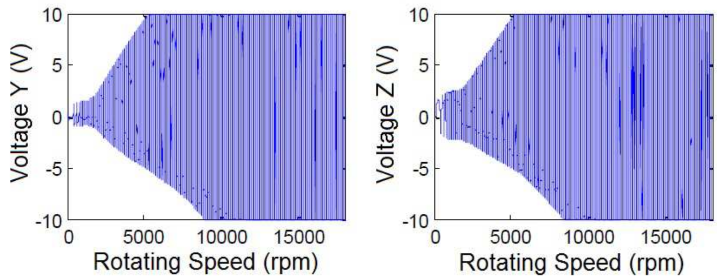


Figure 5: Control voltage in Y and Z directions during the run-up test.

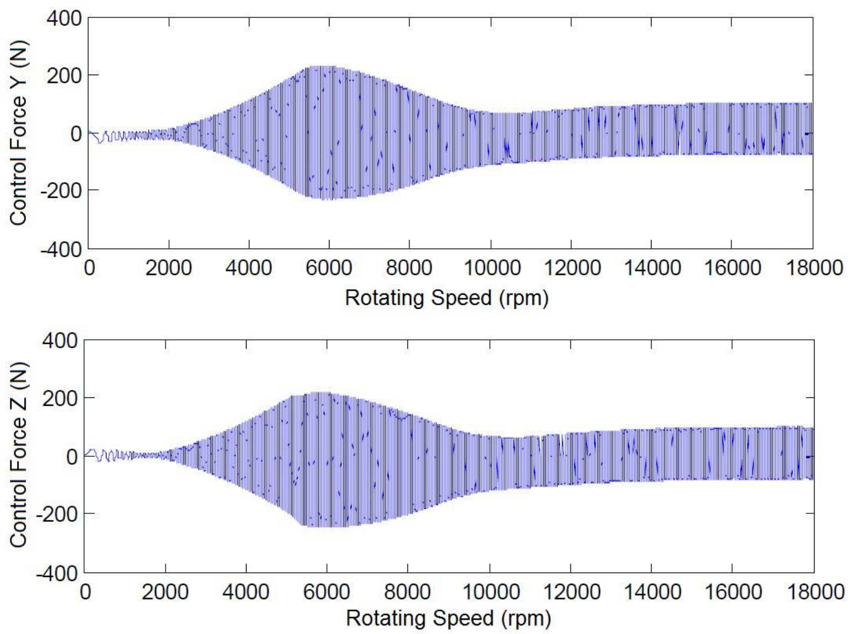
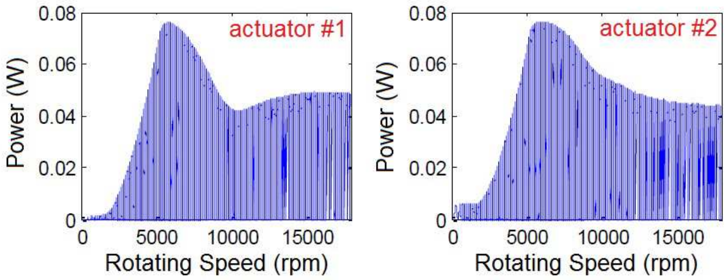
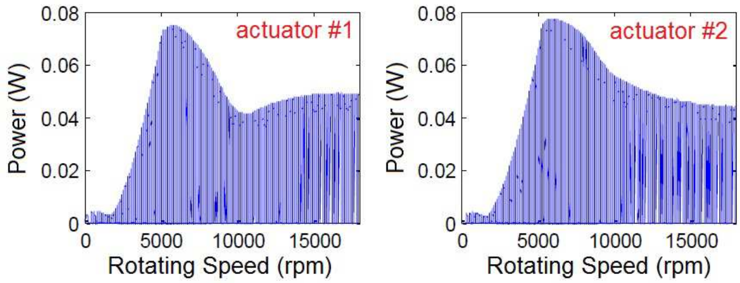


Figure 6: Control forces during run-up test with control voltage limitation of ± 10 V.



(a) Y direction



(b) Z direction

Figure 7: Electric power dissipation in the actuators during run-up test with control voltage limitation of ± 10 V.

Figure 7 presents the power dissipation in the electromagnetic actuators during the run-up test. As one can see, power dissipation remained below 100 mW, which is quite low, thus showing that the Udwadia-Kalaba methodology represents a minimum energy control strategy. Such low power dissipation was due to the low electric currents applied to the actuators.

If there are no limitations in the control voltage, the control system keeps the lateral displacements of the shaft constrained to the value of 1 μm (orbit radius) in the whole speed range of the run-up test, as shown in Fig. 8. In this case, control forces continually increased up to the value of $\sim$1,800 N at the rotating speed of 18,000 rpm, and power dissipation in the actuators reached a maximum value of 10 W, which is still a low value for such applications.

5 Conclusion

The Udwadia-Kalaba methodology was applied to control the lateral motions of a shaft supported by a hybrid bearing: hydrodynamic tilting-pad bearing with electromagnetic actuators embedded in the bearing pads. The numerical results showed that the methodology represents a minimum energy control solution and system was successfully constrained in a run-up test as long as control voltages were not saturated in ± 10 V. After a certain rotating speed, control voltages began to saturate and system constraint was no longer guaranteed. Despite such limitations, the control system managed to reduce the maximum lateral displacement of the shaft in 11% at the critical rotating speed. The results also showed that, if there was no such voltage saturation, the control system would constrain the lateral vibrations of the shaft in the whole range of tested rotating speeds, although requiring much higher control forces and electric power.

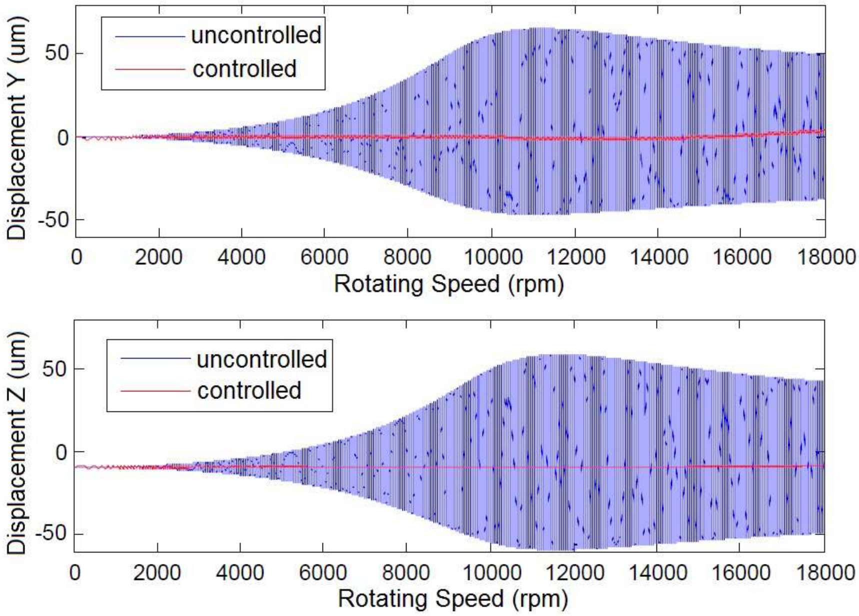


Figure 8: Shaft lateral displacement during run-up test: comparison between uncontrolled and controlled conditions with unlimited control voltage.

Acknowledgements

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